PHYSICAL REVIEW D **80,** 104033 (2009)

Wormholes in Einstein-Born-Infeld theory

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(Received 9 October 2009; published 23 November 2009)

Spherically symmetric thin-shell wormholes are studied within the framework of Einstein-Born-Infeld theory. We analyze the exotic matter content, and find that for certain values of the Born-Infeld parameter the amount of exotic matter on the shell can be reduced in relation to the Maxwell case. We also examine the mechanical stability of the wormhole configurations under radial perturbations preserving the spherical symmetry. In addition, in the Appendix the repulsive or attractive character of the wormhole geometries is briefly discussed.

DOI: 10.1103/PhysRevD.80.104033 PACS numbers: 04.20.Gz, 04.50.−h

I. INTRODUCTION

Traversable Lorentzian wormholes [1,2] are topologically nontrivial solutions of the equations of gravity which would imply a connection between two regions of the same universe, or of two universes, by a traversable throat. In the case that such geometries actually exist they could show some interesting peculiarities as, for example, the possibility of using them for time travel [3,4]. A basic difficulty with wormholes is that for the flareout condition [5] to be satisfied at the throat requires the presence of matter which violates the energy conditions (''exotic matter'') [1,2,5,6]. It was recently shown [7], however, that the amount of exotic matter necessary for supporting a wormhole geometry can be made infinitesimally small. Thus, in subsequent works special attention has been devoted to quantifying the amount of exotic matter [8,9], and this measure of the exoticity has been pointed as an indicator of the physical viability of a traversable wormhole [10]. Theories beyond Einstein-Maxwell framework have been explored with interesting results in this sense [11].

A central aspect of any solution of the equations of gravitation is its mechanical stability. The stability of wormholes has been thoroughly studied for the case of small perturbations preserving the original symmetry of the configurations. In particular, Poisson and Visser [12] developed a straightforward approach for analyzing this aspect for thin-shell wormholes, that is, those which are mathematically constructed by cutting and pasting two manifolds to obtain a new manifold [13]. In these wormholes the associated supporting matter is located on a shell placed at the joining surface; so the theoretical tools for treating them is the Darmois-Israel formalism, which leads to the Lanczos equations [14,15]. The solution of the Lanczos equations gives the dynamical evolution of the wormhole once an equation of state for the matter on the shell is provided. Such a procedure has been subsequently followed to study the stability of more general spherically symmetric configurations (see, for example, Refs. [16]), and an analogous analysis has also been carried out in the case of cylindrical symmetry (see [17]).

In order to avoid an infinite energy density associated to the electron, Born and Infeld [18] introduced a fundamental field strength b which together with the electron charge e determines a characteristic length $r_0 = \sqrt{e/b}$. The associated electromagnetic theory is nonlinear, and among its consequences one can find a regular gravitational field for a fundamental electrically charged particle. From a theoretical point of view, this feature would be enough to suggest a revision of well-known gravitational effects of charged objects within the wider framework of nonlinear electrodynamics. Besides, Born-Infeld (BI) type actions were later recovered in the context of low-energy string theory [19]. For these reasons, in the last years a renewed attention was devoted to spherically symmetric gravitational fields in the framework of Einstein gravity coupled to Born-Infeld or other nonlinear electrodynamics [20,21].

In particular, wormholes within the framework of Einstein gravity and nonlinear electrodynamics were considered, for example, in Refs. [22]. In the present work we study wormholes of the thin-shell type associated to spherically symmetric solutions of general relativity and Born-Infeld theory of electromagnetism. In Sec. II we introduce Einstein-Born-Infeld spherically symmetric geometry, and starting from it, in Sec. III we mathematically construct the associated thin-shell wormholes. In Sec. IV we evaluate the amount of exotic matter required for the existence of the wormholes, and in Sec. V we study the mechanical stability under perturbations preserving the spherical symmetry. In the Appendix we briefly discuss the attractive or repulsive character of the gravitational field associated to wormholes with different values of the parameters. Throughout the paper we use natural units, so $G = c = 1$.

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II. THE THEORY

Let us begin with a review of the main characteristics of the spherically symmetric solutions found in the framework of Einstein-Born-Infeld (EBI) theory. For a 4-dimensional manifold $(\mathcal{M}_4, g_{\mu\nu})$ with nonvanishing cosmological constant Λ, the action takes the following form

$$S = \int_{\mathcal{M}_4} d^4x\sqrt{|g_4|}\left(\frac{1}{2\kappa^2}(R - 2\Lambda) + \mathcal{L}(F)\right), \tag{1}$$

where $g_4 = \det g_{\mu\nu}$, and $\mathcal{L}(F)$ represents the Born-Infeld Lagrangian

$$\mathcal{L}(F) = 4b^4\left(1 - \sqrt{1 + \frac{F_{\mu\nu}F^{\mu\nu}}{2b^2}}\right), \tag{2}$$

with b a parameter which has dimension of length or mass, the so called Born-Infeld parameter. By taking the limit $b \to \infty$, $\mathcal{L}(F)$ reduces to the Maxwell Lagrangian

$$\mathcal{L}(F) = -F_{\mu\nu}F^{\mu\nu} + \mathcal{O}(F^4). \tag{3}$$

By varying the action above with respect to the gauge field A_μ and the metric tensor $g_{\mu\nu}$ the field equations for the spacetime metric and the electromagnetic field are obtained:

$$D_\mu\left(\frac{F^{\mu\nu}}{\sqrt{1 + \frac{F^2}{2b^2}}}\right) = 0, \tag{4}$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = T^{\mathrm{BI}}_{\mu\nu}, \tag{5}$$

$$T^{\mathrm{BI}}_{\mu\nu} = \frac{1}{2}\mathcal{L}(F)g_{\mu\nu} + \frac{1}{\sqrt{1 + \frac{F^2}{2b^2}}}(g_{\mu\nu}F^2 - 2F_{\kappa\mu}F^{\kappa}_{\nu}). \tag{6}$$

It has been proven that for any d-dimensional static spherically symmetric spacetime the metric can be written in terms of the hypergeometric function [23]. In the particular case of 4-dimensional spacetime the metric reads

$$ds^2 = -g(r)dt^2 + \frac{1}{g(r)}dr^2 + r^2 d\Omega_2^2 \tag{7}$$

where the function $g(r)$ takes the form

$$g(r) = 1 - \frac{2m}{r} - \frac{\Lambda r^2}{3} + \frac{2br^2}{3}\left(1 - \sqrt{1 + \frac{q^2}{b^2r^4}}\right) + \frac{4q^2}{3r^2}\,{}_2F_1\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{q^2}{b^2r^4}\right]. \tag{8}$$

Here m is an integration constant related with the Arnowitt-Deser-Misner (ADM) mass of the configuration and q is the charge of the system. The solution above (with nonzero cosmological constant) was found by Fernando and Krug [24]. For large r the metric represents a correction to the Reissner-Nördstrom anti-de Sitter (AdS) black hole:

$$g(r) = 1 - \frac{2m}{r} + \frac{q^2}{r^2} - \frac{\Lambda r^2}{3} - \frac{q^4}{40b^2r^6}. \tag{9}$$

In the case $\Lambda = 0$ and in the limit $b \to \infty$ this metric reduces to the standard Reissner-Nordström geometry. The last term in (9) represents the first Born-Infeld correction to the Reissner-Nördstrom AdS black hole in the large b-limit. Instead, near the origin ($r = 0$) the metric presents a completely different behavior when compared with the Reissner-Nördstrom geometry; for small r we have

$$g(r) = 1 - \frac{2m - a}{r} + 2b\left(-q + \frac{br^2}{3} + \frac{b^2r^4}{10}\right), \tag{10}$$

$$a^2 = q^3\frac{b}{\pi}\Gamma^4\left[\frac{1}{4}\right], \tag{11}$$

that is, close to the origin the leading term in the metric is given by $(2m - a)/r$. In fact, when $2m = a$ the metric is smoothed at $r = 0$, so the nonlinear Born-Infeld source helps to regularize the metric at the position of the charge. The same analysis can be carried out for the only nonzero component of the stress tensor, obtaining that the electric field is also finite at $r = 0$. This fact is not surprising because the Born-Infeld theory was developed in order to have a finite self-energy associated to a pointlike charge.

Besides, by demanding $g(r = r_{\mathrm{hor}}) = 0$ we can obtain the mass in terms of the horizon position r_{hor} (see Fig. 1). In order to keep the main ideas as clear as possible we only show one case in which we contrast the extremal Reissner-Nördstrom (RN) geometry with the BI one. Note that for the same values of m and q the BI geometry and the Maxwell one differ considerably: while in the RN case we have only one horizon, in the BI case there can exist none, one or two horizons depending on the value of the BI parameter (see Fig. 1). To be precise, for a fixed value of m and q, by increasing the parameter b in the range [1, 10] we

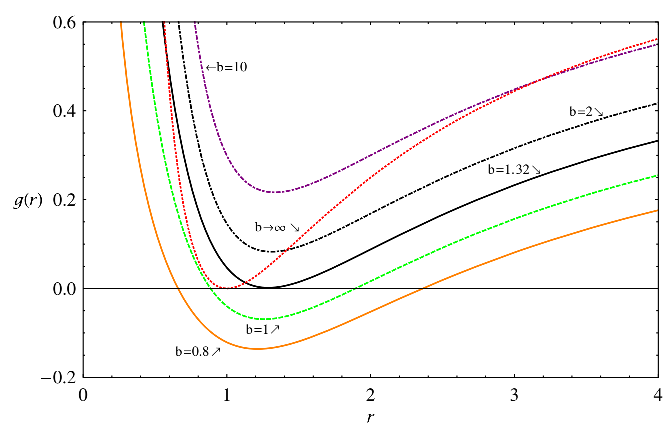

FIG. 1 (color online). We show the position of the horizons for $m = 1$ and $q = 1$. The RN geometry (dot-dashed line) corresponds to the extremal case, with only one event horizon, while the other lines show how the position of the horizons changes with the parameter $b \in [1, 10]$.

could have one, two or zero horizons. In conclusion, EBI black holes seem to be quite interesting because they include a wide variety of geometries.

III. THIN-SHELL WORMHOLE CONSTRUCTION

Starting from the metric given by (7) we build a spherically symmetric thin-shell wormhole in the Einstein-Born-Infeld theory. We take two copies of the spacetime and remove from each manifold the 4-dimensional regions described by

$$\mathcal{R}_{1,2} = \{x/r_{1,2} \leq a, a > r_{\rm hor}\}. \tag{12}$$

The resulting manifolds have boundaries given by the timelike hypersurfaces

$$\Sigma_{1,2} = \{x/r_{1,2} = a, a > r_{\rm hor}\}. \tag{13}$$

Then we identify these two timelike hypersurfaces to obtain a geodesically complete new manifold $\mathcal{M}$ with a matter shell at the surface $r = a$, where the throat of the wormhole is located. This manifold is constituted by two regions which in the case $\Lambda = 0$ are asymptotically flat (see Fig. 2). To study this type of wormhole we apply the Darmois-Israel formalism [14] to the case of Einstein-Born-Infeld theory. We can introduce the coordinates $\xi^i = (\tau, \theta, \chi)$ in Σ, with τ the proper time on the throat. Though we will first focus in static configurations, in the subsequent analysis of the mechanical stability of the configuration we must allow the radius of the throat to be a function of the proper time; then in general we have that the boundary hypersurface reads:

$$\Sigma\colon\ \mathcal{F}(r, \tau) = r - a(\tau) = 0. \tag{14}$$

The field equations projected on the shell Σ are obtained within the Darmois-Israel formalism; this leads to the Lanczos equations [14]

$$\langle K_{ab}\rangle - \langle K\rangle h_{ab} = -\kappa^2 S_{ab}, \tag{15}$$

where the bracket $\langle . \rangle$ stands for the jump of a given quantity across the hypersurface Σ. The tensor h_{ab} is the induced metric on Σ, and the extrinsic curvature tensor K_{ab} is defined as follows:

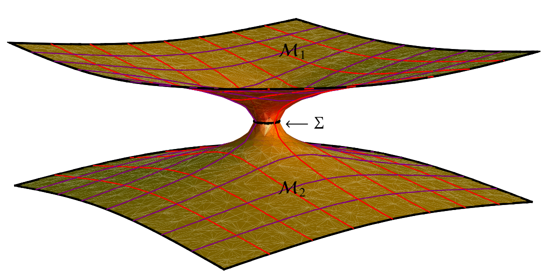

FIG. 2 (color online). We show the wormhole geometry obtained after performing the cut and paste procedure. The shell on Σ is located at the throat radius $r = a$.

$$K_{ab}^{\pm} = -n_c^{\pm}\left(\frac{\partial^2 X^c}{\partial \xi^a \partial \xi^b} + \Gamma^c_{de}\frac{\partial X^d}{\partial \xi^a}\frac{\partial X^e}{\partial \xi^b}\right)_{r=a}, \tag{16}$$

where $n_c^{\pm}$ are the unit normals ($n_c n^c = 1$) to the surface Σ. After some algebraic manipulations, the nonzero components S_a^b of the surface energy-momentum tensor of the shell turn out to be given by

$$S_\tau^\tau = \frac{1}{2\pi a}(\sqrt{\dot{a}^2 + g(a)}), \tag{17}$$

$$S_\phi^\phi = S_\theta^\theta = \frac{1}{8\pi a}\left(\frac{2\dot{a}^2 + ag'(a) + a\ddot{a} + 2g(a)}{\sqrt{\dot{a}^2 + g(a)}}\right), \tag{18}$$

where the dot means a derivative with respect to the proper time and the prime with respect to a. From these equations we read the energy density $\sigma = -S_\tau^\tau$ and the transverse pressure $p = S_\theta^\theta = S_\phi^\phi$ in terms of the throat radius $a(\tau)$, first and second derivatives of $a(\tau)$ and the function $g(a)$ which depends on the parameters of the system. If we explicitly write $g(r)$ and take the limit $b \to \infty$ in both Eqs. (17) and (18) we recover the expression for the energy density σ and pressure p found in the work by Eiroa and Romero (see Ref. [16]) with the Lanczos equations for the case associated to the standard RN solution.

It is easy to see from Eqs. (17) and (18) that the energy conservation equation is fulfilled:

$$\frac{d(A\sigma)}{d\tau} + p\frac{dA}{d\tau} = 0, \tag{19}$$

where A is the area of the wormhole throat. The first term in Eq. (19) represents the internal energy change of the shell and the second the work by internal forces of the shell. The dynamical evolution of the wormhole throat is governed by the Lanczos equations, and to close the system we must supply an equation of state $p = p(\sigma)$ that relates p and σ.

In the next section we will study how the exotic matter amount is related with the BI parameter, that is, we will mainly analyze the differences between the amount of exotic matter in a Born-Infeld wormhole and in a RN one.

IV. AMOUNT OF EXOTIC MATTER

Motivated by the nonlinear structure of the BI theory, we will evaluate the amount of exotic matter and the energy conditions. Essentially, we want to know if it is possible that for certain values of the BI parameter b the amount of exotic matter located at the shell could be reduced in relation with the Maxwell case, that is, if this quantity is larger or smaller than that corresponding to a RN wormhole.

The weak energy condition (WEC) states that for any timelike vector U^μ it must be $T_{\mu\nu}U^\mu U^\nu \geq 0$; the WEC also implies, by continuity, the null energy condition (NEC), which means that for any null vector k^μ it must be $T_{\mu\nu}k^\mu k^\nu \geq 0$ [2]. In an orthonormal basis the WEC reads $\rho \geq 0$, $\rho + p_l \geq 0 \quad \forall\ l$ while the NEC takes the form $\rho + p_l \geq 0 \quad \forall\ l$. In the case of thin-shell wormholes the radial pressure p_r of matter on the shell is zero, and within the relativistic theory of gravitation the surface energy density must fulfill $\sigma < 0$; thus both energy conditions would be violated. The sign of $\sigma + p_t$ where p_t is the transverse pressure is not fixed, but it depends on the values of the parameters of the system. In this section we restrict to static configurations. The surface energy density σ_0 and the transverse pressure p_0 for a static configuration ($a = a_0$, $\dot{a}_0 = 0$, $\ddot{a}_0 = 0$) are given by

$$\sigma_0 = -\frac{1}{2\pi a_0}\sqrt{g(a_0)}, \tag{20}$$

$$p_0 = \frac{1}{4\pi a_0}\left(\frac{a_0 g'(a_0) + g(a_0)}{\sqrt{g(a_0)}}\right). \tag{21}$$

The most usual choice for quantifying the amount of exotic matter in a Lorentzian wormhole is the integral [9]:

$$\Omega = \int (\rho + p_r)\sqrt{|g_4|}d^3x. \tag{22}$$

We can introduce a new radial coordinate $R = \pm(r - a_0)$ with $\pm$ corresponding to each side of the shell. Then, because in our construction the energy density is located on the surface, we can also write $\rho = \delta(R)\sigma_0$, and because the shell does not exert radial pressure the amount of exotic matter reads

$$\Omega = \int_0^{2\pi}\int_0^{\pi}\int_{-\infty}^{+\infty} \delta(R)\sigma_0\sqrt{|g_4|}dR \sin\theta d\theta d\phi = 4\pi a_0^2\sigma_0. \tag{23}$$

Replacing the explicit form of σ_0 and g_4, we obtain the exotic matter amount as a function of the parameters that characterize the configurations:

$$\Omega = -2a_0\left(1 - \frac{2m}{a_0} - \frac{\Lambda a_0^2}{3} + \frac{2ba_0^2}{3} - \frac{2ba_0^2}{3}\left(1 + \frac{q^2}{b^2a_0^4}\right)^{1/2} + \frac{4q^2}{3a_0^2}\,{}_2F_1\left[\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{q^2}{b^2a_0^4}\right]\right)^{1/2}.$$

In the case $\Lambda = 0$ and in the limit $b \to \infty$, we obtain the amount of exotic matter for the wormholes associated to the Reissner-Nordström ($q \neq 0$) and Schwarzschild ($q = 0$) geometries (Fig. 3). For small charge values ($q \approx 0.01$), for the RN geometry as well as for the BI geometry (with $b = 1$) one obtains the same behavior (see Fig. 3(a)). However, by comparing the extremal RN geometry with $q = 1$ and the BI geometry with the same charge (with $b = 1$) the exotic amount of matter turns out to be less for the latter geometry than in the former case. So, even when both geometries have the same charge and the horizon radii are of the same order of magnitude ($r_{\rm hor}^{\rm BI} \approx 2r_{\rm hor}^{\rm RN}$), for a given wormhole radius the BI source helps to reduce the amount of exotic matter. In other words, when the charge appears in a nontrivial manner it could help to minimize the exoticity of a wormhole configuration (see Fig. 3(b)).

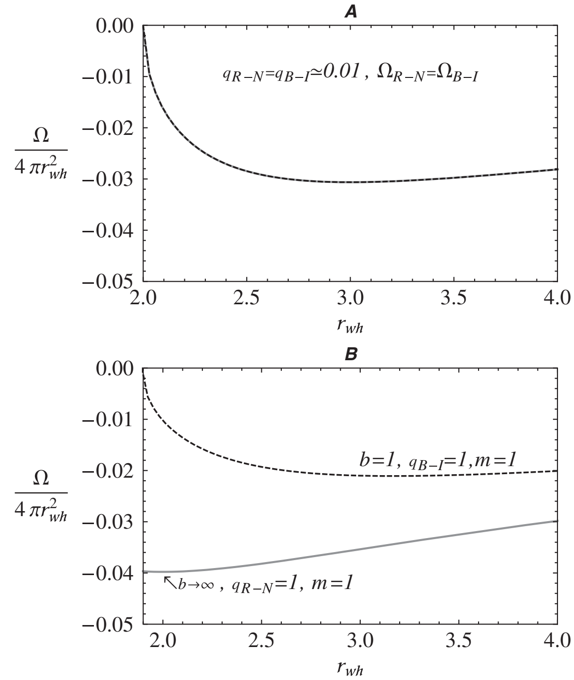

FIG. 3. We show the energy densities corresponding to wormholes associated to the RN geometry (solid line) and the BI spacetime (dashed line) in terms of the wormhole radius $r_{\rm wh}$. Here we take $r_{\rm wh} > r_{\rm hor}^+$ with $r_{\rm hor}^+$ indicating the largest event horizon of both the RN and the BI cases.

V. STABILITY ANALYSIS

In this section we study the mechanical stability of the wormholes under small perturbations preserving the symmetry of the original configuration [12]. The dynamical evolution of the wormhole is determined by Eqs. (17) and (18), or by any of them and Eq. (19), and to complete the system we must add an equation of state that relates p with σ, i.e, $p = p(\sigma)$. From Eq. (17) we have

$$\dot{a}^2 + g(a) = (2\pi a\sigma(a))^2. \tag{24}$$

We first note that the energy conservation equation can be written as

$$\dot{\sigma} = -2(\sigma + p)\frac{\dot{a}}{a} \tag{25}$$

which can be integrated to give

$$\frac{a(\tau)}{a(\tau_0)} = \exp\left[-\frac{1}{2}\int_{\sigma_0}^{\sigma}\frac{d\sigma}{\sigma + p(\sigma)}\right]. \tag{26}$$

From Eq. (19), if the equation of state $p = p(\sigma)$ is given, then one can obtain $\sigma = \sigma(a)$.

Following the procedure introduced by Poisson and Visser, the analysis of the stability of the configuration can be reduced to the analogous problem of the stability of a particle in a 1-dimensional potential $V(a)$ [12]. This is easy to see if we write Eq. (24) as

$$\dot{a}^2 = -V(a), \tag{27}$$

$$V(a) = g(a) - (2\pi a\sigma(a))^2. \tag{28}$$

Then, to study the stability we expand up to second order the potential $V(a)$ around the static solution a_0 (for which $\dot{a} = 0$, $\ddot{a} = 0$). For a stable configuration it is $V(a_0) = 0$ and $V'(a_0) = 0$ (where, as before, the prime means a derivative with respect to the radius). Then, Eq. (27) takes the following form:

$$\dot{a}^2 = -V''(a_0)(a - a_0)^2 + \mathcal{O}[(a - a_0)^3]. \tag{29}$$

To compute the derivatives it is convenient to define the parameter

$$\eta(\sigma) \equiv \frac{\partial p}{\partial \sigma}, \tag{30}$$

which for ordinary matter would represent the squared speed of sound: $v_s^2 = \eta$. Here, however, we simply consider η as a parameter entering the equation of state (see below). Then, we obtain the second derivative of the potencial for the metric (7):

$$V''(a_0) = g_0'' - \frac{g_0'^2}{2g_0} - \frac{1 + 2\eta_0}{a_0^2}[2g_0 - a_0 g_0'] \tag{31}$$

where we use the definitions $\eta_0 := \eta(\sigma_0)$, $g_0 := g(a_0)$, $g_0' := g'(a_0)$, and $g_0'' := g''(a_0)$ for short. The wormhole is stable if and only if $V''(a_0) > 0$ while for $V''(a_0) < 0$ a radial perturbation grows (at least until a nonlinear regime is reached) and the wormhole is unstable. By using that the function $g(a_0)$ is always positive for $a_0 > r_{\rm hor}$, we only have to analyze the sign of $V''(a_0)$ for determining which are the values of the parameters (m, q, b, a_0) that make the wormhole stable. Then, after some simple manipulations, the stability conditions can be written as follows:

$$\text{If } \ 2g_0 > a_0 g_0', \qquad 1 + 2\eta_0 < \frac{a_0^2}{2g_0}\left(\frac{2g_0'' - g_0'^2}{2g_0 - a_0 g_0'}\right). \tag{32}$$

$$\text{If } \ 2g_0 < a_0 g_0', \qquad 1 + 2\eta_0 > \frac{a_0^2}{2g_0}\left(\frac{2g_0'' - g_0'^2}{2g_0 - a_0 g_0'}\right). \tag{33}$$

Note that the above expressions are in agreement with those found in [12,25]. Our starting point is to investigate how the regions of stability for the BI wormhole change with the parameter b. First (see Fig. 4), we check that when $b \to \infty$ the stability regions correspond to the RN wormholes (see Fig. 4(a)). By taking b in the range [1,2] we get that there are different types of stability zones. Basically, we have a stable wormhole with $0 \leq \eta_0 \leq 1$ (indicating that η_0 could represent the speed of sound), $\eta_0 \leq 0$ (being this characteristic of wormhole matter), and a third case with $\eta_0 \geq 1$, which would correspond to a superluminal sound velocity in the wormhole throat (see Figs. 4(b)–4(d)). When the BI parameter is $b = 1$ (that is, considerably far away from the Maxwell limit) and considering larger values of the charge we obtain that the stability regions are considerably enlarged. An interesting aspect is that for quite similar original manifolds one obtains very different stability domains (see Fig. 5). For example, for $q = 2$ (such that there are two horizons in the original manifold, the outer one placed at $r_h^+ \approx 2.26$) stability is achieved with $0 \leq \eta_0 \leq 1$; however, for $q = 1.5$ (also two horizons,

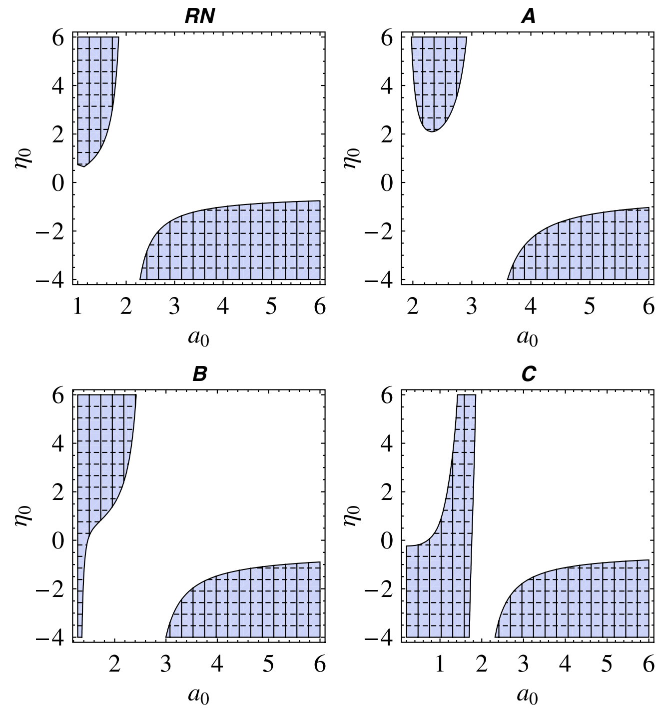

FIG. 4 (color online). We show the stability regions in four different cases: extremal RN with $m = 1$, $q = 1$ (upper left panel), EBI with $m = 1$, $q = 1$, $b = 1$ (upper right panel), EBI with $m = 1$, $q = 1$, $b = 1.32$ (lower left panel), and EBI with $m = 1$, $q = 1$, $b = 2$ (lower right panel).

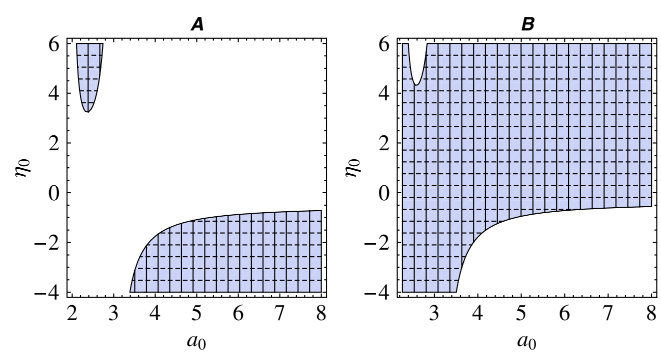

FIG. 5 (color online). We show the stability regions in two extreme cases, corresponding to small charge $q = 0.01$ (left panel) and large charge $q = 2$ (right panel). In both cases we take $m = b = 1$.

the outer at $r_h^+ \approx 5.54$) stability requires $\eta_0 \geq 2$ (see Fig. 5(b)). Later on, we have also examined the stability regions corresponding to the metrics (9) and (10). In the former case, by considering the first correction to RN geometry (that is, $b^{-1} \approx \mathcal{O}(10^{-2})$) we find that essentially there is no appreciable difference between the RN and BI regions of stability when the charge values are in the range $q \in [0.01, 3]$. On the other hand, for the metric (10), with a fixed BI parameter ($b = 1$) and varying $0.01 \leq q \leq 3$ we arrive to the conclusion that, when the condition $2m < a$ holds, only regions of stability with $\eta_0 \leq 0$ are admissible.

VI. CONCLUSION

The generalization of Maxwell electromagnetism to a nonlinear theory in the way proposed by Born and Infeld introduces a new parameter, which allows for more freedom in the framework of determining the most viable charged wormhole configurations. If wormholes could actually exist, one would be interested in those which are stable—at least under the most simple kind of perturbations—and which, besides, require as little amount of exotic matter as possible. Of course, the case could be that a given change of the theory leads to a worse situation, i.e., that configurations turn out to be more unstable or require more matter violating the energy conditions as the departure from the standard theory becomes relevant. However, this seems not to be the case with Born-Infeld electrodynamics coupled with Einstein's gravity: Here we have examined the mechanical stability and exotic matter content of thin-shell wormholes within Einstein-Born-Infeld theory, and—as long as large values of the charge are considered—we have found that for small values of the Born-Infeld parameter, corresponding to a situation far away from the Maxwell limit, both the size of the stability regions in parameter space are enlarged, and the amount of exotic matter is reduced in relation with the standard case. Thus our results suggest that in a physical scenario different from that consistent with present-day observation (as in the early universe, when nonlinear effects could be more relevant) charged wormholes could have been more likely to exist.

ACKNOWLEDGMENTS

M. G. R. and C. S. are supported by the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET).

APPENDIX: THE GRAVITATIONAL FIELD

The wormholes studied could be both attractive or repulsive. To characterize this aspect of the configurations we analyze the force on a test particle at rest in the geometries described above. For this, we evaluate the radial acceleration given by

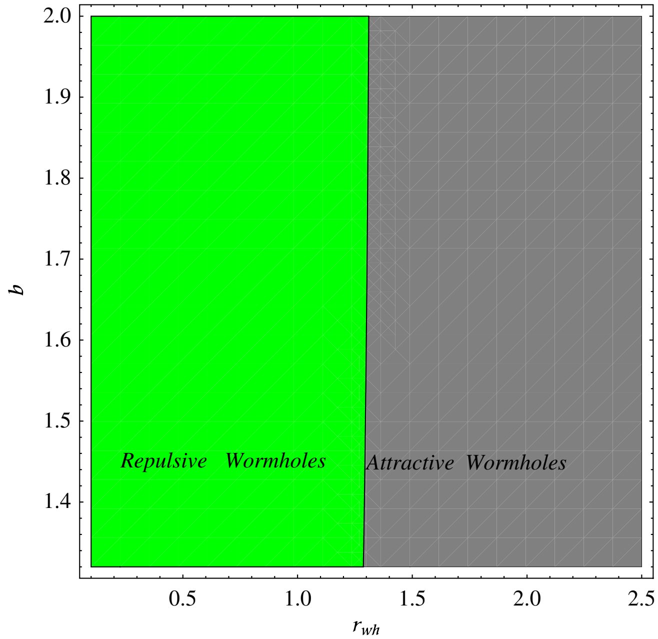

FIG. 6 (color online). We show the regions where the wormholes are attractive or repulsive for $q = m = 1$. The dashed zone corresponds to attractive wormholes while the undashed regions are associated to repulsive ones.

$$\frac{d^2 r}{d\tau^2} = -\Gamma^r_{tt}\left(\frac{dt}{d\tau}\right)^2. \tag{A1}$$

The sign of the acceleration of a particle initially at rest is then given by minus the sign of the component Γ^r_{tt} of the

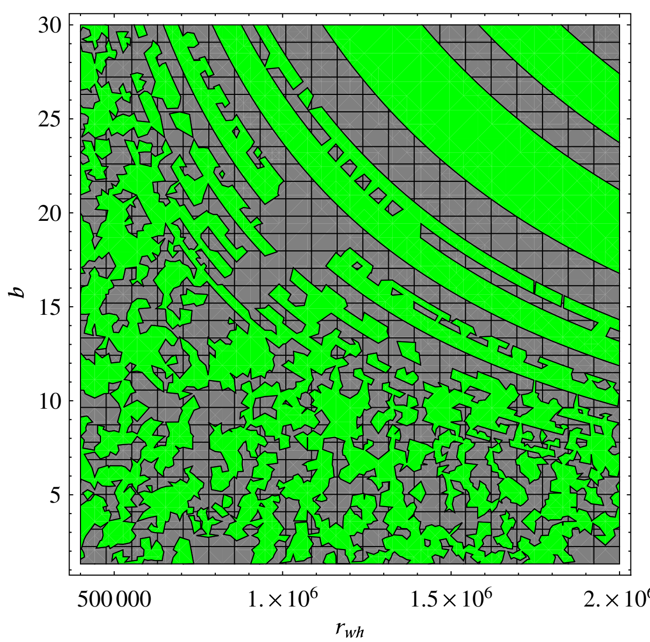

FIG. 7 (color online). We show the regions where the wormholes are attractive or repulsive for $q = m = 1$. The dashed zone corresponds to attractive wormholes while the undashed regions are associated to repulsive ones.

connection, which for the metric considered is equal to $g(r)g'(r)/2$. Thus we have an attractive gravitational field for $g' > 0$, and a repulsive field for $g' < 0$ (of course, we consider only the possibility $g > 0$). For example, we can verify numerically that for $q = 1$, $m = 1$, both in the RN ($b \to \infty$) case and in the case $b = 1$ (very far from the RN solution) we have $g' > 0$ for $r > r_{\rm hor}$. In general, however, we should take care about the sign of $g(r)$. To avoid this problem in a point which is not the central aspect of our analysis, we shall simply restrict a detailed study to the case of no horizons in the original manifold.

We therefore take $q = 1$, $m = 1$, and choose the parameter b appropriately. For b within the interval [1.33, 3] we numerically find (see Fig. 6) that for $0 < r_{\rm wh} < 1.3$ the gravitational field of the configurations turns to be repulsive, while for $1.3 < r_{\rm wh} < 100$ the wormholes are attractive. This could suggest that for these values of the parameters the wormholes are always attractive as long as $r_{\rm wh} > 1.3$. However, a careful analysis shows that this is not true. For very large wormhole radii (typically $r_{\rm wh} \sim 5 \times 10^5 - 2 \times 10^6$) a kind of "islands" in the parameter space appears in which the gravitational field is repulsive. Moreover, the size of these "islands" is increased for larger values of b (see the case $b \sim 30$ in Fig. 7), though always far from the Maxwell limit. One may also wonder about the complementary situation, that is, possible attractive wormholes for very small radii. But for the values of q and m considered this is not the case. As a final comment we mention that by comparing the cases with values of the parameters such that the exotic matter amount associated to Born-Infeld theory turns out to be smaller than the corresponding to the RN wormholes, we obtain that the field associated to these wormhole configurations is attractive.

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