Int J Game Theory
DOI 10.1007/s00182-015-0471-0

Axiomatizing core extensions on NTU games

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Accepted: 15 March 2015

Abstract We study solution concepts for NTU games, where the cooperation (or negotiation) of the players can be obtained by means of non-trivial families of coalitions (e.g. balanced families). We give an axiomatization of the *aspiration core* on the domain of all NTU games as the only solution that satisfies non-emptiness, individual rationality, a generalized version of consistency and independence of individual irrelevant alternatives. If we consider solutions supported by partitions, our axioms characterize the *c-core* [Guesnerie and Oddou in, Econ Lett 3(4):301–306, 1979; Sun et al. in, J Math Econ 44(7–8):853–860, 2008], and if we consider solutions supported only by the grand coalition, our axioms also characterize the classical *core*, on appropriate subdomains. The main result of this paper generalizes Peleg's core axiomatization [J Math Econ 14(2):203–214, 1985] to non-empty solutions that are supported by non-trivial families of coalitions.

Keywords Core · Axiomatizations · Aspiration core · Consistency

JEL Classification C71

1 Introduction

The most interesting cooperative game questions can be summarized as follows: (I) which coalitions are formed?, and (II) how are their values distributed between their members? The fundamental concept of a cooperative equilibrium is the core which

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Published online: 25 March 2015

always assumes that the grand coalition forms. However, the power of the core concept is limited by the fact that the non-emptiness of the core may be assured only in certain ideal environments where the grand coalition formation is reasonable. A natural non-empty extension of the core is the *aspiration core* introduced by Cross (1967) [see also Albers (1979), Bennett (1983), Bejan and Gómez (2012) and Cesco (2012)]. The idea behind the aspiration core is to search those outcomes generated by non-trivial families of coalitions called *balanced families* that no coalition can improve. This solution takes on the two problems simultaneously, stressing the evident relations between questions (I) and (II).

In the context of games with transferable utility (TU games), the aspiration core has been recently characterized by Bejan and Gómez (2012) and Cesco (2012) who presented axiomatizations of the aspiration core on the entire class of TU games extending the core axiomatization given by Peleg (1986).[1] The contribution of our paper is to offer an axiomatization of the aspiration core on games without transferable utility (NTU games) extending the core axiomatizations given by Peleg (1985). We give an axiomatization of the aspiration core on the domain of all NTU games as the only solution that satisfies non-emptiness, individual rationality, some appropriately-modified version of consistency (reduced game property) and independence of individual irrelevant alternatives. Quoting Peleg (1985), "...we may consider a solution to be 'acceptable' if its axiomatization is very similar to that of the core", then our aspiration core axiomatization posits the aspiration core as an acceptable non-empty solution for NTU games. When we consider solutions supported only by the grand coalition, our axioms also characterize the classical core on an appropriate subdomain. Furthermore, if we consider solutions supported by partitions, our axioms give the first axiomatization on NTU games of the *c-core* (Guesnerie and Oddou 1979; Sun et al. 2008; Kóczy and Lauwers 2004). Many core axiomatizations [see, for example, Peleg (1985)] work on the class of games with non-empty core, so there is some circularity when they use the core to define their domain of games. It is important to highlight that our aspiration core axiomatization works on the entire class of NTU games, then such circularity does not occur in our axiomatization.[2]

The traditional consistency axiom and the corresponding reduced game (Davis and Maschler 1965; Peleg 1985) are defined in a framework in which it is assumed that the grand coalition forms. We use a modified reduced game and its corresponding consistency axiom introduced by Moldovanu and Winter (1994) [see also, Hokari and Kibris (2003), Bejan and Gómez (2012)] for solutions supported by non-trivial families of coalitions. The difference between the traditional reduced game and the modified reduced game arises in the way that the coalition of all the remaining players has to cooperate with the departing players. In the traditional reduced game, the remaining coalition has to get together with *all* the departing players while in the modified reduced game, the remaining coalition can do it with *any* subgroup of the departing players that it wishes.

[1] Cesco (2012) works with a solution concept called *M-core* which is equivalent to the aspiration core.

[2] An alternative axiomatization of the core on the entire class of NTU games is presented by Hwang and Sudhölter (2001), but their axioms characterize the empty solution outside the domain of games with non-empty core.

Axioms of independence of irrelevant alternatives have been studied by several authors, for example Aumann (1985), Peleg et al. (2012), among others. In general, if an alternative is prescribed as a solution to a problem, and this remains as a feasible outcome in a game where some feasible payments of some coalitions are removed, independence of irrelevant alternatives requires that such alternative be in the solution of the problem in which the feasible payments were removed. In this work, we use a version of this axiom that only considers the case in which some feasible payments of individual coalitions are removed. Therefore, the axiom is called *independence of individual irrelevant alternatives*.

This paper has the following organization. In Sect. 2, we give basic definitions and notations. In Sect. 3, the axioms are presented. Section 4 includes our main axiomatization results. Section 5 shows the independence of the axioms. The Appendix contains two proofs.

2 Definitions and notation

2.1 NTU games

Let M be a finite set of m players.[3] A *coalition* is a non-empty subset of M. The notations $A \subset B$ and $A \subseteq B$ mean that A is a proper subset of B and A is a subset of B, respectively. The cardinality of a set A is denoted $|A|$. For each coalition $S \subseteq M$, $\mathbf{R}^S$ denotes the $|S|$-dimensional Euclidean space with coordinates indexed by elements of S. For each $x \in \mathbf{R}^M$; x^S denotes its projection into $\mathbf{R}^S$. If $S = \{i\}$, we will denote x^i instead of $x^{\{i\}}$. Write $y^S \geq x^S$ if $y^i \geq x^i$ for each $i \in S$; $y^S > x^S$ if $y^S \geq x^S$ and $y^S \neq x^S$; $y^S >> x^S$ if $y^i > x^i$ for each $i \in S$. A set $A \subseteq \mathbf{R}^S$ is *comprehensive* if $x \in A$ and $x \geq y$ imply $y \in A$. A set $A \subseteq R^S$ is *Pareto-bounded* if $A \cap (x^S + \mathbf{R}^S_+)$ is bounded for every x^S in $\mathbf{R}^S$. The boundary of A is denoted by ∂A. A set $A \subseteq \mathbf{R}^S$ is *non-levelled* if $x^S, y^S \in \partial A$ and $y^S \geq x^S$ imply $x^S = y^S$. The non-levelled assumption is equivalent to the assumption that no segment of the boundary of A in $\mathbf{R}^S$ is parallel to the coordinate axes.

A *non transferable utility game* (*NTU game*) is a pair (N, V), where N is a coalition and V is the *characteristic function* which associates with each coalition $S \subseteq N$ a subset $V(S)$ of $\mathbf{R}^S$, such that:

(i) $V(S)$ is non-empty and closed,
(ii) $V(S)$ is comprehensive,
(iii) $V(S)$ is Pareto-bounded.
(iv) $V(S)$ is non-levelled.

For each $S \subseteq N$, $V(S)$ is the set of feasible payoffs of S. Denote by Γ the set of all NTU games (N, V).

Possible payoffs of a game (N, V) are described by vectors $x \in \mathbf{R}^N$ that assign a payoff x^i to each $i \in N$. The generating collection of $x \in \mathbf{R}^N$ is defined as $GC(x) =$

[3] All the results present in this paper apply to the case where M is infinite. In this case the coalitions are non-empty and finite subsets of M.

$\{S \subseteq N | x^S \in V(S)\}$. A payoff vector x is an *aspiration* of the game (N, V) if $x^S \notin int V(S)$ for each $S \in N$ and $\bigcup_{S \in GC(x)} S = N$ [see, Bennett and Zame (1988)].

2.2 Feasibility

Feasibility is defined by taking into account payoffs generated by overlapping structures of coalitions that may coexist if the players can divide their "time/resources" (not just the grand coalition). Such approach has been considered in Cross (1967) [see also Albers (1979), Bennett (1982) and Bennett (1983)] and recently, studied by Bejan and Gómez (2012) and Cesco (2012). We are interested in payoffs generated by families of coalitions satisfying the following two requirements;

(a) *Each player has all of his "time" split in the coalitions in which he participates.*
(b) *The amount of "time" that a player contributes to a given coalition is the same for all members of that coalition.*

This idea is captured by the classical notion of balanced family with its set of balancing weights. Given $N \subseteq M$, a family of coalitions $\mathcal{B} \subseteq 2^N$ is called a *balanced family of* N if there exists a set of positive real numbers $(\lambda_S)_{S \in \mathcal{B}}$ satisfying [4]

$$\sum_{\substack{S \in \mathcal{B}: \\ S \ni i}} \lambda_S = 1, \quad \text{for all } i \in N.$$

The numbers $(\lambda_S)_{S \in \mathcal{B}}$ are the *balancing weights for* $\mathcal{B}$. A balanced family indicates which coalitions are formed, and its balancing weights are interpreted as the fraction of "time" that each coalition is active. If $S \in \mathcal{B}$, then each $i \in S$ devotes λ_S of his "time" to S. Since $\sum_{S \in \mathcal{B}: S \ni i} \lambda_S = 1$, each player distributes all his "time" among the coalitions that contain him. Let $\Lambda(N)$ denote the set of all balanced families of N, and let $\mathcal{B}$ denote an arbitrary element in $\Lambda(N)$.

Define the *set of payoff vectors generated by a balanced family* $\mathcal{B} \in \Lambda(N)$ as [5]

$$V(\mathcal{B}) = \{x \in \mathbf{R}^N : x^S \in V(S) \ \forall S \in \mathcal{B}\}.$$

Definition 1 The set of *feasible payoff vectors* of (N, V) is

$$X_\Lambda(N, V) = \bigcup_{\mathcal{B} \in \Lambda(N)} V(\mathcal{B})$$

Classical axiomatization literature defines the feasibility condition assuming that the grand coalition forms, then it works with the set

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[4] As usual, 2^N denotes the set of all the subsets of N.

[5] $V(\mathcal{B}) = \{x \in \mathbf{R}^N : x^S \in V(S) \ \forall S \in \mathcal{B}\} = \bigcap_{S \in \mathcal{B}} (V(S) \times \mathbf{R}^{N \setminus S})$. Then, non-emptiness of $V(\mathcal{B})$ follows from the fact that $V(S)$ is non-empty and comprehensive, and $\mathcal{B}$ is finite.

$$X(N, V) = V(N).$$

Clearly, $X(N, V) \subseteq X_\Lambda(N, V)$.

If many coalitions may be formed but the players cannot divide their time in order to participate in more than one coalition, the following subset of $X_\Lambda(N, V)$ should be considered. A family of coalitions $\mathcal{P} \subseteq 2^N$ is a partition of N if $\cup_{S \in \mathcal{P}} S = N$ and for every $S, S' \in \mathcal{P}$ such that $S \neq S'$, $S \cap S' = \emptyset$. Let $\Pi(N)$ denote the set of all the partitions of N, and let $\mathcal{P}$ denote an arbitrary element in $\Pi(N)$. If $\mathcal{P} \in \Pi(N)$, then $\mathcal{P}$ is a balanced family with balancing weights $\lambda_S = 1$ for all $S \in \mathcal{P}$. For every NTU game (N, V) let

$$X_\Pi(N, V) = \bigcup_{\mathcal{P} \in \Pi(N)} V(\mathcal{P}).$$

Remark 1 Since $\Pi(N) \subseteq \Lambda(N)$, $X(N, V) \subseteq X_\Pi(N, V) \subseteq X_\Lambda(N, V)$, for each game (N, V).

2.3 Efficiency

Definition 2 The set of *efficient payoff* vectors for every (N, V) is defined as

$$E_\Lambda(N, V) = \{x \in X_\Lambda(N, V) : \nexists y \in X_\Lambda(N, V) \text{ such that } y >> x\}$$

Classical literature considers the set

$$E(N, V) = \{x \in X(N, V) : \nexists y \in X(N, V) \text{ such that } y >> x\}$$

as the set of efficient (or weak Pareto optimal) payoff vectors of (N, V) [see, for example, Peleg and Sudhölter (2007)].

According to the notation used we denote

$$E_\Pi(N, V) = \{x \in X_\Pi(N, V) : \nexists y \in X_\Pi(N, V) \text{ such that } y >> x\}.$$

As $X(N, V) = V(N)$ satisfies properties (i), (ii) and (iii) in the NTU game definition, then $E(N, V)$ is non-empty and coincides with the boundary of $V(N)$. The next lemma states that $X_\Lambda(N, V)$ also satisfies properties (i), (ii) and (iii) in the NTU game definition, and because of that, $E_\Lambda(N, V)$ is non-empty and coincides with the boundary of $X_\Lambda(N, V)$.

Lemma 1 *For each NTU game (N, V), the set $X_\Lambda(N, V)$ is non-empty, closed, comprehensive, and Pareto-bounded.*

Proof See Appendix. □

Remark 2 The proof of Lemma 1 can be easily adapted to prove that $X_\Pi(N, V)$ is non-empty, closed, comprehensive, and Pareto-bounded. Then, non-emptiness of $E_\Pi(N, V)$ is obtained similarly to non-emptiness of $E_\Lambda(N, V)$.

2.4 Solution concepts

Fix a family of games $\Gamma_0 \subseteq \Gamma$. A *solution on* Γ_0 is a mapping σ that assigns to every game $(N, V) \in \Gamma_0$ a set $\sigma(N, V) \subseteq X_\Lambda(N, V)$.

The core (Aumann 1961) is defined as

$$C(N, V) = \{x \in X(N, V) : \forall S \subseteq N, x^S \notin int V(S)\}.$$

It is well know that the core can be empty. The subdomain of NTU games with non-empty core is denoted by

$$\Gamma^C = \{(N, V) \in \Gamma : C(N, V) \neq \emptyset\}$$

The subdomain of *balanced NTU games* is

$$\Gamma^B = \{(N, V) \in \Gamma : X_\Lambda(N, V) = X(N, V)\}$$

Scarf (1967) showed that $\Gamma^B \subseteq \Gamma^C$, *i.e.*, every balanced game has a non-empty core. The power of the core concept is limited by the fact that the non-emptiness of the core cannot always be assured. As an answer to this limitation, it is natural to consider solutions in which generating families are not only the grand coalition. The generating families may be taken from $\Pi(N)$ or $\Lambda(N)$, setting up two different solutions.

The *c-core* (Sun et al. 2008) [see also Guesnerie and Oddou (1979), Bennett and Zame (1988) and Kóczy and Lauwers (2004)] is defined as

$$cC(N, V) = \{x \in X_\Pi(N, V) : \forall S \subseteq N x^S \notin int V(S)\}$$

This definition leads to a new family of games, those with a non-empty c-core . The subdomain of NTU games with non-empty c-core is denoted by

$$\Gamma^{CC} = \{(N, V) \in \Gamma : cC(N, V) \neq \emptyset\}$$

It is clear that $\Gamma^C \subset \Gamma^{CC}$.

The *aspiration core or balanced aspiration set* (Cross 1967) [see also Albers (1979), Sharkey (1993), Hokari and Kibris (2003), Bejan and Gómez (2012) and Cesco (2012)] is defined as,

$$AC(N, V) = \{x \in X_\Lambda(N, V) : \forall S \subseteq N\ x^S \notin int V(S)\}$$

Remark 3 Considering the new feasibility definition, the aspiration core may be defined as $AC'(N, V) = \{x \in X_\Lambda(N, V) : \forall S \subseteq N$ and $\forall\ \mathcal{B}_S \in \Lambda(S)$ $x^S \notin int V(\mathcal{B}_S)\}$. However, it is easy to check that the two definitions are equivalent.

Remark 4 Sharkey (1993) [see also Cross (1967) and Bennett (1982)] shows that $AC(N, V) \neq \emptyset$ for every $(N, V) \in \Gamma$.[6]

Proposition 1 (a) *For each game* $(N, V) \in \Gamma$, $C(N, V) \subseteq cC(N, V) \subseteq AC(N, V)$.
(b) *For each game* $(N, V) \in \Gamma^B$, $AC(N, V) = cC(N, V) = C(N, V)$.[7]

Proof It is obvious. □

3 The axioms

Let Γ_0 be an arbitrary subset of Γ. Next, we focus in the following properties of solutions on Γ_0.

Non-emptiness (NE) A solution σ on Γ_0 satisfies NE if for every $(N, V) \in \Gamma_0$, $\sigma(N, V) \neq \emptyset$.

Individual Rationality (IR) A solution σ on Γ_0 satisfies IR if for every $(N, V) \in \Gamma_0$, every $x \in \sigma(N, V)$, and every $i \in N$, $x_i \notin int V(\{i\})$.

Efficiency (EF) A solution σ on Γ_0 satisfies *EF* if for every $(N, V) \in \Gamma_0$, $\sigma(N, V) \subseteq E_\Lambda(N, V)$.

We now present two versions of reduced games and their corresponding consistency axioms. First, the *DM-reduced game* (Davis and Maschler 1965; Peleg 1985) defined in a framework which assumes that the grand coalition forms.

Definition 3 Given a game (N, V), a non-empty coalition S, and $x \in X_\Lambda(N, V)$, the DM-*reduced game with respect to S and x* is the game $(S, V_{S,x})$ where:

$$V_{S,x}(T) = \begin{cases} \{y^S \in \mathbf{R}^S : (y^S, x^{N/S}) \in V(N)\} & \text{if } T = S \\ \cup_{Q \subseteq N \setminus S}\{y^T \in \mathbf{R}^T : (y^T, x^Q) \in V(T \cup Q)\} & \text{if } T \in 2^S \setminus \{S, \emptyset\} \\ \emptyset & \text{if } T = \emptyset. \end{cases}$$

DM-consistency (DM-CON) A solution σ on Γ_0 satisfies DM-CON if for every game $(N, V) \in \Gamma_0$, every $\emptyset \neq S \subseteq N$, and every $x \in \sigma(N, V)$, $(S, V_{S,x}) \in \Gamma_0$ and $x^S \in \sigma(S, V_{S,x})$.

Second, the *modified reduced game* used in axiomatizations of solutions that regard other generating families different to the grand coalition [see, among others, Moldovanu and Winter (1994); Hokari and Kibris (2003), Bejan and Gómez (2012)]. In this case, the grand coalition does not have a special treatment.

Definition 4 Given a game (N, V), a non-empty coalition S, and $x \in X_\Lambda(N, V)$, the modified *reduced game with respect to S and x* is the game $(S, V^m_{S,x})$ where:

$$V^m_{S,x}(T) = \begin{cases} \cup_{Q \subseteq N \setminus S}\{y^T \in \mathbf{R}^T : (y^T, x^Q) \in V(T \cup Q)\} & \text{if } T \in 2^S \setminus \{\emptyset\} \\ \emptyset & \text{if } T = \emptyset \end{cases}$$

[6] Cross (1967) considers TU games only. Bennett (1982) gives a more general proof of this fact.

[7] In TU games, this equality holds in the class of games with non-empty core, but it is not true for NTU games.

It can be checked that a (modified) reduced game of a NTU game is a NTU game.

MDM-consistency (MDM-CON) A solution σ on Γ_0 satisfies MDM-CON if for every game $(N, V) \in \Gamma_0$, every $\emptyset \neq S \subseteq N$, and every $x \in \sigma(N, V)$, $(S, V^m_{S,x}) \in \Gamma_0$ and $x^S \in \sigma(S, V^m_{S,x})$.

Remark 5 Peleg (1985) proves that the core satisfies MD-CON on Γ. A similar proof can be given to show that the core satisfies MDM-CON on Γ, therefore we omit it.

Now, we will consider an axiom about independence of irrelevant alternatives. It states that if an element x is prescribed in the solution of a game (N, V), and x remains as a feasible outcome in a game where some feasible payoffs of individual coalitions are removed, then x must be in the solution of the game in which the payoffs have been removed.

Independence of Individual Irrelevant Alternatives (I-IIA) A solution σ on Γ_0 satisfies I-IIA if for every pair of games $(N, V), (N, V^*) \in \Gamma_0$ such that $V^*(S) = V(S)$ for all $S \subseteq N$ with $|S| \neq 1$ and $V^*(\{i\}) \subseteq V(\{i\})$ for all $i \in N$, and every $x \in \sigma(N, V) \cap X_\Lambda(N, V^*)$, $x \in \sigma(N, V^*)$.

The axioms of independence of irrelevant alternatives have often been controversial. Whether or not these axioms are reasonable depends on how we view the solution. If we regard it as an expected or average outcome, this kind of axioms are not very convincing. By removing parts of the feasible sets, we decrease the range of possible outcomes and so, the average may change even if it itself remains feasible. But in NTU games, viewing the solution as an average is fraught with difficulty. An alternative is to view the solution as a group decision or arbitrated outcome, i.e., a reasonable compromise in view of all possible alternatives open to the players. In that case, axioms of independence of irrelevant alternatives do sound quite convincing and even compelling.

4 Axiomatizations

Lemma 2 *The aspiration core satisfies MDM-CON on* Γ.[8]

Proof Let $(N, V) \in \Gamma$, $x \in AC(N, V)$, and $S \in 2^N \backslash \{\emptyset\}$. The proof that the reduced game $(S, V^m_{S,x}) \in \Gamma$ proceeds similarly to the proof of Lemma 12.4.3 in Peleg and Sudhölter (2007) and therefore it is omitted. We need to prove that $x^S \in AC(S, V^m_{S,x})$.

First, we prove that $x^S \in X_\Lambda(S, V^m_{S,x})$. Let $\mathcal{B} \in \Lambda(N)$ with balanced weights $(\lambda_H)_{H \in \mathcal{B}}$ such that $x \in V(\mathcal{B})$. Let us define,

$$\mathcal{B}^S := \{T \subseteq S : T = H \cap S \neq \emptyset \text{ for some } H \in \mathcal{B}\},$$

and

$$\hat{\lambda}_T := \sum_{\substack{H \in \mathcal{B}: \\ T = H \cap S}} \lambda_H \quad \text{for each } T \in \mathcal{B}^S$$

[8] This lemma is stated in Hokari and Kibris (2003) as a final remark. We give a complete proof in this paper.

Then, for each $i \in S$

$$\sum_{\substack{T \in \mathcal{B}^S: \\ i \in T}} \hat{\lambda}_T = \sum_{\substack{T \in \mathcal{B}^S: \\ i \in T}} \sum_{\substack{H \in \mathcal{B}: \\ T = H \cap S}} \lambda_H = \sum_{\substack{H \in \mathcal{B}: \\ i \in H}} \lambda_H = 1$$

Therefore, $\mathcal{B}^S \in \Lambda(S)$ with balanced weights $(\hat{\lambda}_T)_{T \in \mathcal{B}^S}$. We now prove that $x^T \in V^m_{S,x}(T)$ for every $T \in \mathcal{B}^S$. Given $T \in \mathcal{B}^S$, there exists $H \in \mathcal{B}$ such that $T = H \cap S$. As $x \in V(\mathcal{B})$ and $H \in \mathcal{B}$, $(x^T, x^{H \setminus T}) = x^H \in V(H)$. Since $H \setminus T \subseteq N \setminus S$, $x^T \in V^m_{S,x}(T)$.

Second, we prove that $x^T \notin int V^m_{S,x}(T)$ for all $T \subseteq S$. Assume, on the contrary, there exists $T \subseteq S$ such that $x^T \in int V^m_{S,x}(T)$. Since $x^T \in int V^m_{S,x}(T)$, there exists $y^T \in V^m_{S,x}(T)$ such that $y^T >> x^T$. Then, there exists $Q \subseteq N \setminus S$ such that $(y^T, x^Q) \in V(T \cup Q)$. By comprehensiveness and non-levelness of $V(T \cup Q)$, $(x^T, x^Q) \in int V(T \cup Q)$, which contradicts that $x \in AC(N, V)$. Hence, $x^T \notin int V^m_{S,x}(T)$ for all $T \subseteq S$. □

Proposition 2 *The aspiration core satisfies NE, IR, MDM-CON and I-IIA on* Γ.

Proof By Remark 4, the aspiration core satisfies NE. The fact that the aspiration core satisfies IR follows from its definition. By Lemma 2, the aspiration core satisfies MDM-CON on Γ. Then, we only need to prove that the aspiration core satisfies I-IIA. Let $(N, V) \in \Gamma$ and let $(N, V^*) \in \Gamma$ such that $V^*(S) = V(S)$for all $S \subseteq N$ with $|S| \neq 1$ and $V^*(\{i\}) \subseteq V(\{i\})$ for all $i \in N$, and let $x \in AC(N, V) \cap X_\Lambda(N, V^*)$. Then, $x^S \notin int V^*(S)$ for all $S \subseteq N$ and $x \in X_\Lambda(N, V^*)$. Therefore, $x \in AC(N, V^*)$.

□

We proceed to formulate our main result.

Theorem 1 *There exists a unique solution on* Γ *that satisfies NE, IR, MDM-CON and I-IIA, and it is the aspiration core.*

We have already proven that the aspiration core satisfies NE, IR, MDM-CON and I-IIA on Γ. Thus, we only need to demonstrate uniqueness. Before, we state three useful lemmata.

Lemma 3 *Let* σ *be a solution on* Γ_0 *that satisfies IR and MDM-CON. Then, for every* $(N, V) \in \Gamma_0$, $\sigma(N, V) \subseteq E_\Lambda(N, V)$.

Proof Let $(N, V) \in \Gamma_0$ and let $x \in \sigma(N, V)$. Suppose, on the contrary, $x \notin E_\Lambda(V, V)$. Then there is $y \in X_\Lambda(N, V)$ such that $y >> x$. Let $\mathcal{B}' \in \Lambda(N)$ such that $y \in V(\mathcal{B}')$. There are two cases to consider:

Case (i) If $N \notin \mathcal{B}'$. Let $S \in \mathcal{B}'$, let $j, i \in N$ such that $i \in S$ and $j \notin S$, and let $T = \{i, j\}$. Since $y^S \in V(S)$ and $V(S)$ is comprehensive, $(y^i, x^{S \setminus \{i\}}) \in V(S) = V_{\{i\} \cup (S \setminus \{i\})}$. Hence, and since $S \setminus \{i\} \subseteq N \setminus T$, $y^i \in V^m_{T,x}(\{i\})$. So $x^i \in int V^m_{T,x}(\{i\})$. Furthermore, by MDM-CON, $(x^T, \mathcal{B}^T) \in \sigma(T, V^m_{T,x})$ which contradicts that σ satisfies IR.

Case (ii) If $N \in \mathcal{B}'$. Since $y \in V(\mathcal{B}')$, $y \in V(N)$. As $V(N)$ is comprehensive, $x \in V(N)$. Now, let $i \in N$ and let $T = \{i\}$. By MDM-CON,

$$x^i \in \sigma(T, V^m_{T,x}) \tag{1}$$

Since $V(N)$ is comprehensive, $y \in V(N)$ and $y >> x$, $(y^i, x^{N\setminus\{i\}}) \in V(N)$. Then, $y^i \in V^m_{T,x}(\{i\})$. Hence, $x^i \in int V^m_{T,x}(\{i\})$ which contradicts that σ satisfies IR and (1).

Then, there is not $y \in X_\Lambda(N, V)$ such that $y >> x$. Therefore, $x \in E_\Lambda(N, V)$. □

Lemma 4 *If σ is a solution on Γ_0 that satisfies IR and MDM-CON, then $\sigma(N, V) \subseteq AC(N, V)$ for every $(N, V) \in \Gamma_0$.*

Proof Let $(N, V) \in \Gamma$. Assume, on the contrary, that there is $x \in \sigma(N, V) \setminus AC(N, V)$. Then, there exists $S \subseteq N$ such that $x^S \in int V(S)$. There are two cases to consider:

Case (i) If $S \neq N$. Let $i \in S$, let $j \notin S$ and let $T = \{i, j\}$. By MDM-CON, $x^T \in \sigma(T, V^m_{T,x})$. Since $x^S \in int V(S) = int V(S \setminus \{i\}) \cup \{i\})$ and $S \setminus \{i\} \subseteq N \setminus T$, then $x^i \in int V^m_{T,x}\{i\}$ which contradicts that σ satisfies IR.

Case (ii) If $S = N$. Then, $x \in int V_N \subseteq int X_\Lambda(N, V)$, which contradicts Lemma 3. □

Lemma 5 *If σ is a solution on Γ_0 that satisfies NE, IR and MDM-CON, then $\sigma(N, V) = AC(N, V)$ for every $(N, V) \in \Gamma_0$ such that*

$$| AC(N, V) |= 1. \tag{2}$$

Proof Let $(N, V) \in \Gamma_0$ such that statement (2) holds. By Lemma 4, $\sigma(N, V) \subseteq AC(N, V)$. Since that σ satisfies NE and (2) holds, $\sigma(N, V) = AC(N, V)$ □

The following lemma is central to the proof of our main theorem. Details of the proof can be found in the appendix.

Lemma 6 *Let $(N, V) \in \Gamma$, and let $x \in AC(N, V)$. Then, there exists $(N, Z) \in \Gamma$ satisfying the following properties:*

$$AC(N, Z) = \{x\} \tag{*}$$

$$V(S) = Z(S) \quad \textit{for all } S \subseteq N \quad \textit{with } |S| \neq 1 \quad \textit{and} \quad V(\{i\}) \subseteq Z(\{i\}) \quad \textit{for all } i \in N \tag{**}$$

Proof See Appendix. □

Proof of Theorem 1 By Proposition 2, the aspiration core satisfies NE, IR, MDM - CON and I–IIA on Γ, so the only additional point to clarify is uniqueness. If a solution σ, satisfies NE, IR , MDM-CON and I–IIA, by Lemma 4, $\sigma(N, V) \subseteq AC(N, V)$ for all $(N, V) \in \Gamma$. To see the reverse inclusion, let $(N, V) \in \Gamma$ and let $x \in AC(N, V)$. By Lemma 6, there exists a game $(N, Z) \in \Gamma$ that satisfies (*) and (**). By Lemma 5, $x \in \sigma(N, Z)$. As $x \in X_\Lambda(N, V)$, $V(S) = Z(S)$ for all $S \subseteq N$ with $|S| \neq 1$ and $V(\{i\}) \subseteq Z(\{i\})$ for all $i \in N$, and σ satisfies I–IIA, then $x \in \sigma(N, V)$ □

Remark 6 Peleg (1985) obtains a similar axiomatization for the core in term of NE, IR, DM-CON and the converse consistency axiom. A solution σ on Γ_0 satisfies the modified converse consistency axiom **(MDM-CC)** if for every $(N, V) \in \Gamma_0$ and every $x \in X_\Lambda(N, V)$, if $T \in \{S \subseteq N : |S| = 2\}$ implies $(T, V^m_{T,x}) \in \Gamma_0$ and $x^T \in \sigma(T, V^m_{T,x})$, then $x \in \sigma(N, V)$. The aspiration core does not satisfy MDM-CC [see, Hokari and Kibris (2003)].

If many coalitions can be formed, under the only requirement that each player participates in at least one coalition a more relaxed feasibility condition must be considered. Given a game $(N, V) \in \Gamma$ and $x \in \mathbf{R}^n$, define

$$P_i(x) = \{S \subseteq N : i \in S \text{ and } x^S \in V(S)\}.$$

A vector payoff $x \in \mathbf{R}^n$ is a *feasible vector** of (N, V) if $P_i(x)$ is non-empty for each $i \in N$. The *set of feasible vectors** of (N, V) is denoted by $X_\Delta(N, V)$. It is clear that $X_\Lambda(N, V) \subseteq X_\Delta(N, V)$ for each game (N, V).

Fix a family of games $\Gamma_0 \subseteq \Gamma$. A *solution* on* Γ_0 is a mapping σ that assigns to every game $(N, V) \in \Gamma_0$ a set $\sigma(N, V) \subseteq X_\Delta(N, V)$.

The *aspiration set* (Bennett 1982) is defined as

$$ASP(N, V) = \{x \in X_\Delta(N, V) | \forall S \subseteq N \; x^S \notin int V(S)\}.$$

Remark 7 Since $X(N, V) \subseteq X_\Pi(N, V) \subseteq X_\Lambda(N, V) \subseteq X_\Delta(N, V)$,

$$C(N, V) \subseteq cC(N, V) \subseteq AC(N, V) \subseteq ASP(N, V)$$

As $AC(N, V) \neq \emptyset$ for each game $(N, V) \in \Gamma$, then $ASP(N, V) \neq \emptyset$ for each game $(N, V) \in \Gamma$.

Axioms of Non-emptiness, Individual Rationality, Independence of Individual Irrelevant Alternatives, MDM-Consistency and MDM converse consistency for solutions* on Γ_0, are defined in a similar way to the previous section considering the new feasibility condition. These axioms are denoted by NE*, IR*, I-IIA*, MDM-CON* and MDM-CC* in the new feasibility setting, respectively.

Similar results to Theorem 1 can be obtained in this context.

Theorem 2 *There exists a unique solution* on Γ, that satisfies NE*, IR*, MDM-CON* and I-IIA*, and it is the aspiration set.*

Proof It is clear that the aspiration set satisfies NE* and IR* and I-IIA* on Γ. According to Moldovanu and Winter (1994), the aspiration set satisfies MDM-CON* on Γ^C. The uniqueness is obtained reasoning as in Theorem 1 and, therefore, it is omitted. □

Remark 8 Moldovanu and Winter (1994) prove a similar axiomatization of the aspiration set as the only solution that satisfies using NE*, IR*, MDM-CON*, MDM-CC* and another axiom called "unanimity".[9] The independence of the axioms in Moldovanu and Winter axiomatization is still an open problem, but it is not in our present axiomatization of the aspiration set.

In order to obtain similar axiomatizations to those presented in Theorems 1 and 2 for the core and c-core, we now present two axioms that make special reference to the families of coalitions that are considered, in order to support the solutions.

Supported by N. A solution σ on Γ_0 is *supported by* N if for every game $(N, V) \in \Gamma_0$, $\sigma(N, V) \subseteq X(N, V)$.

Supported by Π. A solution σ on Γ_0 is *supported by* Π if for every game $(N, V) \in \Gamma_0$, $\sigma(N, V) \subseteq X_\pi(N, V)$.

The notion of supported by Π (by N) states that only the partitions (the grand coalition) are considered as feasible generating families.

Theorem 3 *There exists a unique solution (solution*) on Γ^C, supported by N, that satisfies NE, IR, MDM-CON and I–IIA (NE*, IR*, MDM-CON* and I–IIA*), and it is the core.*

Proof It is clear that the core satisfies NE and IR and I–IIA on Γ^C. By Remark 5, the core satisfies MDM-CON on Γ^C. The uniqueness is obtained reasoning as in Theorem 1 and taking into account that we are considering solutions supported by N. □

Remark 9 Peleg (1985) proves a similar axiomatization of the core on Γ^C using DM-CON and DM-converse consistency instead of MDM-CON and I–IIA. Theorem 4 is the first axiomatization of the core on NTU games that considers MDM-CON instead of DM-CON.[10] By previous observations, we have that MDM-CON and I–IIA are equivalent to DM-CON and DM-CC for solutions on Γ^C, supported by N, satisfying NE and IR.

If we want to consider solutions supported by Π, a modified axiom of independence of individual irrelevant alternatives must be defined as follows;[11]

Π-Independence of Individual Irrelevant Alternatives (Π -I–IIA) A solution σ on Γ_0 satisfies Π-I–IIA if for every pair of games $(N, V), (N, V^*) \in \Gamma_0$ such that $V^*(S) = V(S)$ for all $S \subseteq N$ with $|S| \neq 1$ and $V^*(\{i\}) \subseteq V(\{i\})$ for all $i \in N$, and every $x \in \sigma(N, V) \cap X_\Pi(N, V^*)$, $x \in \sigma(N, V^*)$.

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[9] In that paper the aspiration core is called the *semi-stable demand correspondence.* A solution satisfies *unanimity* if it coincides with the core for any two-person strictly super-additive game.

[10] In the context of TU games, Bejan and Gómez (2012) show an axiomatization of the core using MDM-CON instead of MD-CON. They prove that the DM-reduced games and the modified reduced games with respect to a payoff vector in the core are equal on TU games. This observation is not true on NTU games.

[11] The c-core does not satisfies I–IIA in the way this axiom has been formulated.

Theorem 4 *There exists a unique solution (solution*) on Γ^{CC}, supported by Π, that satisfies NE, IR, MDM-CON and Π-I–IIA (NE*, IR*, MDM-CON* and Π- I–IIA*), and it is the c-core.*

Proof The proof is obtained reasoning as in Proposition 2 and Theorem 1, and using the fact that we are considering solutions supported by Π, therefore it is omitted. □

Remark 10 This is the first axiomatization of the c-core on NTU games in the literature.

5 Independence of the axioms

The following examples show that each of the axioms used in Theorem 1 are logically independent of the remaining axioms. Such examples can be easily adapted to show that the axioms used in Theorems 2 are also independent.[12]

Example 1 Let $\sigma(N, V) = \emptyset$ for all $(N, V) \in \Gamma$. Then σ satisfies IR, MDM-CON, and I–IIA but not NE.

Example 2 Let $\sigma(N, V) = X_\Lambda(N, V)$ for all $(N, V) \in \Gamma$. Then σ satisfies NE, MDM-CON and I–IIA, but not IR.

Example 3 Let $\sigma(N, V) = \{x \in X_\Lambda(N, V) : x^i \notin int V(\{i\})$ for all $i \in N\}$ for all $(N, V) \in \Gamma$. Then σ satisfies NE, IR and I-IIA, but not MDM-CON.

Example 4 Given a game $(N, V) \in \Gamma$ and $x \in \mathbf{R}^n$ we have defined

$$P_i(x) = \{S \subseteq N : i \in S \text{ and } x^S \in V(S)\}.$$

A vector payoff $x \in \mathbf{R}^n$ is called a partnered payoff if, for each i in N the set $P_i(x)$ is non-empty for each $i \in N$ and for each pair of players i and j in N the following requirement is satisfied:[13]

$$\text{If } P_i(x) \subseteq P_j(x) \quad \text{then } P_j(x) \subseteq P_i(x).$$

The *set of stable demand vectors* (Moldovanu and Winter 1994) [see also Bennett and Zame (1988)] is defined by,[14]

$$SSD(N, V) = \{x \in \mathbf{R}^n : x \text{ is a partnered payoff and for all } S \subseteq S, x^S \notin int V(S)\}.$$

The partnered aspiration core is defined by,

$$PAC(N, V) = SSD(N, V) \cap AC(N, V)$$

[12] These examples do not use the fact that the universal of players, M, is finite. Then, the independence of the axioms also holds in the case in which M is infinite.

[13] This terminology and notation are due to Reny and Wooders (1996).

[14] The stable demand vectors are called bargaining aspirations by Bennett and Zame (1988).

It is clear that the partnered aspiration core satisfies IR on Γ. The fact that the partnered aspiration core satisfies NE on Γ is proven in Reny and Wooders (1996).[15] Moldovanu and Winter (1994) prove that the set of *stable demand vectors* satisfies MDM-CON* on Γ. Furthermore, we prove that the aspiration core satisfies MDM-CON on Γ. Then, the partnered aspiration core satisfies MDM-CON on Γ.

To show that the partnered aspiration core does not satisfy I–IIA on Γ, we consider the games (N, V) and (N, V^*) defined by:

$$N = \{1,2\};\quad V(\{i\}) = \{x^i \in \mathbf{R}^{\{i\}} : x^i \leq 2\},\quad \text{for } i = 1,2;$$
$$V(\{1,2\}) = \{x^{\{1,2\}} \in \mathbf{R}^{\{1,2\}} : x^1 + x^2 \leq 4\},\quad V^*(\{i\}) = \{x^i \in \mathbf{R}^{\{i\}} : x^i \leq i\}$$
$$\text{for } i = 1,2;\quad V^*(\{1,2\}) = \{x^{\{1,2\}} \in \mathbf{R}^{\{1,2\}} : x^1 + x^2 \leq 4\}.$$

It follows that: $(2,2) \in X(N,V)$; $(2,2) \in PAC(N,V)$ and $(2,2) \notin PAC(N,V^*)$. Therefore, the partnered aspiration core does not satisfy the I–IIA on Γ.

The following examples show that each of the axioms used in Theorems 1 are logically independent of the remaining axioms. Such examples can be easily adapted to show that the axioms used in Theorems 2 are also independent.[16]

Example 5 Let $\sigma(N,V) = \emptyset$ for all $(N,V) \in \Gamma$. Then σ is supported by N and satisfies IR, MDM-CON, and I–IIA but not NE.

Example 6 Let $\sigma(N,V) = X(N,V)$ for all $(N,V) \in \Gamma^C$. Then σ is supported by N and satisfies NE, MDM-CON and I–IIA, but not IR.

Example 7 Let $\sigma(N,V) = \{x \in X(N,V) : x^i \notin int V(\{i\}) \text{ for all } i \in N\}$ for all $(N,V) \in \Gamma^C$. Then σ is supported by N and satisfies NE, IR and I–IIA, but not MDM-CON.

Example 8 Let $\Gamma^1 = \{(M,V) \in \Gamma : (M,V) \in \Gamma^B\}$ and let $\Gamma^2 = \Gamma^C/\Gamma^1$. Define

$$\sigma(N,V) = \begin{cases} C(N,V) \cap SSD(N,V) & \text{if } (N,V) \in \Gamma^1 \\ C(N,V) \text{ if } (N,V) \in \Gamma^2 \end{cases}$$

It is clear that σ satisfies IR on Γ^C. The fact that σ satisfies NE on Γ^C is proven in Reny and Wooders (1996).[17] As the core satisfies MDM-CON on Γ^C, σ satisfies MDM-CON on Γ^C. It can be checked that σ violates I–IIA.

Acknowledgments The author acknowledges financial support from Universidad Nacional de San Luis, through grant 319502, and from Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), through Grant PIP 112-200801-00655. The author thanks anonymous referees for valuable comments.

[15] Reny and Wooders (1996) prove that the partnered core ($SSD(N,V) \cap C(N,V)$) of a balanced game is non-empty implying that the partnered aspiration core is non-empty for all games.

[16] These examples use the fact that the universal of players, M, is finite. In case that M is infinite, the independence of I–IIA with respect to the other axioms in Theorems 4 and 3 remains an open question.

[17] Reny and Wooders (1996) prove that the partnered core of a balanced game is non-empty.

Appendix

Lemma 1 *For each NTU game (N, V), the set $X_{\Lambda}(N, V)$ is non-empty, closed, comprehensive, and Pareto-bounded.*

Proof First, we prove that for each $\mathcal{B} \in \Lambda(N)$, $V(\mathcal{B})$ is non-empty, closed and comprehensive. As $V(\mathcal{B}) = \{x \in \mathbf{R}^N : x^S \in V(S)\ \forall S \in \mathcal{B}\} = \bigcap_{S\in\mathcal{B}}(V(S) \times \mathbf{R}^{N\setminus S})$, non-emptiness of $V(\mathcal{B})$ follows from the fact that $V(S)$ is non-empty and comprehensive for each $S \in \mathcal{B}$ and $\mathcal{B}$ is finite.

Second, we prove that $V(\mathcal{B})$ is comprehensive. Let $y \in V(\mathcal{B})$ and $y \geq x$. Then $y^S \in V(S)$ for all $S \in \mathcal{B}$. Since $V(S)$ is comprehensive, $x^S \in V(S)$ for all $S \in \mathcal{B}$. Then, $x \in V(\mathcal{B})$. Therefore, $V(\mathcal{B})$ is comprehensive.

Third, we prove that $V(\mathcal{B})$ is closed. By definition of $V(\mathcal{B})$,

$$V(\mathcal{B}) = \bigcap_{S\in\mathcal{B}} (V(S) \times R^{N\setminus S})$$

Since that $V(S)$ is closed for all $S \in \mathcal{B}$, then $V(S) \times R^{N\setminus S}$ is closed for all $S \in \mathcal{B}$. Therefore, $V(\mathcal{B})$ is closed because it is a intersection of closed sets.

Finally, we prove that $V(\mathcal{B})$ is Pareto-bounded. Let $x \in \mathbf{R}^n$, then

$$\begin{aligned} V(\mathcal{B}) \cap (x + \mathbf{R}^n_+) &= \left(\bigcap_{S\in\mathcal{B}} \left(V(S) \times R^{N\setminus S}\right)\right) \cap (x + \mathbf{R}^n_+) \\ &= \bigcap_{S\in\mathcal{B}} \left(\left(V(S) \times R^{N\setminus S}\right) \cap (x + \mathbf{R}^n_+)\right) \\ &= \bigcap_{S\in\mathcal{B}} \left(V(S) \cap (x^S + \mathbf{R}^S_+)\right] \times (x^{N/S} + \mathbf{R}^{N/S}_+)\right) \end{aligned}$$

Since $\cap_{S\in\mathcal{B}}(V(S) \cap (x^S + \mathbf{R}^S_+)]$ is bounded and $\cup_{S\in\mathcal{B}} S = N$, we have that $\cap_{S\in\mathcal{B}}(V(S) \cap (x^S + \mathbf{R}^S_+)] \times (x^{N/S} + \mathbf{R}^{N/S}_+))$ is bounded. Therefore, $V(\mathcal{B}) \cap (x + \mathbf{R}^n_+)$ is bounded.

We now prove that $X_{\Lambda}(N, V)$ is non-empty, closed, comprehensive, and Pareto-bounded. Since $X_{\Lambda}(N, V)$ is a finite union of sets satisfying these properties, $X_{\Lambda}(N, V)$ also satisfies such properties. □

Lemma 6 *Let $(N, V) \in \Gamma$ and let $x \in AC(N, V)$. Then there exists $(N, Z) \in \Gamma$ such that satisfies the following properties:*

$$AC(N, Z) = \{x\} \tag{*}$$

$$V(S) = Z(S) \quad \textit{for all } S \subseteq N \quad \textit{with } |S| \neq 1 \quad \textit{and} \quad V(\{i\}) \subseteq Z(\{i\}) \quad \textit{for all } i \in N \tag{**}$$

Proof Define

$$Z(S) = \begin{cases} V(S) & \text{if } S \subseteq N \quad \text{and} \quad |S| \neq 1 \\ (-\infty, x^i] & \text{if } S = \{i\} \end{cases}$$

Proof of (*). Since $x \in AC(N, V)$, $x \in AC(N, Z)$. If $y \in AC(M, Z)$. Then, $y^i \notin int Z(\{i\})$ for all $i \in N$. Then, $y \geq x$. Assume that $y \neq x$. Then, there exists $i^* \in N$ such that $y^{i^*} > z^{i^*}$. Let $\mathcal{B}^* \in \Lambda(N)$ such that $y^S \in Z(S)$ for all $S \in \mathcal{B}^*$ and let $S^* \in \mathcal{B}^*$ such that $i^* \in S^*$. Then, $y^{S^*} \in Z(S^*)$ and $y^{S^*} > x^{S^*}$. Therefore, as $Z(S^*)$ is non-levelled, $x^{S^*} \in int Z(S^*)$ which contradicts that $x \in AC(N, Z)$. Therefore, $AC(N, Z) = \{x\}$.

Proof of (**). If $S \subseteq N$ and $|S| \neq 1$. Then, $Z(S) = V(S)$.

If $S \subseteq N$ and $S = \{i\}$. Then, $V(\{i\}) \subseteq (-\infty, x^i] = Z(\{i\})$ (because $x^i \notin int V(\{i\})$). □

References

Albers W (1979) Core-and kernel-variants based on imputations and demand profiles. In: Moeschlin O, Pollaschke D (eds) Game theory and related fields. North-Holland, Amsterdam

Aumann R (1961) The core of a cooperative game without side payments. Trans Am Math Soc 98:539–552

Aumann R (1985) An axiomatization of the non-transferable utility value. Econometrica 53(3):599–612

Bejan C, Gómez J (2012) Axiomatizing core extensions. Int J Game Theory 41(4):885–898

Bennett E (1982) Aspirations Approach to Non-Transferable Utility Games. Working Paper No. 523, School of Management, SUNY at Buffalo

Bennett E (1983) The aspiration approach to predicting coalition formation and payoff distribution in sidepayment games. Int J Game Theory 12(1):1–28

Bennett E, Zame W (1988) Bargaining in cooperative games. Int J Game Theory 17:279–300

Cesco J (2012) Nonempty core-type solutions over balanced coalitions in TU games. Int Game Theory Rev 14(3):1–16 1250018

Cross Z (1967) Some theoretic characteristics of economic and political coalitions. J Confl Resolut 11:184–195

Davis M, Maschler M (1965) The kernel of a cooperative game. Nav Res Logist Quart 12(3):223–259

Guesnerie R, Oddou C (1979) On economic games which are not necessarily superadditive: solution concepts and application to a local public good problem with few a agents. Econ Lett 3(4):301–306

Hokari T, Kibris Ö (2003) Consistency, converse consistency, and aspirations in tu-games. Math Soc Sci 45(3):313–331

Hwang YA, Sudhölter P (2001) Axiomatizations of the core on the universal domain and other natural domains. Int J Game Theory 29:597–623

Kóczy LÁ, Lauwers L (2004) The coalition structure core is accessible. Games Econ Behav 48(1):86–93

Moldovanu B, Winter E (1994) Consistent demands for coalition formation. In: Megiddo N (ed) Essays in game theory in honor of Michael Maschler. Springer, New York, pp 129–140

Peleg B (1985) An axiomatization of the core of cooperative games without side payments. J Math Econ 14:203–214

Peleg B (1986) On the reduced game property and its converse. Int J Game Theory 15:187–200

Peleg B, Sudhölter P (2007) Introduction to the theory of cooperative games, 2nd edn. Theory and Decision Library, Series CSpringer-Verlag, Berlin

Peleg B, Sudhölter P, Zarzuelo JM (2012) On the impact of independence of irrelevant alternatives. SERIEs 3:157–159. doi:10.1007/s13209-011-0073-4

Reny P, Wooders M (1996) The partnered core of a game without side payments. J Econ Theory 70:298–311

Sharkey W (1993) A characterization of some aspirations with an application to spatial games. Bellcore Economics Working Paper 95, Bellcore

Scarf H (1967) The core of an N person game. Econometrica 35(1):50–69

Sun N, Trockel W, Yang Z (2008) Competitive outcomes and endogenous coalition formation in an n-person game. J Math Econ 44(7–8):853–860