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Operations Research Transforms the Scheduling of Chilean Soccer Leagues and South American World Cup Qualifiers

Fernando Alarcón
Industrial Engineering Department, University of Chile, Santiago, Chile, falarcon@ing.uchile.cl

Guillermo Durán
Mathematics Department and Calculus Institute, University of Buenos Aires;
National Scientific and Technical Research Council (CONICET), Buenos Aires, Argentina; and
Industrial Engineering Department, University of Chile, Santiago, Chile, gduran@dm.uba.ar

Mario Guajardo
Department of Business and Management Science, Norwegian School of Economics, Bergen, Norway 5045, mario.guajardo@nhh.no

Jaime Miranda
Department of Management Control and Information Systems, School of Economics and Business, University of Chile, Santiago, Chile, jmirandap@fen.uchile.cl

Hugo Muñoz, Luis Ramírez
Chilean Professional Soccer Association (ANFP), Santiago, Chile
{hrmunozb@anfpchile.cl, lramirez@anfpchile.cl}

Mario Ramírez
Canal del Fútbol (CDF), Santiago, Chile, mramireza@cdf.cl

Denis Sauré
Industrial Engineering Department, University of Chile, Santiago, Chile, dsaure@dii.uchile.cl

Matías Siebert
H. Milton Stewart School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332, msiebert6@gatech.edu

Sebastián Souyris
McCombs School of Business, University of Texas at Austin, Austin, Texas 78712, sebastian.souyris@utexas.edu

Andrés Weintraub, Rodrigo Wolf-Yadlin, Gonzalo Zamorano
Industrial Engineering Department, University of Chile, Santiago, Chile
{aweintra@dii.uchile.cl, rwolf@dii.uchile.cl, gonzaloz@dii.uchile.cl}

For the past 12 years, the Chilean Professional Soccer Association (ANFP) has applied operations research (OR) techniques to schedule soccer leagues in Chile. Using integer programming-based methods, the ANFP decides which matches are played in each round, taking into account various objectives, such as holding down costs and ensuring engaging tournaments for the fans. It has scheduled more than 50 tournaments using this approach, resulting in an estimated direct economic impact of about $59 million, including reductions in television broadcaster operating costs, growth in soccer pay-television subscriptions, increased ticket revenue, and lower travel costs for the teams. This application of OR has also had significant noneconomic impacts. First, the incorporation of team requirements and various sporting criteria has improved process transparency and schedule fairness, increasing fans' interest in local professional tournaments; second, because of the high portability of these techniques, they have been used successfully to schedule sports leagues in other countries (examples include volleyball and basketball in Argentina, and the South American qualifiers for the 2018 Soccer World Cup). Furthermore, the models and methods used in this scheduling application have been disseminated widely, helping to promote OR as an effective tool for addressing practical problems. Our outreach activities have reached thousands of high school and university students in four countries and a more general audience of millions of television viewers and Internet users.

Keywords: sports analytics; scheduling; integer programming; soccer.

Association football, also known as football or soccer, is the most popular sport in the world. Although the sport's popularity might vary from region to region, it undoubtedly attracts worldwide attention. Practiced by approximately 200 million registered players, soccer is a central aspect of the cultural identity of many countries, over 200 of which compete to qualify for the World Cup every four years. A report in Deloitte (2016) revealed that the world's top 10 money league teams have over 600 million social media followers worldwide (including Facebook, Twitter, and Instagram). In economic terms, this report estimated their combined revenue during 2015 at $5 billion. This figure is comparable to the revenue generated by the 2014 World Cup.

Soccer is by far the most popular sport in Chile, where it has been professionally played since 1933. The Chilean Professional Soccer Association (ANFP) organizes the professional leagues and all of Chile's official tournaments. It also manages the national soccer team, which represents Chile in international competitions, such as the World Cup and Copa América.

In 2004, prompted by a decline in match attendance and general public interest, ANFP management set out to improve the attractiveness of its First Division tournaments. Driving this effort was the need to schedule the soccer seasons to make the match calendars attractive to fans, and fair and balanced for all the teams in both sporting and economic terms. Because this task proved too difficult using traditional methods, the ANFP teamed up with the operations research (OR) group in the Department of Industrial Engineering at the University of Chile to design a new scheduling methodology.

Using OR techniques, our team developed an algorithmic approach capable of balancing sporting, operational, and economic criteria. Based on integer programming (IP), our OR models produce schedules that satisfy diverse and dynamic criteria, which can vary significantly from year to year, including television (TV) broadcast considerations and team traveling arrangements. Using this new methodology, the ANFP incorporated a series of enhancements into its First Division schedules, boosting fans' interest in the season tournaments. Benefits were also reflected in improved transparency of the scheduling process and greater competition fairness. Given these positive results, the ANFP has employed OR methods to schedule every Chilean First Division season since 2005. It has also used these methods to schedule most of its other leagues, including the Second, Third, and Youth Divisions, and the annual Chile Cup competition in which all the First and Second Division teams participate. Thus, over these past 12 years, the ANFP has scheduled more than 50 soccer tournaments using OR techniques. The estimated direct economic impact is about $59 million, of which 50 percent results from reductions in broadcaster operating costs and growth in soccer pay-TV subscriptions, 41 percent from growth in ticket revenue, and nine percent from lower team travel costs.

In 2015, we proposed using an OR methodology to design a draft schedule for the 2018 Federation Internationale de Football Association (FIFA) World Cup South American qualification stage organized by the South American Football Confederation (CONMEBOL). Our proposal suggested a series of modifications to the method used, which improved upon the previous schedules and won the unanimous approval of all the CONMEBOL countries. The new schedule is now being used to determine which countries will represent South America at the World Cup in Russia in 2018. To the best of our knowledge, this is the first application of OR to soccer scheduling at the international level.

Background: Professional Soccer in Chile circa 2005

The ANFP manages Chile's three men's professional leagues (the First, Second, and Third divisions), the Chile Cup, the youth leagues, and the female leagues.

Over the years, the First Division league has adopted different formats. In the past decade, it comprised between 16 and 20 teams competing on a split-season format consisting of a fall and a spring tournament. In each of these two tournaments, the teams were organized into groups and played each other exactly once, earning three, one, or zero points for each match won, tied, or lost, respectively. At the end of the season, the two top teams in each group advanced to a play-off stage. The Second Division consisted of 14–16 teams that competed annually in a single tournament. The season began with a group phase in which the teams faced rivals exclusively from their own groups, followed by a national phase where each team played every other team twice, once at home and once away. Each season, the worst (best)-performing teams from the First (Second) Division were relegated (promoted) to the Second (First Division), respectively.

Every year at the start of each tournament, the ANFP posted a complete schedule of games over a series of rounds or match dates. Each team played exactly one match on one of the two or three consecutive days making up each round. The schedules incorporated only a few basic format rules (e.g., each team plays every other team exactly once), typically ignoring other important considerations, as the following two examples illustrate. (1) In the First Division, three teams (Colo-Colo, Universidad Católica, and Universidad de Chile) are traditionally much stronger than the other teams. To reduce the likelihood of a weaker team not winning any points for long periods, scheduling that team to play back to back against any two of the three strong teams should be avoided. (2) The teams' home venues are spread across the length of Chile (Figure 1);

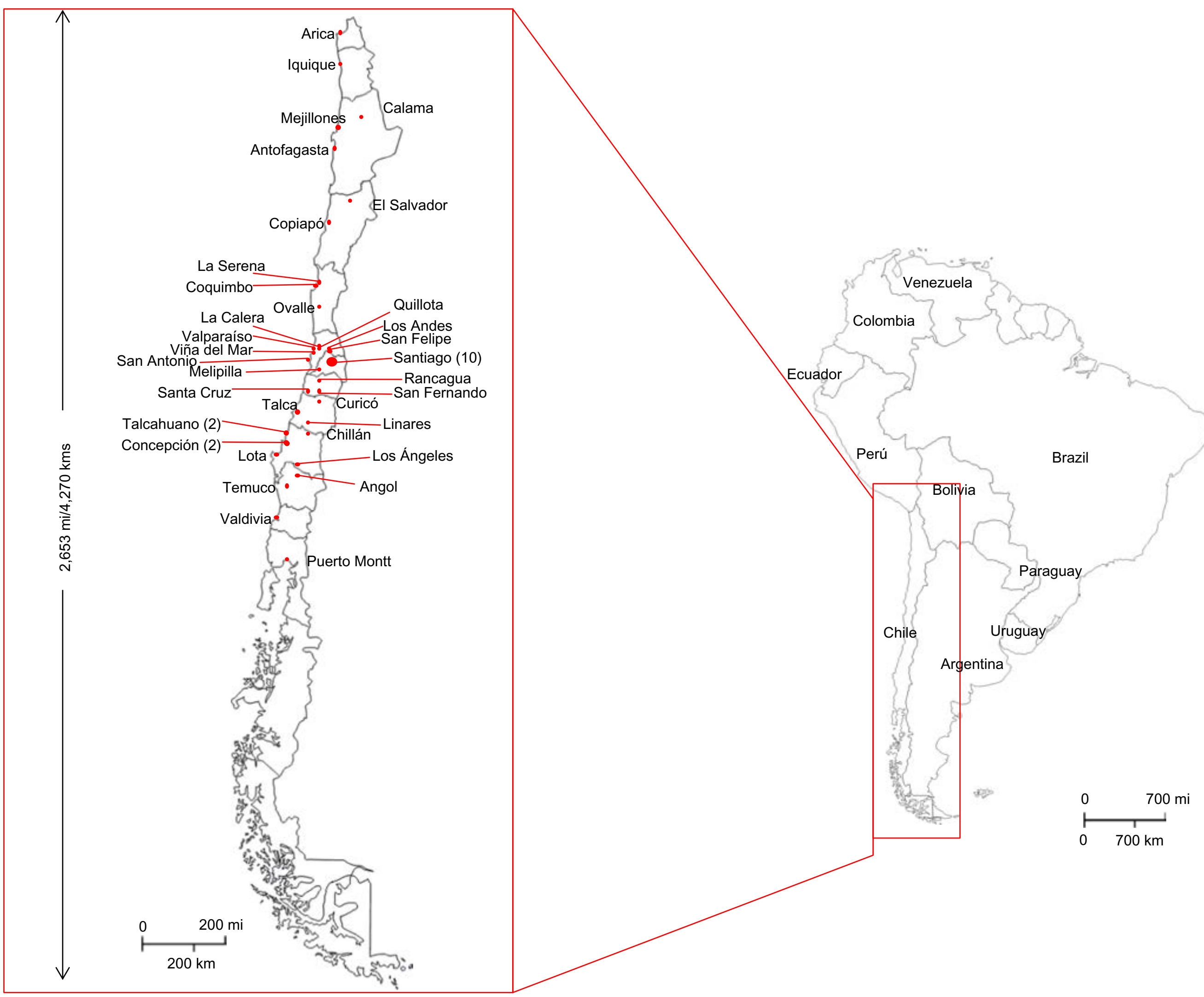

Figure 1: (Color online) On the left, a map of Chile indicates cities in which home venues of professional teams are located (the number of teams is shown in brackets); on the right, a map of South America indicates the 10 countries that are members of CONMEBOL.

therefore, no team should have to travel long distances twice in a short period of time.

The schedules should also consider each team's cash flow. For example, to the extent possible, the weaker teams should be assigned a balanced mix of home and away games against the stronger ones to attract higher attendance; in addition, in any three consecutive games, at least one should be at home so that no team goes for an extended period without ticket revenue (which goes exclusively to the home team).

Unfortunately, although the ANFP acknowledged the necessity to consider these and many other sporting and economic factors (we provide an exhaustive list later in the paper), it did not incorporate them into the schedules. The scheduling method used prior to applying the OR techniques consisted of a preset template of games fulfilling only the basic constraints, with a slot for each team identified simply by a number. Prior to each tournament, the ANFP held a random drawing to assign the slot numbers to the teams. It created the template using the canonical one-factorization method; although this method dates back at least a century, many sports leagues around the world still use it (Goossens and Spieksma 2012). Figure 2 shows a schedule generated using this approach for a single round-robin tournament of six teams. Each column in the figure represents a round, each row within a column represents a match, and the arrows indicate how the teams rotate for the following round's schedule. Although easy to implement manually, the method cannot factor in all the desirable considerations, and the schedule often triggered complaints from team representatives.

In the 2004 season, the deficiencies of the canonical method were particularly evident; the two most popular teams were scheduled to play each other

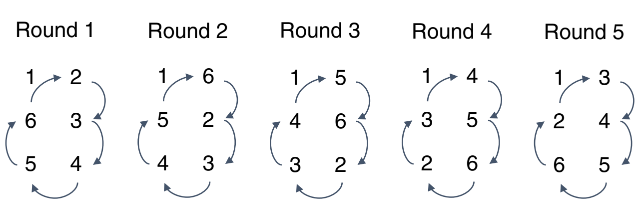

Figure 2: (Color online) This canonical schedule template is an example of scheduling six teams in a single round-robin tournament.

in round one, although public attention was elsewhere. The fall tournament begins late in the Southern hemisphere summer when many Chileans are still on summer vacation. At that time, the newly created paid-subscription channel, Canal del Fútbol (CDF), 80 percent of which belongs to the ANFP and 20 percent to private interests and which owns the exclusive rights to broadcast professional games, had just begun broadcasting a few First Division games from each round. ANFP management was understandably keen to build up CDF's subscription base and the First Division schedule was fundamental to achieving that goal.

This was the scenario in late 2004 when George Nemhauser gave a presentation at a workshop organized at the University of Chile on the use of OR for scheduling the American professional baseball and college basketball leagues (Nemhauser and Trick 1998). Anticipating that such an approach could be applied to the Chilean soccer league tournaments, we set up an interview with Nemhauser by Chile's main daily newspaper. Having caught the attention of the ANFP, we presented its management with a proposal for a new OR-based scheduling methodology, which the Council of Presidents of all the First and Second Division teams in the ANFP accepted.

Sports Scheduling Around the World

Academic interest in sports scheduling dates back to the 1970s when the focus was on reducing travel distances for teams in American professional sports leagues, such as Major League Baseball and the National Basketball Association, because teams in these associations could be on the road for long periods. The traveling tournament problem (Easton et al. 2001), the most studied problem in the field, illustrates the distinctive features and computational complexity associated with the classic approach to sports scheduling. Of its classic benchmarking instances, only those with no more than 10 teams have been solved to optimality.

To the best of our knowledge, as of the end of 2004, there were only two documented real-world cases of using OR for soccer scheduling. The first case was the 1989–1990 season of the Dutch league (Schreuder 1992) in which 18 teams competed in a single round-robin tournament. The underlying model considered various commercial, sporting, and organizational factors.

Constrained by the tournament format, the aim was to satisfy certain conditions by maximizing a weighted sum of soft requirements. The solution method, however, was heuristic, and was able to satisfy only 80 percent of the requirements. The second case was the Argentinean league in which a simulated annealing approach was used to assign teams to a canonical schedule for the 1995 season, considering certain factors relating to TV broadcasting of the games, according to Eduardo Dubuc (Paenza 2006); however, the method was dropped the following year.

Chile's tournament scheduling considerations differ in many respects from those of other soccer leagues, because they incorporate a number of unique spatial factors and stakeholder constraints. Many of them arise from Chile's geography, which stretches approximately 2,600 miles from North to South. Although most rounds are played on weekends, some midweek rounds must also be scheduled to ensure they do not conflict with the national team's FIFA match calendar.

Other distinctive features of Chile's situation include factors relating to the efficient operation of the TV broadcaster and the necessity to boost public interest in the league. The latter aspect raised the challenge of designing a schedule attractiveness metric, which we could incorporate as a criterion in our optimization model.

Main Criteria for Chilean Tournaments

We added a series of sporting and economic-fairness criteria, which we describe later in this section, to the classic round-robin tournament conditions, thus setting our problem apart from others that have been studied in the academic literature.

Geographical considerations. Chile is long and narrow; therefore, cities tend to be relatively far apart. In this spatial context, we focus particularly on the scheduling of away matches, especially two-game sequences, incorporating considerations of sporting fairness, travel-cost savings, reduction of player fatigue, and coordination with international match dates.

Seasonal issues. In Chile's rainy Southern region, games often must be cancelled. To keep cancellations to a minimum, we try to ensure that teams from the South play few home games in the wettest winter months. Conversely, because many Chileans are still on vacation when the fall tournament begins in the late summer, attractive games during this period are scheduled where possible in popular vacation spots.

TV-broadcaster operational considerations. To hold down the travel costs that CDF incurs to broadcast the matches, we include the condition that some matches that are televised live at various times on a weekend round must be scheduled in cities that are relatively close to each other. This allows a single television production truck and crew to cover several games.

Security considerations. Teams with home stadiums in neighboring areas of the same city are constrained not to play at home in the same round to avoid fan brawls on the streets.

Tournament attractiveness. We calculate a schedule's attractiveness through the problem's objective function; we assign each match a value depending on the round in which it is played. Our objective is to schedule the most attractive games in the second half of the season when their outcomes are particularly important; for example, they determine a league title or a spot for an international cup.

Using OR techniques, our group developed a model that efficiently generated schedules incorporating all of the desired requirements. Next, we describe the techniques we used and the model we designed.

OR Approach to Soccer Scheduling

We approached the problem by formulating it as an integer program in which the key decision variables are indexed by (ordered) pairs of teams and a given round. One of the variables takes the value 1 if the first team plays at home against the second team in that round, and 0 otherwise. The conditions we outline above are encoded with the aforementioned variables in the form of linear equalities and inequalities. This ensures that a value assignment fulfilling all the conditions is a feasible tournament schedule.

The model also has additional auxiliary variables that help encode conditions pertaining to consecutive home or away games; it contains about 8,000 binary variables and 3,000 constraints. Because of the complex combinatorial structure characteristic of sports scheduling problems, our attempts to solve the model with an off-the-shelf commercial solver, such as CPLEX, resulted in days of computations, but did not find a feasible solution. For a single round-robin tournament with only eight teams, 31 million schedules are possible (taking into account only the most basic format conditions). This gives us a sense of the computational difficulty of finding a solution. For the First Division tournament with 16–20 teams, the number of schedules is enormous. This, combined with the large number of conditions to be satisfied, made identifying a feasible solution extremely challenging.

To find a feasible first schedule in 2005, we utilized a constraint-programming approach, which had been used successfully in scheduling college basketball leagues (Henz 2001) and in solving the traveling tournament problem (Easton et al. 2003). We obtained a first schedule within three hours of running time.

With one feasible solution already identified, we applied a pattern-based decomposition approach to improve the objective value. Central to this technique is the concept of a home-away (H-A) pattern—that is, an ordered array of "home" and "away" labels containing as many labels as the number of rounds in a tournament. A team is said to follow a given pattern if it plays at home in a given round if and only if the label associated with that round in the pattern is home. Various strategies have been suggested in the literature for constructing these patterns. Examples include the graph-theoretical strategies in de Werra (1988) and Schreuder (1992), and an IP strategy in Nemhauser and Trick (1998).

Our decomposition approach operates in three stages. In the first stage, a set of n patterns (where n is the number of teams) is constructed using an IP model. In the second stage, a group of patterns (possibly all of them) is selected; some are assigned permanently to certain teams associated with more requirements, and the rest are left unassigned. In the third stage, the final model assigns the as-yet unassigned patterns to teams and attempts to find a schedule.

This heuristic approach significantly reduces the number of assignments of values to variables, which are needed to form a feasible schedule. Rasmussen and Trick (2008) provide a detailed survey of pattern-based methods and their application to solving sports scheduling problems.

Not all sets of patterns generated in the first stage lead to feasible solutions. Therefore, the first stage might have to be repeated several times. Furthermore, different pattern sets may lead to different feasible schedules with varying attributes. Therefore, the ability to generate feasible schedules in a matter of minutes is important because of the dynamic nature of the scheduling process. The various stakeholders involved inevitably come up with new ad-hoc conditions, as tentative schedules are generated and new problems not evident in earlier versions become apparent. In practice, the models are typically run about 60 times before a definitive one is selected.

Over the years, improvements in hardware and IP solvers have significantly reduced running times, opening up the possibility of developing more sophisticated solution strategies. This means, for example, that more time can be devoted to generating feasible H-A patterns. This enables the second stage of the approach to start with more patterns than teams, thus reducing the chances that the problem will remain infeasible. Currently, we are using a hybrid approach that combines the fixing of few patterns with the IP model formulation that constructs the final schedule. More specifically, we assign patterns only to some teams, while relaxing all the others so that we can simultaneously construct patterns for the latter with the construction of the schedule. This adds flexibility to the process by allowing a larger feasible set than would be possible with the more restrictive strategy of fixing patterns for all teams, albeit at the expense of potentially longer execution times. Over time, we have adapted to improvements in technology. For example, in 2005, we had to assign patterns to at least 15 of 20 teams to keep execution times down to a matter of minutes. Today, we can assign patterns to only 5 of 20 teams and generate a final solution in about the same amount of time.

We generally adapt our solution strategy to the specifics of the setting. For example, for the Third Division, we fix patterns only partially; that is, we impose H-A conditions for only a subset of rounds. Durán et al. (2007, 2012) include a detailed formulation of the methods developed for the First and Second Divisions. For the sake of completeness, the appendix provides a simplified formulation of the models.

Implementation and Challenges to Decision Making

Using the methodology previously described, a group of faculty and students of the University of Chile generated a sequence of scheduling proposals for the First

Division's fall 2005 tournament, with each proposal satisfying a larger set of conditions. At the end of this process, we presented a final-schedule proposal and an overview of the methodology to the council of presidents of all First and Second Division teams in the ANFP, and the council approved it.

Over the years, we have been able to adapt the OR-based methodology to diverse and ever-changing requirements. This would have been impossible using manual methods or the canonical schedule. Our approach has also demonstrated the versatility and flexibility to accommodate a variety of tournament formats. Since our first efforts with the Chilean league in 2005, we have scheduled 24 First Division tournaments.

One difficulty in scheduling a tournament is aligning the interests of all the involved stakeholders, including team officials, players, coaches, television networks, fans, and public-safety authorities. The ANFP's competitions office has the crucial task of receiving and coordinating all stakeholder requirements. The first scheduling problem we addressed in 2005 consisted of about 20 requirements; that number has since doubled. Several of the new conditions were the result of the incorporation of constraints relating to CDF's operations. On one occasion during the 2006 fall tournament, a CDF executive remarked on how lucky the broadcaster was to be televising a pair of weekend matches involving two strong teams, both of which were playing away games that round in the North of Chile. The relative proximity of the two venues and the correspondingly low TV production truck transfer costs saved the channel a significant amount of money, thanks not to chance but to the OR-generated schedule (Durán et al. 2007).

Other anecdotes reveal how the OR approach has enabled quick responses to unforeseen circumstances. One example is the situation that arose after the schedule for the 2005 spring tournament had already been published: One of the country's most popular teams was scheduled to play an away game in a city that was hosting a major religious festival. This meant that relatively few people would attend the game and that the festival's security demands would reduce the large police presence normally required for such a match. An attempt to manually reschedule the game was unsuccessful; however, using the OR methodology, we quickly found a new solution that altered only 19 of the tournament's 190 games in the published schedule.

A similar situation arose in 2014 during the ODESUR South American Games, hosted in Santiago: The ANFP released a schedule in which the country's three most popular teams were to play several matches during the games in Santiago. When the local authorities found out, they insisted that the ANFP should change the schedule so that they could exclusively focus police resources on the games. We responded quickly by modifying the OR model to incorporate this unforeseen issue, generating a new schedule that minimized the changes to the original one.

Another scheduling difficulty occurred in 2007. The tournament calendar had already been finalized when the ANFP's disciplinary tribunal, which functions autonomously, ruled that a team that had been relegated because of salary payment problems could return to the First Division. A new schedule for 21 teams, not 20, had to be generated within a few days. As in the above examples, without using OR, this would have been extremely difficult to achieve, while keeping all the desirable aspects of the original solution. The ability to handle these types of circumstances has reinforced the ANFP's perception of the OR approach's value to sports scheduling.

Portability

Following our success with the First Division, we extended the application of our OR techniques to the Second and Third Divisions, the annual Chile Cup, and the youth leagues. The annual process of scheduling these tournaments begins with the First Division, and continues with the Second and then the Third. Each division takes two to three weeks and the whole process lasts about two months. The Chile Cup and the youth leagues are handled separately. In a special extension of our work, we recently scheduled the South American qualification stage of the 2018 World Cup to be held in Russia. Figure 3 shows a timeline of the adoption of our OR scheduling methodology by various leagues and tournaments.

Youth Divisions

Our first experience scheduling a league other than the First Division was the 2005 spring tournament of the

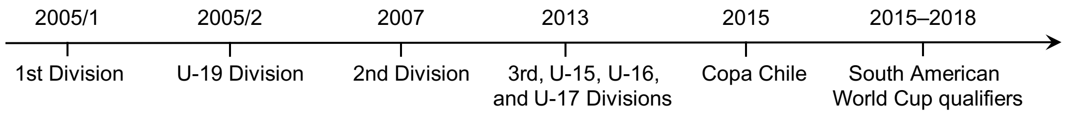

Figure 3: A timeline of the adoption of OR methods for scheduling Chile's professional soccer tournaments and the World Cup's South American qualifiers shows that the use of these methods will extend through at least 2018.

Chilean U-19 (i.e., under 19 years of age) Youth Division, which consisted of 14 teams. All but one of them were owned by First Division teams, most of which have youth teams in various age categories. The U-19 Division competition was organized as a single round-robin tournament played in parallel with the tournament of the First Division. The First Division calendar had already been defined; therefore, the ANFP wanted to schedule as many of the U-19 games as possible such that the First Division teams and their youth counterparts would play on the same date. Using an OR approach for this scheduling problem, we formulated an objective function that maximized the number of such games subject to several constraints particular to the U-19 Division. The overall scheduling problem was less constrained than the problem for the First Division, and we found an optimal solution in a matter of seconds.

In the ensuing years, we have scheduled several U-19 Division tournaments, and those of the younger U-17, U-16, and U-15 Divisions, adapting the OR approach to the formats these leagues have used at various times. Currently, the four calendars are determined simultaneously in such a way that the U-17 and U-19 Division schedules are identical. We simply mirror them to generate the U-15 and U-16 versions, thus achieving a more efficient allocation of team resources. The U-17 and U-19 teams within a given adult team can share the same coach, medical doctor, and other officials. Using the same schedule allows these team personnel to cover two matches on the same trip.

Each of these youth leagues currently consists of 40 teams divided into one main group of 16 teams, and four other groups of six teams each. The main objective for their schedules is to create good trips. We define a good trip as a sequence of two consecutive away games played by a team in the same distant region (in this case the team plays both games without returning to its city). For example, the youth teams from the Northern cities of Iquique and Calama are paired so that they are scheduled to play most pairs of rounds against a pair of teams from another region. This ensures good trips either for the two Northern teams (when they play away) or for their two opponents from the other region (when the Northern teams play them at home). The OR-based calendar generated for the Youth Divisions has scheduled as many as 28 of such good trips per tournament.

Second and Third Professional-Soccer-League Divisions and Chile Cup

After the encouraging results of our first two years of scheduling soccer matches, we were asked to develop an OR application for Chile's Second Division league. The teams in this league are often based in cities outside Santiago; because entertainment options in these cities tend to be limited, soccer is important to the local inhabitants and team sponsors. The Second Division has undergone several modifications since we first scheduled it in 2007. The league's schedule format has at various times been a twice-mirrored quadruple round-robin, a two-phase tournament with zonal and national phases, and a mirrored double round-robin. We applied our OR approach to all of these formats using two IP models detailed in Durán et al. (2012). O.R. has been utilized to schedule all 15 tournaments played by this league since 2007. In its most recent version, the OR model objective was to minimize the number of winter-month games in the rainy Southern regions of the country, and thus reduce the number of matches held in bad weather or cancelled because of it.

In 2013, we began scheduling the Third Division league, which consists of 13 teams that compete in a tournament divided into two stages. The first stage is a double round-robin; the six highest teams in the standings then compete in the second-stage double round-robin for promotion to the Second Division. Only the highest team in the standings from the second stage is promoted. The seven lowest teams in the standings of the first stage compete in a double round-robin tournament to avoid relegation to the Fourth Division. The three lowest teams in these standings are relegated. Unlike the other three divisions, the Fourth Division is an amateur division; therefore, the possibility of relegation heightens the drama and excitement of the Third Division tournaments.

In 2015 we scheduled the edition of the Chile Cup competition. The 32 participating teams were divided into eight groups. In the first or group stage, each group played a double round-robin under one of several possible symmetric formats we used to create schedules. The top two teams in each group advanced to the second stage, a 16-team play-off competition.

South American World Cup Qualifiers

The FIFA World Cup receives more international attention than any other single-sport event in the world, attracting dozens of sponsors, intense media coverage, and billions of viewers.

In the South American qualifiers, 10 national teams (see the map on the right in Figure 1) compete in a double round-robin format to win one of 4.5 berths in the World Cup (four are determined directly and a fifth is determined by an intercontinental play-off). The qualifying tournament consists of 18 rounds spread over two to three years. A main feature of this tournament is that each odd-numbered round (i.e., first, third, fifth, etc.) is followed within a few days by the next (even) round, after which there is a relatively long gap before the next pair of rounds is played. The schedule thus consists of nine of these closely spaced pairs, referred to as double rounds. This arrangement raises some specific issues of scheduling balance and fairness, particularly with regard to double-round breaks—double rounds in which both matches are played either at home or away. Ideally, no team should have a double-round break; that is, in every double round, a team should play one game at home and the other game away.

For all five World Cup qualification tournaments between the 1998 and 2014 editions, CONMEBOL used a mirrored schedule (i.e., the first and second halves of the tournament are identical, except that home-away status is reversed). The minimum number of double-round breaks in this mirrored format is 16 (Durán et al. 2015b). The schedule that CONMEBOL used for every qualifier tournament between 2002 and 2014 had 18 double-round breaks. Given all the local excitement and the economic benefit that home matches entail (e.g., ticket sales, sponsor publicity), a double-round away break is undesirable for fans, team officials, and the media, particularly when long periods separate one double round from the next. For example, in the qualifiers for the 2014 World Cup in Brazil, Chile had an away break after playing at home on November 15, 2011. The team did not play again at home until September 11 of the following year. To make matters worse, the distribution of the double-round breaks among the teams was highly unbalanced (i.e., Bolivia had seven while Argentina and Brazil had none). Furthermore, the distribution of H-A and away-home (A-H) sequences in double rounds was also unbalanced (i.e., Argentina had nine H-A sequences, while Brazil had none). Consider that, for logistical reasons, national teams generally prefer to play the first game of a double round at home. This spares team members, who play in a foreign country and who therefore usually return home shortly before each double round to train with their national team, from having to travel again to play the first match.

As a result of these and other shortcomings, and the inability of the 10 CONMEBOL nations to agree on a new schedule, CONMEBOL's executive committee decided that the schedule for the 2018 World Cup qualifiers would have to be defined by a random drawing, and asked each country to prepare proposals that could serve as a basis for the drawing. In this context, we created a series of templates or generic schedules using OR that, in addition to all the basic and other constraints, allowed us to eliminate all double-round breaks (Durán et al. 2015b), while maintaining a certain symmetry between the first and second halves and balancing the H-A sequences.

One of these schedules, constructed according to a French scheme (Goossens and Spieksma 2012), was selected as Chile's proposal for the 2018 World Cup qualifiers. A key advantage of the schedule was that it ensured all teams play one game at home and one game away in each double round, and greatly improved the balance of H-A and A-H sequences by giving all teams at least four and at most five of these sequences. Although other countries also presented proposals at the meeting, ours was selected unanimously for the 2018 World Cup qualifiers. In July 2015, the random drawing took place in Saint Petersburg at an event that involved the 209 FIFA-affiliated national associations and was broadcast worldwide. Former *Best World Cup Award* players Ronaldo and Forlán drew balls from different pots to define the South American qualifiers schedule. The balls contained the names of the participating teams and numbers indicating the position each team would take in the schedule we constructed. Ronaldo and Forlán might not have understood the OR techniques behind the schedule design, and we cannot score goals the way they do; however, as operational researchers and soccer practitioners came together for the good of the game, the moment was undoubtedly a historic milestone in our scheduling projects.

Argentina's Professional Volleyball and Basketball Leagues and a Norwegian University Squash League

Although our work in Chile has focused on soccer, a few members of our team have also designed spin-off applications in other countries for different sports. One such case is Argentina's professional volleyball league, which has been using O.R. for its schedules since 2007. The league's regular season format is a double round-robin, but with the peculiarity that its 12 teams are grouped into pairs. Each weekend, pair $A = (A_1, A_2)$ visits pair $B = (B_1, B_2)$. On the first day, A_1 plays B_2 and A_2 plays B_1; two days later, these combinations are reversed. Two weekends in the season are devoted exclusively to games between the member teams of each pair. The national champion is determined in a play-off format.

The problem of scheduling this setup is solved in two stages. In the first stage, an IP model defines pairs of teams whose home venues are relatively close to each other. We have used different methods for this, but a simple one is based on minimum perfect matching.

The second stage is essentially an instance of the traveling tournament problem (TTP). We have scheduled both mirrored- and nonmirrored-season formats over the years. In the mirrored format, we use an IP model; for the nonmirrored, we employ tabu-search heuristics.

To the best of our knowledge, this was the first application of the TTP to a real-world sports league reported in the literature. Bonomo et al. (2012) report savings of 15–20 percent in travel distance compared to the manual schedules used in previous years. According to Diego Arribas, the league's competitions manager, "working with the OR group on our schedules since 2007 has enabled us to cut travel costs as well as reducing player fatigue" (INFORMS 2016).

Also in Argentina, the professional basketball league adopted OR for designing its tournaments in 2014 (Durán et al. 2015a). The approach used consisted of both a new scheduling methodology and a new tournament format, which we proposed and which is similar to the one that the NBA uses.

The 20 teams in the First Division of the league compete in a regional stage that consists of two double round-robins, each of which involves 10 teams and two zones. This is followed by a national stage in which all 20 teams compete in a national double round-robin. Again, the national champion is decided using a play-off format.

The variation on the TTP in this case is that the teams define a priori the road trips they want. The problem is therefore one of feasibility rather than one of optimization.

We developed two IP models for this problem, one for game assignment and the second for day assignment. The first model assigns the order of the games; the second specifies the exact day on which each game is played.

More specifically, the first model allows road trips that vary from the ones that the team requested; however, the objective function penalizes such cases by trying to minimize the teams' travel distances. The model solves the regional stage in less than an hour, but it does not find a solution for the national stage. We therefore partition the national stage into parts, usually three or four. The model then proceeds by assigning games to each part until it has scheduled all games in the stage. If it does not find a solution, it iterates this procedure until it finds one. This approach has proven to be efficient, solving the national stage in a few hours.

The teams have preferences regarding the days of the week they play at home; therefore, the objective of the second model is to maximize the number of days on which these preferences are satisfied. This model solves both the regional and national stages in a few seconds.

We have also implemented models that use a similar format for the league's Second Division in which 26 teams play.

Commenting on the results of these scheduling changes, Juan Ignacio "Pepe" Sánchez, gold medalist with the Argentinean national team at the 2004 Olympics in Athens and currently president of the First Division's Bahía Basket, noted that "thanks to the system developed by the OR group, our team has lowered travel time and costs by 40 percent, which has also brought about a major drop in injuries compared to previous years" (INFORMS 2016). The reduction in annual travel costs as a result of the new scheduling system that the two league divisions now use is estimated at $700,000, or about 30 percent of the teams' global travel expense (Durán et al. 2015a).

Another example motivated by our work is the scheduling of the Norwegian School of Economics intramural squash league. Although a smaller-scale problem than those of the professional leagues we discuss above, it involved a number of special considerations because of the limited availability of time slots and number of courts. The OR approach defined a format and a schedule that successfully addressed these constraints.

Quantitative Impact and Benefits

We estimate that the total direct economic impact of applying O.R. to Chilean soccer scheduling from 2005 to 2015 amounts to about $59 million. Figure 4 charts the major sources of this quantitative impact and their relative contributions. Next, we discuss the three main components.

• Impact on ticket revenue: Maximizing the attractiveness of matches to soccer fans has been one of the main criteria underlying the development of the schedules. Although there is no easy way to isolate exogenous effects in measuring the impact of the application of OR on attendance and ticket revenue, detailed calculations of tournament outcomes before and after the adoption of our methodology (Durán et al. 2007, 2012) reveal dramatic improvements. The games between teams with historic rivalries provide a particularly good example. Comparing these classic matches in the tournaments immediately after the introduction of OR modeling, which applied specific criteria to schedule them on appropriate dates, with classic matches immediately before its introduction showed an increase in stadium attendance of 74 percent and in ticket revenue of 142 percent. Similarly, the attendance at summer games scheduled in tourist areas between popular teams and local teams increased between 46 percent and 156 percent. Taking all matches into account, the increased revenue per game averaged about $6,000 for the First Division and $1,100 for the Second Division. For the 3,579 First Division and 2,047 Second Division regular-season games played since we began scheduling them, the estimated ticket revenue impact was $23.9 million.

• Impact on the soccer pay-TV channel: Televising games involves transporting the channel's mobile-broadcasting crews and equipment to the game venues. The application of our OR-designed schedules led to a better allocation of these resources. For example, by scheduling two matches played in the far North of Chile close to each other (e.g., one on a Saturday and the second on Sunday), one television production truck and crew can handle both. Total cost reductions for the pay-TV channel have been about $2.1 million, assuming estimated savings of $20,000 per trip and five good trips scheduled per tournament.

In addition, between 2006 and 2015, the number of games broadcast per round rose from four to nine, and the number of pay-TV premium subscribers increased by 630,000. We cannot state precisely how much of this increase is a result of better schedules; however, we can make a reasonable estimate based on knowing that the largest increases in the number of subscribers were registered for the most attractive classic-game dates. Considering that 90 percent of soccer fans declare themselves to be supporters of one of the three most popular teams playing in these matches, this is not surprising. We assume one-third of these classic-game increases would not have occurred had they not been scheduled for attractive dates. The resulting share corresponds on average to 4.8 percent of the number of subscribers per year, which for the entire 10-year period translates into an estimated economic impact of $27.8 million.

Pay-TV subscription revenue is distributed between the cable operators, the private company that jointly

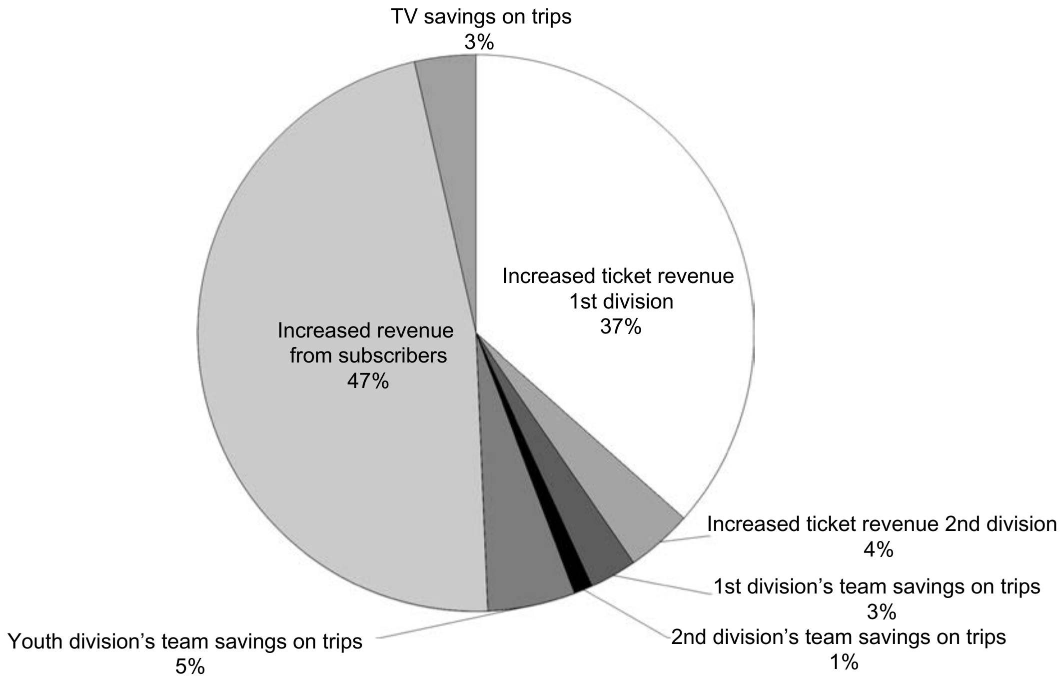

Figure 4: A8 In this pie chart, we categorize the sources of economic impact of OR scheduling on Chilean soccer leagues.

owns the channel with the ANFP, and, most importantly for our purposes, the teams, because they receive more than half of the total revenue. Generally speaking, teams reinvest this income in infrastructure and strengthening their player lineups for the following season. Thus, the improvement in the channel's financial performance as a result of better scheduling also has positive effects on the league's economic bottom line.

- Team travel savings: The third component in the total quantitative impact of the OR-based schedules is the cost reductions the teams achieve because of the better travel sequences that result from a better ordering of home and away games. For example, teams from the far North of Chile have reduced their costs (over the previous scheduling method) each time the schedule requires them to play a pair of consecutive away matches in the Center or Southern regions in a midweek round followed (or preceded) immediately by a weekend round, rather than playing on dates separated by matches at home. Team expenditures on travel since we began using our scheduling methodology decreased by about \$5.2 million; the First Division, Second Division, and Youth Division saved \$1.6 million, \$0.6 million, and \$3.0 million, respectively. To arrive at this estimate, we first calculated the cost difference in total transportation and accommodation expenses between a two-game road trip and two separate one-game trips in which a team plays the same opponents. We made separate calculations for each division because the applicable standards of travel comfort vary significantly. We then multiplied this cost difference for each division by the corresponding average number of good trips per tournament and the number of OR-scheduled tournaments. The savings for the Youth Divisions, which are approximately 50 percent per trip, are particularly important because these teams usually have more limited budgets.

The global quantitative impact from all sources is currently about \$6.4 million per year. As our methods continue to be applied and new improvements are incorporated, we expect this number to rise by about \$7 million each year (assuming a 10 percent increase above the current annual impact).

Qualitative Impact and Benefits

Unlike conventional businesses, soccer has a large social component; attractive tournaments produce fan enjoyment that exceeds the ticket revenues or subscriber fees actually paid. Thus, the qualitative impact of OR scheduling is equal to, if not greater than, the quantitative impact. The benefits have been particularly significant in the following areas:

- greater transparency because all concerned parties are informed of the scheduling criteria;
- greater fairness because all of the teams' concerns are considered;
- greater attractiveness because the matches scheduled, especially toward the end of the season, are more exciting for fans and the media;
- greater public order as a result of better allocation of police resources, which are limited, and fewer street clashes between fans of classic rivals in the same city;
- greater credibility in the eyes of players, team administrators, fans, and the media;
- less player fatigue as a result of better travel sequences;
- fewer rescheduling of matches once the season is underway;
- better performance in international tournaments because league schedules are well coordinated with international match dates. Chilean teams have improved their performance in the group stage of the Copa Libertadores, the main South American soccer tournament, by 37 percent over the pre-OR scheduling period; two teams reached the Copa Sudamericana final for the first time and one of these teams won it; and
- better stadium operations because the scheduling model incorporates constraints to avoid dates reserved by the teams' stadiums for maintenance or other events, such as concerts, political rallies, or other sports competitions; for example, in 2015, when Chile hosted the FIFA U-17 Soccer World Cup, some stadiums were unavailable for certain rounds of the national soccer leagues.

Outreach

In addition to the quantitative and qualitative effects we discuss above, our scheduling work has had a significant educational impact through outreach activities and the creation of content used by schools, universities, and the media. These activities have reached thousands of high school and college students in Chile as a part of public educational programs. Moreover, because of our interaction with the ANFP during the past 12 years, students have benefited greatly from the experience of applying the methodological tools we provide for classroom teaching to what we believe is an interesting practical application.

In the private sector, CDF has created a short video explaining in layperson's terms how the Chilean soccer league is scheduled. Shown several times daily, the video is part of an educational series broadcast on both CDF's premium pay-TV channel and on a basic CDF channel that all Chilean cable operators provide for free. The video targets a potential audience of about 12 million people out of a national population of 18 million.

Outreach based on our work is increasingly extending beyond Chile's borders. The Argentinean Ministry of Education has disseminated content from our scheduling project to a broad audience. Since 2014, the project has been used as a case study in an OR course at the University of Buenos Aires. This material has also been used in a master's program at the University of Buenos Aires, an MBA program at the University of Texas at Austin, and an undergraduate course at the Norwegian School of Economics.

Conclusions

The use of OR by our team in the Chilean soccer leagues, the South American World Cup qualifiers, and other sports leagues in various countries, as we report in this paper, constitutes a unique example of the successful and sustained use of OR in sports scheduling, as well as an exemplary story of continued collaboration between the private sector and academia in general, and in particular the University of Chile whose faculty and students have been generating usable sports schedules for 12 years. By combining analytical tools with highly popular sports competitions, our work has demonstrated the many benefits OR can bring to decision making.

While the application of OR techniques to sports scheduling has resulted in significant economic savings, its qualitative impact on sports fans, the media, and public institutions are often more significant. One of these factors is transparency. The OR approach enables the scheduling method to incorporate objective criteria that are clear to all stakeholders. We believe this contribution is particularly important in light of the recent FIFA corruption cases (Ruiz et al. 2015). Other important qualitative dimensions improved by the adoption of OR scheduling are schedule fairness, schedule attractiveness, and public order.

Our work has also brought about a transformation in the ANFP decision-making culture. The adoption of the new tools encountered some initial resistance in the organization; however, ANFP officials are now fully convinced of their value. Remarkably, despite three major leadership changes at the Association over the life of the project, the use of OR techniques has continued. Arturo Salah, the current president of the ANFP, recently commented that officials at the ANFP "could not even imagine trying to schedule the Chilean league seasons without using OR" (INFORMS 2016).

Interestingly, the implementation of OR techniques in real-world soccer leagues has increased considerably since that day in 2004 when the ANFP's management decided that it had to improve its schedules in the face of falling match attendance. Since the isolated Dutch and Argentinean cases of the 1990s (Paenza 2006), the literature has reported implementations by the soccer leagues of Germany and Austria (Bartsch et al. 2006), Denmark (Rasmussen 2008), Belgium (Goossens and Spieksma 2009), Norway (Flatberg et al. 2009), Honduras (Fiallos et al. 2010), Brazil (Ribeiro and Urrutia 2012), and Ecuador (Recalde et al. 2013). Nevertheless, many other leagues continue to use the canonical schedules despite their drawbacks (Goossens and Spieksma 2012).

The cultural transformation at the ANFP is also reflected in its deployment of analytical tools for decision support in solving other problems. For example, it has applied an OR method we developed to resolve the assignment of referees to matches in its tournaments (Alarcón et al. 2014).

As we have seen, the tournament formats for Chile's First, Second, Third, and Youth Divisions, and the Chile Cup have changed several times over the past 12 years. Without analytical tools, these modifications would have been much more difficult to implement. The use of O.R. has given the ANFP the necessary flexibility to manage them. The ANFP is currently setting up an analytics section in conjunction with the OR group of the University of Chile. It will be devoted exclusively to scheduling and statistical analysis of the various competitions that the Association manages.

The almost universal role of sports competitions as general public entertainment provides an opportunity to create awareness of O.R. among broad segments of society in countries worldwide.

Appendix

In this appendix, we present simplified versions of the two IP models we use to schedule the Chilean soccer leagues. The first model generates patterns; the second model assigns the games.

Sets and Parameters (Common to Both Models)

T: set of teams.
R: set of rounds.
R_1: set of rounds, excluding the last one.
R_2: set of rounds, excluding the last two.
$K_{i,j}$: set of rounds in which a match can be scheduled between teams i and j at the venue of team i.
S: set of all rounds k for which a good two-game road trip can be scheduled for rounds k and $k+1$ (i.e., implying either k or $k+1$ is a midweek round).
C: set of three geographical clusters (North, Center, and South).
L_c: set of teams in cluster c.
E: set of teams in far North or far South.
H_i: set of opposing teams suitable for scheduling good trips for team $i \in E$.
h: maximum number of home breaks per team.
a: maximum number of away breaks per team.
n_i: minimum number of good trips that must be scheduled for team $i \in E$.
m_c: minimum number of matches per round in cluster c.
M_c: maximum number of matches per round in cluster c.
N: number of rounds.
P: set of patterns.
TP: set of teams that follow a pattern.
$u_{p,k}$: 1 if label of pattern p for round k is home, otherwise 0 (this is the solution of the pattern-generation model).

Pattern-Generation Model

The following simplified version of the pattern-generation model omits, among other elements, the constraints that generate patterns for specific teams.

Decision Variables

$\alpha_{i,k}$: 1 if pattern i indicates a home game in round k, otherwise 0.
$\beta_{i,k}$: 1 if pattern i indicates a home break in round k, otherwise 0.

$\gamma_{i,k}$: 1 if pattern i indicates an away break in round k, otherwise 0.
$\delta_{i,j,k}$: 1 if pattern i differs from pattern j in round k, otherwise 0.

Constraints

$$\sum_{p\in P}\alpha_{p,k}=\frac{|P|}{2},\quad \forall k\in R; \tag{1}$$

$$\sum_{k\in R}\delta_{p,g,k}\geq 1,\quad \forall p,g\in P,\ p\neq g; \tag{2}$$

$$\delta_{p,g,k}=\delta_{g,p,k},\quad \forall k\in R,\ p,g\in P,\ p\neq g; \tag{3}$$

$$\delta_{p,g,k}\leq\alpha_{p,k}+\alpha_{g,k},\quad \forall k\in R,\ p,g\in P,\ p\neq g; \tag{4a}$$

$$\alpha_{p,k}+\alpha_{g,k}\leq 2-\delta_{p,g,k},\quad \forall k\in R,\ p,g\in P,\ p\neq g; \tag{4b}$$

$$\alpha_{p,k}-\alpha_{g,k}\leq\delta_{p,g,k},\quad \forall k\in R,\ p,g\in P,\ p\neq g; \tag{4c}$$

$$\alpha_{p,k}+\alpha_{p,k+1}\leq 1+\beta_{p,k},\quad \forall k\in R_1,\ p\in P; \tag{5a}$$

$$\beta_{p,k}-\alpha_{p,k}\leq 0,\quad \forall k\in R_1,\ p\in P; \tag{5b}$$

$$\beta_{p,k}-\alpha_{p,k+1}\leq 0,\quad \forall k\in R_1,\ p\in P; \tag{5c}$$

$$1\leq\gamma_{p,k}+\alpha_{p,k}+\alpha_{p,k+1},\quad \forall k\in R_1,\ p\in P; \tag{6a}$$

$$\alpha_{p,k}+\gamma_{p,k}\leq 1,\quad \forall k\in R_1,\ p\in P; \tag{6b}$$

$$\alpha_{p,k+1}+\gamma_{p,k}\leq 1,\quad \forall k\in R_1,\ p\in P; \tag{6c}$$

$$\sum_{k\in R_1}\beta_{p,k}\leq h,\quad \forall p\in P; \tag{7}$$

$$\sum_{k\in R_1}\gamma_{p,k}\leq a,\quad \forall p\in P; \tag{8}$$

$$1\leq\alpha_{p,k}+\alpha_{p,k+1}\alpha_{p,k+2}\leq 2,\quad \forall p\in P,\ k\in R_2; \tag{9}$$

$$\beta_{p,N-1}+\beta_{p,1}+\gamma_{p,N-1}+\gamma_{p,1}=0,\quad \forall p\in P; \tag{10}$$

$$\frac{|R|-1}{2}\leq\sum_{k\in R}\alpha_{p,k}\leq\frac{|R|+1}{2},\quad \forall p\in P. \tag{11}$$

For this model to generate feasible patterns for the second model, the number of patterns must be the same as the number of teams. To generate additional patterns, we run the model multiple times and impose the constraint that the solutions must be different each time.

Constraints (1) state that in each round, half of the patterns, or teams, must play at home. Constraints (2) impose that the pattern for each pair of teams must differ at least once, so the teams can play against each other. Constraints (3) ensure the symmetry feature of the variable δ. Constraints (4a)–(4c) are the logical relationships between variables α and δ. Constraints (5a)–(5c) are the logical relationships between variables α and β. Constraints (6a)–(6c) are the logical relationships between variables α and γ. Constraints (7) and (8) impose the maximum number of home and away breaks per pattern. Constraints (9) states that each pattern has at least one and at most two home labels every three consecutive rounds. Constraints (10) ensure that no team will either start or end the tournament with two consecutive home (away) matches. Constraints (11) determine the balance required between home and away matches for each team.

Objective Function
This model is either run without an objective function or, in some cases, uses the following formulation:

$$\text{Minimize}\quad f=\sum_{i\in T,\,k\in R_1}(\beta_{i,k}+\gamma_{i,k}). \tag{12}$$

This objective function attempts to minimize the total number of breaks.

Game-Assignment Model
The following simplified version of the main linear model receives as input the patterns generated by the model previously presented.

Decision Variables
$x_{i,j,k}$: 1 if team i plays at home against team j in round k, otherwise 0.
$y_{i,k}$: 1 if team i plays at home in rounds k and $k+1$, otherwise 0.
$z_{i,k}$: 1 if team i plays away in rounds k and $k+1$, otherwise 0.
$w_{i,k}$: 1 if team i has a good trip in rounds k and $k+1$, otherwise 0.
$v_{i,p}$: 1 if team i follows pattern p, otherwise 0.

Constraints

$$\sum_{k\in K_{i,j}}x_{i,j,k}+\sum_{k\in K_{j,i}}x_{j,i,k}=1,\quad \forall i,j\in T,\ i\neq j; \tag{13}$$

$$\sum_{j\in T}(x_{i,j,k}+x_{j,i,k})=1,\quad \forall i\in T,\ k\in R; \tag{14}$$

$$\frac{|R|-1}{2}\leq\sum_{k\in R}\sum_{j\in T}x_{i,j,k}\leq\frac{|R|+1}{2},\quad \forall i\in T; \tag{15}$$

$$\sum_{k\in R:\,k<|R|}y_{i,k}\leq h,\quad \forall i\in T; \tag{16}$$

$$\sum_{k\in R:\,k<|R|}z_{i,k}\leq a,\quad \forall i\in T; \tag{17}$$

$$1\leq\sum_{j\in T}(x_{i,j,k}+x_{i,j,k+1}x_{i,j,k+2})\leq 2,\quad \forall i\in T,\ k\in R_2; \tag{18}$$

$$\sum_{k\in S}w_{i,k}\geq n_i,\quad \forall i\in E; \tag{19}$$

$$m_c\leq\sum_{i\in L_c}\sum_{j\in T}x_{i,j,k}\leq M_c,\quad \forall k\in R,\ c\in C; \tag{20}$$

$$\sum_{j\in T}(x_{i,j,k}+x_{i,j,k+1})\leq 1+y_{i,k},\quad \forall i\in T,\ k\in R_1; \tag{21a}$$

$$\sum_{j\in T}(x_{i,j,k}+x_{i,j,k+1})\geq 2y_{i,k},\quad \forall i\in T,\ k\in R_1; \tag{21b}$$

$$\sum_{j\in T}(x_{j,i,k}+x_{j,i,k+1})\le 1+z_{i,k},\quad \forall i\in T,\ k\in R_1; \tag{21c}$$

$$\sum_{j\in T}(x_{j,i,k}+x_{j,i,k+1})\ge 2z_{i,k},\quad \forall i\in T,\ k\in R_1; \tag{21d}$$

$$\sum_{j\in H_i}(x_{j,i,k}+x_{j,i,k+1})\le 1+w_{i,k},\quad \forall i\in E,\ k\in S; \tag{21e}$$

$$\sum_{j\in H_i}(x_{j,i,k}+x_{j,i,k+1})\ge 2w_{i,k},\quad \forall i\in E,\ k\in S; \tag{21f}$$

$$\sum_{p\in P} v_{i,p}=1,\quad \forall i\in TP; \tag{22}$$

$$\sum_{i\in TP} v_{i,p}\le 1,\quad \forall p\in P; \tag{23}$$

$$\sum_{j\in T} x_{i,j,k}=\sum_{p\in P} u_{p,k}v_{i,p},\quad \forall i\in TP,\ k\in R. \tag{24}$$

Constraints (13) state that each team plays against every other team once. Note that for each match, the summation is not over all the tournament rounds. It is only over those in which the two teams can play each other. Constraints (14) state that all teams play exactly one match per round. Constraints (15) determine the minimum and maximum numbers of games a team can play at home. Constraints (16) and (17) set an upper bound on the number of home and away breaks for each team. Constraints (18) state that each team must play either one or two games at home in every three consecutive rounds. Note that constraints (15)–(18) are not required if each team is assigned to a pattern because the patterns already incorporate them. Constraints (19) set a lower bound on the number of good trips for teams located in extreme Northern or Southern cities of the country. Constraints (20) set lower and upper bounds on the number of matches played in each cluster in each round. These enable good trips to be scheduled for the TV broadcaster. Constraints (21a)–(21f) are the logical relationships between x and the other variables. Constraints (22) state that each team following a pattern can only follow one, while constraints (23) impose that each pattern can be assigned to at most one team. It is not an equality constraint, given that there may be more patterns than teams. Constraints (24) state that each team must follow the home-away label for each round indicated by its pattern.

Objective Function

Various optimization criteria have been used in the game-assignment model over the years. One of the main ones is to maximize the attractiveness of a tournament. For this criterion, we define a parameter $a_{i,j,k}$, which indicates the weight assigned to the scheduling of a match between teams i and j in round k for all triples (i,j,k) in set A. The higher the value of $a_{i,j,k}$, the more attractive it is to schedule match i versus j in round k. Usually, the highest value of this parameter is assigned to pairs of teams that are expected to battle in the final rounds for the top places or to avoid relegation, so that the more decisive games are concentrated toward the end of the tournament. This is captured in the following objective function:

$$\text{Maximize}\quad f=\sum_{(i,j,k)\in A} a_{i,j,k}\cdot x_{i,j,k}.$$

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Fernando Alarcón earned an undergraduate degree in industrial engineering and an MSc in operations management from the University of Chile. In 2015 he was appointed senior manager for equipment at Hapag-Lloyd AG, the successor company to CSAV and currently the world's 4th largest shipping company, assuming responsibility for steering the reefer fleet in Latin America. He has also conducted research on sports scheduling and created applications for their implementation. His professional interests include logistics, scheduling and technology.

Guillermo Durán is associate professor at the University of Buenos Aires (Argentina) and is also a professor at the University of Chile and the UDELAR (Uruguay). Since December 2011 he has been director (competitive appointment) of the Calculus Institute at the University of Buenos Aires, the institution's applied mathematics centre. Guillermo is an independent research associate at CONICET (National Scientific and Technical Research Council, Buenos Aires, Argentina) where the research topics include graph theory, combinatorial optimization and applied O.R. He is an OR consultant for a variety of public and private organizations in Chile and Argentina.

Mario Guajardo is assistant professor in the Department of Business and Management Science at the Norwegian School of Economics (NHH). He received his PhD in management science at NHH in 2013. He graduated from the University of Chile as a civil industrial engineer and earned a master's degree in operations management at the same institution. His research interests include operations research, collaborative logistics, natural resource supply chains, game theory and sports scheduling. He has collaborated on several internationally recognized projects with impact in industry and government.

Jaime Miranda is assistant professor in the Faculty of Economics and Business at the University of Chile. He completed a doctorate in engineering systems at the University in 2014 after earning an undergraduate degree in industrial engineering and a master's degree in operations management at the same institution. His areas of research include O.R., sport scheduling, class scheduling, routing, and staffing. He has collaborated on various high-impact studies in industry and the public sector in Chile and has participated in the development of numerous decision-support systems based on optimization models.

Hugo Muñoz is a lawyer and earned his master's degree in procedural law from the National University of Rosario Argentina. For the last 22 years, he has been professor of theory of process at various universities in Chile and visiting professor at universities in Latin America. He is an associate professor at the Central University of Chile and chair holder at the National University Andres Bello, Santo Tomas and

Bernardo O'Higgins of Chile. Since January 2016 is director of the ANFP.

Luis Ramírez studied architecture at the Catholic University of Valparaiso and the National Autonomous University of Mexico (UNAM). He is the technical development director of the Chilean Football Federation as well as competitions manager at the Chilean Professional Football Association (ANFP). In the latter post he is responsible for the management, organization, and development of all Chile's professional and youth soccer leagues, which represent 11 different categories including women's football. He is a member of the CONMEBOL Beach Soccer Commission and the FUTSAL Commission.

Mario Ramírez graduated in forestry engineering from the University of Chile and earned an MSc in natural resources economics at Michigan State University. He is vice-president corporate affairs of CDF, Chile's pay TV sports channel. He has occupied a wide variety of positions in the Chilean forestry sector. Starting as an engineer in the Economics Department of the Chilean Forest Service, Mario has since served as vice-president exports at Forestal SA, opening the Far East markets for Chilean Radiata pine logs; president of CORMA, the business association representing the Chilean forest industry; vice-president sales for Forestal Bio-Bio, a company oriented to log export markets and co-winner together with other Chilean forestry companies of the 1998 Franz Edelman Award; and part owner and director of Cambium SA, which provides a full range of forestry development services in Chile and Uruguay, consulting services on U.S. pension fund investments in forestry, and wind power evaluations in Chile.

Denis Sauré is assistant professor in the Department of Industrial Engineering at the University of Chile. Prior to joining the University, he was assistant professor at the University of Pittsburgh. He holds an MSc and PhD from Columbia University. His research interests lie in the general area of stochastic modeling and its applications to service operations and revenue management. His research focuses on data-driven approaches to decision making under uncertainty and their applications in the retail industry, online advertising and service systems in general.

Matías Siebert is a first-year doctoral student in operations research at the Georgia Institute of Technology. He holds an undergraduate degree in industrial engineering and an MSc in operations management, both from the University of Chile. While enrolled in the master's program, he was employed as a consultant at the UNTEC Foundation for Technology Transfer, applying operations research techniques in the mining industry. For five of his student years at the University of Chile, Matías also worked on the development of the season schedules for the country's professional soccer leagues.

Sebastián Souyris conducts research in the broad area of data-driven optimization, statistics, and economic modeling in operations and management science. He has worked in academia and industry developing and implementing analytic methods to solve problems in sustainability, vehicle routing, sports scheduling, and media revenue management. Sebastian is a PhD candidate in supply chain and operations management at the McCombs School of Business at The University of Texas at Austin. He earned his MPhil degree in operations management at New York University and his BS and MS degrees in industrial engineering at the University of Chile.

Andrés Weintraub is professor in the Department of Industrial Engineering at the University of Chile. He holds an undergraduate degree in electrical engineering from the University of Chile and a PhD in operations research from UC Berkeley. He is the chair of the Complex Engineering Systems Institute, funded by the Chilean government, which comprises 55 researchers from different fields including operations research, transportation, energy and industrial organization. Dr. Weintraub has carried out methodological research and worked on multiple applications used by industry and government. He is an INFORMS Fellow and is currently area editor for practice in *Operations Research*. He is a former president of IFORS, won the Franz Edelman award in 1998, the Canadian OR Society Harold Larnder Prize in 2000, and the Chilean National Prize in Applied Sciences and Technology in 2000.

Rodrigo Wolf-Yadlin earned an undergraduate degree in industrial engineering from the University of Chile and completed an MSc in operations management at the same institution in 2010. He graduated with Maximum Distinction and received the award of Best Graduating Student in Industrial Engineering granted by the Chilean College of Engineers. Rodrigo was employed as an adjunct professor of optimization and operations management at the University of Chile between 2010 and 2014 while also working in the private sector, first in construction and real estate and then on Santiago's Metro system. He is in the last semester of an MBA degree at Columbia Business School and upon completing his studies will stay on in New York City, where he will begin work at the management consulting firm A.T. Kearney.

Gonzalo Zamorano earned an undergraduate degree in industrial engineering from the University of Chile, an MSc in operations research and management from the same university, and an MSc in operations research at the University of Melbourne. He was employed for four years in the University of Chile's Department of Industrial Engineering as a consultant, solving strategic and logistics issues in the country's education sector, developing long term mining planning models and supporting public tenders using optimization models. Gonzalo works for an insurance company where he develops actuarial, optimization, and statistical models, winning 13 public tenders during 2015. His professional interests include management, sports scheduling, and statistics.

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