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ORIGINAL RESEARCH

Stray Capacitance in a Plasma Focus Device: Implications on the Current Derivative Calibration and the Effective Discharge Current

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Abstract The notion that PF discharge circuits should be represented by an equivalent circuit having two loops instead of the traditional single one is presented. This implies that two frequencies must be expected in the currents and voltages in these devices. Also, that the current flowing into the plasma is not the same as the current flowing from the capacitor bank. Finally, the difficulties for calibrating in situ a Rogowski coil are discussed.

Keywords Plasma focus · Pinch · Plasma measurements · Self-compression

Introduction

By far, the more common diagnostics used in Plasma Focus (PF) devices are Rogowski coils, sensing *dI/dt*, the time derivative of the discharge current and resistive (or capacitive) voltage dividers sensing $V(t)$, the voltage drop between the electrodes, or, to be more accurate, as near the electrodes as possible. These diagnostics can be used for gaining more information on the plasma behavior than it is usually found in the literature, as it was discussed in [1–4]. To this end, it is necessary to have a reasonable understanding of all the features present in these signals, so as to not misrepresent their physical meaning. Particularly, both *dI/dt* and $V(t)$ signals frequently present superimposed to their expected time behavior (i.e., moderately damped sinusoidal like curves), smaller amplitude, higher frequency components which are frequently considered to be "noise", that is, voltage variations externally induced into the detectors. However, as it will be shown in what follows, these "noise" components can be explained using a more accurate equivalent circuit of the discharge which improves the knowledge of the behavior of the plasma within the device. Additionally, the analysis of the adequate equivalent circuit which should be used also raises several questions on the accuracy with which current amplitudes are determined particularly in large PF devices, a subject worth being considered.

Equivalent Circuits

Figure 1 shows a typical equivalent circuit currently used for describing PF discharges. C_o is the capacity of the capacitor bank, SG is a spark-gap switch, L_o is the fixed, stray inductance of the bank and connections up to the coaxial electrodes, and $L_p(t)$ is the plasma-electrodes system variable inductance due to the travelling plasma current sheet. This type of circuit does not describe accurately the operation of a PF device, at least because two different (and consecutive) gas gaps should be broken down, the last one consisting in the gas within the electrodes. This second gap cannot break down simultaneously with the first one; intrinsically it breaks down with a (filling gas pressure dependent) statistically varying time delay respect to that of the capacitor bank spark gap [5]. A more realistic equivalent circuit of a PF device operation is shown in

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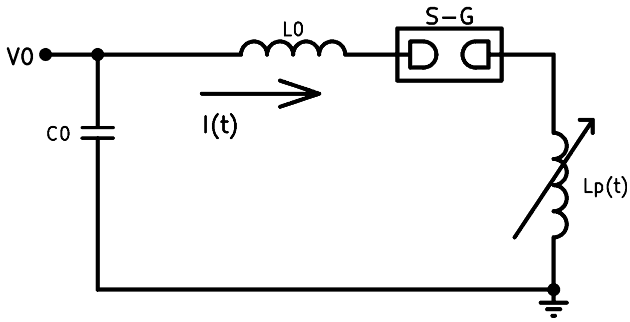

Fig. 1 Simple electrical circuit representing a plasma focus device. C0 is the capacitor bank, L0 the total stray inductance, S-G is the high current switch "spark-gap" and Lp(t) is the plasma-electrodes system variable inductance

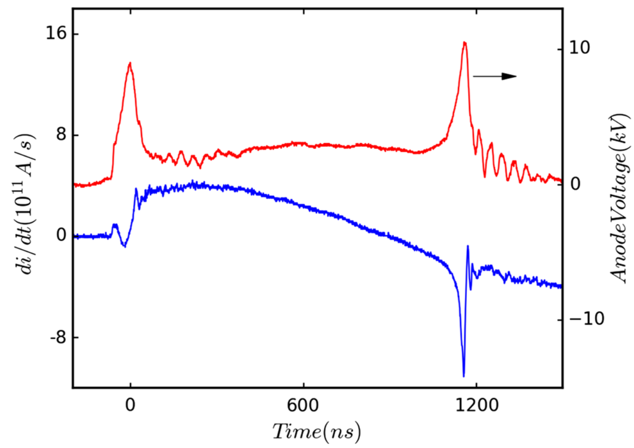

Fig. 3 Current derivative and anode voltage signals in the PACO plasma focus device, for a shot performed in deuterium at 1 mbar, and 31 kV

Fig. 2, in which SG1 represents the bank spark-gap (or spark-gaps, assumed to operate as a single one for simplicity) and SG2 represents the inter-electrodés gas gap. A capacity C_p has been included, accounting for the unavoidable stray capacity of the connections between SG1 and the electrodes. Consistently, L_o was divided into two portions, L_1 (before SG1) and L_2 for the rest of the circuit, such that $L_1 + L_2 = L_o$. One should note that the voltage divider is typically connected in parallel with C_p which keeps grounded this capacitor during the relatively slow charging of the bank, if this divider is purely resistive. If capacitive voltage dividers are used (or no voltage divider at all), the voltage on C_p could rise during the bank charge, but transient glow discharges taken place whenever this voltage exceeds the static breakdown voltage of the inter-electrode gap (typically a few hundred Volts) will limit this voltage. So, the initial rise of $V(t)$ during the "closure" of SG1 corresponds to the fast charging of C_p, which takes place in any device before the start of dI/dt, simply because this voltage rise is needed for the generation of the electric field which produces the breakdown within the electrodes (SG2 closure). As an example, in Fig. 3 we show $V(t)$ and dI/dt signals for a shot in the PACO device [6] operated at 31 kV and 1 mbar Deuterium. The Rogowski signal shows a small amplitude oscillation during the initial rise of $V(t)$ in all the recorded traces, which can be explained with the circuit in Fig. 2. In fact, the charge $q(t)$ flowing into C_p during the initial time interval is given by $q(t) = C_p\, V(t)$ up to the time of the first voltage peak, which normally indicates the start of the electrodes gap breakdown. Hence, the Rogowski should yield a relatively small voltage, proportional to $C_p d^2V/dt^2$, before the breakdown of the gas in the interelectrode space.

Naturally, the stray capacitor C_p remains in the circuit during all the time, and produces other effects in its behavior. Consider the circuit in Fig. 2, assuming for simplicity that $L_p(t)$ is a constant (a short circuit on the inter electrode insulator, say), which, for simplicity, we will consider included in L_2. Consistently, SG2 is also replaced by a short circuit. The equations for such circuit are presented and solved in the "Appendix", which shows that two different frequencies exist, one with a larger time period T_+ and another with a smaller period T_-, both functions of the circuit parameters. One should note that, in the case of a PF device operating normally, the presence of

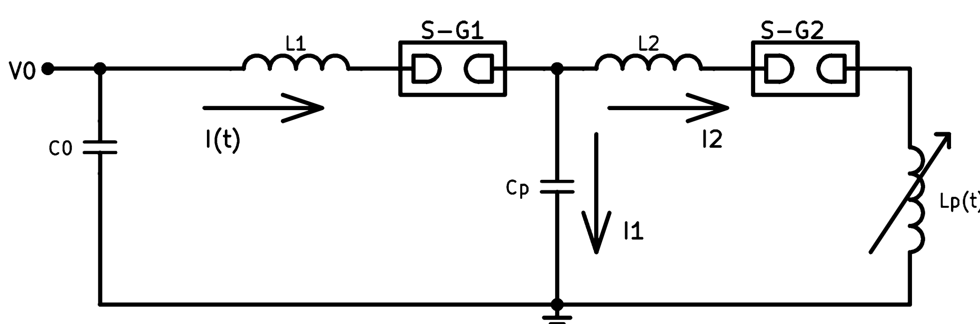

Fig. 2 A more realistic electrical circuit representing a plasma focus device. C0 is the capacitor bank, L1 is the total stray inductance of the transmission line between the capacitor bank and S-G1, S-G1 is the high current switch "spark-gap", Cp is the capacitance of the connections between S-G1 and the discharge chamber, L2 is the corresponding inductance, S-G2 represent the current sheet breakdown and finally, Lp(t) is the plasma-electrodes system variable inductance

two frequencies must always be expected, and in fact they are frequently observed in the *dI/dt* signals recorded. In most of these devices, however, the higher frequency component has rather small amplitudes and becomes rapidly damped, particularly in devices with bank energies of a few kJ or less. Such feature can be seen in the *dI/dt* signal of the PACO device given in Fig. 3, and as another example, in Fig. 4 we show a *dI/dt* trace recorded in the GN1 device [7] from a shot at 1 mbar D_2 and 30 kV which also show small amplitude high frequency components. Perhaps their small amplitude is the reason why such high frequency oscillations are commonly ascribed to "noise". In larger energy devices like Speed 2 (see their Fig. 4 in the Ref. [8]) or PF1000 (see their Fig. 4 in the Ref. [9]), the high frequency components have relatively much larger amplitudes, which can hardly be ascribed to "noise".

The presence of C_p raises a number of questions which may be of interest for PF researchers (and perhaps, also to people working in similar high power pulsed discharges). It is apparent from Fig. 2 that the current circulating in the plasma is not the same as the current circulating in the condenser bank. Hence, Rogowski coils cannot be located in any place in the device if the current in the plasma is the information looked for. For instance, the Rogowski coil in the PACO device was located before SG1, therefore sensing $I(t)$, not $I_2(t)$. Admittedly, the current flowing through C_p might be relatively small in most of the devices, but how small needs confirmation. And naturally enough, this current should not be confused with hypothetical stray currents flowing inside the device on the insulator region. Second, the oscillation period T is used to determine the fixed inductance of the circuit (L_o), assuming that the bank capacity C_o is known and the circuit is that of Fig. 1. As discussed in the "Appendix", this is a reasonable procedure when using a short circuited version of the device and in the cases in which the higher frequency oscillation

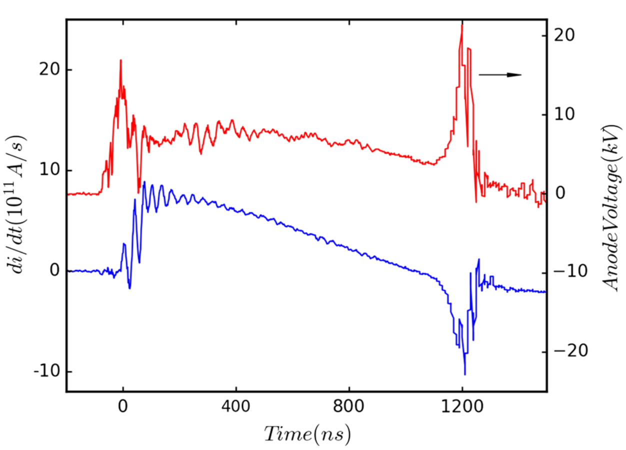

Fig. 4 Current derivative and anode voltage signals for a shot in the GN1 plasma focus device in deuterium at 1.2 mbar and 30 kV

amplitudes are sufficiently small. Otherwise, and even granting that the dependences of T_+ on the circuit parameters is substantially equal to that of T, measuring with a reasonable accuracy T_+ is not trivial because the time position of the ceros in the *dI/dt* signal are modified by the higher frequency oscillation (determining time periods using maxima or minima of the signal is not a good procedure in damped oscillations).

Another consequence deriving from this more accurate circuit description is the calibration of Rogowski coils. Rogowski coils are usually calibrated in situ assuming an equivalent circuit like that in Fig. 1. A common practice is to acquire a *dI/dt* trace at a known V_o charging voltage in a short circuit configuration; use this trace to evaluate an L_o value from the oscillation period T (assuming C_o known), after which the Rogowski proportionality factor, k_R, (such that $dI/dt = k_R V_R(t)$, here V_R is the directly measured signal in Volts) is evaluated from the acquired initial maximum value of the Rogowski trace, V_R("0"), using the equation:

$$k_R V_R(0) = \frac{V_0}{L_0} = \frac{4\pi^2 V_0 C_0}{T^2} \tag{1}$$

On passing, it should be pointed out that the practice of using discharges performed at very high filling pressures with the rationale that "the CS will not move" during one period is not a good practice due to the generation of filamentary CS's in such conditions [5].

The use of Eq. (1) has a difficulty, due to the fact that the closing gaps are not "ideal switches", so that the first maximum in any Rogowski coil is intrinsically smaller than this theoretical estimate [10]. Besides, in many devices the amplitudes of the higher frequency oscillation makes even more difficult to assess an appropriate value of V_R("0"). So another calibration scheme is often used, which consists in integrating the Rogowski signal (let us call such integral $Int[V_R(t)]$) and determine the Rogowski constant from its first maximum, $Int[V_R]_{max}$, as

$$k_R \mathrm{Int}[V_R]_{\max} = \frac{2\pi V_0 C_0}{T}$$

As discussed in the "Appendix", if $C_p \ll C_o$ holds, the above equation can be used for calibrating a Rogowski coil. Hence, provided that T, V_o, C_o and $Int[V_R]_{max}$ could be determined with uncertainties in the few percent range, k_R could be determined with a relative uncertainty in the order of 15%, so actual $I(t)$ values could be obtained with uncertainties smaller than 20%.

An additional question arises concerning the assumed uncertainty for the bank capacity C_o. An uncertainty in the few percent range is reasonable if the bank capacity has been directly measured (with a standard digital multimeter, say). But such measurement can be performed with ease only in small devices, whose banks are formed by one or a few capacitor modules. In the case of larger devices consisting in many modules (or worst still in Marx configurations like that of Speed 2), this measurement is quite difficult to perform. Be it what it may, the crucial point is that without careful measurements of the bank capacity, the evaluations of the current amplitudes circulating in this discharges can have an uncertainty much larger than that previously estimated.

In view of the above, it could be worthwhile then to study the possibility of using the commented oscillations for determining values of C_o. These values can be determined from measured values of the periods of the oscillations if one can measure C_p (which is relatively easy to do with a digital multimeter) and one of the inductances. For the short circuit case, L_2 can be also measured or evaluated geometrically with relatively small uncertainty. Using the equations given in the "Appendix" one obtains

$$\frac{1}{L_1} = \mathrm{C}_p\left(\lambda_+^2+\lambda_-^2 - \lambda_-^2\lambda_+^2 L_2 C_p - \frac{1}{L_2}\right)$$

$$C_0 = \frac{1}{L_1 C_p L_2 \lambda_-^2 \lambda_+^2}$$

Let us evaluate the relative uncertainty arising from this method of determining C_o. The standard error propagation calculation result:

$$\begin{aligned} E(L_1) = & \left(\frac{L_1}{L_2}+\frac{C_p}{C_0}\right)E(L_2) + \left(\frac{C_p}{C_0} + \mathrm{L}_1 C_p\lambda_+^2\right)E\left(\lambda_+^2\right) \\ & + \left(\frac{C_p}{C_0} + \mathrm{L}_1 C_p\lambda_-^2\right)E\left(\lambda_-^2\right) \\ & + \left(\frac{1}{\lambda_+^2}+\frac{1}{\lambda_-^2}\right)\frac{1}{L_2 C_0}E(C_3) \end{aligned}$$

and

$$E(C_0) = E(L_1) + E\left(C_p\right) + E(L_2) + E\left(\lambda_+^2\right) + E\left(\lambda_-^2\right)$$

In these equations, $E(x)$ stands for the relative error of x. Assuming that $C_p << C_o$

$$E(L_1) \approx \frac{L_1}{L_2}E(L_2) + \frac{L_1+L_2}{L_2}\left[E\left(\lambda_+^2\right) + E\left(C_p\right)\right]$$

and

$$\begin{aligned} E(C_0) \approx & \left(1+\frac{L_1}{L_2}\right)E(L_2) + \left(2+\frac{L_1}{L_2}\right)\left[E\left(C_p\right) + E\left(\lambda_+^2\right)\right] \\ & + E\left(\lambda_-^2\right) \end{aligned}$$

Besides, in this limit (see "Appendix"),

$$\lambda_+^2 \approx \frac{L_1+L_2}{C_p L_1 L_2}$$

$$\lambda_-^2 \approx \frac{1}{C_0(L_1+L_2)}$$

It can be seen that λ_+^2 is proportional to $1/T_-^2$ while λ_-^2 is proportional to $1/T_+^2$. Hence, $E\left(\lambda_{+/-}^2\right) = 2E\left(T_{-/+}\right)$ and $T_{+/-}$ are the magnitudes directly measured from the signals.

The relationship between L_1 y L_2 varies with the particular device. However, in order to get an estimation, we assume them to be roughly equal so that

$$E(L_1) \approx E(L_2) + E\left(C_p\right) + 4E(T_-)$$

$$E(C_0) \approx 2E(L_2) + 3E\left(C_p\right) + 2E(T_+) + 6E(T_-)$$

If the primary relative uncertainties were essentially equal and moderately small (4%, say), the uncertainty of L_1 would be 25% and that of C_o a not too satisfying 50%. Worst still, the T_- relative uncertainty is surely larger because it corresponds to an oscillation superimposed to another one with a larger period, and due to the fact that the relative uncertainty of this frequency contributes heavily to that of C_o, this method of determining a C_o value seems to be nearly hopeless.

A much better way for calibrating Rogowski coils in situ can be performed by using the data provided by a voltage divider connected in parallel with L_2 (that is, in its usual position) in a short circuit configuration. In such situation the traces of $V(t)$ and dI/dt should follow closely each other, because $V(t) = L_2 dI_2/dt$, and through a linear correlation between both set of data one can determine with good accuracy a constant k such that $V_R(t) = kV_d(t)$ where $V_d(t)$ is the recorded divider signal. If the calibration constant of the divider is k_d, one obtains $k_R = k_d/(L_2 k)$. Using this procedure, relative uncertainties in the measured currents can be smaller than 10%, assuming that L_2 is separately determined with a few percent uncertainties.

Conclusions

The recognition of the presence of a stray capacitance in PF circuits after the spark gap, in parallel with the discharge chamber is relevant for an appropriate understanding of the characteristics of the measured dI/dt and $V(t)$ signals. High frequency oscillations which are sometimes interpreted as "noise" can be explained and even contribute to a better knowledge of the PF behavior. Furthermore, this recognition helps to analyze the adequate procedure for calibrating in situ Rogowski coils and estimate their uncertainties.

Appendix

The equations ruling the behavior of the circuit given in Fig. 2 in the short-circuit configuration are:

$$\begin{cases} L_2\dot{I}_2 - \dfrac{Q_p}{C_p} = 0 \\ L_1\dot{I} - \dfrac{Q_0}{C_0} + \dfrac{Q_p}{C_p} = 0 \\ \dot{Q}_0 = I \\ \dot{Q}_p = I_1 = I - I_2 \end{cases}$$

This can be rearranged as:

$$\begin{cases} \ddot{Q}_0 + \omega_1^2 Q_0 + \omega_3^2 Q_p = 0 \\ \ddot{Q}_p + \omega_1^2 Q_0 + \omega_2^2 Q_p = 0 \end{cases}$$

$$\omega_1^2 = \frac{1}{L_1 C_0}$$

$$\omega_3^2 = \frac{1}{L_1 C_p}$$

$$\omega_2^2 = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\frac{1}{C_p}$$

Their solutions can be written as linear combinations of $sin(\lambda t)$ and $cos(\lambda t)$ where the eigenvalues λ are obtained from:

$$\begin{vmatrix} \omega_1^2 - \lambda^2 & \omega_3^2 \\ \omega_1^2 & \omega_2^2 - \lambda^2 \end{vmatrix} = 0$$

That is

$$\lambda^4 - \left(\omega_1^2 + \omega_2^2\right)\lambda^2 + \omega_1^2\left(\omega_2^2 - \omega_3^2\right) = 0$$

this yields

$$\lambda_\pm^2 = \frac{1}{2}\left[\left(\omega_1^2 + \omega_2^2\right) \pm \sqrt{\left(\omega_1^2 + \omega_2^2\right)^2 - 4\omega_1^2\left(\omega_2^2 - \omega_3^2\right)}\right]$$

In the limit $C_p << C_o$, which is satisfied in all PF devices, the problem becomes simpler and after a little algebra it can be shown to become

$$\lambda_+^2 \approx \frac{L_1 + L_2}{C_p L_1 L_2}; \quad \lambda_-^2 \approx \frac{1}{C_0(L_1 + L_2)}$$

The resulting frequencies are easily interpreted in terms of simple circuit theory. The root λ_- (=2π/T_+, the larger time period) corresponds to the frequency expected from an LC circuit with a single capacitor (C_o) and the inductances L_1 and L_2 in series (that is, what should be expected if the impedance of C_p were infinite, a reasonable approximation for the lower frequency case) while λ_+ (=2π/T_-, the smaller time period) is the root corresponding to a circuit with a single capacitor (C_p) and both inductances in parallel, that is, just as if C_o have been replaced by a short circuit, also a reasonable approximation for the higher frequency case.

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