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cause the number of events naturally depends on the energy threshold. In our manuscript we focus on the case of neutrinos endowed with non-standard interactions, making it evident due to the presence of the torsion as dynamical field, in particular the relation between the dual of the torsion field as the gradient of the axion field, namely $h_{\alpha} \sim \nabla a$.

The organization of this paper is as follows: in Section 2 we introduce the affine gravity used in this work whereas in Section 3 we recall some of the results of Cirilo-Lombardo (2013). In Section 4, the phenomenological implications of the torsion field computed before Cirilo-Lombardo (2013) with respect to the neutrino oscillation are given. In sections 5 and 6 , the correction to the interaction vertex produced by the torsion field is compared with the experimental values and the bounds to the universal parameters of the model are established. In section 7, the energy window (in the Peccei Quinn sense), the relation of the masses of the interacting fields and possible scenarios are presented. Finally in Section 8 we summarize the obtained results.

## 2. New affine gravity with torsion

In this section we review our model where the axial interaction arises. It is based on a pure affine geometrical construction where the geometrical Lagrangian of the theory contains dynamically the generalized curvature $\mathcal{R}=\operatorname{det}\left(\mathcal{R}^{a}{ }_{\mu}\right)$, namely
$L_{g}=\sqrt{\operatorname{det} \mathcal{R}^{a}{ }_{\mu} \mathcal{R}_{a \nu}}=\sqrt{\operatorname{det} G_{\mu \nu}}$,
characterizing a higher dimensional group manifold, e.g.: $\operatorname{SU}(2,2)$. Then, after the breaking of the symmetry, typically from the conformal to the Lorentz group, e..g.: $S U(2,2) \rightarrow S O(1,3)$, the generalized curvature becomes
$\mathcal{R}^{a}{ }_{\mu}=\lambda\left(e^{a}{ }_{\mu}+f^{a}{ }_{\mu}\right)+R^{a}{ }_{\mu} \quad\left(M_{\mu}^{a} \equiv e^{a v} M_{\nu \mu}\right)$
the original Lagrangian $L_{g}$ taking the following form:
$L_{g} \rightarrow$
$\sqrt{\operatorname{Det}\left[\lambda^{2}\left(g_{\mu \nu}+f_{\mu}^{a} f_{a \nu}\right)+2 \lambda R_{(\mu \nu)}+2 \lambda f_{\mu}^{a} R_{[a \nu]}+R^{a}{ }_{\mu} R_{a \nu}\right]}$,
reminiscent of a nonlinear sigma model or M-brane. Notice that $f^{a}{ }_{\mu}$, in a sharp contrast with the tetrad field $e^{a}{ }_{\mu}$, carries the symmetry $e_{a \mu} f^{a}{ }_{\nu}=f_{\mu \nu}=-f_{\nu \mu}$. See Cirilo-Lombardo $(2010,2011)$ for more mathematical and geometrical details of the theory. Consequently, the generalized Ricci tensor splits into a symmetric and antisymmetric part, namely:
$R_{\mu \nu}=\overbrace{\stackrel{\circ}{R}_{\mu \nu}-T_{\mu \rho}^{\alpha} T_{\alpha \nu}{ }^{\rho}}^{R_{(\mu \nu)}}+\overbrace{\stackrel{\circ}{\nabla}_{\alpha} T_{\mu \nu}^{\alpha}}^{R_{[\mu \nu]}}$
where $\stackrel{\circ}{R}_{\mu \nu}$ is the general relativistic Ricci tensor constructed with the Christoffel connection, $T_{\mu \rho}{ }^{\alpha} T_{\alpha \nu}{ }^{\rho}$ is the quadratic term in the torsion field and the antisymmetric last part $\stackrel{\circ}{\nabla}_{\alpha} T_{\mu \nu}{ }^{\alpha}$ is the divergence of the totally antisymmetric torsion field that introduces its dynamics in the theory. From a theoretical point of view our theory, containing a dynamical totally antisymmetric torsion field, is comparable to that of Kalb-Ramond in string or superstring theory (Green et al., 1988a, 1988b) but in our case energy, matter and interactions are geometrically induced. Notice that ${ }^{*} f_{\mu \nu}$ in $L_{g}$ must be proportional to the physical electromagnetic field, namely $j F_{\mu \nu}$ where the parameter $j$ homogenizes the units such that the combination $g_{\mu \nu}+j F_{\mu \nu}$ has the correct sense. Here we will not go further into details but the great advantage of this model is that it is purely geometric, being matter, energy and interactions geometrically induced: without energy momentum tensor added by hand.

## 3. Cross-section

Torsional effects in affine gravity manifest as a string-flip mechanism. As computed in Cirilo-Lombardo (2013), the cross section for this process reads:

$$
\begin{align*}
& \sigma_{v}^{f l i p}(\beta) \\
& =\left(\frac{j \mu m c}{4 \hbar}\right)\left(\frac{(1-d)^{2}}{\pi^{2} d}\right)^{2} \frac{4 E^{2}}{\left(E+m c^{2}\right)^{2}}[1.09416 \\
& \left.+\operatorname{Ln}\left(\frac{2\left(E^{2}-m^{2} c^{4}\right)}{q_{\min }^{2}}\right)\left(\operatorname{Ln}\left(\frac{2\left(E^{2}-m^{2} c^{4}\right)}{q_{\min }^{2}}\right)-0.613706\right)\right] \tag{2}
\end{align*}
$$

where $j$ and $\mu$ are universal model dependent parameters, carrying units of inverse electromagnetic field and magnetic moment respectively (e.g. $\mu \equiv \zeta \mu_{B}, \zeta=$ constant). The above cross section is in fact in sharp contrast with the string theoretical and standard model cases, depending logarithmically on the energy, even at high energies. Notice that this cross section generalizes in some sense the computation of Bethe (1935). As we can see, for the explicit cross-section formula (2), it is important to note the following:

1. Under the assumption of some astrophysical implications as presented in Gaemers et al. (1989), the logarithmic terms can be bounded with values between 1 and 6 , depending on screening arguments, as generally accepted. This situation of taking the logarithmic energy dependent terms to be constant is at present questioned by the experimental point of view due to the arguments given in the Introduction.
2. The $j$ parameter plays formally (at the cross section level) a role similar to that of the constant $\kappa$ of the string model with torsion. However in our approach, it is related to some physical "absolute field" (as $b$ in the Born-Infeld theory), giving the maximum value that the physical fields can take into the space-time (just as the speed of light $c$ in the relativity theory). In such a case $j$ ("the absolute field") will be fixed to some experimental or phenomenological value.
3. The above results can straightforwardly be applied to several physical scenarios, namely astrophysical neutrinos, dark matter, supernovae explosions, etc.

## 4. Phenomenological implications of the model

In the original version of the standard model (SM), leptons are grouped in three families or flavors:
$\binom{v_{\alpha}}{\alpha}=\binom{v_{e}}{e} ;\binom{v_{\mu}}{\mu} ;\binom{v_{\tau}}{\tau}$.
While the charged leptons are massive (these get their masses via Higgs mechanism (Bhattacharyya, 2011)) neutrinos are not. In the middle of 60s, terrestrial experiments observed a discrepancy between the number of neutrinos predicted by solar theoretical models and measurements of the number of neutrinos passing through the Earth; this discrepancy was called "the solar neutrino problem" (SNP) (Bahcall and Pinsonneault, 1992). A natural explanation for the SNP came from the NOs phenomenon (Pontecorvo, 1957) which allows the flavor transmutation during neutrino propagation into the space. Neutrino oscillations were confirmed by experiments and have shown that neutrinos have non-zero masses ( $m_{i} \sim 1 \mathrm{eV}$ ) (Fukuda et al., 1998; Eguchi et al., 2003). The existence of massive neutrinos opens a new window concerning the nature of neutrinos, Dirac or Majorana. While Dirac neutrinos preserve the lepton number, Majorana ones violate it by two units. Furthermore, if neutrinos are Dirac particles, right handed ones (sometimes called steriles) are essential in order to construct the Dirac mass term $\bar{\nu}_{L} m_{\nu} \nu_{R}$ (Mohapatra and Senjanovic, 1980). On the other hand, if they are Majorana particles, the mass term will be $\bar{\nu}_{L} m_{\nu} v_{L}$ and consequently, the two ways to introduce such gauge invariant term are: via higher dimensional operators (HDOs) (Weinberg, 1979) and spontaneous symmetry breaking (SSB). It is important to note that HDOs are not renormalizable and can be understood as an effective theory where particles with masses $M \gg M_{W}$ have been integrated out. However, in this section we will pay our attention on the effects of Torsion field over the flavor neutrino oscillation.

Let's define the flavor eigenstates, which have a defined flavor $\alpha$

$$
\begin{equation*}
\left|v_{\alpha}\right\rangle=\sum_{i}^{n} B_{\alpha i}\left|v_{i}\right\rangle, \tag{4}
\end{equation*}
$$

where $B_{\alpha i}$ are elements of unitary mixing matrix (PMNS-matrix) and $\left|v_{i}\right\rangle$ are the mass eigenstates, which have a defined mass $m_{i}$. The temporal evolution of the mass (or flavor) eigenstate is led by Schroedinger equation
$\underbrace{i \frac{d}{d t}\left|v_{i}(t)\right\rangle=\hat{H}_{m}\left|v_{i}(t)\right\rangle}_{\text {Mass Basis }} ; \quad \underbrace{i \frac{d}{d t}\left|v_{\alpha}(t)\right\rangle=\overbrace{\hat{B} \cdot \hat{H}_{m} \cdot \hat{B}^{\dagger}}^{\hat{H}_{f}}\left|v_{\alpha}(t)\right\rangle}_{\text {Flavor Basis }}$,
where $\hat{H}_{m}\left(\hat{H}_{f}\right)$ is the Hamiltonian in the mass (flavor) basis. The evolved (in time) states $\left|\nu_{i}(t)\right\rangle$ and $\left|v_{\alpha}(t)\right\rangle$ are
$\left|\nu_{i}(t)\right\rangle=e^{-i \hat{H}_{m} t}\left|\nu_{i}\right\rangle \quad ; \quad\left|v_{\alpha}(t)\right\rangle=e^{-i \hat{H}_{f} t}\left|\nu_{\alpha}\right\rangle$.
In presence of torsion-field the free Hamiltonian $\hat{H}_{0}$ will be modified by an extra $\hat{H}_{T}$ term, as follows:
$\hat{H}_{m}=\underbrace{\left(\begin{array}{cccc}E_{1} & 0 & \ldots & 0 \\ 0 & E_{2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & E_{n}\end{array}\right)}_{\hat{H}_{0}}+\underbrace{\left(\begin{array}{cccc}T_{11} & T_{12} & \ldots & T_{1 n} \\ T_{21} & T_{22} & \ldots & T_{2 n} \\ \vdots & \vdots & \ddots & \vdots \\ T_{n 1} & T_{n 2} & \ldots & T_{n n}\end{array}\right)}_{\hat{H}_{T}}$
Due to the fact that neutrinos masses are very small ( $m_{i} \ll E_{i}$ ), these can be treated as relativistic particles, thus neutrino energy $E_{i}$ can be expressed as:

$$
\begin{equation*}
E_{i}=\sqrt{m_{i}^{2}+\left|\vec{p}_{i}\right|^{2}} \approx\left|\vec{p}_{i}\right|+\frac{m_{i}^{2}}{2\left|\vec{p}_{i}\right|} \tag{8}
\end{equation*}
$$

If neutrino are heavier (non-relativistic), as in the case of heavysterile flavor NOs, the momenta of the two mass eigenstate are slightly different from each other, therefore the Eq. (8) should be treated in a different way (see Cvetic et al., 2015b for a deepest discussion). However, since in this letter we will take care only of the phenomenological aspects, then the relativistic expression suffices. On the other hand, and in order to have a easy phenomenological discussion, we will pay attention to scenarios with only two neutrino families $(n=2)$; in such a case the rotation matrix of $S U(2)$ becomes our mixing matrix
$\hat{B}=\left(\begin{array}{cc}\cos \theta & \sin \theta \\ -\sin \theta & \cos \theta\end{array}\right)$.
Furthermore, $\hat{H}_{m}$ can be written in terms of Pauli matrices in order to use the group theory artillery:

$$
\begin{align*}
\hat{H}_{f} & \equiv \hat{B} \cdot \hat{H}_{m} \cdot \hat{B}^{\dagger}=\left(E_{v}+\frac{m_{1}^{2}+m_{2}^{2}}{4 E_{v}}+\frac{T_{11}+T_{22}}{2}\right) \cdot \mathbb{1}_{2 \times 2}  \tag{10}\\
& -\tilde{\Delta} m_{D}^{2} \cdot\left(\sin 2 \theta \hat{\sigma}_{1}-\cos 2 \theta \hat{\sigma}_{3}\right) \\
& +T_{12} \cdot\left(\sin 2 \theta \hat{\sigma}_{3}+\cos 2 \theta \hat{\sigma}_{1}\right)
\end{align*}
$$

Here $T_{i i} \sim \frac{\mu_{i i}}{r^{2}}$, where $\mu_{i i}$ is the neutrino magnetic moment. It is important to remark that $\mu_{i i}$ is a $2 \times 2$ matrix, however we will pay our attention only in the diagonal terms $\mu_{11}$ and $\mu_{22}$ (we will assume $T_{12}=0$ ) in order to estimate the impact of torsion over flavor NOs. Regarding the $\tilde{\Delta} m_{D}^{2}$ parameter, it can be understood as an "effective" squared mass difference and is given as
$\tilde{\Delta} m_{D}^{2} \equiv\left(\frac{\delta m_{21}^{2}}{4 E_{v}}+\frac{T_{22}-T_{11}}{2}\right) \simeq\left(\frac{m_{2}^{2}-m_{1}^{2}}{4 E_{v}}+\frac{\mu_{22}-\mu_{11}}{2 r^{2}}\right)$.

It is important to note that the first term in Eq. (10) will not be relevant for the NOs probabilities, due to the fact that it can only contribute with a global phase. In order to calculate the transition probabilities we define the flavor eigenstates $(t=0)$ in matrix form as
$\left|v_{\alpha}\right\rangle=\binom{1}{0} \quad ; \quad\left|v_{\beta}\right\rangle=\binom{0}{1}$.

By mean of Eqs. (6), (9), (10), (12) we can write the flavor ${ }^{1}$ state as

$$
\begin{equation*}
\left|v_{\alpha}(t)\right\rangle=\left(\cos \tilde{\Delta} m_{D}^{2} t-i \sin \tilde{\Delta} m_{D}^{2} t\left(\hat{\sigma}_{1} \sin 2 \theta-\hat{\sigma}_{3} \cos 2 \theta\right)\right)\left|v_{\alpha}\right\rangle \tag{13}
\end{equation*}
$$

In consequence, the probability to measure the state $\left|\nu_{\beta}\right\rangle$ at a distance $L$ from the source, is given by
$P\left(v_{\alpha} \rightarrow v_{\beta}\right)=\left|\left\langle v_{\beta} \mid v_{\alpha}(t=L / c)\right\rangle\right|^{2}=\operatorname{Sin}^{2}(2 \theta) \operatorname{Sin}^{2}\left(\frac{\tilde{\Delta} m_{D}^{2} L}{C}\right)$.
In order to study the energy scale, in which flavor NOs due to the torsion could play a relevant role, we should compare both terms presented in Eq. (11); it means, if both terms have the same order of magnitude the torsion effects must be taken into account in NOs, and then the energy scale becomes
$\frac{\delta m_{21}^{2}}{4 E_{v}} \sim \frac{\mu}{2 r^{2}} \Rightarrow E_{v} \sim \frac{\delta m_{21}^{2} r^{2}}{2 \mu}$.
In the context of Eq. (11) we can distinguish the following cases:

- Reactor neutrino experiments (Olive et al., 2014) have shown that $\delta m_{21}^{2} \approx 7.53 \times 10^{-5} \mathrm{eV}^{2}$, whereas $\mu \leq 10^{-11} \mu_{B} \sim$ $10^{-19} \mathrm{eV}^{-1}$ has been reported by the GEMMA expectrometer (Beda et al., 2012). If we choose a $r \sim 10 \mathrm{~km}$ typical of supernova cores (Janka et al., 2007) we require an unrealistic energy $E_{v}$, thus, no relevant effects of torsion are present in the supernova processes.
- Most of the neutrino mass models include at least one heavy sterile neutrinos $N_{i}$ per leptonic family (see Mohapatra and Smirnov, 2006; Mohapatra and Senjanovic, 1980; Cheng and Li, 1980; Foot et al., 1989 for motivations and deeper discussions), these $N_{i}$ could have bigger magnetic moments which depending on the chosen model, ${ }^{2}$ could be proportional or not, to the sterile neutrino mass $m_{N_{i}}$. In the case when $\mu$ is proportional to $m_{N_{i}}$ the scale of energy is still very high ( $\geq G U T$ scale), thus no relevant effects due to the torsion are present in NOs. However, when $\mu$ is independent of $m_{N_{i}}$, the scale of energy admits a fine tuning ${ }^{3}$ ( $m_{N_{2}}-m_{N_{1}} \lll 1$ ) which can push the energy scale to lower values, in such a case the condition in Eq. (15) becomes

$$
\begin{equation*}
E_{v} \sim \frac{\left(m_{N_{2}}-m_{N_{1}}\right)\left(m_{N_{2}}+m_{N_{1}}\right) r^{2}}{2 \mu} . \tag{18}
\end{equation*}
$$

[^0]where $\phi$ is a mixing angle and the fields $W_{L}$ and $W_{R}$ have pure $V \pm A$ interactions. In these models and neglecting neutrino mixing the magnetic moment $\mu$ becomes
\[

$$
\begin{equation*}
\mu=\frac{e G_{F}}{2 \sqrt{2} \pi^{2}}\left[m_{\ell}\left(1-\frac{m_{W_{1}}^{2}}{m_{W_{2}}^{2}}\right) \sin \phi+\frac{3}{4} m_{\nu}\left(1+\frac{m_{W_{1}}^{2}}{m_{W_{2}}^{2}}\right)\right] \tag{17}
\end{equation*}
$$

\]

It is important to note that term proportional to the charged lepton mass $m_{\ell}$ comes from the left-right mixing and can be bigger than the second one in Eq. (1 $\hat{7}$ ). On the other hand, the second term in Eq. (17) is equivalent to the one presented in Eq. (26), whose values are shown in Fig. 1.
3 The fine tuning $m_{N_{2}}-m_{N_{1}} \lll 1$ is fundamental in order to explain baryogenesis via leptogenesis (see Canetti et al., 2013; Fukugita and Yanagida, 1986; Pilaftsis and Underwood, 2005, 2004); in the same framework this fine tuning can be interpreted as a new symmetry in the mass Lagrangian (see Shaposhnikov, 2007; Moreno and Zamora-Saa, 2016).


Fig. 1. Neutrino magnetic moment dependence on mass.

In scenarios of resonant CP violation (Moreno and Zamora-Saa, 2016; Cvetic et al., 2015a; Cvetič et al., 2014a, 2014b, Dib et al., 2015; Zamora-Saa, 2016), crucial for a successful theory of baryogenesis, it is found that
$m_{N_{2}}-m_{N_{1}}=\Gamma_{N} \propto\left|B_{\ell N}\right|^{2} \frac{G_{F}^{2} M_{N}^{5}}{192 \pi^{3}}$,
where $\left|B_{\ell N}\right|^{2}$ are the heavy-light neutrino mixings (for which it stands $\left|B_{\ell N}\right|^{2} \lll 1$; the present limits are shown in Atre et al., 2009) and $G_{F}$ is the Fermi constant, then, the Eq. (18) in term of Eq. (19) is

$$
\begin{equation*}
E_{v} \sim \frac{\left|B_{\ell N}\right|^{2} G_{F}^{2} M_{N}^{6} r^{2}}{192 \pi^{3} \mu} \tag{20}
\end{equation*}
$$

On the other hand during the electroweak epoch, ${ }^{4} t \sim 10^{-36} \mathrm{~s}$ after the big bang, the radio of the observed universe was $r \sim 10^{-2} \mathrm{~m}$ (Ryden, 2016). Then, provided that $M_{N} \sim 1 \mathrm{GeV}$, $\mu \sim 10^{-6} \mathrm{GeV}^{-1}$ as it is suggest in Fig. 1 and $\left|B_{\ell N}\right|^{2} \sim 10^{-10}$, we found an energy scale of $E_{v} \sim 10^{11} \mathrm{GeV}$ which is in agreement with the energy scale of electroweak epoch presented in Fig. 6 of Lineweaver (2003). Therefore, the effects of torsion in NOs could have played a significant role in the origin of baryon asymmetry of the universe via leptogenesis. However, there are extra indications (not related with NOs) that gravity could play a role in Baryogenesis (Lyth et al., 2005; Alexander and Martin, 2005; Alexander and Gates, 2006; Alexander et al., 2006).

## 5. Anomalous momentum and bounds

With the above considerations in mind, it is important to derive the electron anomalous magnetic moment (EAM) within this model in order to remark the role of the torsion from the point of view of the interactions and phenomenologically speaking. This is a key point if we want to know the bounds over the torsion field through the physical limits over the $j$ value. Specifically, from the second order Dirac type equation (derived from the model having into account the commutator of the full covariant derivatives: $\nabla \sim$ $\widehat{P}_{\mu}-e \widehat{A}_{\mu}+c_{1} \gamma^{5} \widehat{h}_{\mu}$ ) we expand up to terms that we are interested, namely (Capozziello et al., 2014)

$$
\begin{align*}
& \sim\left\{\left(\widehat{P}_{\mu}-e \widehat{A}_{\mu}+c_{1} \gamma^{5} \widehat{h}_{\mu}\right)^{2}-m^{2}\right. \\
& \left.-\frac{1}{2} \sigma^{\mu \nu}\left[\left(e-\omega_{1} \frac{\lambda}{d}\right) F_{\mu \nu}\right]\right\} u^{\lambda}=0 \tag{19}
\end{align*}
$$

[^1]where $\omega_{1} \frac{\lambda}{d}$ is the anomalous term. Notice that the gyromagnetic factor is modified as expected. Although the anomalous term is clearly determined from the above equation due to the vertex correction, it is extremely useful in order to compare the present scheme to other theoretical approaches. With these considerations in mind, it is important to derive EAM; specifically, from the last expression, one gets: $\Delta a_{e}=-\frac{\omega_{1}}{e} \frac{\lambda}{d} \equiv \frac{\omega_{1}}{e}\left(1-\frac{1}{d}\right)$. Consequently, we can see that this result is useful in order to give constraints to the theory. The aforementioned correction can be cast in the form
$\Delta a_{e}=\left(\frac{j \mu_{B} G m c^{4}}{4 \hbar^{2}}\right)\left(1-\frac{1}{d}\right)$,
where we have explicitly written the universal geometrical parameter $\omega_{1}$. The experimental precision measurement of this quantity is $\Delta a_{e}^{\exp }=0.28 \times 10^{-12}$ (Peccei and Quinn, 1977). Therefore, the upper bound for the universal field geometric parameter ${ }^{5} j$ is
$j<\frac{4 \hbar^{2}}{\mu_{B} G m c^{4}}\left(\frac{d}{d-1}\right) 0.28 \times 10^{-12}$.
Moreover, in 4-dimensions, we have as maximum limit
$j<1.39 \times 10^{-69} \mathrm{~m}^{2} \equiv 3.4 \times 10^{-56} \mathrm{eV}^{-2}$.

## 6. Torsion field and axion interaction

The particle physics phenomenology suggest that many symmetries of the nature are spontaneously broken, implying the existence of new particles, called Nambu-Goldstone bosons. Within this context, astrophysical objects, like stars or supernovae, can potentially play the role of high energy particle-physics laboratories. In Cirilo-Lombardo (2013) we presented a concrete relation between the axial vector $h_{\alpha}$ and the axion field $a$; furthermore, it was presented the interaction term $L_{1} \approx \frac{C_{f}}{2 f_{a}} \bar{\psi}_{f} \gamma^{\alpha} \gamma_{5} \partial_{\alpha} a \psi_{f}$, where $\psi_{f}$ is a fermion field, $C_{f}$ a model-dependent coefficient of order unity and $f_{a}$ the Peccei-Quinn energy scale related to the vacuum expectation value ${ }^{6}$ (for a more detailed discussion of the interplay between torsion and axion fields, see Chandia and Zanelli, 1997; Mercuri, 2009; Castillo-Felisola et al., 2015a and the references therein). On the other hand, within our unified gravity-model, the interaction coming from the resulting Dirac equation is:
$L_{i n t} \approx j^{\frac{1}{2}} \bar{\psi}_{f} \frac{1-d}{d} \gamma^{\alpha} \gamma_{5} h_{\alpha} \psi_{f}$,

[^2]
 Both the mass and axion decay constant have a direct influence on the cross section values.
therefore, the above interaction is related with $L_{1}$ provided that
$\partial_{\alpha} a \sim h_{\alpha} \quad$ and $\quad \frac{C_{f}}{2 f_{a}} \sim \frac{1-d}{d} j^{\frac{1}{2}}$.
In addition, $C_{f}$ serves to define an effective Yukawa coupling of the form $g_{a f} \equiv \frac{m_{f} C_{f}}{2 f_{a}}$. Relations in Eq. (25), that establish the phenomenological link between torsion and vector/axion, shall be used for the phenomenological analysis in section 7.

## 7. Astrophysical neutrinos, oscillation and possible scenarios

In order to study cross section values we look into the parameter space of $f_{a}, m_{v}$ at different energy values. We adopt the form of the lepton magnetic moment of a hypothetical heavy Dirac neutrino as studied in Dvornikov and Studenikin (2004a, 2004b), Studenikin (2009):
$\mu_{\nu}=\frac{e G_{F} m_{\nu}}{8 \sqrt{2} \pi^{2}} \begin{cases}3+\frac{5}{6} Q, & m_{\ell} \ll m_{\nu} \ll M_{W}, \\ 1, & m_{\ell} \ll M_{W} \ll m_{v},\end{cases}$
where $Q=\frac{m_{v}^{2}}{M_{W}^{2}}$ and $M_{W}$ is the $W$ boson mass. The axion decay constant $f_{a}$ values used in this work range in $10^{6}-10^{24} \mathrm{eV}$ and are in agreement with Raffelt (2008), where a detailed discussion of astrophysical and cosmological limits is presented. For instance, in the early universe time, hot axions are expected to decouple after the QCD epoch if $f_{a} \lesssim 3 \times 10^{7} \mathrm{GeV}$; on the other hand, neutron star axion cooling constraints suggest that $f_{a}>10^{8} \mathrm{GeV}$ (Sedrakian, 2016). Moreover, a very interesting case is the SN 1987A pulse duration, where axion emission might play a major role, by shortening the width of the pulsation. Energetic emissions
are characterized by values of the axion-nucleon Yukawa coupling $g_{a N}$. Free streaming results from low $g_{a N}$ values, whereas for higher $g_{a N}$ values emission corresponds to nucleon Bremsstrahlung and is shortened until it reaches a minimum, matching to the axion mean free path of the order of size of the SN core. Furthermore, the highest $g_{a N}$ range will result into axion trapping and shall be eventually emitted from the so called "axion sphere". It is only after the axions move beyond the neutrino sphere when the supernova signal becomes again unaffected. Strongly coupled axions might interact with in-falling matter from the supernova explosion and might lead to $\gamma$ ray emissions as well (Raffelt and Seckel, 1991). Under the framework of the DFSZ model (Zhitnitsky, 1980), white-dwarf cooling via axion-electron interaction is feasible for similar range of axion parameters to the supernova case, $f_{a} \gtrsim 10^{9} \mathrm{GeV}$. Thus, it is of great interest to explore the impact of axion parameter values, namely the axion decay constant, to the neutrino helicity spin-flip cross sectional values. Fig. 2 shows the resulting cross sections as a function of the neutrino energy $E_{v}$ for a set of fixed mass plots. In addition, each line represents a chosen value of the axion decay constant $f_{a}$. The general trend is that the larger the neutrino masses the larger the cross section values. The cross section seems to be more dramatically dependent on the axion decay constant, presenting the same behavior as for the mass dependence. In Fig. 3 the neutrino energy is fixed where as the mass becomes the free parameter. The result is the same, the $f_{a}$ parameter plays a mayor role in the determination of cross section values.

Moreover, it is worth noticing that a recent estimation on the axion mass has been computed in Borsanyi et al. (2016) in the framework of finite temperature extended lattice QCD and under cosmological considerations. The result is a value of the axion mass in the range of micro-eV, corresponding to a range of


Fig. 3. Set of figures for the cross section dependence on the neutrino mass featuring a fixed neutrino energy value. In each plot $E_{v}$ is fixed whereas each line represents a given $f_{a}$. Both the energy and axion decay constant have a direct influence on the cross section values, similar to the case of a fixed neutrino mass value.
$10^{12} \mathrm{GeV}<f_{a}<10^{14} \mathrm{GeV}$, favoring the highest helicity spin-flip cross-sectional values presented in this work.

## 8. Concluding remarks and outlook

In this report, the cross section for neutrino helicity spin-flip obtained from a new $f(R, T)$ model of gravitation with dynamic torsion field introduced by one of the authors in Cirilo-Lombardo (2013), was phenomenologically analyzed. To this end, due to the logarithmical energy dependence of the cross section, the relation with the axion decay constant $f_{a}$ (Peccei-Quinn parameter) was used. Consequently, the link with the phenomenological energy/mass window is found from the astrophysical and high energy viewpoints. The important point is that, in relation with the torsion vector interaction Lagrangian, the $f_{a}$ parameter gives an estimate of the torsion field strength that can variate with time within cosmological scenarios (Cirilo-Lombardo, 2010, 2011), potentially capable of modifying the overall leptogenesis picture.

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## Do not correct this page. Please mark corrections to sponsor names and grant numbers in the main text.


[^0]:    1 In Eq. (13) we have used the Euler's formula to cast the exponential into the binomial form.
    2 In $S U(2)_{L} \times S U(2)_{R} \times U(1)$ (left-right symmetric models) with direct righthanded neutrino interactions (see Czakon et al., 1999; Kim, 1976) the massive gauge bosons states $W_{1}$ and $W_{2}$ have a dominant left-handed and right-handed coupling
    $W_{1}=W_{L} \cos \phi-W_{R} \sin \phi \quad ; \quad W_{2}=W_{L} \sin \phi+W_{R} \cos \phi$

[^1]:    ${ }^{4}$ During this epoch the baryogenesis processes was started (Gorbunov and Rubakov, 2011a, 2011b).

[^2]:    ${ }^{5}$ Notice the role of the Planck length in the maximum value of $j$.
    6 The spontaneously broken chiral Peccei-Quinn symmetry $U_{P Q}(1)$ provides an axion field with a small mass $m_{a}=0.60 \mathrm{eV} \frac{10^{7} \mathrm{GeV}}{f_{a}}$.

