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A REDUCED-ORDER OBSERVER FOR VELOCITY ESTIMATION OF N-LINK MANIPULATORS

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Abstract

An exponentially nonlinear reduced-order observer for robot manipulators is developed. Velocity estimation of N-link manipulators is performed by integrating only N differential equations. Consequently, computational effort is reduced. Convergence is investigated, and a design criterion is proposed. Simulation results using a two-link manipulator model are presented.

Key Words

Robot manipulators, velocity estimation, reduced-order observer, nonlinear systems

1. Introduction

The issue of controlling the motion of robot manipulators has been studied extensively in the robotics and control literature. Most of the proposed control strategies (such as feedback linearization, compute torque control, PID control, and variable structure control) require not only position information, but also velocity information. Generally, clean position measurements are readily obtained by means of encoders or resolvers, which can give very accurate measurements of the joint displacements. Conversely, velocity sensors are undesirable for many reasons. In the first place, measurements obtained from tachometers are often contaminated by noise. This constrains the use of high-gain controllers, reducing the dynamic performance of the robot. Secondly, tachometers increase the weight of the moving parts, thereby decreasing the robot's efficiency. Therefore, velocity information should be inferred from position information. A trivial way to avoid velocity sensors would be by computing the time-derivative of the position, but this is not recommended because of its noisy nature. One solution to overcome this problem is the development of an observer, which estimates velocity from position and torque measurements, without resorting to noisy numerical differentiation.

In this paper a reduced-order observer for robot manipulators is proposed. As can be confirmed, many proposed observers in literature are full-order observers (see [1-7]). When implementing this kind of observer for an N-link manipulator, $2N$ differential equations must be integrated. Nevertheless, since position is measured, there is no need to estimate it. Based on this concept, a reduced-order observer, which demands only N differential equations to be integrated, has been developed. Consequently, computational burden is substantially diminished [8, 9].

The work is organized as follows. In Section 2, the equations of the observer are formulated. In Section 3, convergence is investigated, showing under what conditions the observer converges exponentially. Section 4 deals with simulation results using a two-link manipulator model. Finally, concluding remarks are presented in Section 5.

2. Reduced-Order Observer

The model of the rigid N-link manipulator resulting from the Lagrange equations is given by:

$$H(q)\ddot{q} = \tau - C(q, \dot{q}0\dot{q} - \tau_g(q) \tag{1}$$

where $q, \dot{q}, \ddot{q} \in R^n$, are the positions, velocities, and accelerations, respectively. $\tau \in R^n$ is the control torque, $H(q) in R^{n\times n}$ is the definite positive inertia matrix, $C(q, \dot{q})$ in ${}^{n\times n}$ is the Coriolis and centripetal matrix, and $\tau_g \in R^n$ are the gravity components.

The state variable representation of (1) is:

$$\dot{x}_1 = x_2 \tag{2}$$

$$\dot{x}_2 = \beta(x_1, x_2) + u(x_1, \tau) \tag{3}$$

$$y = x_1 \tag{4}$$

where $x_1 = q$, $x_2 = \dot{q}$, $\beta = H^{-1}(x_1)[-C(x_1, x_2)x_2]$, and $u(x_1, \tau) = H^{-1}(x_1)[\tau - \tau_g(x_1)]$.

In this section, we will deduce the equations of our velocity observer, which will provide velocity estimation using only position measurements. As a first approach, we consider that the observer dynamic is given by (3), plus a correction term:

$$\dot{\hat{x}}_2 = \beta(x_1, \hat{x}_2) + u(x_1, \tau) + G(x_2 - \hat{x}_2) \tag{5}$$

where $\hat{x}_2 \in R^n$ is the velocity estimate and $G \in R^{n\times n}$ is a constant matrix to be designed such that convergence is guaranteed. Obviously, (5) cannot be the final equation of the observer, because computation of the velocity estimate $\hat{x}_2$ demands direct information of the real velocity x_2 Therefore, (5) must be modified in order to obtain the

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final equations of our reduced-order observer. Accordingly, an additional variable $\upsilon \in R^n$ such that $\upsilon = \hat{x}_2 - Gx_1$ is defined. Then, taking υ as the new vector state, the set of equations for the proposed observer becomes:

$$\hat{x}_2 = \upsilon + Gx_1 \tag{6}$$

$$\dot{\upsilon} = \beta(x_1, \hat{x}_2) + u(x_1, \tau) - G\hat{x}_2 \tag{7}$$

It can be seen that now, the observer dynamic does not explicitly depend on x_2. Furthermore, it is important to note that only N differential equations need to be integrated in order to compute $\hat{x}_2$.

At this point, two questions arise: what are the design conditions that ensure the observer convergence, and what are the constrains for selecting the initial value of υ. These are the subject of the next section.

3. Design of G for Observer Convergence

The dynamic of the velocity estimation error ($e_{x_2} = x_2 - \hat{x}_2$) can be derived by subtracting (5) from (3). Then, it can be written as:

$$\dot{e}_{x_2} = \Delta\beta - Ge_{x_2} \tag{8}$$

where $\Delta\beta = \beta(x_1, x_2) - \beta(x_1, \hat{x}_2)$.

We will design G using the Lyapunov direct method in order to guarantee the convergence of the observer.

Let function V positive definite:

$$V = e_{x_2}^T e_{x_2} \tag{9}$$

be a candidate Lyapunov function. If its time derivative can be put in the form:

$$\dot{V} \leq \sigma \|e_{x_2}\|^2 \tag{10}$$

with $\sigma < 0$, it can be proven that the observer converges exponentially (see Appendix 1) given by:

$$\|e_{x_2}\| \leq \|e_{x_2}(t_0)\| \exp\left(\frac{\sigma}{2}(t - t_0)\right) \tag{11}$$

with initial time t_0.

Now, the problem of convergence has been reduced to finding G such that inequality (10) is verified.

The time derivative of V is:

$$\dot{V} = -e_{x_2}^T Q e_{x_2} + 2\Delta\beta^T e_{x_2} \tag{12}$$

where $Q = (G^T + G)$.

Choosing $G = g_o I_{n\times n}$, with g_o a positive constant and I identity matrix, Q becomes $Q = 2g_o I_{n\times n}$. Then $\dot{V}$ is bounded by:

$$\dot{V} \leq -2g_o \|e_{x_2}\|^2 + 2\|\Delta\beta\|\,\|e_{x_2}\| \tag{13}$$

According to physical considerations, velocity x_2 is limited. It belongs to the ball $B_1(0, r_1) \subset R^n$, whose radius $r_1 > 0$ is the maximum velocity $\|x_{2_{MAX}}\|$ reachable by the real system. Then, given an initial velocity error $e_{x_2}(t_0)$, it can be asserted that for $t = t_0$ the estimated value $\hat{x}_2$ is confined inside a compact set, namely:

$$\hat{x}_2(t_0) \in B_2(0, r_2) \subset R^n \tag{14}$$

where $r_2 \geq \|x_{2_{MAX}}\| + \|e_{x_2}(t_0)\| = r_1 + \|e_{x_2}(t_0)\|$. Therefore, if the partial derivatives $\frac{\partial\beta}{\partial\xi}(x_1, \xi)$ exist and they are continuous in $B_2(0, r_2)$, then β is Lipschitz in $B_2(0, r_2)$ and $\Delta\beta$ can be bounded by:

$$\|\Delta\beta\| \leq M\|e_{x_2}(t_0)\| \tag{15}$$

where $M = \sup_{\xi\in B_2} \|\frac{\partial\beta}{\partial\xi}(x_1, \xi)\|$. Replacing (15) in (13) for $t = t_0$:

$$\dot{V}|_{t=t_0} \leq -2(g_o - M)\|e_{x_2}(t_0)\|^2 \tag{16}$$

Then, if G is designed such that:

$$g_o > M \tag{17}$$

it ensures that at least for $t = t_0$, the inequality (10) is satisfied. This guaranteed that the velocity estimation error will continuously decrease:

$$\|e_{x_2}(t_0)\| > \|e_{x_2}(t_0 + \Delta t)\| \tag{18}$$

Under these circumstances $\hat{x}_2$ remains inside $B_2(0, r_2)$, and consequently, the bound M is still applicable. This ensures that $\|e_{x_2}\|$ will decrease again. Thus $\hat{x}_2$ will not escape from $B_2(0, r_2)$, and M will still be an appropriate bound. Following this reasoning, (13) can be rewritten as:

$$\dot{V} \leq -2(g_o - M)\|e_{x_2}\|^2 \;\forall t \geq t_0 \tag{19}$$

designing G such that $g_o > M$.

In this way exponential convergence is guaranteed. However, a problem still exists. As can be observed in (14), precise knowledge of $e_{x_2}(t_0)$ seems to be needed in order to compute r_2, and consequent calculation of M. If this is so, velocity measurement would be required, at least at $t = t_0$. Fortunately, this obstacle can be easily overcome by setting a bound to $e_{x_2}(t_0)$. Initialize $\hat{x}_2$ with known value $\hat{x}_2(t_0)$, then it can be assured that:

$$\|e_{x_2}(t_0)\| \leq \|\hat{x}_2(t_0)\| + r_1 \tag{20}$$

Hence, redefining r_2 in (14) by using (20) becomes:

$$r_2 = 2r_1 + \|\hat{x}_2(t_0)\| \tag{21}$$

being independent of the initial velocity measurement.

The proposed design criterion described in this section can be summarized as follows:

1. Study the plant and, based on physical considerations, determine the maximum velocity $\|x_{2_{MAX}}\| = r_1$.
2. Define $B_2(0, r_2)$, specifically r_2:
   (a) if some information of $x_2(t_0)$ is available, $\hat{x}_2(t_0)$ must be selected as close as possible to $x_2(t_0)$. Then, r_2 should be computed following (14): $r_2 = r_1 + \|e_{x_2}(t_0)\|$.
   (b) If no information of $x_2(t_0)$ is available, r_2 is calculated according to (21): $r_2 = 2r_1 + \|\hat{x}_2(t_0)\|$, with $\hat{x}_2(t_0)$initialized inside $B_1(0, r_1)$. Furthermore, choosing $\hat{x}_2(t_0) = 0$ generates the smallest

$B_2(0, r_2)$, and consequently the lowest M under these conditions.

3. Compute $M = \sup_{\xi \in B_2} \left\| \frac{\partial \beta}{\partial \xi}(x_1, \xi) \right\|$.
4. Design G verifying inequality (17), which guarantees the observer exponential convergence.

It is important to note that once $\hat{x}_2(t_0)$ has been chosen and G designed, the initial value of v is defined.

4. Simulation Results

In this section the performance of the proposed observer is assessed through computer simulation. The reduced nonlinear observer is tested over a spatial two-link manipulator. The dynamic model of the robot is:

$$H(q)\ddot{q} = \tau - C(q, \dot{q})\dot{q} - \tau_g(q) - f_r(\dot{q})$$

The elements of the inertia matrix are explicitly given by:

$$h_{11} = m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos(q_2)) + I_1 + I_2$$

$$h_{12} = h_{21} = m_2(l_{c2}^2 + l_1 l_{c2}\cos(q_2)) + I_2$$

$$h_{22} = m_2 l_{c2}^2 + I_2$$

See the robot parameters in Appendix 2. The elements of the Coriolis and centripetal torque matrix are:

$$c_{11} = -m_2 l_1 l_{c2}\sin(q_2)\dot{q}_2$$

$$c_{12} = -m_2 l_1 l_{c2}\sin(q_2)(\dot{q}_1 + \dot{q}_2)$$

$$c_{21} = -m_2 l_1 l_{c2}\sin(q_s)\dot{q}_1$$

$$c_{22} = 0$$

The gravity torque vector is comprised by:

$$\tau_{g1} = (m_1 l_{c1} + m_2 l_1)g\sin(q_1) + m_2 l_{c2} g\sin(q_1 + q_2)$$

$$\tau_{g2} = m_2 l_{c2} g \sin(q_1 + q_2)$$

Finally, the ith element of the friction torque vector is described by:

$$fr_i = fv_i\dot{q}_i + fc_i, \; i = 1, 2$$

where the first term represents a viscous friction component and the second term is a non-symmetric Coulomb friction, given by:

$$fc_i = \begin{cases} fcp_i & \text{if } \dot{q}_i > 0 \\ fcn + i & \text{if } \dot{q}_i < 0 \end{cases} \quad i = 1, 2$$

In this example, the robot manipulator is controlled through feedback linearization. To take into account the effect of sensor noise, Gaussian noise with $\sigma = 0.01$ has been added to the measured variables q_1 and q_2. The observer gain g_0 has been selected as 3 to provide fast observer convergence, compared with the dynamics of the plant. The system starts from rest and its initial position is $q(0) = [\frac{\pi}{10} rad \; \frac{\pi}{8} rad]^T$. Appropriate acceleration pulses are applied to the manipulator to obtain the velocity profile displayed in Fig. 1. It is important to note that, to demonstrate the observer behaviour in the presence of incorrect initial conditions, the initial velocity estimate has been deliberately selected as $\hat{x}_2(0) = [1.2rad/sec \; 1.4rad/sec]^T$. The observer effectiveness can be clearly appreciated in Fig. 2, where the velocity estimation error ($e_{x_2} = x_2 - \hat{x}_2$) is depicted.

Figure 1. Actual velocity: (a) link 1, (b) link 2.

Figure 2. Velocity estimation error: (a) link 1, (b) link.

Further simulation has been carried out over many other plants, using diverse values of g_o and initial conditions, and in every case, the observer performance has proven to be good, rendering very satisfactory results.

5. Conclusions

In this work a nonlinear reduced-order observer was presented. It was developed in order to render velocity estimation of an N-link robot manipulator by integrating N differential equations. Since the number of differential equations is divided by two (in comparison with those of a full-order observer), while calculation complexity is not increased, an important reduction of computational burden is achieved. In addition, only one parameter (g_o) must be designed to guarantee a desired speed of convergence.

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Appendix 1

Given the candidate Lyapunov function:

$$V = \|e_{x_2}\|^2$$

If its time derivative can be bounded by:

$$\dot{V} \leq \sigma \|e_{x_2}\|^2$$

with $\sigma < 0$.

Then:

$$\frac{\dot{V}}{V} \leq \sigma$$

such that:

$$V \leq V(t_0) \exp\left(\sigma(t - t_0)\right)$$

It follows:

$$\|e_{x_2}\| \leq \|e_{x_2}(t_0)\| \exp\left(\frac{\sigma}{2}(t - t_0)\right)$$

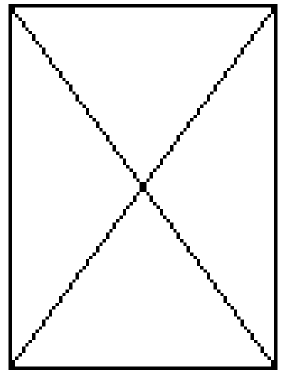
so that the convergence is exponential.

Appendix 2

Table 1
Robot Manipulator Parameters

Description	Notation	Value	Units
Length link 1	l_1	0.45	m
Length link 2	l_2	0.55	m
Center of gravity of link 1	l_{c1}	0.091	m
Center of gravity of link 2	l_{c2}	0.105	m
Mass of link 1	m_1	23.90	kg
Mass of link 2	m_2	4.44	kg
Inertial moment of link 1	I_1	1.27	Kg.m2
Inertial moment of link 2	I_2	0.24	Kg.m2
Gravity	g	9.8	N/s2
Viscous friction link 1	fv_1	2.288	N.s
Viscous friction link 2	fv_2	0.175	N.s
Coulomb friction (+) link 1	fcp_1	8.049	N.m
Coulomb friction (-) link 1	fcn_1	7.14	N.m
Coulomb friction (+) link 2	Fcp_2	1.734	N.m
Coulomb friction (-) link 2	Fcn_2	1.734	N.m

Biographies



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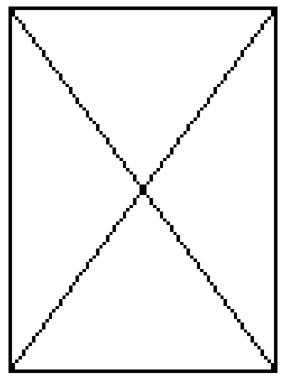

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