Brazilian Journal
of Chemical
Engineering

ISSN 0104-6632
Printed in Brazil
www.abeq.org.br/bjche

Vol. 33, No. 01, pp. 145 - 153, January - March, 2016
dx.doi.org/10.1590/0104-6632.20160331s00002846

ROBUST CONTROL OF WIENER SYSTEMS: APPLICATION TO A pH NEUTRALIZATION PROCESS

S. I. Biagiola, O. E. Agamennoni[1] and J. L. Figueroa*

Instituto de Investigaciones en Ingeniería Eléctrica –IIIE (UNS-CONICET),
Departamento de Ingeniería Eléctrica y de Computadoras,
Universidad Nacional del Sur,
Avda. Alem 1253, (8000) Bahía Blanca, Argentina.
Phone: + 54 291 4595101 ext. 3325, Fax: + 54 291 4595154
E-mail: figueroa@uns.edu.ar

(Submitted: July 18, 2013 ; Revised: February 6, 2015 ; Accepted: February 21, 2015)

Abstract - In this paper, the robustness of a typical control scheme for Wiener systems is studied. These systems consist of the cascade connection of a linear time invariant system and a static nonlinearity. To control this kind of systems, several approaches were discussed in the literature. Most of these control schemes involve transformation of the measured variable as well as the setpoint, by the inverse of the nonlinear gain. The approach followed in this work uses the inverse model of the static nonlinear gain, while the uncertainty in the Wiener model is treated as a partitioned problem. The linear block is considered as a parameter-affine-dependent model and, on the other hand, the nonlinear block uncertainty is analyzed as a conic-sector. The robustness analysis is performed using μ-theory. The results are evaluated on the basis of a simulation of a pH neutralization process.
Keywords: Wiener Models; Process Control; Uncertainty; Robustness.

INTRODUCTION

Many contributions for controller design are based on the assumption of a linear model of the system. However, in some cases it is difficult to represent a given process using a linear model. This situation takes place, for example, when a highly nonlinear system undergoes operating point changes along a wide region. For this reason, in the last decades, there has been much interest in nonlinear model-based control within the chemical engineering community. A critical step in the application of these methods is the development of a suitable model of the process dynamics. In this sense, Sjöberg *et al.* (1995) describe several approaches for model development and this approach can be specially appealing as regards control process applications. In particular, the Wiener model (WM) can be mentioned (Pearson and Pottmann, 2000) due to its wide diffusion and applicability in control. The WM consists of a cascade connection of a linear time invariant (LTI) system followed by a static nonlinearity.

The use of these models has been treated in the literature in various contexts. WMs have proved to be useful for applications in several fields, such as in chemical processes (Kalafatis *et al.*, 1995; Pajunen, 1992; Pearson and Pottmann, 2000; Zhu, 1999), biological processes (Korenberg, 1973; Hunter and Korenberg, 1986), communications (Kang *et al.*, 1998; Kang *et al.*, 1999; Cheong *et al.*, 2005) and control (Norquay *et al.*, 1998; Gerkšič *et al.*, 2000; Lussón Cervantes *et al.*, 2003; Biagiola *et al.*, 2004).

*To whom correspondence should be addressed
[1]Also with Comisión de Investigaciones Científicas de la Pcia. de Bs. As. (CIC)

A highly cited paper regarding Wiener model identification is the contribution by Gómez and Baeyens (2004), in which noniterative algorithms for the identification of both multivariable Hammerstein and Wiener systems were developed. Rational orthonormal bases were considered for the representation of the linear subsystem in the case of the Wiener model.

In most of the control applications of Wiener Models, the underlying strategy involves the inverse of the nonlinearity, such as in the works by Gerkšič *et al.* (2000), Bloemen *et al*. (2001a) and Gómez and Baeyens (2000, 2004).

Although much research effort has been dedicated to Wiener models, to the best of our knowledge, a systematic robustness analysis for this scheme under uncertainty has not been developed in the literature.

Since Wiener models could be approximations of the real process, the robustness of the designed control structure may deserve analysis. In order to apply robust control theory, one needs not only a nominal process model, but also a suitable description of the modeling errors, which are typically in the form of some bounds of parameter variations (Wang and Romagnoli, 2003). The classical robust control analysis and design methodologies are based on linear models (Doyle, 1982). As regards techniques for robust nonlinear control, they usually consist of covering the nonlinearity by an affine convex hull and, then, to perform the analysis on it (Popov, 1962; Bloemen *et al*., 2001b). However, a dedicated controller design and robustness analysis should be developed for processes approximated by Wiener models.

In this article, an analysis of the robustness of closed loop Wiener Systems is performed. The uncertainty in the Wiener model is treated as a partitioned problem. The linear block is considered as a parameter-affine-dependent model, which is suitable for Lyapunov-based analysis. Therefore, the stated stability problem can be dealt with as a sector bounded uncertainty problem and easily converted to a linear fractional uncertainty model. On the other hand, the nonlinear block uncertainty is analyzed as a conic-sector. The robustness analysis is performed using μ-theory.

To illustrate the proposed control strategy, a pH neutralization process is selected herein. This process has been widely recognized in the literature as a challenging problem due to the highly nonlinear and time-varying dynamic nature. From the perspective of system identification, pH processes have often been considered in the literature as having a Wiener structure (Kalafatis *et al*., 1995). Different Wiener representations have been favorably used for the purpose of pH neutralization process control. Among these works, we mention the paper by Kalafatis *et al.* (2005a), in which they evaluated the control of pH processes based on the Wiener model structure, where the static nonlinearity was assumed to represent the titration curve. Moreover, assessment of the conditions under which the pH process behaves like an exact Wiener system was accomplished. A simple linearizing feedforward controller was proposed based on an estimation of the inverse titration curve.

Along the same line, Gómez and Baeyens (2004) illustrated the suitability of the proposed methods in the identification of the pH neutralization process. They recognized this dynamic system as a benchmark drawn from the process control literature.

More research work on this subject is due to Mahmoodi *et al.* (2009). They employed Laguerre filters and polynomial functions as the linear and nonlinear blocks of the Wiener model, respectively. The so-called Wiener-Laguerre model was used to evaluate identification and nonlinear model predictive control of a pH neutralization process.

It must be remarked that, although the study herein developed is in the context of a pH neutralization reactor control, the conclusions can be directly extended to any other application.

The paper is organized in the following way. In the next section the process description is given. A Wiener model (and the related uncertainty) is then developed. The controller is designed and the simulation results are presented. The paper concludes with some final remarks.

PROCESS DESCRIPTION

The control of a pH neutralization processes is a relevant topic in several industries, such as wastewater treatment, pharmaceuticals production, bioprocesses plants, and chemical processing. It is often difficult to achieve a high performance and robust pH control due to their time-varying and severe nonlinear characteristics (Henson and Seborg, 1994). Therefore, pH control is frequently conceived as the unavoidable case study for the assessment of novel modeling and control strategies. Actual research work confirms that this process is still interpreted as a control benchmark (Mahmoodi *et al*., 2009; Wang and Zhang, 2011; Kim *et al.*, 2012).

The process consists of the neutralization reaction between a strong acid (*HA*) and a strong base (*BOH*) in the presence of a buffer agent (*BX*) as described by Galán *et al*. (2004). The neutralization takes place in a continuous stirred tank reactor (CSTR) with a constant volume *V*. In this reactor, an inlet acidic solution of composition $x_{1i}(t)$ and a time-varying volumetric flow $q_A(t)$ is neutralized using an alkaline solution of volumetric flow $q_B(t)$ and known composition made up of base x_{2i} and buffer agent x_{3i}. The nominal values for q_A and q_B are 1 and 0.5 L/min, respectively. Due to the high reaction rates of the acid-base neutralization, chemical equilibrium conditions are instantaneously achieved. Moreover, under the assumptions that the acid, the base and the buffer are strong enough, then the total dissociation of the three compounds takes place.

The process dynamic model can be obtained by considering the electroneutrality condition (which is always preserved) and through mass balances of equivalent chemical species (known as chemical invariants) that were introduced in Gustafsson and Waller (1983). For this specific case, under the previous assumptions, the dynamic behavior of the process can be described considering the state variables: $x_1=[A^-]$, $x_2=[B^+]$ and $x_3=[X^-]$. Therefore, the mathematical model of the process can be written in the following way (Galán, 2000):

$$\dot{x}_1 = \frac{q_A}{V}x_{1i} - \frac{(q_A+q_B)}{V}x_1 \tag{1}$$

$$\dot{x}_2 = \frac{q_B}{V}x_{2i} - \frac{(q_A+q_B)}{V}x_2 \tag{2}$$

$$\dot{x}_3 = \frac{q_B}{V}x_{3i} - \frac{(q_A+q_B)}{V}x_3 \tag{3}$$

$$F(x,\xi) \equiv \xi + x_2 + x_3 - x_1 - \frac{K_w}{\xi} - \frac{x_3}{\left[1+\left(K_x\xi / K_w\right)\right]} = 0 \tag{4}$$

where $\xi = 10^{-pH}$. K_w and K_x are the dissociation constants of the buffer and the water, respectively. The parameters of the system represented by Equations (1)-(4) are addressed in Table 1. Equation (4) was deduced by McAvoy *et al.* (1972), and it takes the standard form of the widely used implicit expression that connects pH with the states of the process.

Now, making $\tilde{x} = x_2 - x_1$, it is possible to reduce the process model to:

$$\dot{\tilde{x}} = \frac{q_B}{V}x_{2i} - \frac{q_A}{V}x_{1i} - \frac{(q_A+q_B)}{V}\tilde{x} \tag{5}$$

$$\dot{x}_3 = \frac{q_B}{V}x_{3i} - \frac{(q_A+q_B)}{V}x_3 \tag{6}$$

$$F(\tilde{x},x_3,\xi) \equiv \xi + \tilde{x} + x_3 - \frac{K_w}{\xi} - \frac{x_3}{\left[1+\left(K_x\xi / K_w\right)\right]} = 0 \tag{7}$$

Table 1: Neutralization Parameters.

Parameter	Value
x_{1i}	0.0012 mol HCl/l
x_{2i}	0.0020 mol NaOH/l
x_{3i}	0.0025 mol $NaHCO_3$/l
K_x	10^{-7} mol/l
K_w	10^{-14} mol^2/l^2
q_A	1 l/m
V	2.5 l

WIENER MODEL

Figure 1 depicts a Wiener model. It consists of a LTI system described in a state space form (this description for the linear block was used by, for example, Bruls *et al.* (1999), Westwick and Verhaegen (1996) and Lussón Cervantes *et al.* (2003)) (A,B,C) followed by a static nonlinearity $N(.)$. That is, the linear model maps the input signal $\overline{q}_B(t)$ into the intermediate variable $v(t)$, and the overall model output is $y(t)=N(v(t))$. In this scheme, the bar over $\overline{q}_B$ means deviation: i.e., $\overline{q}_B = q_B - q_{B,s}$, where $q_{B,s}$ stands for the steady-state value of this physical variable.

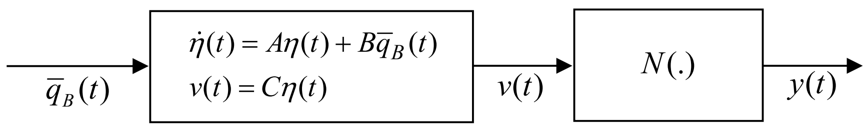

Figure 1: The Wiener model structure.

In this case, the relation between the input and the output is represented by the following model:

$$\dot{\eta}(t) = A\eta(t) + B\bar{q}_B(t)$$
$$v(t) = C\eta(t) \tag{8}$$

where the linear description of the process results from a linearization around the steady state $x_s = \left[(x_{2i}q_B - x_{1i}q_A)/(q_B+q_A) \quad x_{3i}q_B/(q_B+q_A) \right]_s^T$. The output variable $y(t)$ is the deviation variable for the measured pH. Note that Equation (7) can be rewritten as the following third-order polynomial:

$$h(\tilde{x}, x_3, \xi) \equiv \xi^3 + \left[\frac{K_w}{K_x} + x_3 + \tilde{x} \right] \xi^2 + \\ + \left[\tilde{x} - K_x \right] \frac{K_w}{K_x} \xi - \frac{K_w^{\ 2}}{K_x} = 0 \tag{9}$$

Then, the resulting linear model is:

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right] = \left[\begin{array}{cc|c} \dfrac{-(q_B+q_A)}{V} & 0 & \dfrac{(x_{2i}-x_{1i})q_A}{V(q_B+q_A)} \\ 0 & \dfrac{-(q_B+q_A)}{V} & \dfrac{x_{3i}\, q_A}{V(q_B+q_A)} \\ \hline \dfrac{K_x\xi^2 + K_w\xi}{\xi \ln(10) \dfrac{dh}{d\xi}} & \dfrac{K_x\xi^2}{\xi \ln(10) \dfrac{dh}{d\xi}} & 0 \end{array} \right]$$

where

$$\frac{\partial h}{\partial \xi} = 3\xi^3 + 2\left[\frac{K_w}{K_x} + x_3 + \tilde{x} \right]\xi + \left[\tilde{x} - K_x \right] \frac{K_w}{K_x}$$

Note that computation of the following linear model matrix in the steady-state

$$\left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array} \right]_s$$

involves the entries of the matrices (A,B,C) that must be evaluated for $q_B=q_{B,s}$, $q_A=q_{A,s}$, $x_{1i}= x_{1i,s}$, $x_{2i}= x_{2i,s}$, $x_{3i}= x_{3i,s}$, pH= pH$_s$.

The nominal linear model is computed at $q_{B,s}$=0.5. Then, to determine the values for the static nonlinear gain, the values of q_B are varied in the range [0,1]. This also implies that the linear model will differ from the nominal one. To cope with these changes, the following proposals are considered: a) the linear model includes uncertainties in the entries of the matrices (A,B,C) and b) the nonlinear gain is characterized (i.e., no uncertainty is present in it).

The influence of q_B values on the LTI model parameters was determined and the results are shown in Figure 2.

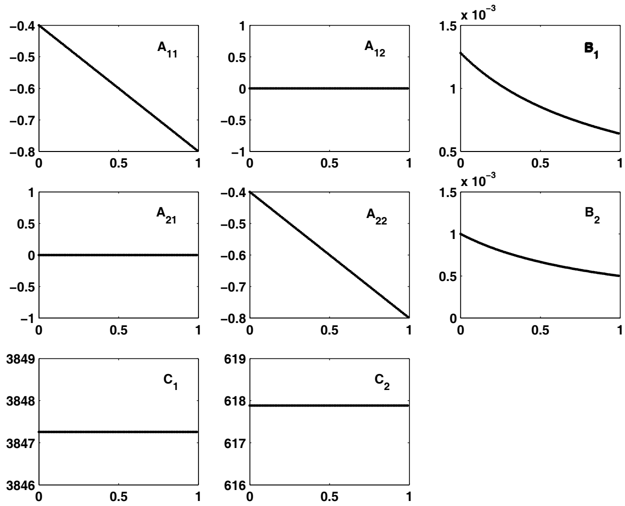

Figure 2: Parameters of the linear model as a function of q_B.

As regards the characterization of the static nonlinear gain, the titration curve is approximated by means of a Piecewise Linear (PWL) function. PWL functions have proved to be a very powerful tool in the modeling and analysis of nonlinear systems (Chua and Ying, 1983). The general formulation of PWL functions allows us to represent a continuous nonlinear function through a set of linear expressions, each of them valid in a certain operation region. To make this approximation, the range of input variables v (i.e., $\aleph$) is partitioned into a set of σ non-empty regions $\aleph^i$, such that $\aleph = \bigcup_{i=1}^{\sigma} \aleph^i$. In each of these regions, the non-linear function is approximated using a linear (affine) representation. These functions allow the development of a systematic and accurate treatment of the approximation.

It can be proved (Julián *et al.*, 1999) that any continuous nonlinear function $N(v)$: $\Re^m \rightarrow \Re^1$ can be uniquely represented using the PWL functions as

$$y = \sum_{i=0}^{\sigma+1} f_i \Lambda(v, \beta_i) \tag{10}$$

where β_i are given parameters that define the partition of the domain of v, and Λ are functions that involve nested absolute values (Julián *et al*., 1999).

Figure 3 depicts the real curve for the nonlinear gain, as well as the PWL approximation. The parameters identification of the PWL model in Eq. (10) is accomplished using the pH data and the $v(t)$ data obtained with Eq. (8). Therefore, for this pH neutralization model description the identified parameters are: β^N=[-3, -1.5, -1.2, 0.4, 1, 3]^T, where the superscript N means that this partition is used to represent the gain N.

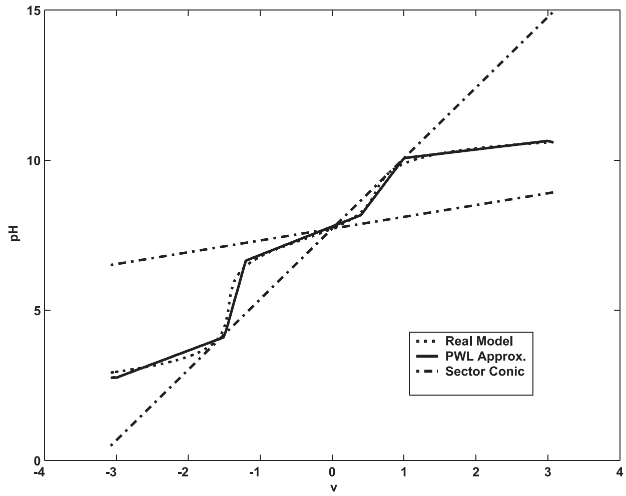

Figure 3: Nonlinear gain.

WIENER SYSTEM CONTROL

Wigren (1990) presented a structure for the control of Wiener systems. In this scheme (see Figure 4), two static nonlinearities are included in the loop. Under the hypothesis that $N(.)$ is invertible, the natural selection for the controller nonlinear functions is $f(.)\equiv N^{-1}(.)$. Note that this is the case of the pH neutralizer.

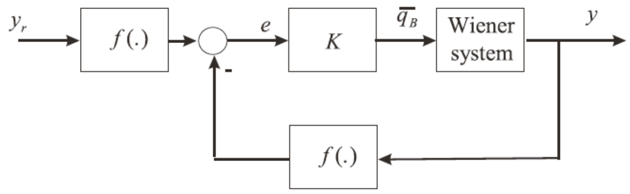

Figure 4: The closed loop scheme for Wiener Model.

Now, the controller design involves two steps: a) the inversion of the nonlinear gain and, b) the computation of a LTI controller in order to compensate for the linear block model of the process.

To compute $f(.)$ we approximate it using a PWL function (Lussón Cervantes *et al*., 2003); Figure 5 shows this function. The partition of the f-domain is defined as β^f= [2.5, 3.8, 6.5, 8, 10]^T, which corresponds to the partition of the pH range.

As mentioned above, the linear controller K should be designed to compensate for the behavior of the linear dynamic block of the process model. This controller could be designed using any of the classical techniques found in the literature. In our case, we use the H_∞ methodology (Gahinet *et al*., 1995). Let us consider the reduced loop (i.e., the nonlinearity is excluded) of Figure 6. In this case, the design signals are $u_c=[u]$, $y_c=\tilde{e}$, $w_c=[v_r \quad n]^T$ and $z_c=[e \quad u]^T$. The design transfer function is then

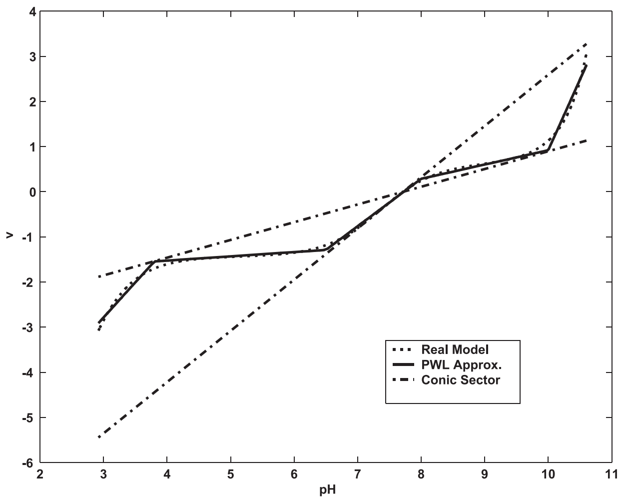

Figure 5: Inverse of the nonlinear gain.

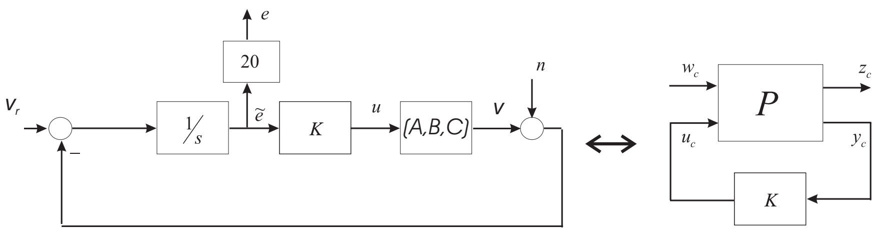

Figure 6: The closed loop scheme for the linear block.

$$\left[\begin{array}{c|c|c} A^P & B_1^p & B_2^p \\ \hline C_1^p & D_{11}^p & D_{12}^p \\ \hline C_2^p & D_{21}^p & D_{22}^p \end{array}\right] = \left[\begin{array}{cc|cc|c} A & 0 & 0 & 0 & B \\ -C & 0 & 1 & -1 & 0 \\ \hline 0 & 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 1 & 0 & 0 & 0 \end{array}\right]$$

The controller (A_K, B_K, C_K) is designed to reduce the norm of the transfer function between w_c and z_c, then the integral action (1/*s*) is included in the controller expression. Note that in this formulation we are minimizing the H_∞ norm between the set point input (v_r) and the measurement noise (*n*) to the weighed integrated error (*e*) and the manipulated variable (*u*) signals. This minimization is performed using the function *hinfsyn* (Gahinet *et al.*, 1995) that implements the algorithm defined by Doyle *et al.* (1989). It is important to note the significance of considering the measurement noise because its existence in the complete scheme (which includes the static nonlinearity) could produce a mismatch between the PWL sector of *N* and the PWL sector of *f*.

The robustness of this controller is tested against all the uncertainty sources: the linear model parameters variation (Fig. 2), the conic sector of the nonlinear gain model (Fig. 3) and the conic sector related to the controller (Fig. 5). The system used for robustness analysis is described in Figure 7. This analysis is performed using μ-tools (Gahinet *et al.*, 1995) with $\Delta = diag\{\Delta_A, \Delta_B, \Delta_N, \Delta_f\}$ (the uncertainty Δ_f in the forward line does not affect the stability) and the following M-Δ structure:

$$\left[\begin{array}{c|c} A^M & B^M \\ \hline C^M & D^M \end{array}\right] = \left[\begin{array}{cc|cccc} A & BC_K & I & I & 0 & 0 \\ -B_K C\tilde{f}\tilde{N} & A_K & 0 & 0 & -B_K\tilde{f} & -B_K \\ \hline I & 0 & 0 & 0 & 0 & 0 \\ 0 & C_K & 0 & 0 & 0 & 0 \\ C & 0 & 0 & 0 & 0 & 0 \\ C\tilde{N} & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

In these expressions, the nominal values $\tilde{N}$ (and $\tilde{f}$) are computed as the average of the conic sector of Fig. 3 and Fig. 5, and the magnitude of the uncertainties depends on the variation of the entries of *A*, *B* and the respective conic sectors. Under these definitions, the closed loop becomes robustly stable (μ=0.74).

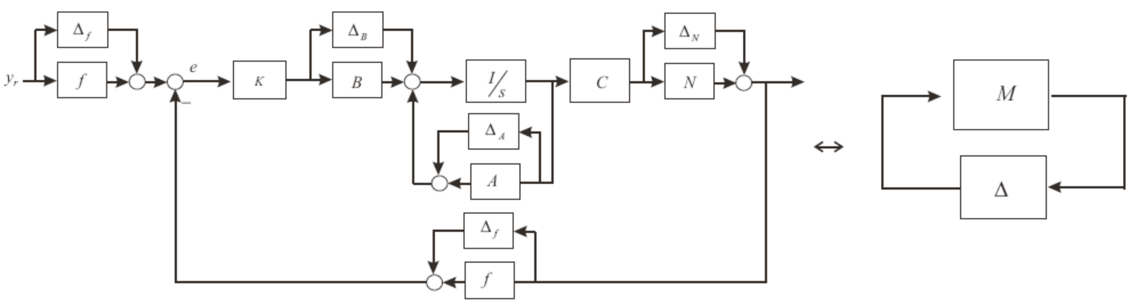

Figure 7: The closed loop scheme for robustness analysis.

Figure 8 shows the simulation results for setpoint changes. Note that the system follows the setpoint with smooth changes in the manipulated variable, even under the wide excursion of the reference signal. An interesting point is that, when we try to follow the same setpoint using only the linear controller (i.e., without considering the block f in the feedback loop), the system becomes unstable due to the significant nonlinearity of the process.

Two additional tests are performed; first we consider the effect of the measurement noise. A Gaussian noise of variance 0.05 is added to the pH output. The simulation results are shown in Figure 9. It is important to mention that, when noise variance increases, the performance of the controller deteriorates and the control system could become unstable.

In what follows, the effect of perturbations is evaluated. When load changes are applied to the process, a fixed Wiener model no longer represents adequately the process. For example, the titration curve changes drastically (Kalafatis *et al*., 2005a; 2005b). To perform a robustness analysis in this case, we consider that q_A varies between 0.8 and 1.2 and that x_{1i} varies between 0.0008 and 0.0014. The effects of these changes on the LTI model parameters and the nonlinear gain are shown in Figure 10 and Figure 11, respectively. From these plots, it is clear that the model uncertainties increase. For this level of uncertainty the system is no longer robustly stable (μ=1.147).

A simulation to evaluate the influence of lower perturbations is shown in Figure 12. In this case, q_A is reduced to 0.9 at t=10 min and x_{1i} is increased to 0.0014 at t=30 min.

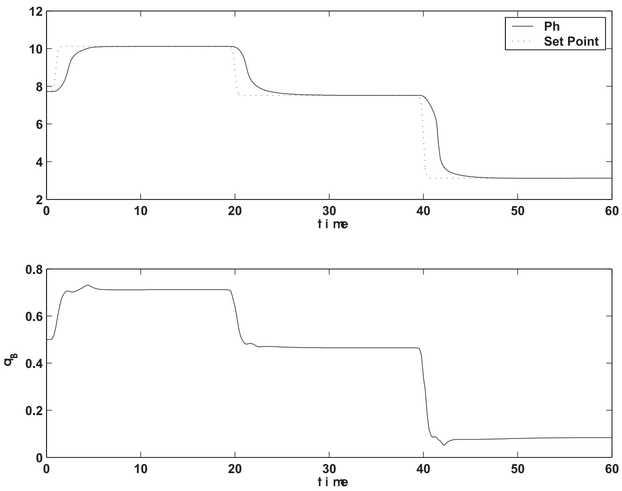

Figure 8: Closed loop simulations.

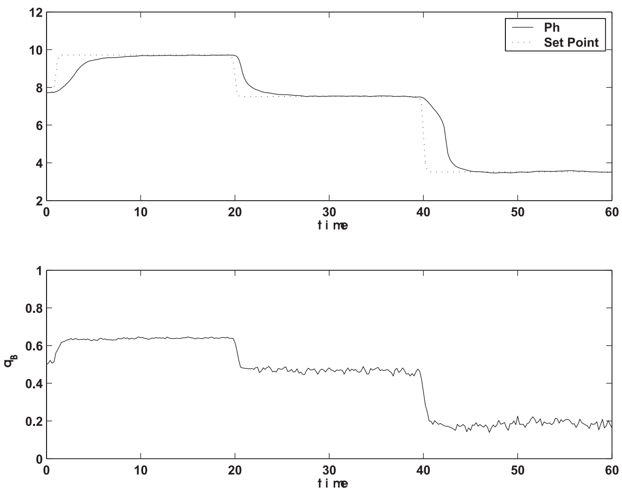

Figure 9: Closed loop simulations for noisy measurement.

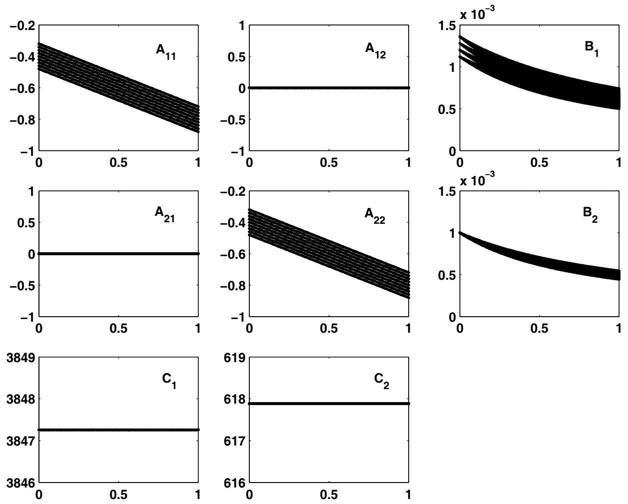

Figure 10: Parameters of the linear model as a function of q_B for different values of q_A and x_{1i}.

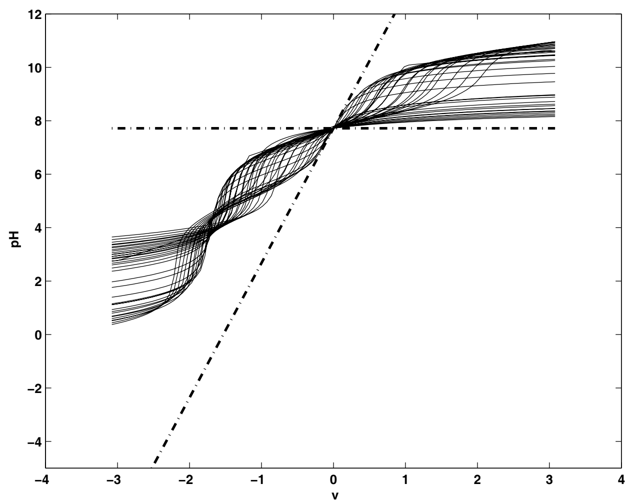

Figure 11: Nonlinear gain for different values of q_A and x_{1i}.

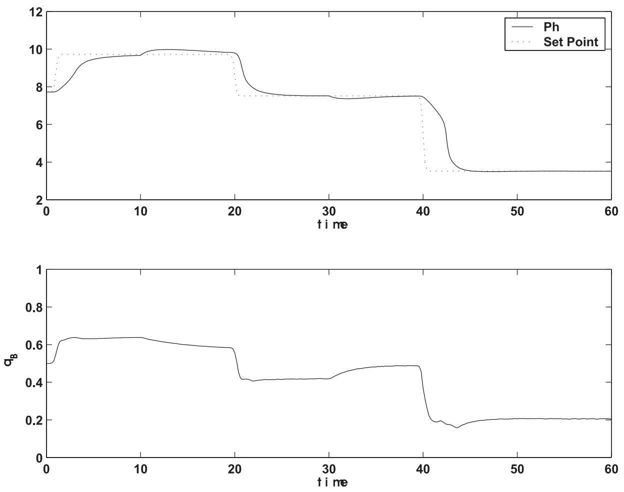

Figure 12: Closed loop simulations under perturbation.

CONCLUSIONS

In this article, controller design as well as robustness analysis of Wiener systems were considered. Wiener systems modeling and control were treated in the context of the more realistic case in which different sources of uncertainty are present. For this purpose, PWL approximating functions were introduced. As regards the modeling and control approaches proposed in this work, PWL functions proved to be an appropriate and simple tool for uncertainty inclusion, nonlinearity modeling and nonlinearity inversion.

A design technique for the controller synthesis was also proposed. It makes use of well-known tools based on H_∞ theory, and it was shown to be a suitable method for the uncertain feedback structure proposed herein. Stability aspects of the closed loop system under uncertainty were also dealt with using μ-theory. The different topics developed were tackled together in an application example of significant complexity.

ACKNOWLEDGMENTS

This work was financially supported by the Agencia Nacional de Promoción Científica y Tecnológica, CONICET, CIC and Universidad Nacional del Sur, Bahía Blanca, Argentina.

REFERENCES

Biagiola, S. I., Agamennoni, O. E. and Figueroa, J. L., H_∞ Control of a Wiener type system. International Journal of Control, 77, 572-583 (2004).

Bloemen, H. H. J., Chou, C. T., van den Boom, T. J. J., Verdult, V., Verhaegen, M. and Backx, T. C., Wiener model identification and predictive control for dual composition control of a distillation column. Journal of Process Control, 11(6), 601-620 (2001a).

Bloemen, H. H. J., van den Boom, T. J. J. and Verbruggen, H. B., Model-based predictive control for Hammerstein–Wiener systems. International Journal of Control, 74, 482-495 (2001b).

Bruls, J., Chou, C. T., Haverkamp, B. R. J. and Verhaegen, M., Linear and non-linear system identification using separable least-squares. European Journal of Control, 5, 116-128 (1999).

Chua, L. O. and Ying, L. P., Canonical piecewise-linear analysis, IEEE transactions on circuits and systems. CAS-30, 125-140 (1983).

Cheong, M. Y., Werner, S., Cousseau, J. and Laakso, T., Predistorter identification using the simplicial canonical piecewise linear function. 12th International Conference on Telecommunications, Cape Town, South Africa (2005).

Doyle, J. C., Analysis of feedback systems with structured uncertainties. IEE Proceedings – Part D, 129(6), 242-250 (1982).

Doyle, J. C., Glover, K., Khargonekar, P. and Francis, B., State-space solutions to standard H_2 and H control problems. IEEE Transactions on Automatic Control, 34, 831-847 (1989).

Gahinet, P., Nemirovski, A., Laub, A. J. and Chilali, M., LMI Control Toolbox, The MathWorks. The MathWorks Inc., Natick, MA, USA (1995).

Galán, O., Robust multi-linear model-based control for nonlinear plants. PhD Thesis, University of Sydney, Australia (2000).

Galán, O., Romagnoli, J. A. and Palazoglu, A., Real-time implementation of multi-linear model-based control strategies: An application to a bench-scale pH neutralization reactor. Journal of Process Control, 14, 571-579 (2004).

Gerkšič, S., Juričic, D., Strmčnic, S. and Matko, D., Wiener model based nonlinear predictive control. International Journal of Systems Science, 31, 189-202 (2000).

Gómez, J. C. and Baeyens, E., Identification of multivariable Hammerstein systems using rational orthonormal bases. Proceedings of the 39th IEEE Conference on Decision and Control, Sydney, Australia, 3, 2849-2854 (2000).

Gómez, J. C. and Baeyens, E., Identification of block-oriented nonlinear systems using orthonormal bases. Journal of Process Control, 14(6), 685-697 (2004).

Gustafsson, T. and Waller, K., Dynamic modelling and reaction invariant control of pH. Chemical Engineering Science, 38, 389-398 (1983).

Henson, M. and Seborg, D., Adaptive nonlinear control of a pH neutralization process. IEEE Trans. Control Syst. Technol., 2, 169-182 (1994).

Hunter, I. W. and Korenberg, M. J., The identification of nonlinear biological systems: Wiener and Hammerstein cascade models. Biological Cybernetics, 55(2-3), 135-144 (1986).

Julián, P., Desages, A. C. and Agamennoni, O. E., High level canonical piecewise linear representation using a simplicial partition. IEEE Transactions on Circuits and Systems, CAS-46, 463-480 (1999).

Kalafatis, A., Arifin, N., Wang, L. and Cluett, W. R., A new approach to the identification of pH processes based on wiener model. Chemical Engineering Science, 50, 3693-3701 (1995).

Kalafatis, A. D., Wang, L. and Cluett, W. R., Linearizing feedforward-feedback control of pH processes based on the Wiener Model. Journal of Process Control, 15, 103-112 (2005a).

Kalafatis, A. D., Wang, L. and Cluett, W. R., Identification of time-varing pH processes using sinusoidal signals. Automatica, 41, 385-691 (2005b).

Kang, H. W., Cho, Y. S. and Youn, D. H., Adaptive precompensation of Wiener systems. IEEE Transactions on Signal Processing, 40(10), 2825-2829 (1998).

Kang, H. W., Cho, Y. S. and Youn, D. H., On compensating nonlinear distortions of an OFDM system using an efficient adaptive predistorter. IEEE Transactions on Communications, 47(4), 522-526 (1999).

Kim, K. K., Ríos-Patrón, E. and Braatz, R. D., Robust nonlinear internal model control of stable Wiener systems. Journal of Process Control, 22, 1468-1477 (2012).

Korenberg, M. J., Identification of biological cascades of linear and static nonlinear systems. 16th. Proc. IEEE Midwest Symposium on Circuit Theory, p. 1-9 (1973).

Lussón Cervantes, A., Agamennoni, O. E. and Figueroa, J. L., A nonlinear model predictive control scheme based on Wiener piecewise linear models. Journal of Process Control, 13, 655-666 (2003).

Mahmoodi, S., Poshtana, J., Jahed-Motlagh, M. R. and Montazeri, A., Nonlinear model predictive control of a pH neutralization process based on Wiener–Laguerre model. Chemical Engineering Journal, 146(3), 328-337 (2009).

McAvoy, T., Hsu, E. and Lowenthal, S., Dynamics of pH in controlled stirred tank reactor. Industrial Engineering and Chemistry Process Design Development, 11(1), 68-70 (1972).

Norquay, S. J., Palazoglu, A. and Romagnoli, J. A., Model Predictive Control Based on Wiener Models. Chemical Engineering Science, 53(1), 75-84 (1998).

Pajunen, G. A., Adaptive control of Wiener type nonlinear systems. Automatica, 28(4), 781-785 (1992).

Pearson, R. K. and Pottmann, M., Gray-box identification of block-oriented nonlinear models. Journal of Process Control, 10, 301-315 (2000).

Popov, V. M., Absolute stability of nonlinear systems of automatic control. Automation and Remote Control, 22, 857-875 (1962).

Sjöberg, J., Zhang, O., Ljung, L., Benveniste, A., Delyon, B., Glorennec, P. Y., Hjalmarsson, H. and Juditsky, A., Nonlinear black-box modeling in system identification: A unified overview. Automatica, 12, 1691-1724 (1995).

Wang, D. and Romagnoli, J. A., Robust model predictive control design using a generalized objective function. Computers and Chemical Engineering, 27, 965-982 (2003).

Wang, Q. and Zhang, J., Wiener model identification and nonlinear model predictive control of a pH neutralization process based on Laguerre filters and least squares support vector machines. J. Zhejiang Univ-Sci C (Comput & Electron), 12(1), 25-35 (2011).

Westwick, D. and Verhaegen, M., Identifying MIMO Wiener systems using subspace model identifications methods. Signal Processing, 52, 235-258 (1996).

Wigren, T., Recursive identification based on the nonlinear Wiener model. PhD Thesis, Uppsala, Sweden (1990).

Zhu, Y., Distillation column identification for control using Wiener model. Proc. American Control Conference, San Diego, California, USA, 2-4 June, 5, p. 3462-3466 (1999).