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Quantum q-field theory: q-Schrödinger and q-Klein-Gordon fields

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Abstract – We show how to deal with the generalized q-Schrödinger and q-Klein-Gordon fields in a variety of scenarios. These q-fields are meaningful at very high energies (TeVs) for $q = 1.15$ high ones (GeVs) for $q = 1.001$ and at low energies (MeVs) for $q = 1.000001$ (PLASTINO A. and ROCCA M., *Nucl. Phys. A*, **948** (2016) 19; PLASTINO A. *et al.*, *Nucl. Phys. A*, **955** (2016) 16). (See the Alice experiment of LHC.) We develop here the quantum field theory (QFT) for the q-Schrödinger and q-Klein-Gordon fields showing that both reduce to the customary Schrödinger and Klein-Gordon QFTs for q close to unity. Further we analyze the q-Klein-Gordon field for $q \geq 1.15$. In this case for $2q - 1 = n$ (n integer ≥ 2) and analytically compute the self-energy and the propagator up to second order.

Introduction. – Classical fields theories (CFT) associated to Tsallis' q-scenarios have been intensely studied recently [1–3]. Associated quantum (QFT) treatments have also been discussed [3]. In this paper we show how to treat the q-Schrödinger and q-Klein-Gordon (KG) fields in a variety of cases. It has been shown in [4,5] that q-fields emerge at 1) very high energies (TeV) for $q = 1.15$, 2) high (GeV) for $q = 1.001$, and 3) low (MeV) for $q = 1.000001$. LHC-Alice experiments show that Tsallis q-effects manifest themselves [6] at TeV energies.

In this effort we develop QFTs associated to q-Schrödinger and q-Klein-Gordon fields. Moreover we study the q-KG field in the case $2q - 1 = n$, n integer ≥ 2. Here we evaluate the self-energy and propagator up to second order thus generalizing results of [3]. In this respect note also recent work on Proca-de Broglies' classical field theory [7].

Motivations for nonlinear quantum evolution equations can be divided up into two types namely A) as basic equations governing phenomena at the frontiers of quantum mechanics mainly at the boundary between quantum and gravitational physics (see [8,9] and references therein). The other possibility is B) to regard nonlinear-Schrödinger-like equations (NLSE) as effective single-particle mean field descriptions of involved quantum many-body systems. A paradigmatic illustration is that of [10]. In earlier applications of nonlinear Schrödinger equations one encounters situations involving a cubic nonlinearity in the wave function.

Referring to A) our present NLSE can be used for a description of dark matter components since the associated variational principle (the one that leads to the NLSE) is seen to describe particles that cannot interact with the electromagnetic field [11]. With reference to B) we remark that the NLSE displays strong similarity with the Schrödinger equation linked to a particle endowed with a time-position–dependent effective mass [12–15] involving particles moving in nonlocal potentials reminiscent of the energy density functional quantum many-body problem's approach [2].

During the last years the search for insight into a number of complex phenomena produced interesting proposals involving localized solutions attached to nonlinear Klein-Gordon and Schrödinger equations *i.e.*, nonlinear generalizations of these equations [1,11]. Following [11] we extend these generalizations here by developing quantum field theories (QFT) associated to the q-Schrödinger and q-Klein-Gordon equations [1].

Here we develop first the classical field theory (CFT) associated to that q-Schrödinger equation deduced in [16] from the hypergeometric differential equation. We define the corresponding physical fields via an analogy with treatments in string theory [17] for defining physical states of the bosonic string. Our ensuing theory reduces to the conventional Schrödinger field theory for $q \to 1$.

Secondly we develop the QFT for that very q-Schrödinger equation (see also [18]). *This* equation is similar but not identical to that advanced in [1].

Its treatment is however much simpler than that employed in [11].

In the third place we develop the QFT for the q-KG Field in several scenarios generalizing results of [3] and showing that the ensuing q-KG field reduces to the customary KG field for $q \to 1$.

A nonlinear q-Schrödinger equation. –

Classical theory. We develop here the CFT for that particular q-Schrödinger equation advanced in [18] from the Hypergeometric Differential Equation. This NLSE is different from the pioneer one proposed in [1], but exhibits better qualitative features. One has

$$i\hbar \frac{\partial}{\partial t}\psi(\vec{x},t)^q = H\psi(\vec{x},t). \tag{1}$$

In the free particle instance one writes

$$H_0 = -\frac{\hbar^2}{2m}\Delta, \tag{2}$$

whose solution reads

$$\psi(\vec{x},t) = [1+(1-q)\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)]^{\frac{1}{1-q}}. \tag{3}$$

Introduce now the action

$$\begin{aligned}\mathcal{S} = &\frac{1}{(4q-2)V}\int\limits_{-\infty}^{\infty}\int\limits_{V}\Big(i\hbar\psi^{\dagger q}\partial_t\phi^\dagger - i\hbar\psi^q\partial_t\phi \\ &-\frac{\hbar^2}{2m}\nabla\psi\nabla\phi - \frac{\hbar^2}{2m}\nabla\psi^\dagger\nabla\phi^\dagger\Big)\mathrm{d}t\ \mathrm{d}^3x,\end{aligned} \tag{4}$$

with V the Euclidian volume. Our action can be rewritten in the fashion

$$\mathcal{S} = \int\limits_{-\infty}^{\infty}\int\limits_{V}\mathcal{L}(\psi,\psi^\dagger,\partial_t\phi,\partial_t\phi^\dagger,\nabla\psi,\nabla\psi^\dagger\nabla\phi,\nabla\phi^\dagger)\mathrm{d}t\ \mathrm{d}^3x. \tag{5}$$

One obtains from (5) the field's motion equations

$$i\hbar\frac{\partial}{\partial t}\psi^q(\vec{k},t) + \frac{\hbar^2}{2m}\Delta\psi(\vec{x},t) = 0, \tag{6}$$

$$i\hbar q\psi(\vec{x},t)^{q-1}\frac{\partial}{\partial t}\phi(\vec{k},t) - \frac{\hbar^2}{2m}\Delta\phi(\vec{x},t) = 0. \tag{7}$$

whose solution is (3). Instead that for (7) reads

$$\phi(\vec{x},t) = \left[1+(1-q)\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)\right]^{\frac{2q-1}{q-1}}. \tag{8}$$

If $q \to 1$ ϕ becomes $\psi^\dagger$ the adjoint of ψ. Now the concomitant canonically conjugated momenta are

$$\begin{aligned}\Pi_\psi &= \frac{\partial\mathcal{L}}{\partial(\partial_t\psi)} = 0; \quad \Pi_{\psi^\dagger} = \frac{\partial\mathcal{L}}{\partial(\partial_t\psi^\dagger)} = 0,\\ \Pi_\phi &= \frac{\partial\mathcal{L}}{\partial(\partial_t\phi)} = -\frac{i\hbar\psi^q}{(4q-2)V};\\ \Pi_{\phi^\dagger} &= \frac{\partial\mathcal{L}}{\partial(\partial_t\phi^\dagger)} = \frac{i\hbar\psi^{\dagger q}}{(4q-2)V},\end{aligned} \tag{9}$$

and the associated Hamiltonian is

$$\mathcal{H} = \Pi_\phi\partial_t\phi + \Pi_{\phi^\dagger}\partial_t\phi^\dagger - \mathcal{L}, \tag{10}$$

that we cast in terms of ψ, ϕ as

$$\mathcal{H} = \frac{\hbar^2}{(8q-4)mV}(\nabla\psi\nabla\phi + \nabla\psi^\dagger\nabla\phi^\dagger). \tag{11}$$

The field energy is

$$E = \int\limits_V \mathcal{H}\mathrm{d}^3x. \tag{12}$$

If we replace the solutions (3) and (8) into (12) one has

$$E = \int\limits_V \frac{\hbar^2}{(8q-4)mV}(4q-2)\frac{p^2}{\hbar^2}\mathrm{d}^3x, \tag{13}$$

or

$$E = \frac{p^2}{2m}, \tag{14}$$

that exactly corresponds to the wave energy (3) as one should expect. The field-momentum density reads

$$\begin{aligned}\vec{\mathcal{P}} = &-\frac{\partial\mathcal{L}}{\partial(\partial_t\psi)}\nabla\psi - \frac{\partial\mathcal{L}}{\partial(\partial_t\phi)}\nabla\phi\\ &-\frac{\partial\mathcal{L}}{\partial(\partial_t\psi^\dagger)}\nabla\psi^\dagger - \frac{\partial\mathcal{L}}{\partial(\partial_t\phi^\dagger)}\nabla\phi^\dagger,\end{aligned} \tag{15}$$

or

$$\vec{\mathcal{P}} = \frac{i\hbar}{(4q-2)V}(\psi^q\nabla\phi - \psi^{\dagger q}\nabla\phi^\dagger), \tag{16}$$

the field-momentum becoming

$$\vec{P} = \int\limits_V \vec{\mathcal{P}}\mathrm{d}^3x. \tag{17}$$

Employing (3) and (8) one finds for the momentum

$$\vec{P} = \frac{i\hbar}{(4q-2)V}\int\limits_V \frac{4q-2}{i\hbar}\vec{p}\mathrm{d}^3x, \tag{18}$$

or

$$\vec{P} = \vec{p}. \tag{19}$$

The probability density is now

$$\rho = \frac{1}{2V}[\psi^q\phi + \psi^{\dagger q}\phi^\dagger], \tag{20}$$

verifying

$$\frac{\partial}{\partial t}\rho + \nabla\cdot\vec{j} = K, \tag{21}$$

where

$$\vec{j} = \frac{\hbar^2(q+1)}{8mVqi}[\phi\nabla\psi - \psi\nabla\phi + \psi^\dagger\nabla\phi^\dagger - \phi^\dagger\nabla\psi^\dagger], \tag{22}$$

is the probability current. K reads

$$K = \frac{\hbar^2(q-1)}{8mVqi}[\psi^\dagger\Delta\phi^\dagger + \phi^\dagger\Delta\psi^\dagger - \phi\Delta\psi - \psi\Delta\phi] \tag{23}$$

that vanishes at $q=1$. However the *physical* fields are those for which $K=0$. For example one lists as physical the solutions (3) and (8) since for them probability is indeed conserved.

Quantum theory. We start with the action

$$\mathcal{S} = -\int \left(i\hbar\psi^q \partial_t \phi - i\hbar\psi^{\dagger q}\partial_t \phi^\dagger + \frac{\hbar^2}{2m}\nabla\psi\nabla\phi + \frac{\hbar^2}{2m}\nabla\psi^\dagger\nabla\phi^\dagger \right)\mathrm{d}t\ \mathrm{d}^3x. \quad (24)$$

We develop first a theory for 1) q close to unity and 2) weak fields ψ. In these condiitions one appeals to the approximation

$$\psi^q \simeq \psi + (q-1)\psi\ln\psi, \quad (25)$$

and since ψ is a weak field

$$\psi \simeq I + (q-1)\eta. \quad (26)$$

Consequently the action (24) becomes

$$\mathcal{S} = -(q-1)\int \left(i\hbar\eta\partial_t\phi - i\hbar\eta^\dagger\partial_t\phi^\dagger + \frac{\hbar^2}{2m}\nabla\eta\nabla\phi + \frac{\hbar^2}{2m}\nabla\eta^\dagger\nabla\phi^\dagger \right)\mathrm{d}t\ \mathrm{d}^3x, \quad (27)$$

where we used

$$\int \eta(\vec{x},t)\mathrm{d}t\ \mathrm{d}^3x = \int \phi(\vec{x},t)\mathrm{d}t\ \mathrm{d}^3x = 0, \quad (28)$$

since the fields are

$$\eta(\vec{x},t) = \frac{1}{(2\pi\hbar)^{\frac{3}{2}}}\int a(\vec{p})e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)}\mathrm{d}^3p, \quad (29)$$

(see [19])

$$\eta^\dagger(\vec{x},t) = \frac{1}{(2\pi\hbar)^{\frac{3}{2}}}\int a^\dagger(\vec{p})e^{-\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)}\mathrm{d}^3p, \quad (30)$$

$$\phi(\vec{x},t) = \frac{1}{(2\pi\hbar)^{\frac{3}{2}}}\int b(\vec{p})e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)}\mathrm{d}^3p, \quad (31)$$

and

$$\phi^\dagger(\vec{x},t) = \frac{1}{(2\pi\hbar)^{\frac{3}{2}}}\int b^\dagger(\vec{p})e^{-\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)}\mathrm{d}^3p. \quad (32)$$

Surprisingly enough the q-Schrödinger field (qSF) reduces to the usual SF of low energies! Creation-destruction operators verify

$$[a(\vec{p}), a^\dagger(\vec{p}')] = [b(\vec{p}'), b^\dagger(\vec{p}')] = \delta(\vec{p}-\vec{p}'). \quad (33)$$

The propagator for the field η is [19]

$$\Delta_\eta(\vec{x},t) = \left(\frac{m}{2\pi i\hbar}\right)^{\frac{3}{2}} t_+^{-\frac{3}{2}} e^{\frac{im\vec{x}^2}{2\hbar t}}, \quad (34)$$

that in terms of energy and momentum reads

$$\hat{\Delta}_\eta(\vec{p},E) = \frac{i\hbar}{E - \frac{\vec{p}^2}{2m} + i0}. \quad (35)$$

These two representations are related via

$$\Delta_\eta(\vec{x},t) = \frac{1}{(2\pi\hbar)^4}\int \hat{\Delta}(\vec{p},E)e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)}\mathrm{d}E\mathrm{d}^3p. \quad (36)$$

The convolution of this propagator with itself with E and $\vec{p}$ as variables is NOT finite. It can be calculated however by appeal to distributions' theory using the relation

$$\hat{f} * \hat{g} = (2\pi\hbar)^4\mathcal{F}(fg). \quad (37)$$

This is so because divergences in the convolution of two phase space functions derive from the multiplication of distributions possessing singularities *at the same* configuration-space point. Keeping in mind that

$$\Delta_\eta^2(\vec{x},t) = \left(\frac{m}{2\pi i\hbar}\right)^3 t_+^{-3} e^{\frac{im\vec{x}^2}{\hbar t}}, \quad (38)$$

(37) yields

$$\frac{1}{(2\pi\hbar)^4}\left(\hat{\Delta}_\eta(\vec{p},E) * \hat{\Delta}_\eta(\vec{p},E)\right) = \int \left(\frac{m}{2\pi i\hbar}\right)^3 t_+^{-3} e^{\frac{im\vec{x}^2}{\hbar t}} e^{-\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)}\mathrm{d}t\ \mathrm{d}^3x. \quad (39)$$

The spatial integral is

$$\int e^{\frac{im\vec{x}^2}{\hbar t}} e^{-\frac{i}{\hbar}(\vec{p}\cdot\vec{x})}\mathrm{d}^3x = \pi^{\frac{3}{2}}\frac{(i\hbar t)^{\frac{3}{2}}}{m^{\frac{3}{2}}} e^{-\frac{i\vec{p}^2 t}{4\hbar m}}, \quad (40)$$

so that the convolution becomes

$$F\hat{\Delta}_\eta(\vec{p},E) * \hat{\Delta}_\eta(\vec{p},E) = \frac{(2\pi\hbar)^4}{8}\left(\frac{m}{i\pi\hbar}\right)^{\frac{3}{2}}\int t_+^{-\frac{3}{2}} e^{\frac{i}{\hbar}\left(E-\frac{\vec{p}^2}{4m}\right)t}\mathrm{d}t. \quad (41)$$

Using the result below (see [20])

$$\mathcal{F}[x_+^\lambda] = ie^{\frac{i\pi\lambda}{2}}\Gamma(\lambda+1)(k+i0)^{-\lambda-1}, \quad (42)$$

one finds

$$\hat{\Delta}_\eta(\vec{p},E) * \hat{\Delta}_\eta(\vec{p},E) = 4\pi^2\hbar^2 m^{\frac{3}{2}}\left(E - \frac{\vec{p}^2}{4m} + i0\right)^{\frac{1}{2}}. \quad (43)$$

A nonlinear q-Klein-Gordon equation. – The classical FT associated to the q-Klein-Gordon equation was developed in [3]. Here we tackle the quantum version whose action is

$$\mathcal{S} = \int \Big\{ \partial_\mu\phi(x_\mu)\partial^\mu\psi(x_\mu) + \partial_\mu\phi^\dagger(x_\mu)\partial^\mu\psi^\dagger(x_\mu) - qm^2\left[\phi^{2q-1}(x_\mu)\psi(x_\mu) + \phi^{\dagger 2q-1}(x_\mu)\psi^\dagger(x_\mu)\right]\Big\}\mathrm{d}^4x. \quad (44)$$

This theory is 1) adequate for very energetic (TeVs) q-particles according to CERN-Alice experiments and 2) non-renormalizable for any $q > 1$. Thus it cannot be dealt neither with dimensional regularization nor with differential one. A way out is provided by the ultradistributions' convolution of Bollini and Rocca [21–24]. Ultradistributions provide a general formalism to treat non-renormalizable theories and gives in the configuration space a general product in a ring with zero divisors (a product of distributions of exponential type). For example we can treat cases with $q \geq 1.15$ as we will do later.

The concomitant theory is tractable here for weak fields and for A) $q \sim 1$ or B) particular q-values. We analyze first the case $q \sim 1$ associated to energies smaller that 1 TeV. We can thus write

$$qm^2\phi^{2q-1} = qm^2\phi + 2(q-1)m^2\phi\ln\phi. \quad (45)$$

Since the field is weak, we have

$$\phi \simeq I + (q-1)\eta, \quad (46)$$
$$\ln\phi \simeq (q-1)\eta, \quad (47)$$

Using (45), (46) and (47) the field's action becomes

$$\mathcal{S} = (q-1)\int\Big\{\partial_\mu\eta(x_\mu)\partial^\mu\psi(x_\mu) + \partial_\mu\eta^\dagger(x_\mu)\partial^\mu\psi^\dagger(x_\mu) - (3q-2)m^2\Big[\eta(x_\mu)\psi(x_\mu) + \eta^\dagger(x_\mu)\psi^\dagger(x_\mu)\Big]\Big\}\mathrm{d}^4x. \quad (48)$$

where we employed

$$\int\eta(x_\mu)\mathrm{d}^4x = \int\psi(x_\mu)\mathrm{d}^4x = 0. \quad (49)$$

Defining

$$3q-2 \neq 0, \qquad \mu^2 = (3q-2)m^2. \quad (50)$$

one has

$$\mathcal{S} = (q-1)\int\Big\{\partial_\mu\eta(x_\mu)\partial^\mu\psi(x_\mu) + \partial_\mu\eta^\dagger(x_\mu)\partial^\mu\psi^\dagger(x_\mu) - \mu^2\Big[\eta(x_\mu)\psi(x_\mu) + \eta^\dagger(x_\mu)\psi^\dagger(x_\mu)\Big]\Big\}\mathrm{d}^4x. \quad (51)$$

The low-energy field is just the usual Klein-Gordon one! For the fields we have

$$\eta(x_\mu) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int\left[\frac{a(\vec{k})}{\sqrt{2\omega}}e^{-ik_\mu x^\mu} + \frac{b^\dagger(\vec{k})}{\sqrt{2\omega}}e^{ik_\mu x^\mu}\right]\mathrm{d}^3k, \quad (52)$$
$$\psi(x_\mu) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int\left[\frac{c(\vec{k})}{\sqrt{2\omega}}e^{-ik_\mu x^\mu} + \frac{d^\dagger(\vec{k})}{\sqrt{2\omega}}e^{ik_\mu x^\mu}\right]\mathrm{d}^3k \quad (53)$$

where $k_0 = \omega = \sqrt{\vec{k}^2 + \mu^2}$.

Field quantization proceeds then along familiar lines:

$$[a(\vec{k}), a^\dagger(\vec{k}')] = [b(\vec{k}), b^\dagger(\vec{k}')] = [c(\vec{k}), c^\dagger(\vec{k}')] = [d(\vec{k}), d^\dagger(\vec{k}')] = \delta(\vec{k} - \vec{k}'). \quad (54)$$

For $3q-2=0$, *i.e.*, $q = \frac{2}{3}$ the low-energy theory is one for a null mass field

$$\mathcal{S} = -\frac{1}{3}\int\left[\partial_\mu\eta(x_\mu)\partial^\mu\psi(x_\mu) + \partial_\mu\eta^\dagger(x_\mu)\partial^\mu\psi^\dagger(x_\mu)\right]\mathrm{d}^4x, \quad (55)$$

where $k_0 = \omega = |\vec{k}|$.

We tackle now the q-KG theory for an integer n such that $2q-1=n$ for m small where the action is

$$\mathcal{S} = \int\Big\{\partial_\mu\phi(x_\mu)\partial^\mu\psi(x_\mu) + \partial_\mu\phi^\dagger(x_\mu)\partial^\mu\psi^\dagger(x_\mu) - \frac{n+1}{2}m^2\Big[\phi^n(x_\mu)\psi(x_\mu) + \phi^{n\dagger}(x_\mu)\psi^\dagger(x_\mu)\Big]\Big\}\mathrm{d}^4x. \quad (56)$$

Now we define i) the free action $\mathcal{S}_0$ and ii) that corresponding to the interaction $\mathcal{S}_I$ as

$$\mathcal{S}_0 = \int\Big[\partial_\mu\phi(x_\mu)\partial^\mu\psi(x_\mu) + \partial_\mu\phi^\dagger(x_\mu)\partial^\mu\psi^\dagger(x_\mu)\Big]\mathrm{d}^4x, \quad (57)$$
$$\mathcal{S}_I = -\frac{n+1}{2}m^2\int\Big[\phi^n(x_\mu)\psi(x_\mu) + \phi^{\dagger n}(x_\mu)\psi^\dagger(x_\mu)\Big]\mathrm{d}^4x. \quad (58)$$

The fields in the interaction representation satisfy the equations of motion for free fields corresponding to the action $\mathcal{S}_0$. This is to satisfy the usual massless Klein-Gordon equation. As a consequence we can cast the fields ϕ and ψ in the fashion

$$\phi(x_\mu) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int\left[\frac{a(\vec{k})}{\sqrt{2\omega}}e^{ik_\mu x_\mu} + \frac{b^\dagger(\vec{k})}{\sqrt{2\omega}}e^{-ik_\mu x_\mu}\right]\mathrm{d}^3k \quad (59)$$
$$\psi(x_\mu) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int\left[\frac{c(\vec{k})}{\sqrt{2\omega}}e^{ik_\mu x_\mu} + \frac{d^\dagger(\vec{k})}{\sqrt{2\omega}}e^{-ik_\mu x_\mu}\right]\mathrm{d}^3k, \quad (60)$$

where $k_0 = \omega = |\vec{k}|$ The quantification of these two fields is i) immediately tractable and ii) the usual one given by

$$[a(\vec{k}), a^\dagger(\vec{k}')] = [b(\vec{k}), b^\dagger(\vec{k}')] = [c(\vec{k}), c^\dagger(\vec{k}')] = [d(\vec{k}), d^\dagger(\vec{k}')] = \delta(\vec{k} - \vec{k}'). \quad (61)$$

The naked propagator corresponding to both fields is the customary one and it is just the Feynman propagator for massless fields

$$\Delta_0(k_\mu) = \frac{i}{k^2 + i0}, \quad (62)$$

where $k^2 = k_0^2 - \vec{k}^2$. The dressed propagator which takes into account the interaction is given by

$$\Delta(k_\mu) = \frac{i}{k^2 + i0 - i\Sigma(k_\mu)}, \quad (63)$$

where $\Sigma(k_\mu)$ is the self-energy. Let us calculate the self-energy for the field ϕ at second order in perturbation theory for which the only non-vanishing diagram corresponds to the convolution of $n-1$ propagators for the field ϕ and one propagator for the field ψ. All remaining diagrams are null. (this is easily demonstrated using the regularization of Guelfand for integrals containing powers of x [20]). Therefore we have for the self-energy the expression

$$\Sigma(k_\mu) = \frac{(n+1)^2m^4}{4}\left(\frac{i}{k^2+i0} * \frac{i}{k^2+i0} * \frac{i}{k^2+i0}\cdots * \frac{i}{k^2+i0}\right). \quad (64)$$

The convolution of n Feynman's propagators of zero mass is calculated directly using the theory of convolution of ultradistributions [21–24]. Here we just give the result that turns out to be rather simple. A detailed demonstration lies beyond this paper's scope. We arrive at

$$\frac{i}{k^2+i0} * \frac{i}{k^2+i0} * \frac{i}{k^2+i0} \cdots * \frac{i}{k^2+i0} = \frac{i\pi^{2(n-1)}k^{2(n-2)}}{\Gamma(n)\Gamma(n-1)}[\ln(k^2+i0)+2\lambda(1)-\lambda(n-1)-\lambda(n)], \tag{65}$$

where $\lambda(z)=\frac{\mathrm{d}\ln\Gamma(z)}{\mathrm{d}z}$. The self-energy is then

$$\Sigma(k_\mu) = \frac{(n+1)^2m^4}{4}\frac{i\pi^{2(n-1)}k^{2(n-2)}}{\Gamma(n)\Gamma(n-1)} \times \ln(k^2+i0)+2\lambda(1)-\lambda(n-1)-\lambda(n)]. \tag{66}$$

For both fields ϕ and ψ the self-energy and the dressed propagator coincide up to second order.

Note that the current of probability is given by

$$\mathcal{J}_\mu = \frac{i}{4m}[\psi\partial_\mu\phi - \phi\partial_\mu\psi + \phi^\dagger\partial_\mu\psi^\dagger - \psi^\dagger\partial_\mu\phi^\dagger] \tag{67}$$

and it is verified that

$$\partial_\mu\mathcal{J}^\mu = 0. \tag{68}$$

This implies that the fields defined in the representation of the interaction are physical fields.

Conclusions. – We have here obtained some results that may be regarded as interesting.

1) We developed the CFT for the particular q-SE advanced in [18].

2) For this CFT we showed that the customary dispersion relations apply. We also introduced the physical fields, *i.e.*, those that the probability current is conserved. The physical states are introduced via analogy with bosonic string theory.

3) We developed the QFT associated to the q-SE of [18]. For weak fields this q-QFT coincides with the ordinary SE-QFT. This result confirms our Nuclear Physics A results. These show that one needs energies of up to 1 TeV in order to clearly distinguish between q-theories and $q=1$ ordinary ones.

4) Using the distribution theory [20] we discussed the convolution of two Schrödinger propagators obtaining a finite result.

5) We developed the QFT associated to the q-KGE generalizing our result of [3].

6) For low energies and q close to 1 this theory coincides with the ordinary KG-QFT.

7) For particular q-values $q=\frac{n+1}{2}$, n integer, we develop the q-KG-QFT.

8) We calculate the convolution of n naked propagators the corresponding self-energy up to the second order and the dressed propagator. This was achieved appealing to the ultradistributions theory.

REFERENCES

[1] Nobre F. D., Rego-Monteiro M. A. and Tsallis C., *Phys. Rev. Lett.*, **106** (2011) 140601.

[2] Ring P. and Schuck P., *The Nuclear Many Body Problem* (Springer, New York) 1980.

[3] Plastino A. and Rocca M. C., *EPL*, **116** (2016) 41001.

[4] Plastino A. and Rocca M., *Nuc. Phys. A*, **948** (2016) 19.

[5] Plastino A., Rocca M. C., Ferri G. L. and Zamora D. J., *Nucl. Phys. A*, **955** (2016) 16.

[6] ALICE Collaboration (Barile F. *et al.*), *EPJ Web of Conferences*, **60** (2013) 13012; ALICE Collaboration (Abelev B. *et al.*), *Phys. Rev. Lett.*, **111** (2013) 222301; ALICE Collaboration (Kharlov Yu. V.), *Phys. At. Nuclei*, **76** (2013) 1497; ALICE Collaboration, *Phys. Rev. C*, **91** (2015) 024609; ATLAS Collaboration, *New J. Phys.*, **13** (2011) 053033; CMS Collaboration, *J. High Energy Phys.*, **05** (2011) 064; CMS Collaboration, *Eur. Phys. J. C*, **74** (2014) 2847.

[7] Nobre F. D. and Plastino A. R., *Eur. Phys. J. C*, **76** (2016) 343.

[8] Bennett C. H., Leung D., Smith G. and Smolin J. A., *Phys. Rev. Lett.*, **103** (2009) 170502.

[9] Plastino A. R. and Zander C., *AIP Conf. Proc.*, **841** (2006) 570.

[10] Pitaevskii L. P. and Stringari S., *Bose Einstein Condensation* (Clarendon Press, Oxford) 2003.

[11] Nobre F. D., Rego-Monteiro M. A. and Tsallis C., *EPL*, **97** (2012) 41001.

[12] Plastino A. R., Rigo A., Casas M., Garcias F. and Plastino A., *Phys. Rev. A*, **60** (1999) 4318.

[13] de Souza Dutra A., Hott M. and Almeida C. A. S., *Europhys. Lett.*, **62** (2003) 8.

[14] Costa Filho R. N., Almeida M. P., Farias G. A. and Andrade J. S. jr., *Phys. Rev. A*, **84** (2011) 050102.

[15] Rego-Monteiro M. A. and Nobre F. D., *Phys. Rev. A*, **88** (2013) 032105.

[16] Plastino A. and Rocca M. C., *Physica A*, **450** (2016) 435.

[17] Green M. B., Schwarz J. H. and Witten E., *Superstring Theory* (Cambridge University Press) 1987.

[18] Plastino A. and Rocca M. C., *Phys. Lett. A*, **379** (2015) 2690.

[19] Kleinert H., *Particles and Quantum Fields*, free web version (2016).

[20] Gel'fand I. M. and Shilov G. E., *Generalized Functions*, Vol. **1** (Academic Press) 1968.

[21] Bollini C. G., Escobar T. and Rocca M. C., *Int. J. Theor. Phys.*, **38** (1999) 2315.

[22] Bollini C. G. and Rocca M. C., *Int. J. Theor. Phys.*, **43** (2004) 59.

[23] Bollini C. G. and Rocca M. C., *Int. J. Theor. Phys.*, **43** (2004) 1019.

[24] Bollini C. G. and Rocca M. C., *Int. J. Theor. Phys.*, **46** (2007) 3030.