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Different vertex parameterizations and propagators for the Δ contribution in π-photoproduction

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Abstract

We evaluate the total cross section for the π-photoproduction process and analyze the behavior of the $\Delta(1232)$ resonance contribution when the photon energy is increased from threshold up to 0.7 GeV. Within this energy range we compare two different parameterizations for the $\gamma N \Delta$ vertex: the normal parity and the covariant multipole decomposition ones. For completeness, we also compare different versions for the Δ propagator: the first is the dressed propagator obtained including one-loop self-energy contributions (EXACT), the second is the complex mass scheme which consists in replacing $m_\Delta \to m_\Delta - \mathrm{i}\Gamma_\Delta/2$ in the bare propagator, and the third is an intermediate approximation between the two previous ones (EXCMS). We conclude that, in order to extend the present calculation to include more energetic resonances in the future and to obtain non-divergent results for the total cross section we will need to use the MD parametrization and the EXCMS propagator.

Keywords: π-photoproduction, resonances, effective Lagrangian approach

1. Introduction

The understanding of the baryon spectrum and the search for the missing nucleon resonances and new exotic states are hot topics in hadronic physics. The production of mesons by hadron-induced reactions has been extensively used in the study of the properties of nucleon resonances (see e.g. the reviews of [1, 2]). In particular, the scattering of pions by free nucleons and nucleus has greatly contributed to the experimental data base. These types of reaction are complicated since the initial and final states are governed by the strong interaction. However, the existence of high duty-cycle electron accelerators opens the possibility of studying reactions induced by the electromagnetic interaction, photoproduction and electroproduction of mesons (π, η, etc) off the free nucleon and off the nucleus allows us to analyze the structure of the nucleon excited states (see [3–65]).

Since the pioneering work of Chew, Goldberger, Low and Nambu [3], extensive work during more than fifty years has shown that, below 0.45 GeV incident photon energy, the dominant mechanisms of the $\gamma N \to \pi N$ reaction are the background and the Δ-resonant excitation. After that, high efforts have been carried out both experimentally (see [4–30] for π and [31–41] for η-photoproduction and references therein) and theoretically (regarding π photoproduction see for example models developed in [42–53], MAID model [54–56], Bonn–Gatchina model [57, 58], chiral Lagrangian based models [59–61], quark model [62], and [63–65] for η-photoproduction) to study the cross sections of π and η mesons photoproduction reactions, arriving in the present to higher incident photon energies of the order of 1.2 GeV. By this reason, to obtain a good theoretical description of the experimental data we need: (i) to add to the Δ resonance the contribution of those ones belonging to the second resonance region; and (ii) to perform a consistent description of the Δ resonance, non-divergent at energies well above its peak.

Actual experiments where nuclei are used as targets require the inclusion of nuclear medium effects on the free nucleon cross section to make a comparison between theoretical results and experimental data. However, before including nuclear effects, it is important that the model used to describe the free nucleon cross section incorporates consistently the resonant and the background terms as well as their non-negligible interference. The study of the spin-$3/2$ fields in hadron physics started very early with the pioneering work of Rarita Schwinger (RS) [66] and there is nowadays an abundant literature on spin-$3/2$ vertices, propagators and associated problems [67–74]. The RS theory has shown several difficulties along the time, when interactions were introduced. In fact, when the RS field propagates in an external electromagnetic field, being the coupling obtained from the minimal substitution in the free Lagrangian, two problems are reported in the literature. One is that, while the free and electromagnetic Lagrangians are fully covariant, the second quantization is not realizable in all reference frames [75]. The other one is the appearance of acausal all order solutions of the equation of motion coming from these Lagrangians [76]. Later, using the RS field to describe the $\Delta(1232)$ resonance in πN scattering, Nath *et al* [68] proposed a consistent πN vertex invariant from the point of view of the contact transformations of the spin-$3/2$ field and its quantization. Soon, similar problems as those mentioned previously were found, but now with the hadronic πN interaction [77–80]. The RS equation of motion describes a 'constrained' dynamical system, and for this reason is supplemented by certain primary and secondary constraints or subsidiary conditions ($\partial^\mu \Psi_\mu = \gamma^\mu \Psi_\mu = 0$, being Ψ_μ the spin-$3/2$ field) that eliminate the redundant degrees of freedom [70–74, 81]. By a consistent treatment we mean that the effective Lagrangian model describing the spin-$3/2$ resonance and its interactions must take into account that field-theoretical constraints for the spin-$3/2$ particle, its resonant character and it must also be gauge-invariant when photonic interactions are included[4]. Theoretical calculations have some problems of formal consistence. In fact, most of the works

[4] The role of gauge invariance and contact interactions for the spin-$3/2$ fields has been discussed e.g. in [82, 83]. As was shown in [73], when radiative corrections are considered it is not possible to keep the electromagnetic gauge invariance and the spin-$3/2$ gauge invariance for the strong $\pi N \Delta$ vertices introduced in [82, 83]. Thus, we prefer to adopt the usual lower order pion derivative $\pi N \Delta$ vertex. Certainly, the calculations done in those references, in spite of the formal problem mentioned above, work well in the resonance region but here we pretend to go to higher energies.

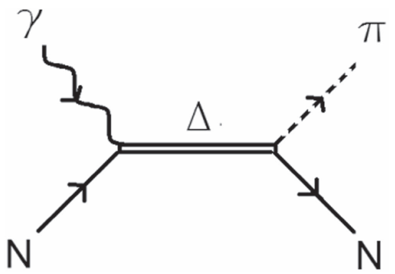

Figure 1. Resonant s-channel diagram for π photoproduction process.

mentioned above treat the vertices and the propagator of the spin-$3/2$ resonances inconsistently because, as mentioned previously and discussed in [84] (see also [85, 86] and references therein) the Lagrangian densities and the amplitude must be invariant under the contact transformation (see discussion on section 2.3 below). Also, non-resonant contributions (background) should be added coherently to the resonance one, without neglecting the interference, which is an inconsistent procedure [44, 51, 87–89]. Additionally, when the incident photon energy is increased, interference between different resonance terms in the scattering probability plays an important role. On the other hand, we remark that the calculation performed in [51], in spite of being consistent from the theoretical point of view, only considers incident photon energies from threshold up to 0.45 GeV, well below the current region of interest.

In order to analyze current experimental data of π-photoproduction the contribution of the resonances of the second region needs to be added to that of the $\Delta(1232)$. We plan to develop in the future a consistent formalism which includes the contribution of these resonances. The arguments mentioned previously motivate us to study consistently the Δ resonance contribution to the cross section with the purpose of obtaining a good behavior, not divergent, at energies well above the peak located at $E_\gamma \sim 0.35$ GeV. This will be done in this paper, which is organized as follows: the formalism is presented in section 2, paying special attention to the Δ vertex and propagator, numerical results and a comparison between them for different versions of the Δ vertex and the propagator are shown in section 3 together some final remarks.

2. Formalism

2.1. Cross section

The total cross section for the π-photoproduction process $\gamma N \rightarrow \pi N$ receives contribution from both, resonant and background terms. As mentioned previously, we will analyze here in detail the resonant contribution, which corresponds to the so called resonant s-channel shown in figure 1.

This resonant contribution to the total cross section will be calculated as:

$$\sigma_R = \frac{|\vec{q}|}{|\vec{k}|}\frac{m_N^2}{32\pi s}\left(\frac{ef_{\pi N\Delta}I_\Delta}{m_\pi}\right)^2 \int_0^\pi \sum_{m_s m_s'} \mathrm{d}\theta^* \sin\theta^* \\ \times |\bar{u}(p_\mathrm{f}, m_s')q_\alpha G^{\alpha\beta}(p_\Delta)\Gamma_{\beta\nu}\epsilon^\nu(\lambda)u(p_\mathrm{i}, m_s)|^2, \tag{1}$$

where the integration is performed over the c.m. scattering angle θ^* between π and N, with $G^{\alpha\beta}$ being the Δ propagator and $\Gamma_{\beta\nu}$ the vector $\gamma N\Delta$ vertex. Here $p_{\rm i}$ and $p_{\rm f}$ are the initial and final nucleon four-momenta, k, $p_\Delta = p_{\rm i} + k$ and $q = p_{\rm i} - p_{\rm f} + k$ are the photon, resonance and pion four-momenta, respectively. In the c.m. of the πN system we can write all the momenta in terms of the photon energy in laboratory frame, E_γ, as follows:

$$k = \begin{pmatrix} E_k \\ 0 \\ 0 \\ E_k \end{pmatrix},\ p_{\rm i} = \begin{pmatrix} E_{\rm i} \\ 0 \\ 0 \\ -E_k \end{pmatrix},\ q = \begin{pmatrix} E_q \\ p_2 \sin\theta^* \\ 0 \\ p_2 \cos\theta^* \end{pmatrix},\ p_{\rm f} = \begin{pmatrix} E_{\rm f} \\ -p_2 \sin\theta^* \\ 0 \\ -p_2 \cos\theta^* \end{pmatrix}, \tag{2}$$

where $E_k = \frac{s - m_N^2}{2\sqrt{s}}$, $E_{\rm i} = \frac{s + m_N^2}{2\sqrt{s}}$, $E_q = \frac{s - m_N^2 + m_\pi^2}{2\sqrt{s}}$, $E_{\rm f} = \frac{s + m_N^2 - m_\pi^2}{2\sqrt{s}}$ and $p_2 = \frac{\sqrt{s^2 + m_N^4 + m_\pi^4 - 2sm_N^2 - 2sm_\pi^2 - 2m_N^2 m_\pi^2}}{2\sqrt{s}}$ with $s = m_N(m_N + 2E_\gamma)$ being m_N and m_π the nucleon and pion masses, respectively. The constants in (1) are $e = \sqrt{4\pi\alpha}$ with $\alpha = 1/137$, $\frac{f_{\pi N\Delta}^2}{4\pi} = 0.317$ [51] and the isospin factor is

$$I_\Delta = \begin{cases} \frac{2}{3} \text{ for } \gamma N \to \pi^0 N,\ N = n, p \\ -\frac{\sqrt{2}}{3} \text{ for } \gamma p \to \pi^+ n \\ \frac{\sqrt{2}}{3} \text{ for } \gamma n \to \pi^- p. \end{cases} \tag{3}$$

For the Dirac spinor we use the normalization $u(p, m_s) = \sqrt{\frac{E + m_N}{2m_N}} \begin{pmatrix} 1 \\ \frac{\sigma\cdot\mathbf{p}}{E + m_N} \end{pmatrix} \chi_{m_s}$ and for the polarization vector we adopt $\epsilon^\lambda(1) = \left(0, -\frac{1}{\sqrt{2}}, -\frac{\rm i}{\sqrt{2}}, 0\right)$, $\epsilon^\lambda(-1) = \left(0, \frac{1}{\sqrt{2}}, -\frac{\rm i}{\sqrt{2}}, 0\right)$.

2.2. Δ resonance propagator

The unperturbed Δ propagator satisfying the Ward–Takahashi identity (assuring the gauge invariance of the radiative amplitude) has been dressed in [90, 91] by the inclusion of a self-energy, giving to it a width corresponding to an unstable particle. In [90] the dressed propagator ${\rm i}G^{\mu\nu}(p)$ has been obtained by solving the Schwinger–Dyson equation $({\rm i}G^{-1})^{\mu\nu}(p) = ({\rm i}G_0^{-1})^{\mu\nu}(p) - \Sigma^{\mu\nu}(p)$, being ${\rm i}G_0^{\mu\nu}(p)$ the bare propagator and $\Sigma^{\mu\nu}(p)$ the self energy correction of the Δ, and considering only the one-loop contribution to the absorptive (imaginary) part of the self energy[5]. Based in this calculation, three different approximations have been discussed in [91]:

(i) <u>The EXACT approximation</u>
Reversing the Schwinger–Dyson equation without additional approximations and calculating the one-loop self-energy corrections 'exactly', i.e. without performing any approximation in the one-loop integral describing the propagation of one pion-one nucleon inside the loop (see equation (6) from [90]) the propagator can be written as

[5] In [92] the authors consider additional terms in the self energy, because they calculate the width of the Δ resonance at leading two-loop order in baryon chiral perturbation theory. It is important to point out that it would be difficult to implement these calculations in our context.

$$
\begin{aligned}
G^{\mu\nu}(p) = {} & \frac{\tilde{m} + \not{p}}{\tilde{m}^2 - p^2}(\mathcal{P}^{3/2})^{\mu\nu} \\
& + \frac{1}{2}\left[\frac{2\tilde{m} - 2\sqrt{p^2} + A_+}{-\tilde{m}^2 + X_+} + \frac{2\tilde{m} + 2\sqrt{p^2} + A_-}{-\tilde{m}^2 + X_-}\right](\mathcal{P}_{11}^{1/2})^{\mu\nu} \\
& + \frac{1}{2\sqrt{p^2}}\left[-\frac{2\tilde{m} - 2\sqrt{p^2} + A_+}{-\tilde{m}^2 + X_+} + \frac{2\tilde{m} + 2\sqrt{p^2} + A_-}{-\tilde{m}^2 + X_-}\right]\not{p}(\mathcal{P}_{11}^{1/2})^{\mu\nu} \\
& + \frac{1}{2}\left[\frac{3\frac{J_3 - \sqrt{p^2}J_4}{1 - J_2}}{-\tilde{m}^2 + X_+} + \frac{3\frac{J_3 + \sqrt{p^2}J_4}{1 - J_2}}{-\tilde{m}^2 + X_-}\right](\mathcal{P}_{22}^{1/2})^{\mu\nu} \\
& + \frac{1}{2\sqrt{p^2}}\left[\frac{3\frac{J_3 - \sqrt{p^2}J_4}{1 - J_2}}{-\tilde{m}^2 + X_+} - \frac{3\frac{J_3 + \sqrt{p^2}J_4}{1 - J_2}}{-\tilde{m}^2 + X_-}\right]\not{p}(\mathcal{P}_{22}^{1/2})^{\mu\nu} \\
& + \frac{\sqrt{3}}{2}\left[\frac{\tilde{m} - \left(\frac{J_1 + \sqrt{3}J_7}{1 - J_2}\right)}{-\tilde{m}^2 + X_+} - \frac{\tilde{m} - \left(\frac{J_1 - \sqrt{3}J_7}{1 - J_2}\right)}{-\tilde{m}^2 + X_-}\right][(\mathcal{P}_{21}^{1/2})^{\mu\nu} + (\mathcal{P}_{12}^{1/2})^{\mu\nu}] \\
& - \frac{\sqrt{3}}{2\sqrt{p^2}}\left[\frac{\tilde{m} - \left(\frac{J_1 + \sqrt{3}J_7}{1 - J_2}\right)}{-\tilde{m}^2 + X_+} + \frac{\tilde{m} - \left(\frac{J_1 - \sqrt{3}J_7}{1 - J_2}\right)}{-\tilde{m}^2 + X_-}\right] \\
& \times \not{p}[(\mathcal{P}_{21}^{1/2})^{\mu\nu} - (\mathcal{P}_{12}^{1/2})^{\mu\nu}], \qquad (4)
\end{aligned}
$$

where

$$
\begin{aligned}
X_\pm &\equiv \frac{2m(J_1 + J_3 \pm \sqrt{3}J_7 \mp \sqrt{p^2}J_4) + 2\sqrt{p^2}(\mp J_3 + \sqrt{p^2}J_4) + J_1^2}{(1 - J_2)^2}, \\
A_\pm &\equiv \frac{3(J_5 \pm \sqrt{p^2}J_6) - 2(J_1 \pm \sqrt{p^2}J_2)}{1 - J_2}, \qquad (5)
\end{aligned}
$$

being $p = p_\Delta$ and the effective mass is given by

$$\tilde{m} \equiv \frac{m + J_1}{1 - J_2}, \qquad (6)$$

with $m = m_\Delta$ the bare Δ mass. The coefficients J_i are functions of $s = p^2$ and depend on the $\pi N\Delta$ coupling constant g, and the $S = 1/2$, $3/2$ spin projectors $\mathcal{P}_{ij}^S$ are defined in [90]. Because the Schwinger–Dyson equation has been reversed without additional approximations and the self energy one-loop integral has been calculated exactly in this case, we follow the notation of [91] and refer to this propagator as the 'EXACT' one.

(ii) The CMS approximation

Neglecting terms of $O(g^4)$ and of $O((m_\Delta - \sqrt{s})g^2)$ which are expected to be very small in the Δ resonance region ($s \approx m_\Delta^2$) in equations (4)–(6) and assuming within the formal limit of massless N and π in the loop contribution that the dressing gives a dependence $g(s) = \frac{\tilde{g}}{\sqrt{s}}$, with adimensional $\tilde{g} \equiv \mathsf{a}g_0$, being $g_0 = \frac{f_{\pi N\Delta}}{m_\pi} = 14.3\ \mathrm{GeV}^{-1}$ the bare $\pi N\Delta$ coupling constant and a a constant to fit, one gets [91]

$$G^{\mu\nu} = \frac{\not{p} + \tilde{m}}{p^2 - \tilde{m}^2}\left\{-g^{\mu\nu} + \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{2}{3\tilde{m}^2}p^\mu p^\nu - \frac{1}{3\tilde{m}}(p^\mu\gamma^\nu - \gamma^\mu p^\nu) - \frac{2(\not{p} - \tilde{m})}{3\tilde{m}^2}[\gamma^\mu p^\nu - p^\mu\gamma^\nu + (\not{p} + \tilde{m})\gamma^\mu\gamma^\nu]\right\}, \tag{7}$$

with the effective mass

$$\tilde{m} \simeq m_\Delta - \mathrm{i}\frac{\Gamma_\Delta(s)}{2}, \qquad \text{where} \qquad \Gamma_\Delta(s) = \Gamma_\Delta^{\mathrm{CMS}}\left(1 + \frac{s^{1/2} - m_\Delta}{m_\Delta}\right), \tag{8}$$

being $\Gamma_\Delta^{\mathrm{CMS}} = \frac{\tilde{g}^2 m_\Delta}{192\pi}$. At the $s \approx m_\Delta^2$ region we obtain the approximated expression

$$\tilde{m} \simeq m_\Delta - \mathrm{i}\frac{\Gamma_\Delta^{\mathrm{CMS}}}{2}, \tag{9}$$

where now Γ^{CMS} is the fitting parameter in place of a. The equation (7) together with (9) are the so-called complex-mass scheme (CMS), which consists in replacing $m_\Delta \to m_\Delta - \mathrm{i}\Gamma_\Delta^{\mathrm{CMS}}/2$ in the unperturbed propagator, ensuring the gauge invariance of the radiative amplitude. The CMS has been used thoroughly in the literature [50, 84, 93–95]. In [50] only the $3/2$ part of the propagator in equation (7) is maintained since the 'spin $3/2$-gauge invariant' vertex introduced in [71] is adopted. Remembering that this kind of Lagrangian interaction has the consistency problems mentioned previously [72–74], in this case the authors fix the resonance widths phenomenologically and not from a self energy calculation. Also, in [93] the same propagator is adopted on the basis of assuming on shell-subsidiary conditions to avoid the $1/2$ contribution together with the conventional couplings. For the width they use a parameterization depending on the considered partial wave. On the other hand, in [94, 95] the authors adopt the conventional pion derivative vertices for the $\pi N\Delta$ vertex but take only the spin-$3/2$ part of the propagator, not allowing the coupling of this kind of vertex to the spin-$1/2$ virtual contribution of the propagator.

(iii) The EXCMS approximation
Following [91], an intermediate approximation for the propagator can be adopted. It corresponds to the exact expression (4) but with the assumption (9) for the effective mass. This will be called 'EXCMS' dressed propagator.

2.3. Vertex parameterizations

As discussed in [68, 84], each of the terms in the Δ Lagrangian density (kinetic and interaction terms) depend on an arbitrary parameter. A through the expression $\Lambda_{\mu\nu}(A) = g_{\mu\nu} + \frac{1}{2}(1 + 3A)\gamma_\mu\gamma_\nu$. The Lagrangian is invariant under the contact transformation on the spin-$3/2$ field, $\Psi_\Delta^\mu \to \Psi_\Delta^\mu + a\gamma^\mu\gamma_\alpha\Psi_\Delta^\alpha$, $A \to A' = \frac{A - 2a}{1 + 4a}$, where A and a ($a \neq -1/4$) are arbitrary parameters. This invariance assures that spurious spin-$1/2$ components are removed from the field describing an on-shell Δ particle. However, the propagation of an off-shell Δ particle unavoidably carries an spin-$1/2$ component (see [72, 85, 86] and references therein). Transition (physical) amplitudes involving the Δ resonance calculated from A-dependent Feynman rules should be, however, independent of A [84]. A common mistake in some calculations is to take the simplest form of the propagator for the Δ corresponding to $A = -1$ and, simultaneously, the simplest form of the $\pi N\Delta$ and/or $\gamma N\Delta$ vertex with a different value of this parameter (for example $A = -1/3$). Thus, to avoid this problem and because physical amplitudes should be independent of this parameter, we need to use a set of A-independent Feynman rules [84] to derive the amplitudes. The form of the Lagrangian terms were extensively discussed in [85, 96] and we only give here the vertices and the propagator of the Δ, consistent with all symmetries, necessary to write the amplitudes involving the resonance.

We treat the $\pi N\Delta$ interaction within the chiral effective Lagrangian approach. One can classify the interaction in orders depending on the derivatives of the fields. The conventional and lowest order interaction is the pion field derivative, adopted already in the past in several works [67, 68, 97]. In the first two papers is also discussed the fixing of a second parameter z on which the interaction Lagrangian depends and is compatible with the contact symmetry of the free Δ Lagrangian. Here we adopt the prescriptions for the strong conventional $\pi N\Delta$ vertex presented with details in [72]. Then the $\gamma N\Delta$ vertex, which should be self-gauge invariance since it is not possible to obtain it from minimum coupling, will be obtained from the Sachs or parity conserving parameterizations presented below. Other authors adopt a second order strong vertex with an additional derivative in the Δ field [71, 98], based in a symmetry of the massless free Δ Lagrangian, which is supported in the assumption that this spin-$3/2$ gauge invariant interaction is consistent from the formal point of view while the conventional is not. Also the $\gamma N\Delta$ vertex is derived on the same lines. Nevertheless, we have shown that this vertex does not work better than the conventional one from the phenomenological point of view in [72], that the spin-$3/2$ gauge invariance presents certain coexistence problems with the electromagnetic gauge invariance in [73] and quite recently that this interaction is also formally inconsistent [74].

Based on the previous arguments, we analyze here two different prescriptions consistent with the choice of the propagator for the $\gamma N\Delta$ vertex [99, 100]. They are:

(i) Normal parity

A standard 'normal parity' (NP) parametrization of the vertex given by

$$\Gamma^{\mathrm{NP}}_{\mu\nu} = \left[\frac{G_1(k^2)}{2m_N}(g_{\mu\nu}\not{k} - k_\mu\gamma_\nu) - \frac{G_2(k^2)}{2m_N^2}(g_{\mu\nu}P\cdot k - k_\mu P_\nu)\right]\mathrm{i}\gamma_5, \tag{10}$$

with $P = (p_{\mathrm{i}} + p_\Delta)/2$. For real photons only $G_1 \equiv G_1(0)$ and $G_2 \equiv G_2(0)$ play a role.

(ii) Covariant multipole decomposition

A 'covariant multipole decomposition' (MD) analogous to the Sachs choice [101] of the form

$$\Gamma^{\mathrm{MD}}_{\mu\nu} = G_M(k^2)K^M_{\mu\nu} + G_E(k^2)K^E_{\mu\nu}, \tag{11}$$

with

$$\begin{aligned} K^M_{\mu\nu} &= -\frac{3}{2M_N\Sigma M}\epsilon_{\mu\nu\alpha\beta}P^\alpha k^\beta, \\ K^E_{\mu\nu} &= -K^M_{\mu\nu} - \frac{6}{M_N\Sigma M(\Delta M)^2}\epsilon_{\mu\lambda\alpha\beta}P^\alpha k^\beta\epsilon_\nu{}^\lambda{}_{\gamma\delta}p^\gamma_\Delta k^\delta \mathrm{i}\gamma_5. \end{aligned} \tag{12}$$

Using the identities

$$\begin{aligned} \epsilon_{\mu\nu\alpha\beta}P^\alpha k^\beta &= \mathrm{i}[(\not{P}\not{k} - P\cdot k)\mathrm{i}\sigma_{\mu\nu} + \not{k}(\gamma_\mu P_\nu - P_\mu\gamma_\nu) \\ &\quad - \not{P}(\gamma_\mu k_\nu - \gamma_\nu k_\mu) + (P_\mu k_\nu - k_\mu P_\nu)]\gamma_5, \\ \epsilon_{\mu\lambda\alpha\beta}\epsilon_\nu{}^\lambda{}_{\gamma\delta} &= -\begin{vmatrix} g_{\mu\nu} & g_{\mu\gamma} & g_{\mu\delta} \\ g_{\alpha\nu} & g_{\alpha\gamma} & g_{\alpha\delta} \\ g_{\beta\nu} & g_{\beta\gamma} & g_{\beta\delta} \end{vmatrix}, \end{aligned} \tag{13}$$

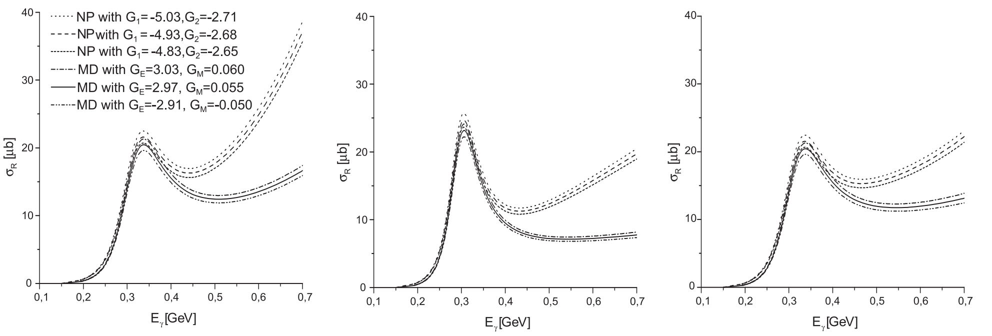

Figure 2. Resonant s-channel contribution to the total cross section σ_R, normalized to $\frac{|\vec{q}\,|}{|\vec{k}\,|}\frac{m_N^2}{32\pi s}\left(\frac{ef_{\pi N\Delta}I_\Delta}{m_\pi}\right)^2$. We compare the MD and NP parameterizations of the $\gamma N\Delta$ vertex for three different approximations of the Δ propagator: (a) CMS; (b) EXACT; (c) EXCMS. We show the results for different sets of parameters (see text for details).

and assuming a real Δ and thus the validity of the on-shell constrains (i.e., $p_\Delta^2 \simeq m_\Delta^2$, $\bar{\Psi}_\Delta^\mu \gamma_\mu \simeq 0$, $\bar{\Psi}_\Delta^\mu p_{\Delta,\mu} \simeq 0$, being Ψ_Δ^μ the Δ field), together with $\bar{\Psi}_\Delta^\mu \not{p}_\Delta = \bar{\Psi}_\Delta^\mu m_\Delta$, $k^2 = 0$, the transversality condition $k \cdot \epsilon = 0$ and the four-momentum conservation at the vertex (which allows to write $p_\Delta \cdot k = \frac{1}{2}(m_\Delta + m_N)(m_\Delta - m_N)$) after some simple algebra we get

$$\begin{aligned}\Gamma_{\mu\nu}^{\mathrm{MD}} \simeq \bar{\Psi}_\Delta^\mu \Bigg\{ &-\frac{3m_\Delta}{2m_N \Sigma M}[G_M(0) - G_E(0)](g_{\mu\nu}\not{k} - k_\mu\gamma_\nu) \\ &- \frac{3}{m_N}\left[-\frac{(G_M(0) - G_E(0))}{2\Sigma M} + \frac{G_E(0)}{\Delta M}\right](g_{\mu\nu}P\cdot k - k_\mu P_\nu)\Bigg\} \mathrm{i}\gamma_5, \end{aligned} \tag{14}$$

with $\Sigma M \equiv m_\Delta + m_N$ and $\Delta M \equiv m_\Delta - m_N$. Comparison with equation (4) from [99] indicates that

$$G_1^{\mathrm{SCADRON}}(0) = -\frac{G_1(0)}{2M_N}, \qquad G_2^{\mathrm{SCADRON}}(0) = \frac{G_2(0)}{2M_N^2}.$$

On the other hand, comparison with equation (1) gives

$$\begin{aligned} G_1(0) &= -\frac{3m_\Delta}{\Sigma M}[G_M(0) - G_E(0)], \\ G_2(0) &= -6M_N\left[\frac{(G_M(0) - G_E(0))}{2\Sigma M} - \frac{G_E(0)}{\Delta M}\right]. \end{aligned} \tag{15}$$

This result indicates that we can arrive to the NP parametrization by starting from the MD one and assuming a real Δ. The relations shown in equation (15) establish the connection between both parameterizations of the vertex[6]. Adopting $G_M(0) = 2.97 \pm 0.06$ and $G_E(0) = 0.055 \pm 0.005$ (see page 15 on [51]) we obtain $G_1(0) = -4.93 \pm 0.10$ and $G_2(0) = -2.68 \pm 0.03$.

[6] We mention that they agree with those from Jones and Scadron. Using the Dalitz–Sutherland values given in equations (48) and (50) from [99], $G_M(0) = 2.97$ and $G_E(0) = 0.03$, we obtain $G_1^{\mathrm{SCADRON}}(0) = 2.68\ \mathrm{GeV}^{-1}$, $G_2^{\mathrm{SCADRON}}(0) = -1.84\ \mathrm{GeV}^{-2}$ or, equivalently $G_1(0) = -5.03$ and $G_2(0) = -3.24$.

3. Results and final remarks

We have evaluated numerically the resonant s-channel contribution to the total cross section σ_R given in equation (1). Because we are not interested in comparing with the experimental data[7] but only between different versions of the vertex and propagator for the Δ resonance contribution in π-photoproduction, we present in figure 2 our results for $\sigma_R \bigg/ \left(\frac{|\vec{q}\,|}{|\vec{k}\,|} \frac{m_N^2}{32\pi s} \left(\frac{e f_{\pi N \Delta} I_\Delta}{m_\pi} \right)^2 \right)$, allowing us to not have to distinguish between the processes indicated in equation (3). We compare the MD and NP parameterizations of the $\gamma N \Delta$ vertex for three different approaches of the Δ propagator presented in the previous section: (a) CMS, (b) EXACT and (c) EXCMS. With the aim of providing an estimation of the uncertainties in our calculation associated with the error bars in the parameters, we present the plots corresponding to its central values ($G_M = 2.97$, $G_E = 0.055$ and $G_1 = -4.93$, $G_2 = -2.68$) and also those given the lowest ($G_M = 2.91$, $G_E = 0.050$ and $G_1 = -4.83$, $G_2 = -2.65$) and highest ($G_M = 3.03$, $G_E = 0.060$ and $G_1 = -5.03$, $G_2 = -2.71$) cross section around the central value. To perform our comparison we have used $m_N = 0.938$ GeV, $m_\Delta = 1.211$ GeV and $\Gamma_\Delta^{\mathrm{CMS}} = 0.088$ GeV [91] (these values are consistent with those published in [102]).

Our results clearly indicate that the NP and MD parameterizations lead always to similar results in the resonance region, i.e. below $\sim$0.45 GeV incident photon energy. However, above this region the NP behaves much worse than the MD because it strongly diverges when the photon energy is increased. This can be easily understood after the analysis performed in section 2.3 where we have shown that NP can be obtained starting from MD vertex parametrization by assuming an on-shell Δ resonance. Otherwise, comparison of the different approximations for the $\Delta(1232)$ propagator indicate that the validity of the CMS is restricted, as expected, to the low energy region where the approximations indicated in section 2.2 hold and, at higher energies, a divergent behavior is shown. This problem is solved by using the EXACT propagator which shows a good behavior at high energies but would require a readjustment of the parameters (particularly, the resonance mass m_Δ and the coupling constant) to locate the peak at the right place. Finally, an improved behavior both at low and high energies is obtained when using the intermediate approximation EXCMS for the propagator, because it preserves the CMS characteristics by the use of equation (9), which allows to use the same set of parameters adjusted in our previous works in the context of the CMS approximation, and also the non-divergent character of the EXACT calculation above the peak because of the use of equation (4). Our plots also indicate clearly that the uncertainties associated to our parameters do not change at all the conclusions mentioned previously.

Summarizing, we have analyzed the contribution of the $\Delta(1232)$ resonance to the total cross section of the π-photoproduction process with the aim of discovering which is the best model to describe its behavior when the photon energy is increased from threshold up to 0.7 GeV. We have compared two different parameterizations for the $\gamma N \Delta$ vertex (the NP and MD ones) and three different versions of the Δ propagator (CMS, EXACT and the intermediate EXCMS approximation). Our results show that in order to develop a formalism to include more energetic resonances in the future and to obtain non-divergent results for the total cross section we will need to use the MD parametrization and the EXCMS propagator.

[7] Experiments measure the total resonant plus background contribution. Thus, to perform a comparison with experimental data would require us the evaluation of the background graphs shown in figure 1 from [53], which escapes the objectives of the present work.

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References

[1] Sato T and Lee T-S H 2009 *J. Phys. G: Nucl. Part. Phys.* **36** 073001
[2] Suzuki N, Sato T and Lee T-S 2009 *Phys. Rev.* C **79** 025205
[3] Chew G F, Goldberger M L, Low F E and Nambu Y 1957 *Phys. Rev.* **106** 1345
[4] Menze D, Pfeil W and Wilcke R 1977 ZAED compilation of pion photoproduction dataUniversity of Bonn
[5] Sandorfi A M *et al* (Legs Collaboration) 1994 *Few Body Syst. Suppl.* **7** 317
Sandorfi A M *et al* (Legs Collaboration) 1994 LEGS data release No. L7a8.0
[6] Arndt R A, Briscoe W J, Workman R L and Strakovsky I I *SAID database* http://gwdac.phys.gwu.edu
[7] DeStaebler H C, Erickson E F, Hearn A C and Schaerf C 1965 *Phys. Rev.* **140** B336
[8] Govorkov B B *et al* 1966 *Yad. Fiz.* **4** 507
[9] Bacci C *et al* 1967 *Phys. Rev.* **159** 1124
Bacci C *et al* 1972 *Phys. Lett.* B **39** 559
[10] Fischer G *et al* 1970 *Nucl. Phys.* B **16** 93
[11] Hemmi Y *et al* 1973 *Phys. Lett.* B **43** 79
Hemmi Y *et al* 1973 *Nucl. Phys.* B **55** 333
[12] Genzel H 1974 *Z. Phys.* A **268** 43
[13] Yoshioka M *et al* 1980 *Nucl. Phys.* B **168** 222
[14] Saxon D H *et al* 1980 *Nucl. Phys.* B **162** 522
[15] Mazzucato E *et al* 1986 *Phys. Rev. Lett.* **57** 3144
[16] Fuchs M *et al* 1996 *Phys. Lett.* B **368** 20
[17] Beck R *et al* 1997 *Phys. Rev. Lett.* **78** 606
[18] Ahrens J 2000 *Phys. Rev. Lett.* **84** 5950
[19] Riska D O and Brown G E 2001 *Nucl. Phys.* A **679** 577
[20] Kossert K *et al* 2004 *Eur. Phys. J.* A **19** 391
[21] Bartholomy O *et al* (CB-ELSA Collaboration) 2005 *Phys. Rev. Lett.* **94** 012003
[22] Bartalini O *et al* (GRAAL Collaboration) 2005 *Eur. Phys. J.* A **26** 399
[23] Beck R 2006 *Eur. Phys. J.* A **28S1** 173
[24] Dugger M *et al* 2007 *Phys. Rev.* C **76** 025211
[25] Schumann S *et al* 2010 *Eur. Phys. J.* A **43** 269
[26] Crede V *et al* (CBELSA/TAPS Collaboration) 2011 *Phys. Rev.* C **84** 055203
[27] Hornidge D *et al* (A2 and CB-TAPS Collaborations) 2013 *Phys. Rev. Lett.* **111** 062004
[28] Dieterle M *et al* (A2 Collaboration at MAMI) 2014 *Phys. Rev. Lett.* **112** 142001
[29] Schumann S *et al* (A2 Collaboration at MAMI) 2015 *Phys. Lett.* B **750** 252
[30] Gardner S *et al* (A2 Collaboration at MAMI) 2016 *Eur. Phys. J.* A **52** 333
[31] Price J W 1995 *Phys. Rev.* C **51** 2283
[32] Krusche B *et al* 1995 *Phys. Rev. Lett.* **74** 3736
[33] Dytman S A *et al* 1995 *Phys. Rev.* C **51** 2710
[34] Renard F *et al* 2002 *Phys Lett.* B **528** 215
[35] Crede V *et al* (CB-ELSA Collaboration) 2005 *Phys. Rev. Lett.* **94** 012004
[36] Nakabayashi T *et al* 2006 *Phys. Rev.* C **74** 035202
[37] Kuznetsov V *et al* 2007 *Phys. Lett.* B **647** 23
[38] Bartalini O *et al* (GRAAL Collaboration) 2007 *Eur. Phys. J.* A **33** 169
[39] Krusche B, Jaegle I and Mertens T 2007 *Eur. Phys. J.* A **31** 485
[40] Sumihama M *et al* (LEPS Collaboration) 2009 *Phys. Rev.* C **80** 052201
[41] McNicoll E F *et al* (Crystal Ball at MAMI Collaboration) 2010 *Phys. Rev.* C **82** 035208
McNicoll E F *et al* 2011 *Phys. Rev.* C **84** 029901 (erratum)

[42] Nozawa S, Blankleider B and Lee T-S H 1990 *Nucl. Phys.* A **513** 459
[43] Sato T and Lee T-S H 1996 *Phys. Rev.* C **54** 2660
[44] Sato T and Lee T-S H 2001 *Phys. Rev.* C **63** 055201
[45] Matsuyama A, Sato T and Lee T-S H 2007 *Phys. Rep.* **439** 193
[46] Kamano H, Juliá-Díaz B, Lee T-S H, Matsuyama A and Sato T 2009 *Phys. Rev.* C **80** 065203
[47] Kamano H, Nakamura S X, Lee T-S H and Sato T 2013 *Phys. Rev.* C **88** 035209
[48] Nakamura S X, Kamano and Sato T 2015 *Phys. Rev.* D **92** 074024
[49] Kamano H, Nakamura S X, Lee T-S H and Sato T 2016 *Phys. Rev.* C **94** 015201
[50] Fernández-Ramírez C, Moya de Guerra E and Udías J M 2006 *Ann. Phys.* **321** 1408
[51] Mariano A 2007 *J. Phys. G: Nucl. Part. Phys.* **34** 1627
[52] Xiao L-Y, Cao X and Zhong X-H 2015 *Phys. Rev.* C **92** 035202
[53] Quirós J J, Tamayo D F, Jaramillo D E and Barbero C 2015 *Nucl. Part. Phys. Proc.* **267–269** 230
[54] Drechsel D *et al* 1999 *Nucl. Phys.* A **645** 145
[55] Drechsel D *et al* 2007 *Eur. Phys. J.* A **34** 69
[56] Darwish E M, Hemmdan A and El-Shamy N T 2016 *Makara J. Sci.* **20** 55–60
[57] Anisovich A V *et al* 2005 *Eur. Phys. J.* A **24** 111
[58] Anisovich A V *et al* 2010 *Eur. Phys. J.* A **44** 203
[59] Haberzettl H *et al* 1998 *Phys. Rev.* C **58** R40
[60] Gasparyan A and Lutz M F M 2010 *Nucl. Phys.* A **848** 126
[61] Mai M *et al* 2012 *Phys. Rev.* D **86** 094033
[62] Zhao Q *et al* 2002 *Phys. Rev.* C **65** 065204
[63] Choi K S, Nam S I, Hosaka A and Kim H C 2009 *J. Phys. G: Nucl. Part. Phys.* **36** 015008
[64] Rönchen D *et al* 2014 *Eur. Phys. J.* A **50** 101
Rönchen D *et al* 2015 *Eur. Phys. J.* A **51** 63 (erratum)
[65] Rönchen D *et al* 2015 *Eur. Phys. J.* A **51** 70
[66] Rarita W and Schwinger J 1941 *Phys. Rev.* **60** 61
[67] Peccei R D 1969 *Phys. Rev.* **181** 1902
[68] Nath L M, Etemadi B and Kimel J D 1971 *Phys. Rev.* D **3** 2153
[69] Benmerrouche M *et al* 1989 *Phys. Rev.* C **39** 2339
[70] Pascalutsa V 1999 *Phys. Rev.* D **58** 096002
[71] Pascalutsa V and Timmermans R 1999 *Phys. Rev.* D **60** 042201
[72] Mariano A, Barbero C and Badagnani D 2012 *J. Phys. G: Nucl. Part. Phys.* **39** 035005
[73] Badagnani D, Barbero C and Mariano A 2015 *J. Phys. G: Nucl. Part. Phys.* **42** 125001
[74] Badagnani D, Mariano A and Barbero C 2017 *J. Phys. G: Nucl. Part. Phys.* **44** 025001
[75] Johonson K and Surdashan E C G 1961 *Ann. Phys.* **13** 126
[76] Velo G and Zwanziger D 1969 *Phys. Rev.* **186** 267
[77] Hagen C R 1971 *Phys. Rev.* D **4** 2204
[78] Singh L P 1973 *Phys. Rev.* D **7** 1256
[79] Hasumi A, Endo R and Kimura T 1979 *J. Phys. A: Math. Gen.* **12** L217
[80] Cox W 1989 *J. Phys. A: Math. Gen.* **22** 1599
[81] Kirchbach M and Ahluwalia D V 2002 *Phys. Lett.* B **529** 124
[82] Pascalutsa V 2001 *Phys. Lett.* B **503** 85
[83] Krebs H *et al* 2009 *Phys. Rev.* C **80** 028201
[84] El-Amiri M, López Castro G and Pestieau J 1992 *Nucl. Phys.* A **543** 673
[85] Barbero C, López Castro G and Mariano A 2008 *Phys. Lett.* B **664** 70
[86] Mariano A, Barbero C and López Castro G 2011 *Nucl. Phys.* A **849** 218
[87] Fogli G L and Nardulli G 1979 *Nucl. Phys.* B **160** 116
[88] Kamalov S S and Yang S N 1999 *Phys. Rev. Lett.* **83** 4494
[89] Mariano A 2007 *Phys. Lett.* B **647** 253
[90] Barbero C, Mariano A and López Castro G 2012 *J. Phys. G: Nucl. Part. Phys.* **39** 085011
[91] Barbero C and Mariano A 2015 *J. Phys. G: Nucl. Part. Phys.* **42** 105104
[92] Gegelia J *et al* 2016 *Phys. Lett.* B **763** 1
[93] Garcilazo H and Moya de Guerra E 1993 *Nucl. Phys.* A **562** 521
[94] Shyam R 1999 *Phys. Rev.* C **60** 055213

[95] Liu B-C 2012 *J. Phys. G: Nucl. Part. Phys.* **39** 105107
[96] López Castro G and Mariano A 2001 *Nucl. Phys.* A **697** 440
[97] Feuster T and Mosel U 1997 *Nucl. Phys.* A **612** 375
[98] Scholten O *et al* 1996 *Phys. Lett.* B **384** 13
[99] Jones H F and Scadron M D 1973 *Ann. Phys.* **81** 1
[100] Barbero C, López Castro G and Mariano A 2014 *Phys. Lett.* B **728** 282
[101] Ernst F, Sachs R and Wall F 1960 *Phys. Rev.* **119** 1105
[102] Olive K A *et al* 2014 Review of particle physics *Chin. Phys.* C **38** 090001