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Last giant impact on Uranus
(Research Note)

Constraints on oligarchic masses in the trans-Saturnian region

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ABSTRACT

Context. Modern models of the formation of ice giants attempt to account for the formation of Uranus and Neptune within the protoplanetary disk lifetime. These models assume a higher initial surface density well above that of the minimum mass solar nebula model and/or the formation of all giant planets in an inner compact configuration. Other effects include planetesimals migration due to gas drag and the small size of the accreted planetesimals, which accelerates the accretion rate. However, at present, none of these models account for the spin properties of the ice giants.
Aims. Stochastic impacts by large bodies are, at present, the usually accepted mechanisms able to account for the obliquity of the ice giants. We attempt to set constraints on giant impacts as the cause of Uranus's current obliquity of 98° and on the impactor masses.
Methods. Since stochastic collisions among embryos are assumed to occur beyond oligarchy, we model the angular momentum transfer to proto-Uranus by the last stochastic collision (GC) between the protoplanet and an oligarchic mass at the end of Uranus's formation. We take a minimum impactor mass $m_{\rm i}$ of 1 $m_\oplus$.
Results. We find that an oligarchic mass $m_{\rm i} \sim 1\ m_\oplus \leq m_{\rm i} \leq 4.5\ m_\oplus$ would be required at the GC to reproduce the present rotational properties of Uranus. An impact with $m_{\rm i} > 4.5\ m_\oplus$ is not possible, unless the impact parameter of the collision is very small and/or the angle between the spin axis of Uranus prior and after the GC is higher than 130°. This result is valid if Uranus formed in situ or between 10−20 AU and does not depend on the occurrence of the GC after or during the possible migration of the planet. This result is very similar to one obtained for Neptune from its rotational properties.
Conclusions. If the stage of stochastic impacts among oligarchs has occurred and if the present rotational status of Uranus is the result of such processes, the 4.5 $m_\oplus$ mass limit must be understood as an upper constraint on the oligarchic masses in the trans-Saturnian region at the end of ice giants' formation. This result may be used to set constraints on planetary formation scenarios.

Key words. planets and satellites: general – planets and satellites: formation

1. Introduction

It has long been known that dynamical times in the trans-Saturnian minimum mass solar nebula model (MMSN) are so long that core growth takes more than 15 Myr. Observations of young, solar-type stars suggest that circumstellar disks dissipate on a timescale of a few Myr (e.g. Briceño et al. 2001). Different models and scenarios have been proposed to account for the formation of the ice giants within the protoplanetary disk lifetime. Modern models shorten the timescale for giant planet formation if taking a higher initial surface density into account well above that of the MMSN and/or the formation of all giant planets in an inner compact configuration (e.g. Dodson-Robinson & Bodenheimer 2010; Benvenuto et al. 2009; Thommes et al. 2003; Tsiganis et al. 2005). Other effects include planetesimals migration due to gas drag and the small size of the accreted planetesimals, which accelerates the accretion rate (e.g. Benvenuto et al. 2009; Goldreich et al. 2004). However, modern scenarios of the formation of Uranus and Neptune have several difficulties to overcome, and inconsistencies among the different models and scenarios are still present.

Oligarchic masses in the trans-Saturnian region in the frame of the MMSN are much lower than the present solid cores of Uranus and Neptune if the ice planets had formed in situ or between 10−20 AU. Collisions among oligarchs take a very long time to form the ice giants' cores. To avoid stochastic giant collisions, an initial surface density ∼5−10 times that of the MMSN would be required to produce oligarchs with masses similar to the cores of Uranus and Neptune (e.g. Dodson-Robinson & Bodenheimer 2010). But within ten MMSN, Jupiter falls like a stone into the Sun due to type III migration (Crida 2009). On the other hand, a solid surface density five to ten times the MMSN would lead to the formation of about five ice giants instead of two, which occurred with the three other giants; i.e., whether they were ejected or if they were simply spread out and all retained is a matter of debate (Goldreich et al. 2004; Dodson-Robinson & Bodenheimer 2010; Ford & Chiang 2007; Levison & Morbidelli 2007). In the last case, then, where are they? Thommes et al. (2003) show that even in a disk ten times

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the MMSN, oligarchs do not have time to reach their isolation mass in the outer solar system, and even an Earth mass at the orbit of Uranus by 10 Myr seems to be implausible.

Although Benvenuto et al. (2009) and Goldreich et al. (2004) find that the small size of the accreted planetesimals reduces the timescale for giant planet formation, Morbidelli et al. (2009) find that asteroids were born big suggesting that the minimal size of the planetesimals was ~100 km. Actually, it is not known whether the ice planets formed in situ or well inside the 20 UA, and/or if the initial mass of the nebula was that of the MMSN or much higher. Moreover, the mass of the planetesimals from which Uranus and Neptune accreted remains a matter of debate. It is necessary to look for independent ways of setting constraints on models of ice giant planet formation.

The origin of the rotational properties of the planets in the solar system is one of the fundamental questions in cosmogony. Several models have been proposed to account for a net L_z component of the planetary spin (Lissauer & Kary 1991; Lissauer et al. 1997; Schlichting & Sari 2007) but the problem of the obliquity (the angle between the rotational axis of a planet and the perpendicular to its orbital plane) of the ice planets remains open. A random component of planetary rotation may come from stochastic impacts with large bodies and may be in any direction (Lissauer & Safronov 1991; Chambers 2001). In particular, the large obliquity of Uranus (98°) has been attributed to a great tangential collision with another protoplanet at the end of the epoch of accretion (e.g., Safronov 1969; Korycansky et al. 1990; Slattery et al. 1992; Parisi et al. 2008, and references therein). Greenberg (1974) & Kubo-Oka & Nakazawa (1995) have investigated the tidal evolution of satellite orbits and examined the possibility that the orbital decay of a retrograde satellite leads to the high obliquity of Uranus, but the high mass required for the hypothetical satellite makes this very implausible. An asymmetric in-fall or torques from nearby mass concentrations during the collapse of the molecular cloud core leading to the formation of the solar system could twist the total angular momentum vector of the planetary system. This twist could generate the obliquities of the outer planets (Tremaine 1991). This model has disadvantages in that the outer planets must form before the infall is complete and that the conditions for the event that would produce the twist are rather strict. The model itself is difficult to be quantitatively tested.

In the Nice model of Tsiganis et al. (2005), close encounters among the giant planets produce large orbital eccentricities and inclinations that were subsequently damped to the current value by gravitational interactions with planetesimals. The obliquity changes because of the change in the orbital inclinations. Since the inclinations are damped by planetesimals interactions on timescales that are much shorter than the timescales for precession due to the torques from the Sun, especially for Uranus and Neptune, the obliquity returns to low values, if it is low before the encounters (Lee et al. 2007). Boué & Laskar (2010) report numerical simulations showing that Uranus's axis might be tilted during the giant planet instability phase described in the Nice model, provided that the planet has an additional satellite and a temporary steep inclination and the satellite is ejected after the tilt. However, the required satellite is too massive. For Saturn, a good case can be made for spin-orbit resonances (Hamilton & Ward 2004), but giant impacts in the late stages of the formation of Uranus and Neptune remain the plausible explanation for the obliquities of the ice giants (Lee et al. 2007; Parisi et al. 2008; Parisi & del Valle 2011).

We follow the same procedure for Uranus as was developed for Neptune in Sect. 2 of Parisi and del Valle (2011). The angular momentum transferred to Uranus by the last stochastic collision (hereafter, GC) between the protoplanet and an oligarchic mass is computed in Sect. 2. We obtain in this way an upper constraint on the oligarchic masses to impact Uranus. The conclusions of the results are presented in Sect. 3.

2. The spin of Uranus: angular momentum transfer to Uranus by the last giant collision

Beyond oligarchy, the final stage of planet formation consists of close encounters, collisions, and accretion events among oligarchs. Strong impacts deliver spin angular momentum to the final planet in a random-walk fashion (Lissauer & Safronov 1991). The planetary spin accumulated by successive collisions with a distribution of small or/and large planetesimals requires ad-hoc assumptions about unknown properties of the planetesimal disk, such as the mass distribution of the bodies, the velocities distribution, and the regime of growth. We avoid the necessity of quantifying these unknown parameters by modelling what happened to the planet just before it acquires its present rotational status, which is our available data.

The last off-centre giant collision (GC) between proto-Uranus and the last colliding oligarch is computed assuming that the present spin properties of Uranus are acquired by the GC. From angular momentum conservation, we get the following relation between the impactor mass $m_{\rm i}$ and its incident speed $\upsilon_{\rm i}$ (Parisi et al. 2008), assuming that the impact is inelastic (Korycansky et al. 1990):

$$\upsilon_{\rm i} = \frac{2m_{\rm U}{R_{\rm U}}^2}{5m_{\rm i}b}\left(1+\frac{m_{\rm i}}{m_{\rm U}}\right) \times \sqrt{\Omega^2 + \frac{\Omega_0^2}{\left(1+\frac{m_{\rm i}}{m_{\rm U}}\right)^2\left(1+\frac{m_{\rm i}}{3m_{\rm U}}\right)^4} - \frac{2\Omega_0\Omega\cos\alpha}{\left(1+\frac{m_{\rm i}}{m_{\rm U}}\right)\left(1+\frac{m_{\rm i}}{3m_{\rm U}}\right)^2}}\,, \qquad (1)$$

where b is the impact parameter of the collision, $\mathbf{\Omega}$ the present spin angular velocity of Uranus, $\mathbf{\Omega}_0$ the spin angular velocity that Uranus would have today if the GC had not occurred, and α the angle between $\mathbf{\Omega}$ and $\mathbf{\Omega}_0$. We get Uranus data from the JPL homepage. The current radius of Uranus $R_{\rm U}$ is taken as 25 362 km and the mass of Uranus after the GC, $(m_{\rm i} + m_{\rm U})$, is taken as its current mass of 8.68103×10^{25} kg. The spin period of Uranus is $T = 17.28$ h, thus $\Omega = 1.01002\times10^{-4}$ s^{-1} ($\Omega = 2\pi/T$). Uranus's current obliquity is 98°. In the single stochastic impact approach, $\alpha = 98°$. At the end of the formation of Uranus, its gas envelope extends until the accretion radius (e.g. Bodenheimer & Pollack 1986; Pollack et al. 1996), whereas its core contains a mass $m_{\rm C}$ of 7.4737×10^{25} kg in a small radius $R_{\rm C}$ of 1.8×10^4 km. In this situation, a collision onto the core is necessary for an inelastic collision to occur and to impart the required angular momentum (Korycansky et al. 1990). Since b is an unknown quantity, we take its most probable value: $b = (2/3)R_{\rm C}$ (Parisi et al. 2008, and references therein).

We calculate the lower bound of $\upsilon_{\rm i}$ ($\upsilon_{\rm im}$) corresponding to a body within the Hill radius $R_{\rm H}$ of Uranus and undergoing a free fall towards Uranus's core:

$$\upsilon_{\rm im} = \sqrt{\frac{2Gm_{\rm C}}{R_{\rm C}} - \frac{2Gm_{\rm C}}{R_{\rm H}}}. \qquad (2)$$

The second term of Eq. (2) is negligible, and then $\upsilon_{\rm im}$ is ~22 km s^{-1}.

We compute $\upsilon_{\rm i}$ as a function of T_0 ($T_0 = 2\pi/\Omega_0$) through Eq. (1) for different values of $m_{\rm i}$. We took six values of α: 0°, 30°, 70°, 98°, 130°, and 170°. For α between 180° and 360°,

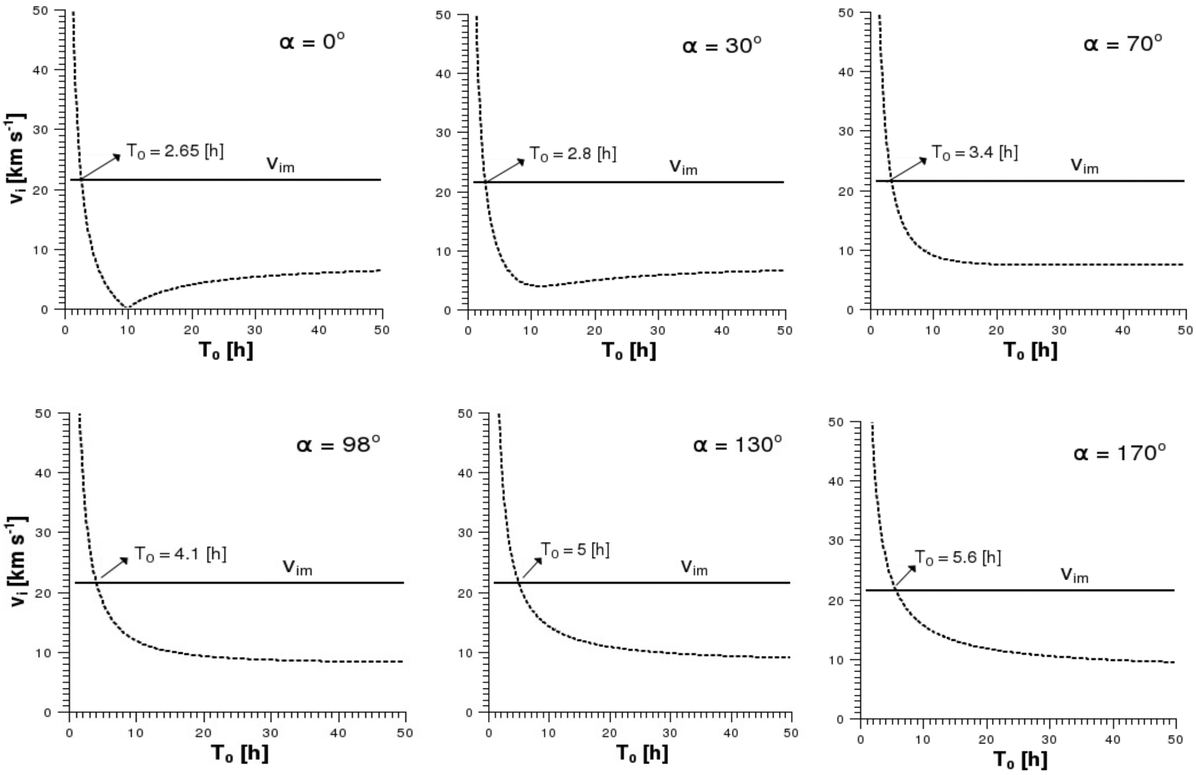

Fig. 1. The impactor incident speed v_i as a function of the initial period T_0 is depicted by dotted lines. $m_i = 4\ m_\oplus$ and the impact parameter b is 2/3 R_C. $\alpha = 0°, 30°, 70°, 98°, 130°$, and $170°$. The lower constraint on v_i (v_{im}) is shown by a solid line.

Table 1. Maximum allowed value of T_0 for $m_i = 4\ m_\oplus$.

T_{0M} [h]	2.65	2.8	3.4	4.1	5.0	5.6
α [°]	0	30	70	98	130	170

the results would be the same as for the interval [0°,180°] since Eq. (1) is an even function of α. In Fig. 1, we show the results for $m_i = 4\ m_\oplus$. For each α, the permitted values for the impactor speed v_i are those between v_{im} and ~40 km s^{-1} (Parisi & del Valle 2011). The intersection of v_i with v_{im} gives the maximum allowed value of T_0, T_{0M}, tabulated in Table 1. The curves shift up and to the right as α increases, and then T_{0M} increases with α.

We calculate the break-up speed Ω_{0b} given by (Parisi & del Valle 2011):

$$\frac{Gm_U}{R_{U0}^2} = \Omega_{0b}^2 R_{U0}, \tag{3}$$

where $R_{U0} = R_U/(1 + m_i/3m_U)$. From Eq. (3), we get the period T_{ob} $(2\pi/\Omega_{0b})$ tabulated in Table 2. If $T_0 < T_{ob}$, the planet breaks up since centrifugal forces exceed gravitational forces. Then, the condition $T_{0M} > T_{ob}$ must be satisfied to reproduce the present rotational properties of Uranus.

In Fig. 1, for $\alpha = 98°$, T_{0M} is 4.1 h ~ T_{ob}. If m_i is higher, $T_{0M} < T_{ob}$. From Eq. (1), the curves of Fig. 1 would shift to the left side as m_i increases (Parisi & del Valle 2011). Then, for an impactor mass $m_i > 4\ m_\oplus$, T_{0M} diminishes, while T_{ob} increases (see Table 2). This implies that an impactor mass $m_i > 4\ m_\oplus$ for $\alpha \leq 98°$ would refute the GC hypothesis since T_{0M} is less than T_{ob} unless $b \ll 2/3\ R_C$. In Fig. 1, for $\alpha = 130°$, T_{0M} is 5 h. If m_i is 4.5 $m_\oplus$ and α is 130°, T_{0M} is 4 h, and T_{ob} is 4.34 h. This implies that an impactor mass $m_i > 4.5\ m_\oplus$ for $\alpha \leq 130°$ would refute the GC hypothesis since T_{0M} is less than T_{ob} unless $b \ll 2/3\ R_C$. For $\alpha = 170°$, T_{0M} is 5.6 h (see Table 1) and then T_{0b} is less than T_{0M} for an impactor mass less or equal to around 6 $m_\oplus$ (see Table 2). It means that if α is 170°, the mass of the impactor must be less than 6 $m_\oplus$ to reproduce the spin properties of Uranus. However, the case of $\alpha = 170°$ is very implausible. The cases with $\alpha > 130°$ are very unlikely and may then be discarded.

It should be noted that Fig. 3 of Parisi & del Valle (2011) is very similar to Fig. 1 of this work. However, the maximum allowed initial period T_{0M} for Neptune is 0.5 h higher than for Uranus for each α (see Fig. 3 of Parisi & del Valle 2011; and Fig. 1 of this work). The method consists of comparing the permitted maximum initial period of these figures (T_{0M}) with the break-up period (T_{0b}) tabulated in Table 1 of Parisi & del Valle (2011) and in Table 2 of this work, and T_{0b} must be less than T_{0M} to reproduce the present rotational properties of the planet, and then, to get the constraints. For each oligarchic mass and α, proto-Neptune T_{0b} is about 0.5 h lower than for Uranus since it is a bit more massive and then supports the rotation better. It gives a difference of about one hour between Neptune and Uranus cases in comparing T_{0M} and T_{0b}. Looking at Fig. 3 of Parisi & del Valle (2011), the range of permitted T_0 is about one to two hours, and then a difference of an hour between Uranus and Neptune cases is close to the range of permitted periods. This difference leads to different outcomes for both planets.

Since the actual obliquity of Neptune is 29.58° and that of Uranus 98°, we assume a maximum probable value for α of 60° for Neptune and 130° for Uranus. For the Neptunian case with

Table 2. Break-up period of proto-Uranus for different impactor masses.

$m_{\rm i}$ [$m_\oplus$]	1	2	3	4	5	6	7	8
$T_{\rm ob}$ [h]	3.15	3.41	3.72	4.11	4.60	5.24	6.10	7.23

α lower or equal to 60°, we find that $T_{\rm 0b}$ is lower than $T_{\rm 0M}$ if the impactor mass is lower than or equal to 4 $m_\oplus$. For the Uranian case with α lower or equal to 130°, $T_{\rm 0b}$ is lower than $T_{\rm 0M}$ if the impactor mass is lower than or equal to 4.5 $m_\oplus$. These results imply that the maximun allowed value for the oligarchic mass is then 4 $m_\oplus$ from the rotational properties of Neptune and 4.5 $m_\oplus$ from those of Uranus.

3. Conclusions

We conclude that an impact between proto-Uranus and an oligarch with a mass higher than 4.5 $m_\oplus$ could not have occurred since it cannot reproduce the present rotational and physical properties of Uranus, unless the impact parameter of the collision is very small and/or the angle between the spin axis of Uranus prior and after the GC is higher than 130°. The formation of Uranus as the result, for instance, of collisional accretion between two similar oligarchs with masses ~7 $m_\oplus$ seems to be unlikely. This result is very similar to that obtained for Neptune from its rotational properties (Parisi & del Valle 2011).

The model here presented is independent of unknown parameters, such as the mass and distribution of the planetesimals, the location at which Uranus was formed, the occurrence of the stochastic impact during or after the possible migration of the planet, the solar nebula initial surface mass density, and the regime of growth.

If the stage of stochastic impacts among oligarchs has occurred and if the present rotational status of Uranus is the result of such processes, the 4.5 $m_\oplus$ mass limit must be understood as an upper constraint on the oligarchic masses in the trans-Saturnian region at the end of ice giant formation. These results may be used to set constraints on planetary formation scenarios.

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