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A topological optimization algorithm applied to the design of composites patch repair of mixed-mode cracked plate

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Abstract

In this work, a topology optimization algorithm is developed and implemented to get an optimum composite patch shape. Typically, the design process consists of an iterative analysis, where the best solution is obtained from a comparative study. In this way, we propose a topology optimization algorithm applied to obtain the optimum composite patch shape. The algorithm is implemented in MatLab, and uses the commercial finite element code Abaqus/Standard. A numerical example is analysed to show the capability of the proposed method. The obtained results are compared with numerical results reported by other researchers, revealing the potential of the developed algorithm.

Keywords

Topology optimization, cracked plate, finite element method, composite patch

Introduction

The use of externally bonded composite patches for repairing cracks and defects in aircraft structures achieves great success these last years, what fostered researchers to study further this domain. Baker[1] was the pioneer in the use of bonded composite patches to repair damaged aircraft and marine structures. The use of the adhesively bonded composite patches as a repair method has many advantages compared to the conventional mechanically fastened repair methods.[2] The adhesively bonded patch repair reduces stresses in the cracked region by doing load transfer through patch into the panel and thereby prevents the crack tip from opening and, thereby, it from growing.

In the last two decades, greater developments have taken place in computational mechanics, especially in the area of finite element analysis. Several authors have studied the influence of composite patch on the performance of externally bonded patch repair in the case of a metallic panel having a crack and they have recommended the best one based on stress intensity factor (SIF) reduction.[3–7]

Some researchers[8–15] have focused on the analysis of the effect of composite patch repair on composite panels with circular cut-out. In these types of problems, the authors use the reduction of the stress concentration factor (SCF) as a design parameter. In all these works, the design process consists of an iterative analysis, where the best solution is obtained from a comparative study.

Recently, several papers have described the effects of the adhesive disband on the repair efficiency.[16–20] They concluded that both mode I and mode II SIF are increased by the presence of the adhesive disband.

Mathias et al.[21] developed an application of a biology-based method, known as the genetic algorithm, to the optimization of composite patch bonded on a metal structure in order to reduce the stress level in a given area under some constraints such as a maximum surface of the patches. This method deduced the best patch topology by determining the optimal material density distribution. Such algorithms have been used in many

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cases of composite structure optimizations.[22–26] Brighenti[27] showed that, by adopting the optimal patch shape, the SIF can be reduced to about 40% and 60% with respect to that related to a square or rectangular shape patch. However, the obtained patch shape was very irregular and not able to be applied to real cases.

Errouane et al.[28] proposed a numerical model for the optimization patch repair of aluminium plate, in which design variables are considered, such as patch height, width and thickness, in addition to adhesive thickness, large reduction of patch material volume. Nevertheless, their model is only valid for patches with rectangular shape. Therefore, the optimization is limited to defining their relationship aspect.

In the present work, a topological algorithm applied to the design of the composite patch shape is developed. Topology optimization methods enable designers to find the best structural layout for a required structural performance. The importance of this type of optimization lies on the fact that the choice of the appropriate topology of a structure in the conceptual phase is generally the most decisive factor for a novel product efficiency.[29]

Over the last three decades, many mathematical and heuristic optimization methods have been developed.[30–33] The works related to the optimization method have dealt with the optimization of structures with an isotropic material, even in some commercial programs of FEA,[34,35] topology optimization modules are included. However, so far there does not exist a topological algorithm applied to orthotropic materials, therefore, we had to propose and implement a new topology optimization criterion. The proposed algorithm is programmed in MatLab, and the FEA is solved with the commercial software Abaqus/Standard. The performance of the algorithm is assessed through with a problem, showing that the obtained patch designs are adequate to be applied to real practical cases. Furthermore, the results are compared with studies presented by other authors.

Description of the proposed algorithm

The topological optimization algorithm proposed in this work is able to solve the problem of the shape design of composite patches applied to repair cracked structures. Figure 1 presents a flowchart of the topological optimization algorithm developed in this work.

The algorithm is implemented in the MatLab language and is based on a FEA in order to obtain the structure stress field. Thus, the finite element model implemented in the software Abaqus is applied to the study of the SIF evolution at the crack front, and the stress field of the composite patches.

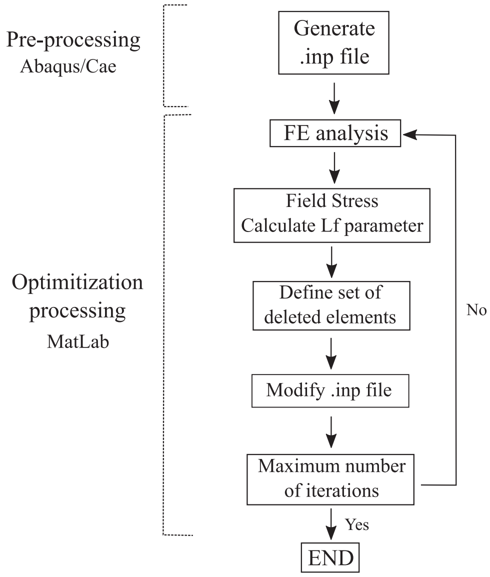

Figure 1. Flowchart of the topological optimization algorithm implemented in a MatLab.

The first step is to generate the typical Abaqus input file, *.inp*, where the different parts of the problem are defined, assign their material properties, mesh topology and boundary conditions. This first input file is generated from the Abaqus/CAE interface.

Secondly, the MatLab program resorts to the Abaqus solver to obtain the stress field of the composite patches.

Typically, the topological optimization algorithms are applied to isotropic materials, and the criterion to delete the elements is based on Von Mises stress.[34] However, this criterion is not available to orthotropic. Therefore, in order to define the load state of each element of the composite patches, we calculate the parameter L_f of the fracture criterion of Tsai-Hill.[36] This parameter is given from this equation (1)

$$L_F = \left(\frac{\sigma_{11}}{X}\right)^2 - \frac{\sigma_{11}\sigma_{22}}{Y^2} + \left(\frac{\sigma_{22}}{Y}\right)^2 + \left(\frac{\sigma_{12}}{S}\right)^2 \quad (1)$$

where X, Y are the lamina longitudinal and transverse strengths, and S is the shear strength of the lamina. From the results of the FEA are extracted σ_{11}, σ_{22} are the lamina longitudinal and transverse stress, and σ_{12} is the shear stress.

Note that, the election of the Tsai Hill failure criterion is strictly arbitrary; we could use another criterion like Tsai Wu or Hoffman. In the optimization algorithm, the failure criterion is not used to describe a physics process; it is only an equivalent parameter that describes the level of the element solicitation.

In the next step, the set of elements of the composite patches is defined, which has the lowest values of the parameter L_f (the least requested elements). This set of elements is incorporated within the Abaqus input file and is assigned new material properties with null stiffness. Therefore, these elements are fictionally deleted from the finite element model for the next iteration. Note that, in this step the new patch shape obtained with the optimization process is defined.

In order to simplify the model, the number of elements deleted in each iteration is fixed as a 0.75% of the initial patch area. This percent is obtained from a calibration process previously realized. The calibration process is based on the following criterion: if the percent of elements elimination is high, the geometry of the patch presents holes inside; but if this percentage is low, the number of iterations and the computational cost increase.

Finally, the algorithm verifies that the number of iterations is lower than the maximum number of iterations. If this condition is satisfied, the FEA has repeated with the new input file, and the analysis is finished.

Geometry and boundary conditions

The example deals with cracked aluminium plates with mixed mode condition repaired with double-sided composite patches. The obtained results are compared with studies presented by other authors.

The problem consists of a cracked aluminium alloy 2014 T6 panel, submitted to a load tensile of 121.11 MPa. The panel is a rectangular plate of $160 \times 39 \times 3.175\,\mathrm{mm}^3$, with a central crack $2a$ of 10 mm of length. The crack is inclined at an angle β of 45° with the horizontal axis. Figure 2 shows the geometrical and boundary conditions of this case.

The material properties of the aluminium panel are the Young's modulus of $E = 73.1\,\mathrm{GPa}$ and the Poisson's ratio $\nu = 0.30$.[6,37] The patch is made of four layers of unidirectional carbon/epoxy composite laminate, which are parallel to the load direction. The material properties for CFRP patches used in the analysis are: $E_1 = 135$ GPa, $E_2 = E_3 = 9$ GPa, $nu_{23} = 0.02$, $nu_{13} = nu_{12} = 0.30$, $G_{23} = 8$ GPa, $G_{12} = G_{13} = 5\,\mathrm{GPa}$, $X = 1500\,\mathrm{MPa}$, $Y = 50\,\mathrm{MPa}$ and $S = 70\,\mathrm{MPa}$. In all analysed cases, the initial dimensions of the composite patch are $32.5 \times 32.5\,\mathrm{mm}^2$.

The patch is bonded symmetrically to the panel using AV138/HV998 adhesive material. Its mechanical properties are the Young's modulus of $E = 4.59\,\mathrm{GPa}$ and the Poisson's ratio $\nu = 0.47$.[6,37]

Finite element analysis

The finite element model consisted of three parts including the cracked panel, the adhesive, and the composite patch (see Figure 3). Due to symmetry in the plane of the plate thickness, only half part of the repaired plate is modelled, in order to reduce the computational cost.

It is assumed that the patch is perfectly bonded to the panel by adhesive[6,37,38]) and modelled by means of tie constraints, which are available in ABAQUS/Standard[34] library. Appropriately, the nodes are coupled at the respective interfaces to reflect the

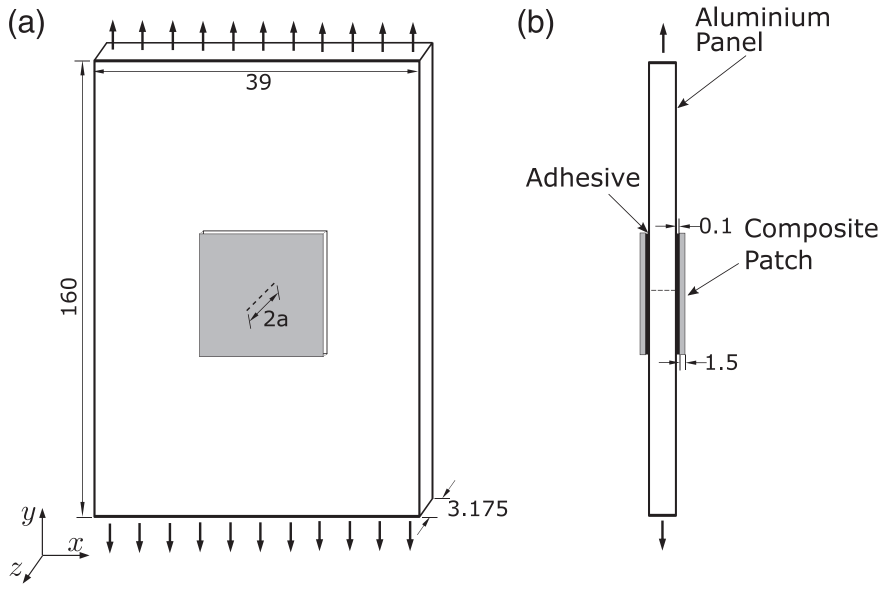

Figure 2. Geometry and boundary condition of the repaired aluminium panel with double-sided composite patch (all dimensions are in mm). (a) Front view. (b) Side view.

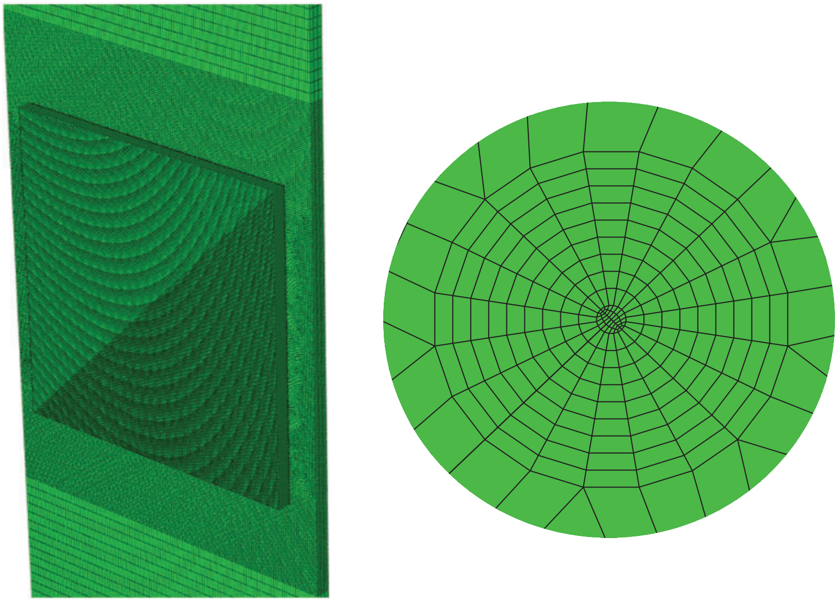

Figure 3. Finite element model of repaired cracked aluminium panel with double-sided patches and near the crack tip.

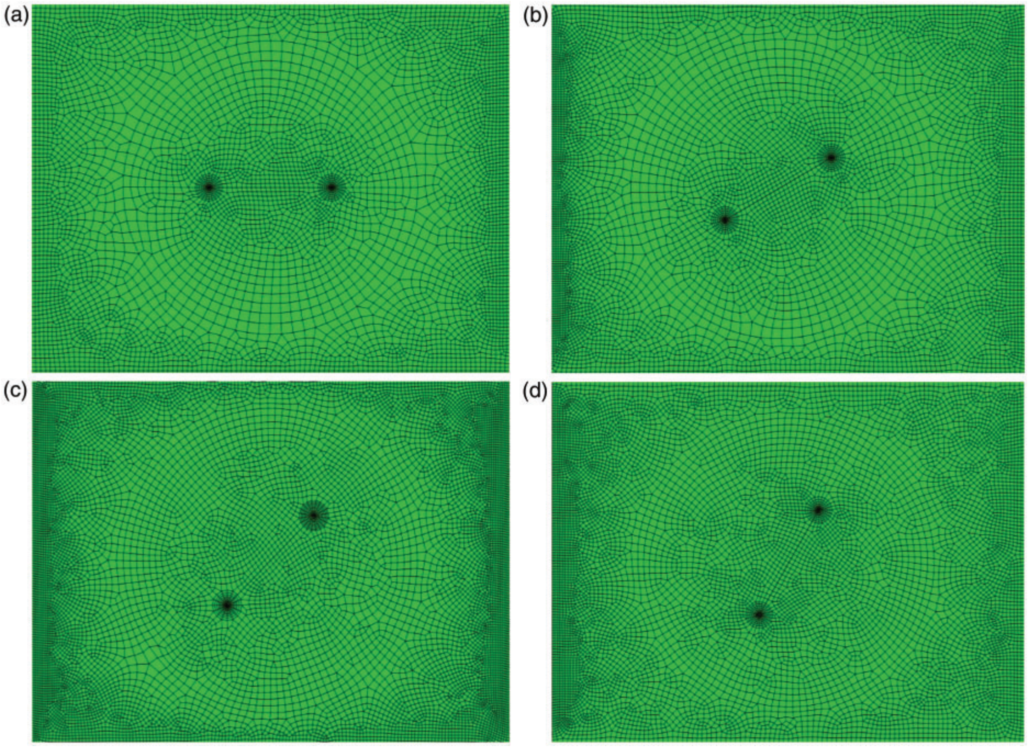

Figure 4. Mesh of the aluminium plate for different crack inclination angles. (a) $\beta = 0^\circ$. (b) $\beta = 30^\circ$. (c) $\beta = 45^\circ$. (d) $\beta = 60^\circ$.

perfectly bonded behaviour. During coupling, all three degrees of freedom are coupled at each node.

All parts are meshed with eight-node brick elements (C3D8R).[34] The mesh is refined near the crack tip area with elements dimension of 0.0025 mm. The aluminium panel, the adhesive and the composite patch, are discretized with 123054, 26569 and 26569 elements, respectively. Figure 4 shows the meshes of the aluminium plate used in each analysis.

Mesh validation

The finite element meshes are validated with the numerical results presented by Ramji et al.[6] There are considered a cracked aluminium plate repaired with a square composite patch, with dimensions: 22 × 22, 24 × 24, 26 × 26 and 28 × 28 mm^2. The crack inclination angle considered is 45°.

The values of the SIF obtained are presented in Table 1. We can see that the difference between our model and the results presented in Ramji et al.[6] are lower than 1.5%.

Results and discussions

In order to quantitatively compare the effective patch shape for the cracked plate, a parameter R is used. This parameter was introduced by Ramji et al.,[6] and it is defined as

$$R = \sqrt{\left(\frac{K_I^U - K_I^R}{K_I^U}\right)^2 + \left(\frac{K_{II}^U - K_{II}^R}{K_{II}^U}\right)^2} \tag{2}$$

where K_I^U and K_{II}^U are the unrepaired mode I and mode II SIF values, while K_I^R and K_{II}^R represents the values of the SIF for the repaired panel. This parameter combines both mode I and mode II SIF reduction into one value so that comparison becomes easier and straight forward. Higher the R value, better patch performance in relation to SIF reduction.

Table 1. The values of the SIF for cracked aluminium plate repaired with square double-sided composite patches (values of SIF are presented in MPa·mm$^{1/2}$).

Patch [mm^2]	Present study		Ramji et al.[6]		Difference (%)	
	K_I	K_{II}	K_I	K_{II}	ΔK_I	ΔK_{II}
22 × 22	82.50	75.25	82.80	76.30	0.36	1.39
24 × 24	77.50	70.50	78.80	71.25	0.90	0.85
26 × 26	76.25	67.25	76.00	67.46	0.94	0.31
28 × 28	75.00	65.50	74.53	64.86	0.62	0.97

SIF: stress intensity factor.

Influence of the composite patch size

In this section, the problem described in Figure 2 is analysed, with a crack inclination angle of 45°.

Figure 5 presents the values with the parameter R in function of the area of the composite patches. The solutions are compared with the results presented by other authors[6] for different patch shapes. As we can see the results obtained with the proposed algorithm present the highest values of R for all patch sizes. Therefore, the patch shapes designed with our model are the best solution for this example. It is evident that R values increase when increasing patch area.

Furthermore, it can be found that the obtained results present a completely linear dependence with the patch area, while the results presented by Ramji et al.[6] show different behaviours. This difference between the analyses is due to the fact that the study[6] performed in the patch geometry is fixed and only its proportions change, while in the results of our model the areas and the geometry of the patch change.

Figure 6 presents the composite patches shapes of these sizes: 616, 706 and 804 mm^2. In this figure, the deleted elements are presented in red, while the final patch shape is defined by the green area. In Figure 6(c) we can see that the octagonal shape is

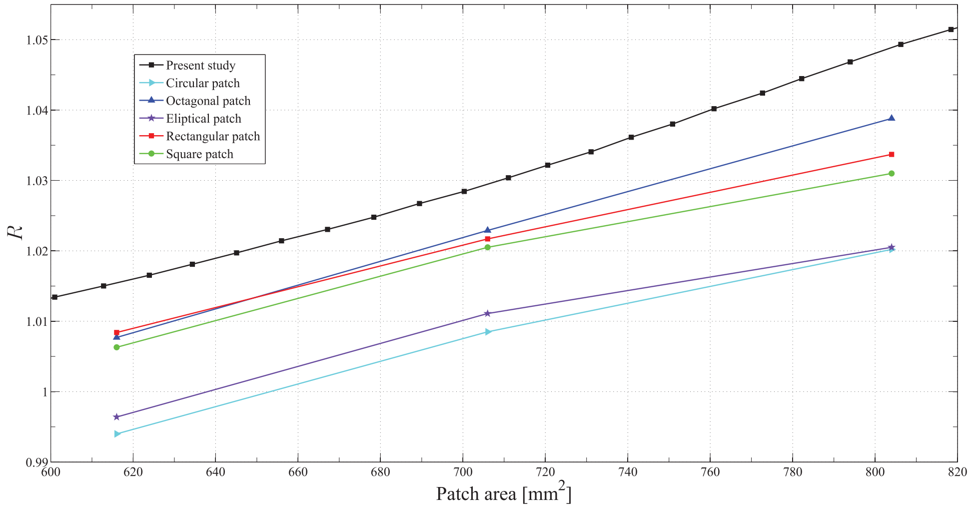

Figure 5. Variation of parameter R with respect to the area of the composite patch, are compared our solutions and the results obtained by Ramji et al.[6]

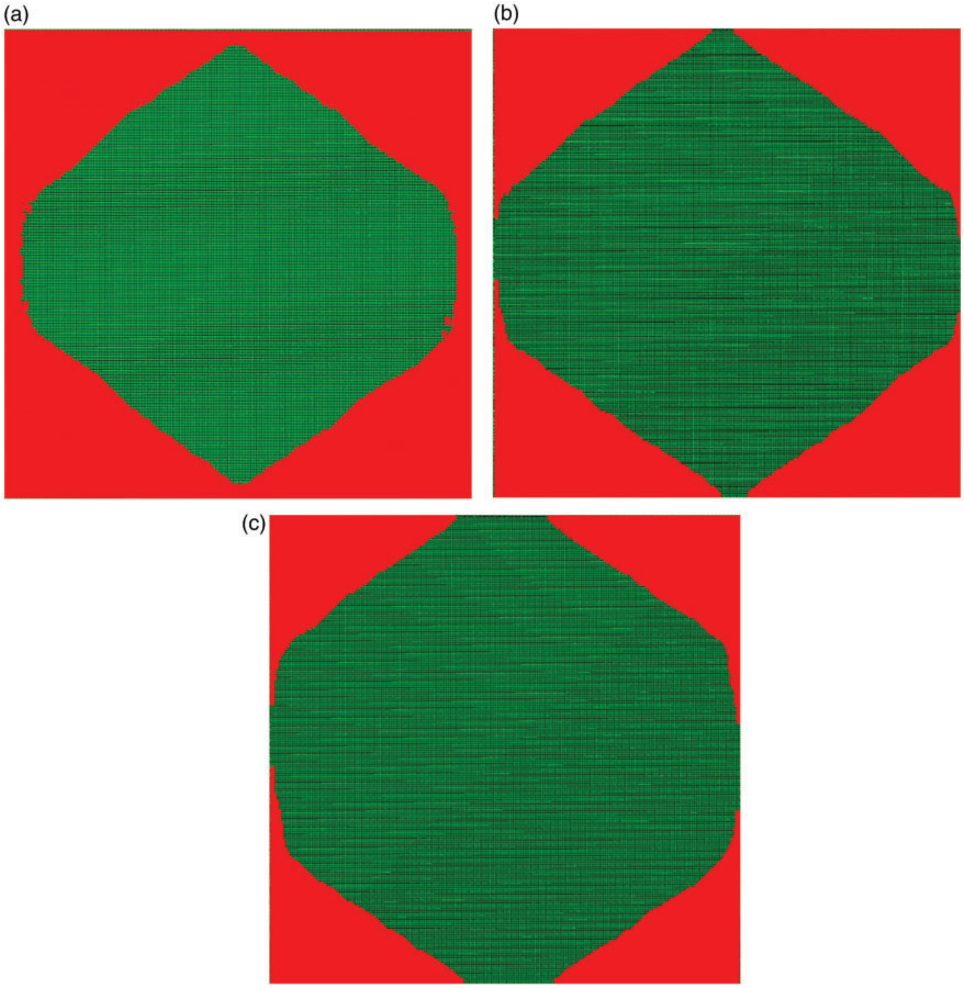

Figure 6. The shape of the composite patch obtained for different patch sizes. (a) 616 mm^2. (b) 706 mm^2. (c) 804 mm^2.

obtained from the biggest patch size analysed. This case is in agreement with the results presented by Ramji et al.,[6] but with other proportions of sides. Although for smaller sizes the hexagonal patch shape is more efficient, this geometry was not studied in Ramji et al.[6]

Influence of the crack inclination

Furthermore, we have analysed the influence of the crack inclination β for the patch shape obtained with the proposed model, corresponding to a patch area of 616 mm^2. We are considering four crack inclination angles: 0°, 30°, 45° and 60°.

Figure 7 presents the variation of the parameter R in relation to crack inclination angles β. Our results are compared with the solution obtained by Ramji et al.[6] for the same problem. We can see that the values of R obtained with the proposed model are higher than the results presented by Ramji et al.[6] for all analysed angles. Therefore, the patch shapes designed with the developed topological algorithm are better than the patch shapes considered by the other authors.[6]

Figure 8 presents the patch shape for each crack inclination angle β studied. We can see that the hexagonal patch shape is the best solution for all cases. The hexagons are symmetrical with respect to the vertical and horizontal axes. Note that, the patch shapes are hexagons but with a small change in the relationship of sides, in function with the crack inclination angle. Also, the hexagonal patch shapes have rounded corners making it more resistant against debonding.

Furthermore, it can be observed, from Figure 7, that R is higher for the crack inclination of 30° with the hexagonal patch shape, while in Ramji et al.[6] the maximum efficiency is obtained for a crack inclination of 45°.

Therefore, the hexagonal patch shape made of CFRP is recommended in cases of the inclined cracked panel repairs.

The results of simulations show that the proposed algorithm is able to obtain valid patch shapes for

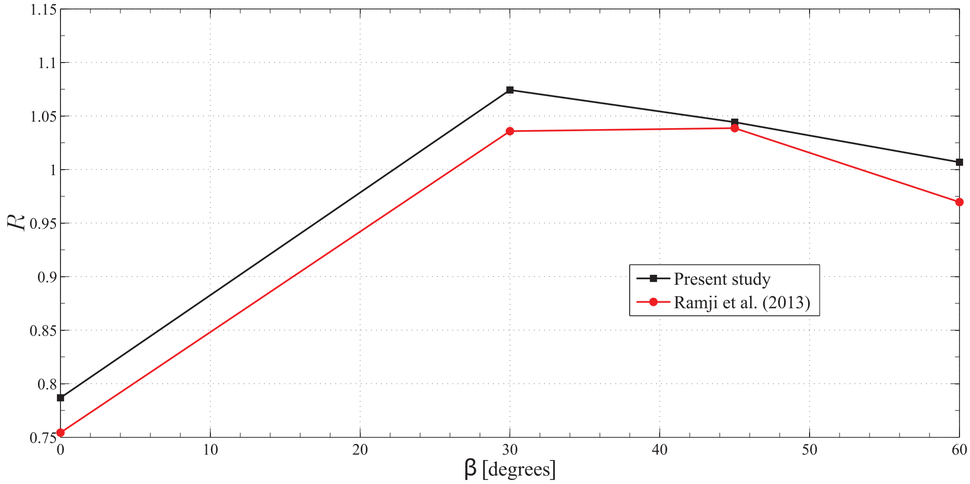

Figure 7. Variation of parameter R with respect to the crack inclination angle β.

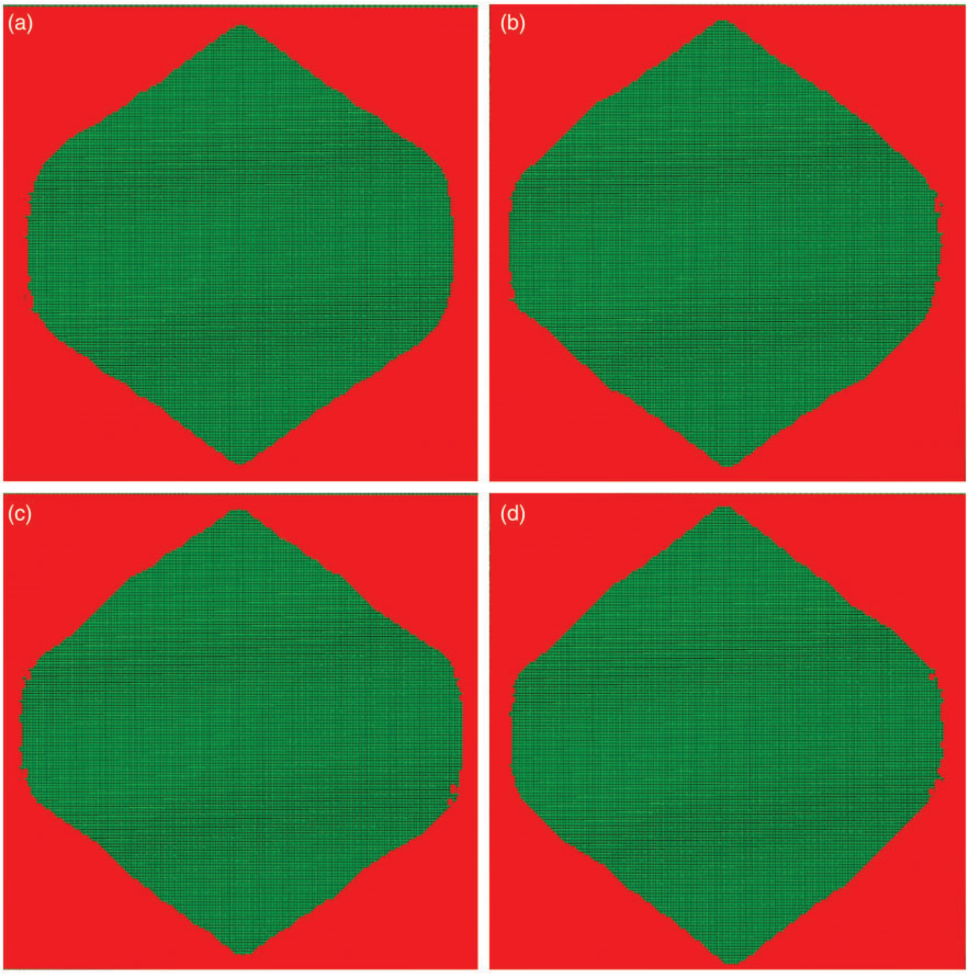

Figure 8. The shape of the composite patch obtained for crack inclination angle β. (a) $\beta = 0^{\circ}$. (b) $\beta = 30^{\circ}$. (c) $\beta = 45^{\circ}$. (d) $\beta = 60^{\circ}$.

practical cases, where the patches geometries are regulars and it do not have holes inside.

Conclusions

In this work, a topological algorithm applied to the design of shape of composite patch is developed. The algorithm is programmed in MatLab, and the FEA is solved with the commercial software Abaqus/Standard.

The performance of the algorithm is assessed by analysing the problem of a cracked aluminium plate repaired with double-sided composite patches. The obtained results are compared with the study presented by Ramji et al.[6]

The results of the simulations show that the octagonal patch shape made of CFRP with the maximum permissible area is recommended in cases of inclined cracked panel repairs, it is in agreement with the results presented in Ramji et al.[6] However, the hexagonal patch shape made of CFRP is recommended in cases of repairs with smaller patch sizes. We can conclude that the shapes of the patches obtained with our model are able to be applied to practical cases.

The simple and reliable implemented optimization procedure seems to be suitable to tackle several problems related to the design of the best topology for patch repairs in order to achieve any given mechanical performance of the cracked structures.

Declaration of Conflicting Interests

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References

1. Baker A. Repair of cracked or defective metallic aircraft components with advanced fibre composites: an overview of Australian work. *Compos Struct* 1984; 2: 153–181.
2. Ting T, Jones R, Chiu W, et al. Composite repairs to rib stiffened panels. *Compos Struct* 1999; 47: 737–743.
3. Kumar AM and Hakeem S. Optimum design of symmetric composite patch repair to centre cracked metallic sheet. *Compos Struct* 2000; 49: 285–292.
4. Rachid M, Serier B, Bouiadjra BB, et al. Numerical analysis of the patch shape effects on the performances of bonded composite repair in aircraft structures. *Compos Part B Eng* 2012; 43: 391–397.
5. Bouiadjra BB, Bouanani MF, Albedah A, et al. Comparison between rectangular and trapezoidal bonded composite repairs in aircraft structures: a numerical analysis. *Mater Des* 2011; 32: 3161–3166.
6. Ramji M, Srilakshmi R and Prakash MB. Towards optimization of patch shape on the performance of bonded composite repair using {FEM}. *Compos Part B Eng* 2013; 45: 710–720.
7. Iváñez I and Braun M. Numerical analysis of surface cracks repaired with single and double patches of composites. *J Compos Mater*. DOI: 20170021998317722044..
8. Soutis C and Hu F. Design and performance of bonded patch repairs of composite structures. *Proc Inst Mech Eng G J Aerospace Eng* 1997; 211: 263–271.
9. Gong XJ, Cheng P, Rousseau J and Aivazzadeh S. Effect of local stresses on static strength and fatigue life of patched composite panels. In: 16th International conference on composite materials. Kyoto, Japan; 8–13 July 2007.
10. Campilho RDSG, De Moura M, Ramantani D, et al. Tensile behaviour of three-dimensional carbon-epoxy adhesively bonded single-and double-strap repairs. *Int J Adhes Adhes* 2009; 29: 678–686.
11. Her SC and Chao M. Adhesively bonded patch repair of composite laminates. *J Adhes Sci Technol* 2011; 25: 2569–2585.
12. Cheng P, Gong XJ, Hearn D, et al. Tensile behaviour of patch-repaired CFRP laminates. *Compos Struct* 2011; 93: 582–589.
13. Kashfuddoja M and Ramji M. Whole-field strain analysis and damage assessment of adhesively bonded patch repair of CFRP laminates using 3d-DIC and FEA. *Compos Part B Eng* 2013; 53: 46–61.
14. Kashfuddoja M and Ramji M. Design of optimum patch shape and size for bonded repair on damaged carbon fibre reinforced polymer panels. *Mater Des* 2014; 54: 174–183.
15. Caminero M, Lopez-Pedrosa M, Pinna C, et al. Damage monitoring and analysis of composite laminates with an open hole and adhesively bonded repairs using digital image correlation. *Compos Part B Eng* 2013; 53: 76–91.
16. Qing XP, Beard SJ, Kumar A, et al. A real-time active smart patch system for monitoring the integrity of bonded repair on an aircraft structure. *Smart Mater Struct* 2006; 15: N66.
17. Ouinas D, Bouiadjra BB and Serier B. The effects of disbonds on the stress intensity factor of aluminium panels repaired using composite materials. *Compos Struct* 2007; 78: 278–284.
18. Ouinas D, Bouiadjra BB, Achour T, et al. Influence of disbond on notch crack behaviour in single bonded lap joints. *Mater Des* 2010; 31: 4356–4362.
19. Bouiadjra BB, Oudad W, Albedah A, et al. Effects of the adhesive disband on the performances of bonded composite repairs in aircraft structures. *Mater Des* 2012; 37: 89–95.
20. Bouiadjra BB, Ouinas D, Serier B, et al. Disbond effects on bonded boron/epoxy composite repair to aluminium plates. *Comput Mater Sci* 2008; 42: 220–227.
21. Mathias JD, Balandraud X and Grediac M. Applying a genetic algorithm to the optimization of composite patches. *Comput Struct* 2006; 84: 823–834.

22. Kogiso N, Watson LT, Gürdal Z, et al. Genetic algorithms with local improvement for composite laminate design. *Struct Multidiscipl Optim* 1994; 7: 207–218.
23. Kogiso N, Watson LT, Gürdal Z, Haftka RT, et al. Design of composite laminates by a genetic algorithm with memory. *Mech Compos Mater Struct* 1994; 1: 95–117.
24. Le Riche R and Haftka R. Improved genetic algorithm for minimum thickness composite laminate design. *Compos Eng* 1995; 5: 143–161.
25. Todoroki A and Haftka RT. Stacking sequence optimization by a genetic algorithm with a new recessive gene like repair strategy. *Compos Part B Eng* 1998; 29: 277–285.
26. Lin CY and Hajela P. Genetic algorithms in optimization problems with discrete and integer design variables. *Eng Optimiz* 1992; 19: 309–327.
27. Brighenti R. Patch repair design optimisation for fracture and fatigue improvements of cracked plates. *Int J Solids Struct* 2007; 44: 1115–1131.
28. Errouane H, Sereir Z and Chateauneuf A. Numerical model for optimal design of composite patch repair of cracked aluminum plates under tension. *Int J Adhes Adhes* 2014; 49: 64–72.
29. Eschenauer HA and Olhoff N. Topology optimization of continuum structures: a review. *Appl Mech Rev* 2001; 54: 331–390.
30. Bendsoe MP and Kikuchi N. Generating optimal topologies in structural design using a homogenization method. *Comput Methods Appl Mech Eng* 1988; 71: 197–224.
31. Bendsoe MP and Sigmund O. *Topology optimization: theory, methods, and applications*. London: Springer Science & Business Media, 2013.
32. Xie Y and Steven G. *Evolutionary structural optimization*. London: Springer, 1997.
33. Huang X and Xie Y. Evolutionary topology optimization of continuum structures including design-dependent self-weight loads. *Finite Elem Anal Des* 2011; 47: 942–948.
34. ABAQUS/Standard. *Abaqus standard v6.13 user's manual*. Version 6.13th ed. Richmond: ABAQUS Inc., 2013.
35. software A (2009) *User's documentation*. Version 12.1 ed. PA: ANSYS, Inc., 2009.
36. Ansys (2009) User's documentation. Version 12.1 ed. Canonsburg, PA: ANSYS, Inc.
37. Agarwal BD, Broutman LJ and Chandrashekhara K. *Analysis and performance of fiber composites*. New York: John Wiley & Sons, 2006.
38. Ramji M and Srilakshmi R. Design of composite patch reinforcement applied to mixed-mode cracked panel using finite element analysis. *J Reinf Plast Compos* 2012; 31: 585–595.
39. Benyahia F, Aminallah L, Albedah A, et al. Experimental and numerical analysis of bonded composite patch repair in aluminum alloy 7075 T6. *Mater Des* 2015; 73: 67–73.