Journal of Sound and Vibration 384 (2016) 362–363

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Commentary on "T.G. Ritto, M.R. Escalante, Rubens Sampaio, M.B. Rosales, Drill-string horizontal dynamics with uncertainty on the frictional force, *Journal of Sound and Vibration* 332 (2013) 145–153"

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ARTICLE INFO

Article history:
Received 24 May 2016
Received in revised form
4 July 2016
Accepted 10 August 2016
Handling Editor: M.P. Cartmell
Available online 25 August 2016

ABSTRACT

The goal of this article is to clarify some points of the formulation presented in the "T.G. Ritto, M.R. Escalante, Rubens Sampaio, M.B. Rosales, Drill-string horizontal dynamics with uncertainty on the frictional force, *Journal of Sound and Vibration* 332 (2013) 145–153".

1. Introduction

The motive of this work is to respond to comments raised in a discussion [1], questioning the formulation presented in "T.G. Ritto, M.R. Escalante, Rubens Sampaio, M.B. Rosales, Drill-string horizontal dynamics with uncertainty on the frictional force, *Journal of Sound and Vibration* 332 (2013) 145–153" [2].

As is well known, variational formulation incorporates the dynamical boundary conditions in a functional, and the natural boundary conditions in the definition of the admissible functions. Also, variational formulations give scalar equations, since this is one of the ways to write equations in a Hilbert space framework. Another way is to write the equations in the distribution sense, in this case some of the boundary conditions may appear as delta functions [3,4]. If the equations are going to be discretized via Finite Elements, variational formulations are convenient.

Let us answer each pointed raised by [1]. For the sake of clarity some statements will be repeated.

1. Considering the discretized system obtained by means of the FEM. Any finite element book or structural dynamics book [5–9] shows the steps to go from $-EA\partial^2 u/\partial x^2$ to $\mathbf{Kx}$, for instance. As it is well known, using linear shape functions, the finite element stiffness matrix might be written as

$$\mathbf{K}^{(e)} = \frac{EA}{l^{(e)}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \tag{1}$$

DOI of original article: http://dx.doi.org/10.1016/j.jsv.2016.07.017
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http://dx.doi.org/10.1016/j.jsv.2016.08.018
0022-460X/

where A is the area of the cross-section, E is the elasticity modulus, and $l^{(e)}$ is the size of the finite element. Note that after integrating the continuous equation, the dimensions of the terms of the discretized system are all in N. Also, it is a common assumption to add, a posteriori, a proportional matrix $C=\alpha M+\beta K$ to model the damping. For a more complete explanation and a step-by-step procedure, the reader should consult an appropriate textbook [5–9].

2. The Dirac delta function δ is commonly used to represent a discontinuity. For instance, a system might have a discontinuity in time $\ddot{x}+\dot{x}+x=\delta(t-5)$, see [9], for example, or the discontinuity might be in space, $\delta(x-5)$, which is the case under analysis. This is well known notation employing standard theory.

3. On the assumptions of the model. There are many ways to model this specific dynamical system, and some assumptions must be made. If these assumptions do not represent well the real system, they must be changed. As stated [2], this article was the first attempt to model the horizontal drill-string dynamics. The industrial view is that it is acceptable to consider a harmonic force at the bit, for instance. Note that recently a more complete model for the horizontal drill-string dynamics was proposed by [10], where a kinematic condition is imposed on the left of the column.

To conclude, it should be noted that there is a series of publication inspired by the problem of drill-string oil-well dynamics. In 2007, [11], the coupled axial/torsional vibrations of drill-strings were analyzed. After that, as a result of more research, other papers have been published [12–17]. Different formulations were considered in the cited publications. The most complete model is found in [12]. Finally, the first models proposed for the horizontal drill-string dynamics were published in 2013 [2] and 2015 [10]. These models were extensively verified, and, recently, the results of the torsional dynamical model of the drill-string were compared with drill-string dynamical field data (5 km drill-string). For some conditions, the torsional model presented excellent results comparing with the field data.

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