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Restoring the SU(4) Kondo regime in a double quantum dot system

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Abstract

We calculate the spectral density and occupations of a system of two capacitively coupled quantum dots, each one connected to its own pair of conducting leads, in a regime of parameters in which the total couplings to the leads for each dot Γ_i are different. The system has been used recently to perform pseudospin spectroscopy by controlling independently the voltages of the four leads. For an odd number of electrons in the system, equal coupling to the leads $\Gamma_1 = \Gamma_2$, equal dot levels $E_1 = E_2$ and sufficiently large interdot repulsion U_{12} the system lies in the SU(4) symmetric point of spin and pseudospin degeneracy in the Kondo regime. In the more realistic case $\Gamma_1 \neq \Gamma_2$, pseudospin degeneracy is broken and the symmetry is reduced to SU(2). Nevertheless, we find that the essential features of the SU(4) symmetric case are recovered by appropriately tuning the level difference $\delta = E_2 - E_1$. After this tuning, the system behaves as an SU(4) Kondo one at low energies. Our results are relevant for experiments which look for signatures of SU(4) symmetry in the Kondo regime of similar systems.

Keywords: quantum dots, SU(4) symmetry, Kondo effect, emergent symmetry

(Some figures may appear in colour only in the online journal)

1. Introduction

The Kondo effect is one of the most studied phenomena in strongly correlated condensed matter systems [1] and continues to be a subject of great interest. The effect is characterised by the emergence of a many-body singlet ground state formed by the impurity spin and the conduction electrons in the Fermi sea. The binding energy of this singlet is of the order of the characteristic Kondo temperature T_K below which the effects of the 'screening' of the impurity spin manifest themselves in different physical properties. The first observed manifestation of the Kondo effect was the logarithmic increase of the resistivity as the temperature is decreased in systems of magnetic impurities in metals [2]. The same type of behaviour is present in systems with orbital degeneracy but no spin degeneracy [3]. In recent decades, the research has moved to nanoscopic systems with semiconducting [4–7] or molecular [8–10] quantum dots (QDs), with a single 'impurity', in which different parameters like on-site energy and hybridisation of the impurity with the conduction electrons can be controlled very well.

In recent years there has been research on Kondo systems in which in addition to the spin degeneracy, there is also degeneracy in other 'orbital' degrees of freedom such that the complete symmetry of the system is very high, corresponding to the SU(4) Lie group [11–31]. Some examples are quantum dots in carbon nanotubes [15–22], silicon nanowires [23], and organic molecules deposited on Au(1 1 1) [25, 26].

More recently a double QD with strong interdot capacitive coupling, and each QD tunnel-coupled to its own pair of leads has been experimentally studied [28, 32]. The occupation of one QD or the other plays the role of the orbital degree of freedom, and behaves as a pseudospin. The pseudospin regime of the Kondo transport in similar devices had been demonstrated experimentally previously [33, 34]. The occupations,

the tunneling matrix elements (coupling to the leads) and the voltages at the four leads can be controlled independently. In this work we concentrate on the regime in which starting with the ground state for an even number of electrons in both QDs, assumed a singlet, an additional electron or hole is added to one QD resulting in a spin doublet ground state. For two added electrons the system also displays some rich physics [35]. The changes in the occupation, Kondo temperature and other properties when the same gate voltage is applied to both QDs is discussed in [36].

Theoretically, the system was studied, among others, by Trocha using scaling, slave bosons and equations of motion out of equilibrium [37], by Büsser *et al* [27], who proposed it for controlling spin-polarised currents, and is now a subject of intense research [28–31, 38, 39]. Comparing experiment with numerical-renormalisation-group (NRG) calculation, Keller *et al* found evidence of SU(4) Kondo behaviour [28]. However, using renormalised perturbation theory (RPT) with parameters obtained from NRG, Nishikawa *et al* concluded that the experimental system is not in the SU(4) regime, particularly because of the relatively small value of the interdot repulsion U_{12} in comparison with other parameters [30].

In addition, in spite of the great tunability of the parameters, it is difficult to reach the SU(4) condition $\Gamma_1 = \Gamma_2$, where $\Gamma_i = \Gamma_{Si} + \Gamma_{Di}$ and $\Gamma_{\nu i}$ is the coupling of the source ($\nu = S$) or drain ($\nu = D$) lead with dot i. The total coupling of dot i, Γ_i, corresponds to the line width of the local spectral density of dot i in the absence of Coulomb repulsion and is inferred from experiment with the aid of theory [28]. Instead, the energy of the relevant partly occupied level at each dot E_i is easier to control directly by the different applied voltages, as described in the supplementary material of [32].

The purpose of the present work is to study to what extent the loss of SU(4) symmetry caused by unequal couplings $\Gamma_1 \neq \Gamma_2$ can be restored by tuning the energy difference $\delta = E_2 - E_1$ in a regime of parameters in which intrasite U_i and intersite U_{12} repulsions are much larger than the Γ_i. This is related to the concept of *emergent symmetry*, i.e. the fact that new symmetries not realised in the Hamiltonian describing the system can emerge at low energies [40].

We use the non-crossing approximation (NCA), which has proved in the past an excellent technique to deal with large Coulomb interactions and has the advantage of been easily extensible to the non-equilibrium case of finite bias voltages [23, 29, 41–44]. It reproduces well the scaling relations with temperature T and bias voltage in the Kondo regime [45] and was, for example, also successfully used to interpret experimental results on a controlled crossover between SU(4) and SU(2) Kondo states driven by a magnetic field in a nanoscale Si transistor [23], as well as quantum phase transitions involving singlet and triplet states in molecular quantum dots [9]. Using NCA we have previously studied the conductance of the system of two capacitively coupled quantum dots in the general case of different finite bias voltages V_i applied to each dot [29]. We have discussed the conditions to observe an SU(4) → SU(2) crossover under an applied pseudo-magnetic field δ, and non-trivial crossed effects of changes in the conductance through one QD as a voltage being applied to the other [29]. Recently the general non-equilibrium case has been studied using equations of motion [39]. However, as in most previous theoretical studies of the system, $\Gamma_1 = \Gamma_2$ was assumed. An alternative to studying the non-equilibrium case for small V_i might be to use RPT [30, 46–48], but its extension to the two-dot case and finite V_i seems difficult because of the presence of many parameters [30].

In this paper we calculate the spectral densities ρ_i of each dot. They can be addressed experimentally in a situation with very asymmetric coupling to the source and drain leads for each dot i, changing only the voltage to the less coupled lead. In fact, a ratio $\Gamma_{Si}/\Gamma_{Di} = 10$ or 0.1 is enough for the differential conductance dI/dV to represent accurately ρ_i [29, 44], and a ratio 12 has been used in some experiments [32]. The main effect of the different total couplings of both dots $\Gamma_1 \neq \Gamma_2$ is to introduce an effective pseudo-Zeeman splitting δ_{eff}. This can be understood by a straightforward generalisation of the scaling treatment of Haldane [49] for the simplest impurity Anderson model (corresponding to the one-dot case) [37]. This δ_{eff} can be compensated by tuning the gate voltages so that $\delta = E_2 - E_1 = -\delta_{\text{eff}}$ leading to an SU(4) behaviour at low energies.

This paper is organised as follows. The model is explained in section 2. In section 3 we explain the main differences in the spectral densities in the regimes of SU(4) or SU(2) symmetry and the 'transition' between them. Section 4 describes the effect of different total couplings to the leads $\Gamma_2 \neq \Gamma_1$. In section 5 we describe how tuning the energy levels can compensate the effect of different couplings in a restricted energy range. Section 6 contains a summary and a discussion.

2. Model

The system is described by an Anderson model which contains as localised configurations a singlet $|s\rangle$ with an even number of particles in each dot and two spin doublets $|i\sigma\rangle$ ($i = 1$ or 2) with one additional electron (or hole) in QD i. In general, it is cumbersome to express these states in fully second quantised form. One exception is the singlet-triplet model used to describe the observed quantum-phase transition in C_{60} devices [9], for which the 'translation' is explicitly given in [50].

There are four conduction bands which correspond to separate source and drain leads for each dot. The Hamiltonian is

$$H = E_s|s\rangle\langle s| + \sum_{i\sigma} E_i|i\sigma\rangle\langle i\sigma| + \sum_{i\nu k\sigma} \epsilon_{\nu k} c^\dagger_{\nu k i\sigma} c_{\nu k i\sigma} + \sum_{i\nu k\sigma} (V_i^\nu |i\sigma\rangle\langle s| c_{\nu k i\sigma} + \text{H.c.}), \quad (1)$$

where $c^\dagger_{\nu k i\sigma}$ creates conduction states at the source ($\nu = S$) or drain ($\nu = D$) lead, and V_i^ν is the hopping between the lead ν and dot i, assumed independent of k. Since other charge configurations are excluded, the model assumes infinite on-site repulsions U_i and interdot repulsion U_{12}.

The tunnel couplings of each QD to the leads are $\Gamma_{\nu i} = 2\pi \sum_k |V_i^\nu|^2 \delta(\omega - \epsilon_{\nu k})$, and we take the unit of energy

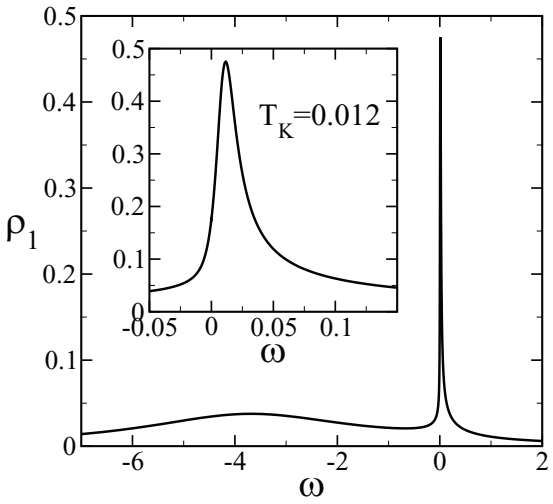

Figure 1. Density of states at each dot as a function of energy at low temperature $T = 5.10^{-3} = 0.42\,T_K$ in the SU(4) case $E_1 = -4$ and $\Gamma_1 = \Gamma_2 = 1$. The inset is a detail close to the Fermi level.

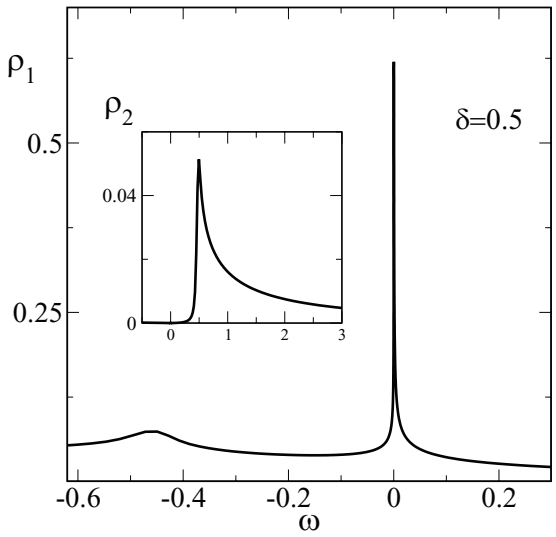

Figure 2. Density of states of dot 1 (main figure) and dot 2 (inset) as a function of energy at low temperature $T = 1.8 \times 10^{-4}$ for $E_1 = -4$, $E_2 = E_1 + \delta$ with $\delta = 0.5$ and $\Gamma_1 = \Gamma_2 = 1$.

$\Gamma_1 = \Gamma_{S1} + \Gamma_{D1} = 1$ unless otherwise stated. We assume (without loss of generality) $\Gamma_2 \leqslant \Gamma_1$

3. The SU(4) → SU(2) crossover

For future comparison, in this section we review briefly the well-known effect of Zeeman or pseudo-Zeeman splitting on the spectral density of the SU(4) Anderson model [11, 14, 17, 21, 23], and the effects of temperature. In particular, in [17] four different theoretical approaches were used to study both the linear and nonlinear conductance for different values of the symmetry-breaking field.

In figure 1 we show the spectral density of states per spin at each dot $\rho_1 = \rho_2$ (the spin subscript is dropped) in an SU(4)-symmetric case $E_1 = E_2 = -4$ and $\Gamma_1 = \Gamma_2 = 1$. We take the half band width $D = 10$ for all the calculations presented here. The density ρ_i corresponds to the operators $|i\sigma\rangle\langle 0|$ (see [43] for details). The density of states shows two peaks. The charge-transfer one is broad. Its half width at half maximum is near Γ_1 (two times that in the non-interacting case [44]). The Kondo peak near the Fermi level has a half width at half maximum of the order of the Kondo temperature T_K. As a consequence of the increase of degeneracy, the Kondo effect is stronger than for the usual SU(2) case. We remind the reader that the Kondo temperature for the infinite-U SU(N) Anderson model is $T_K \approx D\exp[\pi E_1/(N\Delta)]$, where D is half the band width and $\Delta = \Gamma_1/2$ [1]. The NCA reproduces correctly this result [51].

From half the width of the spectral density we obtain $T_K^{SU(4)} = 0.012$. In comparison with the SU(2) case with the same T_K, the Kondo resonance is displaced to higher energies and the maximum is clearly above the Fermi energy, which we set as the origin of energies ($\epsilon_F = 0$). In fact, the SU(4) case is characterised by a high derivative of $\rho_i(\omega)$ at the Fermi level, leading to a large thermoelectric power [24].

Although the symmetry is broken immediately when even a tiny pseudo-Zeeman splitting δ is introduced, the changes in physical quantities like conductances for each dot and occupations are not appreciable until δ becomes of the order of $T_K^{SU(4)}$ [14]. This is expected and has been discussed previously in the context of NRG calculations of thermodynamic properties [52]. In particular, the Kondo temperature $T_K(\delta)$ obtained from the width of the Kondo peak displays initially a plateau and then decreases strongly for $\delta > T_K^{SU(4)}$. We have noticed previously that our NCA results for $T_K(\delta)$ can be very well represented by a simple equation derived from a variational wave function

$$T_K = \{(D+\delta)D\exp[\pi E_1/(2\Delta)] + \delta^2/4\}^{1/2} - \delta/2. \qquad (2)$$

times a factor of the order of 1 (0.606 for the parameters used) [21].

For $\delta > T_K^{SU(4)}$ the changes in the spectral density at low temperatures are dramatic, as shown in figure 2 (the interacting part of the NCA self energy remains well behaved at the relevant frequencies[3]). The density of the dot that corresponds to the lowest lying level (1 in our convention) displays two peaks: one at energy near $-\delta$ which corresponds to a splitting of the SU(4) Kondo peak due to a pseudomagnetic field, in analogy to the splitting of the ordinary SU(2) Kondo peak under a magnetic field [53, 54], and another one at the Fermi energy, which corresponds to the SU(2) Kondo peak associated with the remaining spin degeneracy. With increasing δ, the SU(2) Kondo peak narrows (following equation (2)), and shifts towards the Fermi energy. The density of the dot that corresponds to the highest lying level (ρ_2) displays only a peak near $+\delta$ but no peak near the

[3] The interacting part of the self energy, which for our problem is diagonal in the dot index can be defined as $\Sigma_i^{int}(\omega) = \omega - E_i + i\Delta - 1/[G_i(\omega)]$, where $G_i(\omega)$ is the Green function for dot i. We have verified that $\mathrm{Im}[\Sigma_1^{int}(\omega)]$ is positive ($\Sigma_1^{int}(\omega)$ is causal) for all parameters used in figures 2 and 3. Instead, $\Sigma_2^{int}(\omega)$ is slightly negative at the smallest temperature used ($T = 0.000\,18$) for frequencies between –0.36 and 0.31. However, as it is clear from figure 3, the spectral density of dot 2 is negligible in this region. Thus, this failure of causality does not affect the peak in ρ_2.

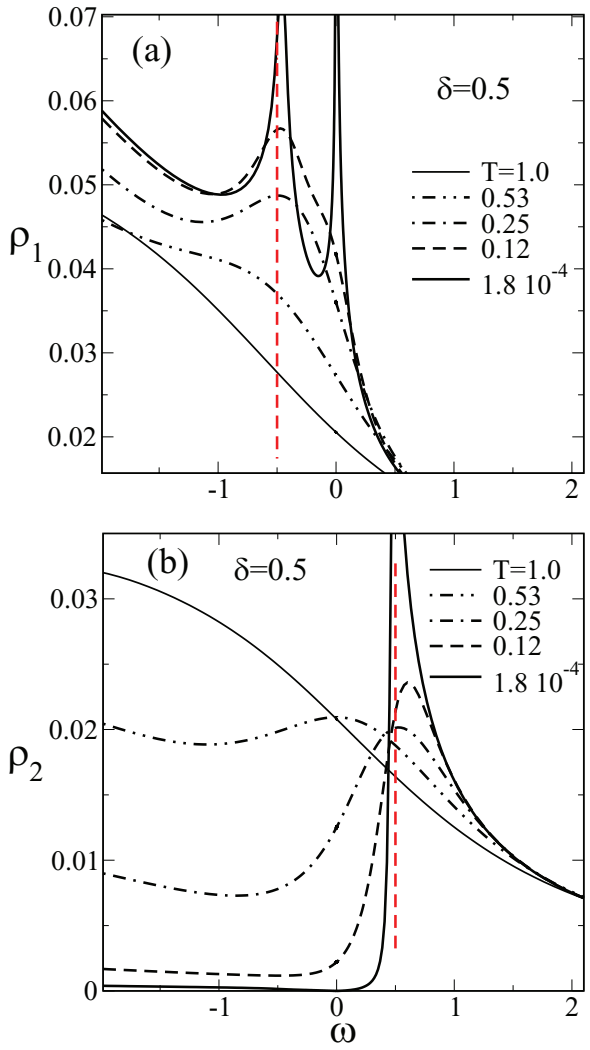

Figure 3. Density of states of (a) dot 1 and (b) dot 2 as a function of energy for different temperatures. Parameters as in figure 2.

Fermi energy. We find that the width of both inelastic peaks is of the order of $T_K^{SU(4)}$ for small δ (but $\delta > T_K^{SU(4)}$ in order to ensure that the inelastic peaks are split from the SU(2) Kondo peak) and increases with increasing δ. This behaviour is reminiscent of the evolution of the peaks of the ordinary SU(2) Kondo model under an applied magnetic field, which has been studied by Bethe ansatz techniques [54]. We note that the simplest mean-field slave-boson approximation used in similar problems [24, 37, 55] renormalises in the same fashion the widths of the peaks related to both levels, and therefore does not reproduce correctly the width of the inelastic peaks in the spectral densities.

It is interesting to see the evolution of the side peaks at $\pm\delta$ with temperature. This is shown in figure 3. At high temperatures $T > \delta$ both spectral densities are similar. As the temperature is lowered below δ the side peaks start to develop. In addition, as the total occupation of dot 1, $n_1 = n_{1\uparrow} + n_{1\downarrow}$, increases and that of dot 2 (n_2) decreases, the charge transfer peak of dot 1 (2) increases (decreases). In the figure, due to the restricted energy range, only the tail of this peak is visible, but the above mentioned effect is clear. The changes in n_i as a function of δ were studied before [14, 30]. At temperatures of the order of $T_K(\delta)$ and below ($T_K(0.5) \sim 4.4 \times 10^{-4}$ in the figure), the Kondo peak develops in ρ_1. The width of both side peaks is of the order of $T_K(0) = T_K^{SU(4)}$.

It is important to recall that in general, in the presence of both Zeeman and pseudo-Zeeman splitting, the spectral density at the Fermi level at zero temperature for dot i and spin σ is related to the corresponding occupation by the Friedel sum rule generalised for orbital degeneracy [56]. We assume that the Γ_i and the unperturbed densities of conduction states are independent of energy. Since $T_K(\delta)$ is always much smaller than typical scales of variations of these parameters, this assumption is realistic. In this case, the Friedel sum rule simplifies to [21, 56]

$$\rho_{i\sigma}(\epsilon_F) = \frac{1}{\pi\Delta}\sin^2(\pi n_{i\sigma}). \tag{3}$$

At $\delta = 0$ the four occupations $n_{i\sigma}$ are slightly below 1/4 (the total occupation is below 1 because of a finite small occupation of the singlet $|0\rangle$). For finite δ and high temperatures in comparison with δ, also all $n_{i\sigma}$ are slightly below 1/4. As the temperature decreases below δ, with δ large in comparison with $T_K(0)$, $n_{1\sigma}$ increases towards 1/2 while $n_{2\sigma}$ decreases towards 0. The Friedel sum rule implies that at $T = 0$, $\pi\Delta\rho_{i\sigma}$ is slightly below 1/2 in the SU(4) case, while well inside the SU(2) regime, $\pi\Delta\rho_{1\sigma} \to 1$ (or slightly below) and $\pi\Delta\rho_{2\sigma} \to 0$. The NCA has an error of the order of 15% in the Friedel sum rule, but the tendencies are well reproduced [21].

4. Effect of different couplings for degenerate levels

Starting from degenerate levels $E_1 = E_2 = E_d$, the main effect expected from different total hybridisations $\Gamma_1 \neq \Gamma_2$ is to generate an effective pseudo-Zeeman splitting $\delta_{eff} = E_2^* - E_1^*$, where E_i^* are renormalised energy levels [37]. This can be seen generalising the theory used by Haldane based on poor man's scaling to find E^* for the case of one level [49]. One proceeds by integrating out the states near the top (with energy $+ D$) and bottom (energy $-D$) of the conduction band. The localised state can be empty with energy e_0 or occupied with energy $e_{i\sigma}$. After renormalisation, the energy necessary to add one localised particle is $E_{i\sigma}^* = e_{i\sigma} - e_0$. The renormalisation is caused by the possible processes of destroying an electron in the localised level and creating it in the conduction band, or vice versa. When being integrating out, each state near the bottom of the conduction band contributes to lowering the energy of the empty state e_0 by

$$\sum_{i\sigma} \frac{V_{i\sigma}^2}{E_d + D}. \tag{4}$$

Similarly, the states near the bottom of the conduction band lower the energy of the occupied state $e_{i\sigma}$ by

$$\frac{V_{i\sigma}^2}{D - Ed}. \tag{5}$$

Scaling down to a cutoff C one obtains

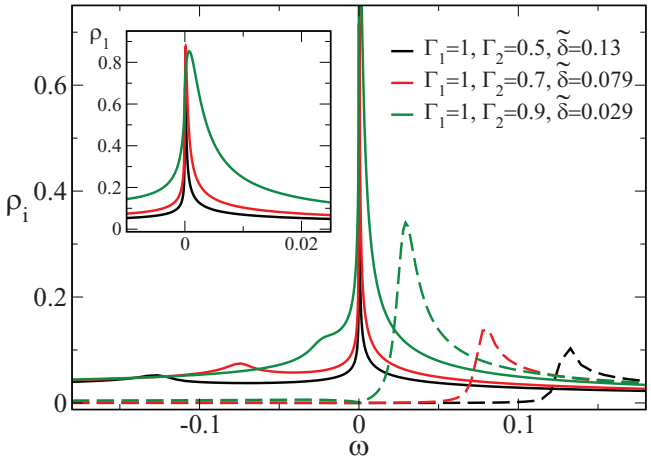

Figure 4. Density of states of dot 1 (full lines) and dot 2 (dashed lines) as a function of frequency for low temperatures ($T = 5.10^{-5}$), $E_1 = E_2 = -4$, $\Gamma_1 = 1$, and several values of Γ_2. The inset shows a detail near the Fermi energy.

$$E^{*}_{i\sigma} = \frac{1}{2\pi}\left[\sum_{j\sigma'} \Gamma_{j\sigma'} - \Gamma_{i\sigma}\right] \ln\left(\frac{D}{C}\right). \tag{6}$$

In our case, in which the couplings are independent of spin, this leads to an effective splitting

$$\delta_{\text{eff}} = \frac{1}{2\pi}(\Gamma_2 - \Gamma_1) \ln\left(\frac{D}{C}\right). \tag{7}$$

This equation has been obtained before by Trocha [37].

In figure 4 we display the spectral density of states for three cases with $\Gamma_1 > \Gamma_2$. A comparison with the results of the previous section indicates that the effects of different total coupling to the leads for both dots are similar to those of a splitting of the energy levels. The Kondo peak at the Fermi energy narrows and displaces towards the Fermi energy and a side peak appears for each dot, at negative (positive) energies for the more (less) coupled dot. For the case $\Gamma_2 = 0.9$, only 10% less than Γ_1, the side peak in ρ_1 appears as a shoulder to the left of the Kondo peak rather than being well separated, because the effective splitting $\delta_{\text{eff}} \approx 0.029$ is of the order of the Kondo temperature for $\Gamma_i = 1$, $T_K^{SU(4)} = 0.012$. The results shown in this figure are consistent with previous calculations using NRG (figure 5 of [17]) in which the total density of states is presented [17]. The difference is that the side peaks look broader in the NRG results. This may be caused by the well known lack of resolution of NRG calculations at finite energies [57, 58].

The positions of the side peaks allow us to infer the values of the effective splitting within the NCA. They are listed inside figure 4 as $\tilde{\delta}$ and represented in figure 5 together with the result of equation (7) with a cutoff $C = |E_d/2|$. In his scaling calculation, Haldane used a cutoff of the order of Γ_i [49], while in a recent detailed study of the prefactor of the Kondo temperature of the SU(4) case, Filipone *et al* used $C = |E_d|/\alpha$ with α of the order of 1 [31]. We obtain a better agreement with the NCA results using the latter choice. The good agreement between both approaches (in spite of the corresponding limitations of each one) seems to confirm the physical picture of the main effect of different couplings.

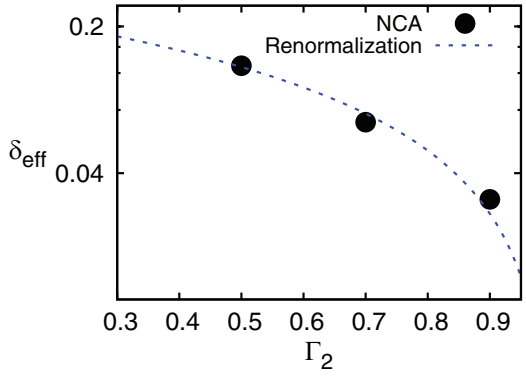

Figure 5. Effective splitting as a function of Γ_2. The dashed line is the result from equation (7) with $C = |E_d/2|$. Other parameters as in figure 4.

5. Restoring SU(4) symmetry

After the results of the previous section, the question arises as to whether on introducing a real splitting $\delta = E_2 - E_1$ such that it compensates the effect of different couplings (so that $\delta + \delta_{\text{eff}} = 0$), the SU(4) symmetry can be restored in the low-energy properties tested by conductance measurements. Clearly the symmetry remains broken at the Hamiltonian level, so that one cannot expect a higher symmetry at all energies. Therefore, we search for indications of a low-energy *emergent symmetry* [40].

In figure 6 we show the temperature dependence of the total occupations (adding both spins) at each dot for a splitting such that $\delta + \delta_{\text{eff}} \approx 0$ according to the results of the previous section. At high temperatures, of the order of Γ, $n_1 \approx n_2$, although n_2 (the occupation of the less hybridised doublet lying at lower energy) is slightly larger. As the temperature is lowered by two orders of magnitude, the situation is similar, although $n_2 - n_1$ first increases slightly and then decreases. At temperatures below $T_K^{SU(4)}$, $n_2 - n_1$ changes sign and increases in magnitude, signaling a complete loss of SU(4) symmetry for $T \to 0$. However, it is possible to tune δ so that the condition $n_2 = n_1$ (implied by SU(4) symmetry) is satisfied at any given temperature. Conversely, for a given δ, T can be varied so that $n_2 = n_1$ at $T = T_{\text{occ}}(\delta)$. In fact, the choice $\delta = -\delta_{\text{eff}}$ with δ_{eff} extracted from the position of the satellite peaks is a good initial guess, but tuning δ, $T_{\text{occ}}(\delta)$ can be reduced by orders of magnitude, as shown in figure 7. This tuning is very time consuming for our numerical procedure used to solve the self-consistent set of NCA equations (for details see for example [43]) because the whole procedure has to be repeated for several 'guessed' values of δ near δ_c, where δ_c is defined by $T_{\text{occ}}(\delta_c) = 0$. In addition, the NCA cannot reach arbitrarily small temperatures. It is interesting to note that we find that $T_{\text{occ}}(\delta)$ has a nearly exponential dependence near δ_c. As δ is varied between -0.13 and -0.131, T_{occ} decreases from 10^{-2}(of the order of $T_K^{SU(4)}$) to 10^{-4}. Note that for sufficiently negative δ ($\delta < \delta_c$), n_2 remains larger than n_1 and there is no crossing point with $n_1 = n_2$.

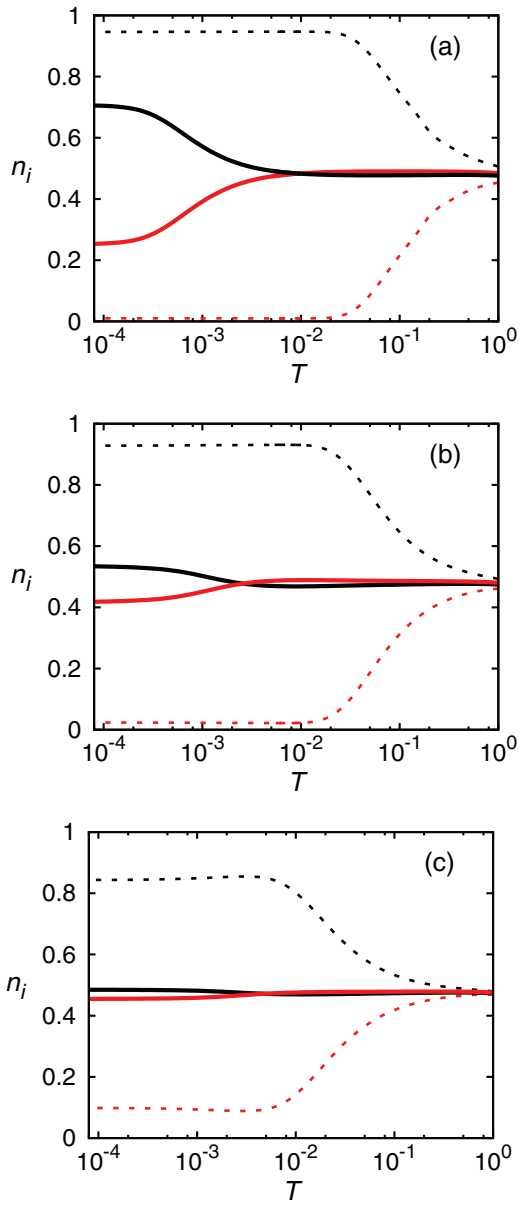

Figure 6. Total occupations of dot 1 (black, larger at lower temperature) and dot 2 (red) as a function of temperature for $E_1 = -4$, $E_2 = E_1 + \delta$, $\Gamma_1 = 1$, and (a) $\Gamma_2 = 0.5$, $\delta = -0.13$, (b) $\Gamma_2 = 0.7$, $\delta = -0.08$ and (c) $\Gamma_2 = 0.9$, $\delta = -0.03$. The case $\delta = 0$ (dashed lines) is shown for comparison.

If $n_1 = n_2$ at $T = 0$ (for $\delta = \delta_c$), the Friedel sum rule equation (3) also implies that the spectral densities at the Fermi energy are equal: $\rho_1(0) = \rho_2(0)$. This is difficult to test in a conductance measurement, because the conductance through dot i is proportional to the asymmetry factor $A_i = 4\Gamma_{Si}\Gamma_{Di}/(\Gamma_{Si} + \Gamma_{Di})^2$, and these factors are not easy to be determined precisely [28, 32]. However, as we have explained, the line shape of the spectral densities are very different in the SU(4) and SU(2) regimes, not only because of the presence of the satellite peaks but also due to the different shape of the Kondo peak, which in turn implies, for example, a different temperature dependence of the equilibrium conductances [17, 24, 28].

In figure 8 we show the evolution of the low-temperature densities of states with δ, as the crossing point $n_1 = n_2$

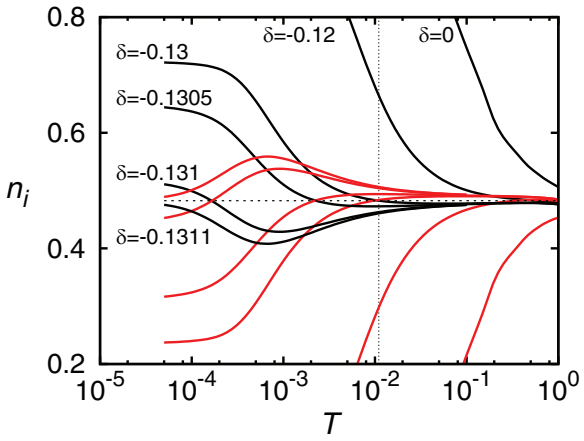

Figure 7. Total occupations of dot 1 (black, larger at low T) and dot 2 (red) as a function of temperature T for $\Gamma_2 = 0.5$, and several values of $\delta = 0,\ -0.12,\ -0.13,\ -0.1305,\ -0.131,\ -0.1311$. Other parameters as in figure 6.

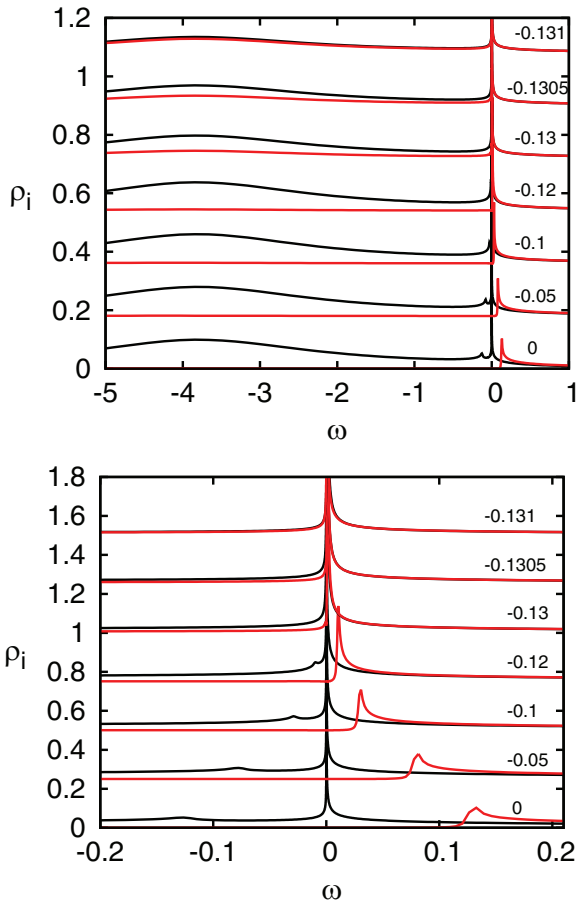

Figure 8. Spectral density of dot 1 (black, larger at low ω) and dot 2 (red) as a function of energy ω for several values of $-\delta$: 0, 0.05, 0.1, 0.12, 0.13, 0.1305 and 0.1309. The upper figure corresponds to an extended energy range. The temperature is $T \approx 5.10^{-5}$. Other parameters as in figure 7. The curves for $\delta \neq 0$ are shifted with a vertical offset for clarity.

is approached lowering δ from 0. The first rather obvious change is that as n_1 decreases and n_2 increases, the weight of the corresponding charge-transfer peaks near $\omega = E_d = -4$ changes roughly proportionally to n_i until they become almost

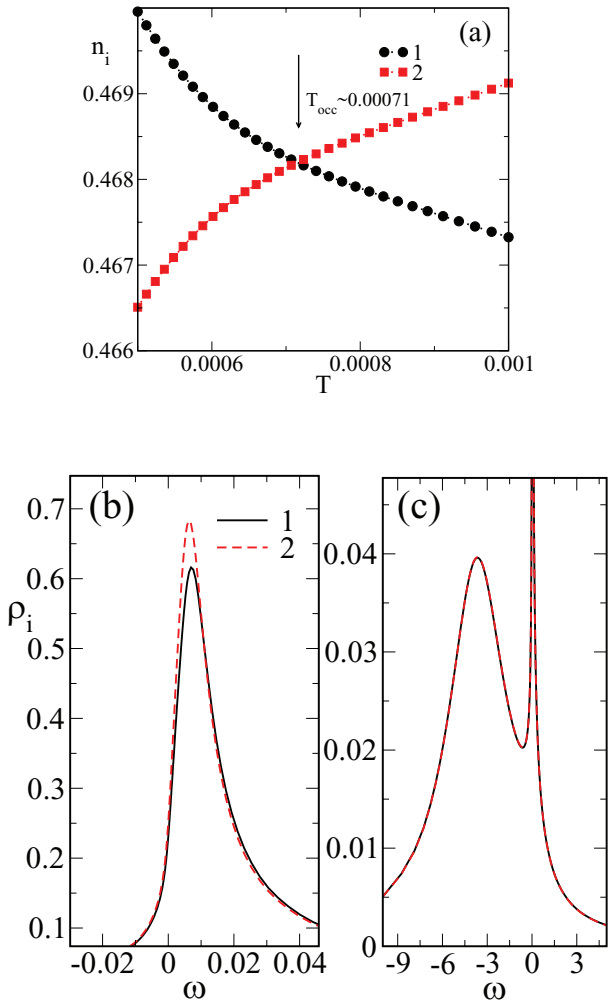

Figure 9. (a) Occupation as a function of temperature and (b), (c) spectral density at $T = T_{\rm occ}(\delta)$ as a function of energy of dot 1 (black) and dot 2 (red), for $\Gamma_1 = 1$ and $\Gamma_2 = 0.9$, $E_1 = -4$, and $\delta = -0.031$.

coincident. The changes near the Fermi energy are more subtle and they resemble the opposite of those reported in section 3: the side peaks move towards the Fermi energy, the Kondo resonance in $\rho_1(\omega)$ broadens and displaces partially to higher energies, a Kondo resonance appears in $\rho_2(\omega)$ and both densities tend to merge.

As explained at the beginning of this section, we do not expect that for any parameters, $\rho_1(\omega)$ and $\rho_2(\omega)$ coincide for all energies. In figure 9 we compare both densities at $T = T_{\rm occ}(\delta)$ for a 10% mismatch in the Γ_i. The charge-transfer peaks look identical, but the maxima of the Kondo resonances differ by about 10%, being higher for dot 2 (the lowest lying and least hybridised level). The magnitude of $\rho_i(0)$, the densities at the Fermi level are also slightly different being $\rho_2(0) > \rho_1(0)$ by around 10%. This might be an effect of finite temperature or due to the inaccuracy of the NCA to reproduce the Friedel sum rule equation (3). In any case, the shape of both densities are characteristic of the SU(4) regime and they are quite similar.

Depending on the particular property that is being studied, the tuning of the parameters is slightly different to get the effective SU(4) symmetry for this property. This is illustrated in figure 10, where the densities of states are compared for two conditions different from $n_1 = n_2$ discussed above. At the top of figure 10, the parameters are tuned in such a way that the densities coincide at the Fermi level: $\rho_1(0) = \rho_2(0)$. This condition renders the densities of states very similar in the whole energy range. The occupations are slightly different, $n_1 = 0.470$ and $n_2 = 0.465$, signaling a deviation from the Friedel sum rule, equation (3) is valid at $T = 0$. The maximum of the Kondo resonance of the doublet 2 (that with the lowest energy) is higher, although both maxima lie nearly at the same position, and the shape of the resonance corresponds to the SU(4) Kondo effect. However, the half widths at half maximum are slightly different: $T_{\rm K1} = 0.0098$ and $T_{\rm K2} = 0.0091$ for doublets 1 and 2, respectively.

At the bottom of figure 10, the parameters are chosen to get the same value of the maximum of the Kondo resonance $\rho_1^{\rm max} = \rho_2^{\rm max}$. When this condition is satisfied, the weight of the charge transfer peak (and the corresponding occupation n_i) for each doublet differs, being larger for dot 1. The resonance in ρ_2 is displaced slightly to the right with respect to ρ_1. However, both densities of states near the Fermi level are very similar. This implies that in suitable conductance experiments, the conductance through both dots $G_i(V)$ are proportional. These experiments correspond to asymmetric arrangements such that the coupling to the source and drain leads differs by a factor of an order of 10 or more, and $G_i(V) = {\rm d}I_i/{\rm d}V_i$ is measured, where I_i is the current through dot i and V_i is the voltage of the lead (source or drain) less coupled to dot i [29, 44]. This is a situation similar to that in scanning-tunneling-spectroscopy experiments. In the conditions at the bottom of figure 10, $G_1(V)/G_2(V) = A_1/A_2$, where the constant asymmetry factors are $A_i = 4\Gamma_{\rm Si}\Gamma_{\rm Di}/(\Gamma_{\rm Si} + \Gamma_{\rm Di})^2$.

6. Summary and discussion

We have considered an Anderson model that describes two capacitively coupled quantum dots, each one connected to a drain and a source lead in the Kondo regime with one electron (or hole) added to a singlet configuration. We have investigated the possibility that the SU(4) symmetry, lost at the Hamiltonian level when the total couplings to the leads are different ($\Gamma_1 \neq \Gamma_2$), can be restored at low energies as an emergent symmetry [40], by changing the difference of on-site energies $\delta = E_2 - E_1$. We find that for small temperatures (specifically lower that the Kondo temperature of the SU(4) Kondo effect $T_{\rm K}^{\rm SU(4)}$), it is possible to tune δ such that the Kondo resonances for each dot sensed by suitably chosen conductance experiments are proportional. Specifically at this value of δ, the conductance through each dot $G_i(V)$ in a configuration of voltages similar to those used in scanning-tunneling-spectroscopy experiments, have the same line shape within experimental errors and reflect the characteristic shape of the SU(4) Kondo resonance in the spectral density. However, for this value of δ, the total occupations n_i for each dot are slightly different, indicating the absence of full SU(4) symmetry at large energies.

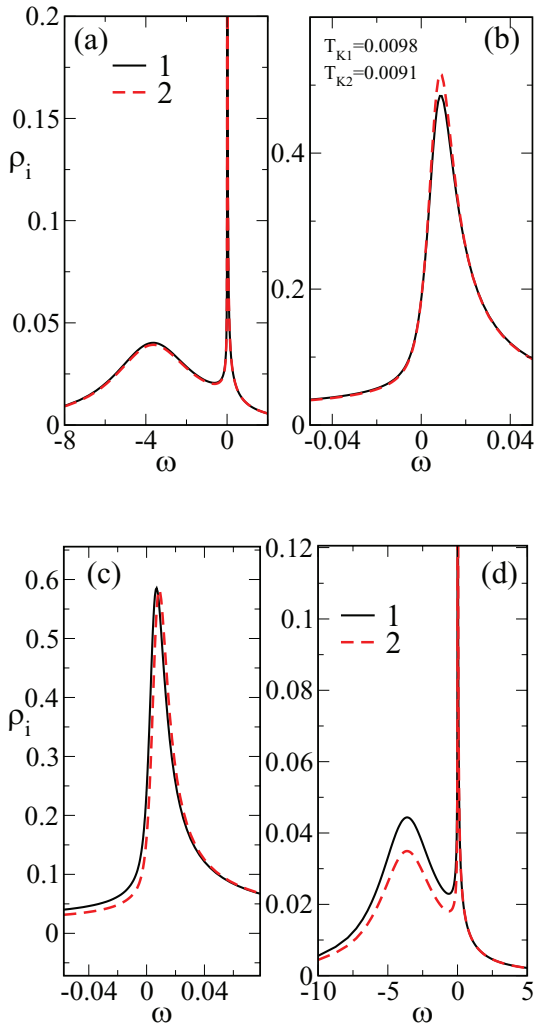

Figure 10. Spectral density of dot 1 (black, solid) and dot 2 (red, dashed) as a function of energy for $\Gamma_1 = 1, \Gamma_2 = 0.9, E_1 = -4$ and (a), (b): $\delta = -0.0305$ and $T = 0.37\ 10^{-2}$, (c), (d): $\delta = -0.029$ and $T = 0.25\ 10^{-2}$.

The temperature dependence of the conductances at not too high temperatures also corresponds to the SU(4) regime rather that the SU(2) one, since it is given by the energy and temperature dependence of the densities ρ_i. However, slight differences in the line shape of both conductances can appear as a function of temperature, because both spectral densities and occupations do not have exactly the same temperature dependence.

The present study assumed infinite intradot (U_i) and interdot (U_{12}) Coulomb repulsions, which in practice means that only two configurations with N and $N + 1$ (or $N - 1$) particles are included, and all other configurations lie high in energy and can be neglected. This can be adjusted experimentally for low enough Γ_i. When in addition to the singlet with N particles and the four states with $N + 1$ ($N - 1$) particles, configurations with $N + 2$ ($N - 2$) particles are important, the system has SU(4) symmetry only for $\Gamma_1 = \Gamma_2$ and $U_{12} = U_i$. In this case the sector with $N \pm 2$ particles is six-fold degenerate. In practice, $U_{12} < U_i$ reduces the symmetry to SU(2) × SU(2) if $U_1 = U_2$ and to SU(2) if in addition either $U_1 \neq U_2$ or $\Gamma_1 \neq \Gamma_2$. In these conditions, restoring SU(4) symmetry at low energies tuning the on-site energies E_i is not possible.

From the theoretical point of view it remains to study more accurately with alternative techniques, to what extent the SU(4) symmetry is kept at the lowest energies. The NCA is not reliable at temperatures well below the Kondo one, where the Friedel sum rule is not reproduced with a deviation of about 15% at very low temperatures. Combining NRG and RPT, the low-energy Fermi-liquid properties and the symmetry of the low-energy effective Hamiltonian might be studied in detail. A difficulty for numerical studies is the fine tuning in δ required to obtain a manifestation of SU(4) symmetry in a given property.

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