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Transverse vibrations of circular annular plates with edges elastically restrained against rotation, used in acoustic underwater transducers

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Abstract

Simply supported or clamped boundary conditions are rather ideal situations difficult to satisfy from a physical viewpoint. This paper considers a more “moderate” restriction: the case of edges elastically restrained against rotation for which no exact solution appears in the open literature. Eigenvalues corresponding to a wide range of the intervening geometric and mechanical parameters are determined. Good agreement is obtained with frequency coefficients determined two decades ago by means of a variational method. Obviously the problem is of basic interest in many ocean engineering applications: from the design of certain underwater acoustic transducers to pump and compressor elements passing through the design of naval vehicles and ocean structures.

Keywords: Vibrations; Circular plates; Underwater transducer

1. Introduction

The problem of transverse vibrations of thin, elastic, circular, annular plates requires the solution of transcendental equations involving regular and modified Bessel functions as explained in Leissa’s classical treatise (Leissa, 1969).

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Extensive, accurate data is available for classical boundary conditions: simply supported, clamped, clamped and free while a limited amount of information exists for the intermediate situation between simply supported and clamped. In many instances the annular plate is supported by a structure of considerable rigidity and one can safely assume that the transverse displacement of the plate at the edges is null but, on the other hand, a certain degree of rotation is allowed. It is assumed in the present work that each plate edge possesses a flexibility coefficient (see Fig. 1) in such a manner that the boundary conditions are:

$$W(r = a,\theta) = 0$$
$$W(r = b,\theta) = 0 \tag{1}$$

$$\frac{\partial W(r = a,\theta)}{\partial r} = \phi_a D[\frac{\partial^2 W}{\partial r^2} + \nu\left(\frac{1\partial W}{r\,\partial r} + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2}\right)]_{r=a}$$
$$\frac{\partial W(r = b,\theta)}{\partial r} = -\phi_b D[\frac{\partial^2 W}{\partial r^2} + \nu\left(\frac{1\partial W}{r\,\partial r} + \frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2}\right)]_{r=b} \tag{2}$$

Equations (2) are the constitutive relations which define ϕ_a and ϕ_b. In practice ϕ_a and ϕ_b can be determined using a numerical approach by studying the type of connection between the plate and its supporting structure or experimentally, measuring by accurate optical means, the angle of edge rotation under an applied radial moment under static or dynamic conditions.

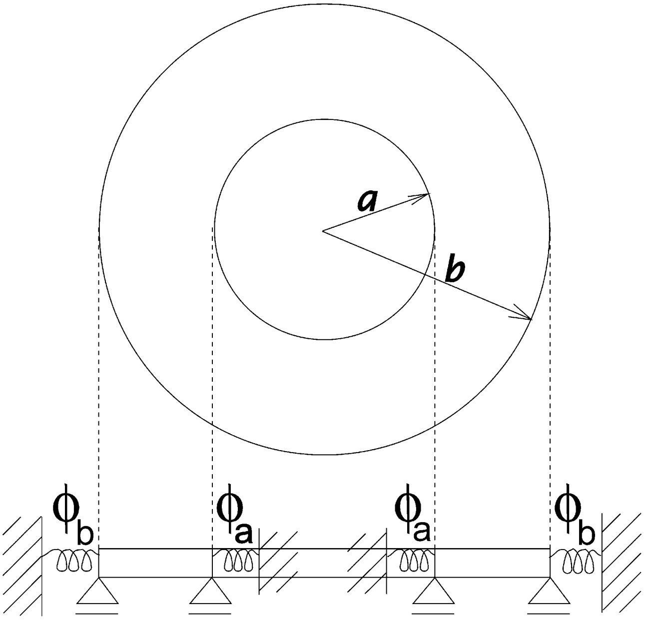

Fig. 1. Mechanical system under study.

Obviously, determining accurate natural frequencies and mode shapes of such basic structural elements is of fundamental interest in many ocean engineering situations where they are employed: from transducers to naval vehicles passing through mechanical applications in engines, pumps, compressors, etc.

2. Basic theory

In the case of normal modes of vibration the displacement amplitude $W(r,\theta)$ must satisfy the partial differential equation:

$$\nabla^4 W(r,\theta) - \frac{\rho h \omega^2}{D} W(r,\theta) = 0 \tag{3}$$

and the governing boundary conditions (1) and (2).

In terms of Bessel functions the solution is (Leissa, 1969)

$$W(r,\theta) = \sum_{n=1}^{\infty} [A_n J_n(kr) + B_n Y_n(kr) + C_n I_n(kr) + D_n K_n(kr)]\cos(n\theta) \tag{4}$$

Table 1
The fundamental frequency coefficient and the first two corresponding to antisymmetric modes of circular, annular plates in the case of ideal boundary conditions, Ω_{0n}

	Inside (r=a)		Outside (r=b)	
Case I	Simply supported		Simply supported	
Case II	Simply supported		Clamped	
Case III	Clamped		Simply supported	
Case IV	Clamped		Clamped	
	a/b	n=0	n=1	n=2
Case I	0.1	14.438	16.763	25.966
	0.3	21.035	23.287	30.270
	0.5	40.011	41.769	47.070
	0.7	110.0389	111.419	115.564
Case II	0.1	22.584	25.209	35.392
	0.3	33.652	35.807	42.666
	0.5	63.856	65.373	70.036
	0.7	174.241	175.363	178.717
Case III	0.1	17.837	19.440	26.758
	0.3	30.036	31.459	36.295
	0.5	59.898	61.064	64.705
	0.7	168.645	169.614	172.536
Case IV	0.1	27.280	28.915	36.617
	0.3	45.346	46.643	51.138
	0.5	89.250	90.230	93.321
	0.7	248.402	249.164	251.480

Table 2
The fundamental frequency coefficient and the first two corresponding to antisymmetric modes of circular, annular plates in the case where the inner edge is simply supported while the outer edge is elastically restrained, Ω_{0n}

	Inside (r=a)		Outside (r=b) $b/\phi D$	
Case V	SS		1	
Case VI	SS		10	
Case VII	SS		30	
Case VIII	SS		100	
	a/b	n=0	n=1	n=2
Case V	0.1	15.612	17.897	27.024
	0.3	22.525	24.704	31.533
	0.5	42.070	43.769	48.918
	0.7	113.426	114.772	118.818
Case VI	0.1	19.555	21.911	31.262
	0.3	28.248	30.310	36.886
	0.5	51.566	53.097	57.796
	0.7	132.929	134.128	137.747
Case VII	0.1	21.248	23.731	33.463
	0.3	31.137	33.226	39.887
	0.5	57.622	59.120	63.728
	0.7	150.122	151.251	154.664
Case VIII	0.1	22.133	24.706	34.721
	0.3	32.778	34.906	41.683
	0.5	61.573	63.077	67.703
	0.7	164.451	165.558	168.906

and the secular determinant is obtained substituting (4) in (1) and (2), see Appendix. The procedure is greatly facilitated by the use of MAPLE (we used MAPLE V5, from 1991). The eigenvalue k expresses the identity

$$k = \sqrt{\sqrt[4]{\frac{\rho h}{D}}\sqrt{\omega}} \tag{5}$$

where ρ is the density of the plate material, h is the plate thickness, D is the flexural rigidity and ω the circular natural frequency of the structural system.

3. Numerical results and conclusions

The eigenvalues have been determined for $\nu = 1/3$and b/a=0.1, 0.3, 0.5 and 0.7.

The natural frequency coefficients, $\Omega_{0n} = \sqrt{\rho h/D}\ \omega_{0n}a^2$, have been evaluated fixing the adimensionless flexibility $a/\phi D$ and varying the ratio of inner to outer radius, a/b. It was considered of practical use to obtain the fundamental frequency coef-

Table 3
The fundamental frequency coefficient and the first two corresponding to antisymmetric modes of circular, annular plates in the case where the outer edge is simply supported while the outer inner is elastically restrained, Ω_{0n}

	Inside (r=a) $a/\phi D$		Outside (r=b)	
Case IX	1		SS	
Case X	5		SS	
Case XI	10		SS	
Case XII	30		SS	
	a/b	n=0	n=1	n=2
Case IX	0.1	15.987	17.813	26.175
	0.3	23.304	25.220	31.479
	0.5	42.793	44.403	49.322
	0.7	114.066	115.391	119.375
Case X	0.1	17.183	18.807	26.474
	0.3	26.684	28.257	33.628
	0.5	48.917	50.282	54.529
	0.7	125.798	126.987	130.576
Case XI	0.1	17.475	19.081	26.585
	0.3	27.978	29.470	34.585
	0.5	52.315	53.586	57.561
	0.7	134.861	135.969	139.320
Case XII	0.1	17.708	19.309	26.690
	0.3	29.228	30.669	35.593
	0.5	56.513	57.709	61.451
	0.7	150.361	151.373	154.441

ficient, Ω_{00}, and the first two coefficients corresponding to antisymmetric modes, Ω_{01} and Ω_{02}.

The values of Ω_{00} previously determined in the literature (Avalos and Laura, 1979) were compared with those obtained in the present study and excellent agreement was observed. Since the Galerkin method was used in that study they constitute upper bounds with respect to the ones obtained in the present investigation which are evaluated using the exact approach. The cases of classical boundary conditions are treated in Table 1 and agree with those obtained recently (Vera et al., 1998).

Table 2 deals with the case where r=a defines a simply supported edge while $b/\phi D$ varies.

Table 3 depicts the eigenvalues for the situation where the outer edge is simply supported while $a/\phi D$ varies.

The case of an inner edge clamped while the outer edges are elastically restrained against rotation is dealt with in Table 4.

Finally, Table 5 shows natural frequency coefficients for the annular plate with a clamped outer edge while $a/\phi D$ varies for the inner boundary.

It is observed that for the cases under study the variation of the frequency coefficients is significant for $10 \leq a/\phi D \leq 100$ or $10 \leq b/\phi D \leq 100$ when $a/b > 0.5$. From a

Table 4
The fundamental frequency coefficient and the first two corresponding to antisymmetric modes of circular, annular plates in the case where the inner edge is clamped while the outer edge is elastically restrained, Ω_{0n}

	Inside (r=a)		Outside (r=b) $b/\phi D$	
Case XIII	C		1	
Case XIV	C		5	
Case XV	C		10	
Case XVI	C		30	
	a/b	n=0	n=1	n=2
Case XIII	0.1	19.086	20.638	27.846
	0.3	31.619	32.993	37.698
	0.5	62.079	63.209	66.747
	0.7	172.227	173.171	176.032
Case XIV	0.1	21.927	23.428	30.551
	0.3	35.641	36.930	41.402
	0.5	68.313	69.362	72.666
	0.7	183.844	184.729	187.418
Case XV	0.1	23.554	25.0629	32.252
	0.3	38.254	39.5164	43.912
	0.5	72.995	74.0024	77.184
	0.7	194.202	195.045	197.609
Case XVI	0.1	25.605	27.165	34.569
	0.3	41.944	43.206	47.596
	0.5	80.700	81.673	84.747
	0.7	215.471	216.255	218.642

practical viewpoint when the flexibility parameter is less than 10 the edge behaves as a quasi-simply supported edge and when it is larger than 100 it approaches the clamped case.

4. Application of the results to ocean technologies

The present study was undertaken motivated by the need of improving the performance of certain economic underwater acoustic transducers. Obviously the technological applications of the results obtained in the present investigation are very large in number: from portions of ship decks to printed circuit boards passing through nuclear reactor elements and offshore platforms.

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Table 5
The fundamental frequency coefficient and the first two corresponding to antisymmetric modes of circular, annular plates in the case where the inner edge is elastically restrained while the outer edge is clamped, Ω_{0n}

	Inside (r=a) $a/\phi D$		Outside (r=b)	
Case XVII	1		C	
Case XVIII	5		C	
Case XIX	10		C	
Case XX	30		C	
	a/b	n=0	n=1	n=2
Case XVII	0.1	24.625	26.612	35.712
	0.3	36.299	38.145	44.269
	0.5	66.950	68.354	72.702
	0.7	178.606	179.677	182.918
Case XVIII	0.1	26.314	28.001	36.175
	0.3	40.617	42.099	47.241
	0.5	74.284	75.473	79.209
	0.7	191.924	192.893	195.833
Case XIX	0.1	26.742	28.394	36.347
	0.3	42.389	43.776	48.612
	0.5	78.663	79.759	83.219
	0.7	202.834	203.735	206.473
Case XX	0.1	27.086	28.723	36.511
	0.3	44.165	45.486	50.090
	0.5	84.386	85.400	88.607
	0.7	222.770	223.582	226.052

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Appendix: Frequency determinant

The following equation is the frequency determinant:

$$\begin{vmatrix} J_n(ka) & I_n(ka) \\ J_n(kb) & I_n(kb) \\ \left. \phi_a D\frac{\mathrm{d}^2J_n(kr)}{\mathrm{d}r^2} + (\frac{\phi_a D\nu}{r}-1)\frac{\mathrm{d}J_n(kr)}{\mathrm{d}r}\right|_{r=a} & \left. \phi_a D\frac{\mathrm{d}^2I_n(kr)}{\mathrm{d}r^2} + (\frac{\phi_a D\nu}{r}-1)\frac{\mathrm{d}I_n(kr)}{\mathrm{d}r}\right|_{r=a} \\ \left. \phi_b D\frac{\mathrm{d}^2J_n(kr)}{\mathrm{d}r^2} + (\frac{\phi_b D\nu}{r}+1)\frac{\mathrm{d}J_n(kr)}{\mathrm{d}r}\right|_{r=b} & \left. \phi_b D\frac{\mathrm{d}^2I_n(kr)}{\mathrm{d}r^2} + (\frac{\phi_b D\nu}{r}+1)\frac{\mathrm{d}I_n(kr)}{\mathrm{d}r}\right|_{r=b} \end{vmatrix}$$

$$
\begin{vmatrix}
\cdots & Y_n(ka) & \cdots & K_n(ka) \\
\cdots & Y_n(kb) & \cdots & K_n(kb) \\
\cdots & \phi_a D\frac{d^2Y_n(kr)}{dr^2} + \left(\frac{\phi_a D\nu}{r} - 1\right)\frac{dY_n(kr)}{dr}\Big|_{r=a} & \cdots & \phi_a D\frac{d^2K_n(kr)}{dr^2} + \left(\frac{\phi_a D\nu}{r} - 1\right)\frac{dK_n(kr)}{dr}\Big|_{r=a} \\
\cdots & \phi_b D\frac{d^2Y_n(kr)}{dr^2} + \left(\frac{\phi_b D\nu}{r} + 1\right)\frac{dY_n(kr)}{dr}\Big|_{r=b} & \cdots & \phi_b D\frac{d^2K_n(kr)}{dr^2} + \left(\frac{\phi_b D\nu}{r} + 1\right)\frac{dK_n(kr)}{dr}\Big|_{r=b}
\end{vmatrix} **** = 0
$$

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