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RESEARCH PAPER

Crack identification using model reduction based on proper orthogonal decomposition coupled with radial basis functions

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Abstract This work deals with the crack identification using model reduction based on the proper orthogonal decomposition method. The proposed inverse problem consists of the estimation of the crack length and its position in a plate using boundary displacements as input data. Genetic algorithm and particle swarm optimization were applied for the minimization of the error function expressed as the difference between the boundary displacements of the actual crack and those of the estimated crack. It was found that the proposed approach is able to accurately estimate crack size and detect its location. The stability of the identification algorithm was tested against measurement uncertainty by introducing a white Gaussian noise in the input data. The approach showed high stability for noise levels lower than 5 %. The efficiency of the approach using small number of sensor points was also demonstrated.

Keywords Crack identification · Inverse problem · Proper orthogonal decomposition · Radial basis functions · Genetic algorithm · Particle swarm optimization

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1 Introduction

Lifetime of structures is limited by the initiation and propagation of cracks. Therefore, evaluation of the integrity of these structures is crucial throughout the civil, mechanical, and aerospace engineering communities. It could be experimentally examined using, for instance, non-destructive testing (NDT) techniques such as acoustic or ultrasonic methods, magnetic field methods, radiography, eddy-current methods or thermal field methods (Ness et al. 1996). Unfortunately, these methods present some restrictions such as the requirement of a priori knowledge of the vicinity of the damage and that the portion of the structure being inspected is readily accessible. The need for quantitative global damage detection methods that can be applied to complex structures has led to the development of several numerical methods, including vibration based approaches, to identify crack parameters (size, position and orientation), as summarized in (Doebling et al. 1998). These methods examine changes in the vibration characteristics of the structure. The basic idea is that modal parameters including notably frequencies, mode shapes, and modal damping, are functions of the physical properties of the structure, i.e. mass, damping, and stiffness. Therefore, changes in these physical properties, such as reductions in stiffness resulting from the onset of cracks or loosening of connection, will cause detectable modifications in the modal properties.

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Because changes in modal properties or in derived quantities are used as indicators of damage, the process of vibration-based damage detection is reduced to some form of a pattern recognition problem. It is evident that these methods make use of inverse problem techniques, motivated by the need to overcome the lack of information concerning the properties of the system.

Some theories and approaches of inverse crack identification in the case of elastostatic problems are presented in (Bonnet and Constantinescu 2005; Bui 2007; Stavroulakis 2000). In these problems, defect parameters are unknown but the displacements along the boundaries are accessible. Inverse methods based on boundary data, which aim to identify the crack parameters using experimentally measured or numerically evaluated boundary displacements, have found broad applications (Alessandri and Mallardo 1999; Amoura et al. 2010; Burczynski and Beluch 2001; Hattori and Sáez 2013; Mellings and Aliabadi 1995; Vossou et al. 2007). Predictions of unknown defect parameters using inverse identification approaches are based on a function that compares actual and calculated input data, at selected sensor points. This function, named the fitness function, is minimized with an optimization method.

Classical optimization methods are fast, but they suffer from big limitations related to the continuity of the objective function, the Hessian of the objective function which might not be positive-defined in all points during optimization, and the substantial possibility of getting a local optimum which strongly depends on starting point (Venter 2010). The Genetic Algorithm (GA) (Gen and Cheng 2000) and the Particle Swarm Optimization (PSO) (Kennedy 2010) are well-known optimization techniques that are free from the mentioned restrictions. Contrary to classical methods where the gradient of the objective function is computed, GA and PSO depend only on the fitness function and the search domain.

As crack identification is an iterative operation, numerical simulation methods are costly in terms of computing time. Model reduction techniques are alternative methods that reduce computing time without accuracy lost. In fact, they were widely employed in the last decade, mainly because of their speed (Schilders et al. 2008). Among model reduction techniques is proper orthogonal decomposition (POD), which is a powerful and elegant method allowing approximation of a problem (making a low dimensional descriptions of high-dimensional processes) making use of the most appropriate set of approximation functions (Chatterjee 2000). The POD approach has been applied for the detection and location of damage in beam structures using vibration data (Galvanetto and Violaris 2007; Lanata and Del Grosso 2006; Shane and Jha 2011). This approach, coupled with the Radial Basis Functions (RBF), was also employed in elastostatic inverse problems for determining the Young modulus, yield stress and hardening coefficient of material, using instrumented indentation tests data (Bolzon et al. 2011; Buljak and Maier 2011; Hoang et al. 2013). The POD-RBF network has been compared to other methods in (Bolzon and Buljak 2011) and its accuracy has been proved (Buljak 2011; Buljak and Maier 2011; Rogers et al. 2012).

This work deals with crack identification using model reduction based on POD-RBF approach combined with GA and PSO methods, making use of structure boundary displacements. POD technique was used to reduce the dimensions of the matrix containing the boundary displacements calculated by Finite Element Method (FEM) and corresponding to different crack configurations. RBF method allowed the calculation of the boundary displacements related to unknown cracks, more quickly than using Finite Element Analysis (FEA). GA and PSO were used as optimization methods in the identification procedure.

To our best knowledge, the proposed approach has never been applied for structure crack identification. It aimed to be a non-destructive and continuous lifetime monitoring method. It reduces computational cost, simplifies crack identification algorithms and gives the possibility to execute inverse computation with models based on experimental input data. This brings out a new dimension to the inverse problem field, particularly in the crack identification case.

2 Problem description

In elastostatic based inverse crack identification methods, the deformation of the studied structure is used to predict crack parameters, i.e. size and position, based in the fact that crack changes the behavior of the structure under loading. Boundary displacements are affected by variations in crack's size and position and each given crack is related to a unique boundary displacement field. Inverse identification of cracks is done by comparing boundary displacements of unknown crack to the ones of known cracks which are introduced as input data.

In elastostatic problems, mechanical behavior is governed by elastic linear equations satisfying continuity and compatibility conditions, which depends on the elastic properties of the material (Bonnet and Constantinescu 2005; Stavroulakis 2000).

In this study, a 40 mm × 40 mm plate (Fig. 1) was modeled under tension with FEM, using ABAQUS code. Uniform stress was imposed at the upper and lower sides of the specimen. Each plate edge was discretized with 80 quadrilateral elements. Material's Young modulus and Poisson coefficient were respectively $E = 210$ GPa and $\nu = 0.3$.

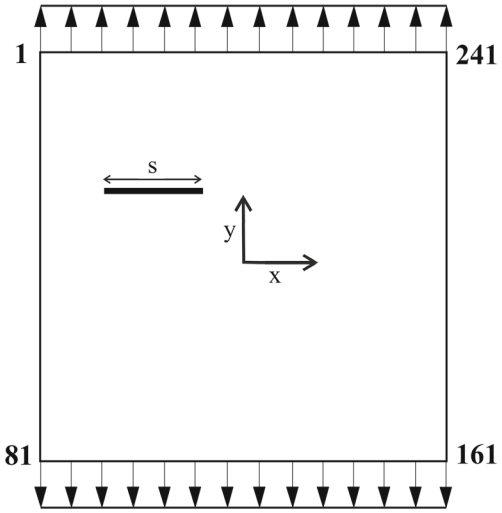

Fig. 1 Cracked plate specimen under tension

3 POD-RBF as model reduction method

POD was applied to determine the boundary displacement of an elastic structure containing an unknown crack. It is a powerful statistical method used for data analysis as model order reduction technique in different fields (Liang et al. 2002a, b; Chatterjee 2000; Schilders et al. 2008). The mathematical procedure of POD adopted herein is consistent with the approach called Principal Component Analysis (PCA), described with details by Liang et al. (2002a, b). The model reduction described by Buljak and Maier (2011) was here adopted and is then briefly summarized below.

The main purpose of the POD method is to propose a set of orthogonal vectors $\mathbf{\Phi}$ called POD basis vectors, to reassemble the snapshot matrix $\mathbf{U}$ in an optimal way. These so-called snapshots are the boundary nodal displacements corresponding to a set of known cracks used as input data, stored in matrix $\mathbf{U}$ as:

$$\mathbf{U} = \begin{bmatrix} u_1^1 & u_1^2 & \cdots & u_1^S \\ u_2^1 & u_2^2 & & u_2^S \\ \vdots & \vdots & \ddots & \vdots \\ u_N^1 & u_N^2 & \cdots & u_N^S \end{bmatrix}, \tag{1}$$

where S is the number of sensor points and N represents the number of snapshot vectors $\mathbf{U}_i$, which represent the boundary displacement field of each crack configuration. The orthogonal vectors $\mathbf{\Phi}$ are related to $\mathbf{U}$ linearly:

$$\mathbf{U} = \mathbf{\Phi}\cdot\mathbf{A}, \tag{2}$$

where $\mathbf{A}$, the amplitude matrix collecting the coefficients of the new basis combination, is computed as:

$$\mathbf{A} = \mathbf{\Phi}^{\mathrm{T}}\cdot\mathbf{U}, \tag{3}$$

due to the orthogonality of $\mathbf{\Phi}$. Optimal basis vectors are defined by the performance of the POD method, also known as the singular value decomposition operation (Buljak and Maier 2011):

$$\mathbf{\Phi} = \mathbf{U}\cdot\mathbf{V}\cdot\mathbf{\Lambda}^{-1/2}, \tag{4}$$

where $\mathbf{V}$ is the matrix storing the normalized eigenvectors of the covariance matrix $\mathbf{C}$, and $\mathbf{\Lambda}$ the diagonal matrix storing its eigenvalues. The matrix $\mathbf{C}$ is given by the following equation (Buljak and Maier 2011):

$$\mathbf{C} = \mathbf{U}^{\mathrm{T}}\cdot\mathbf{U}, \tag{5}$$

Low dimensional approximation $\hat{\mathbf{\Phi}}$ of high accuracy is extracted from $\mathbf{\Phi}$, by preserving only K ($K \ll N$) columns that correspond to the largest eigenvalues. Since the eigenvalues of the covariance matrix $\mathbf{C}$ are stored in a descending order, POD directions that hold little information can be discarded without influencing the accuracy of the representation. This is known as the truncation of the POD basis, and is accomplished by choosing the fraction of system that can be neglected in later calculations. Consequently, the amplitude matrix $\hat{\mathbf{A}}$ is given by:

$$\hat{\mathbf{A}} = \hat{\mathbf{\Phi}}^{\mathrm{T}}\cdot\mathbf{U}, \tag{6}$$

since:

$$\mathbf{U} = \hat{\mathbf{\Phi}}\cdot\hat{\mathbf{A}}. \tag{7}$$

To determine the boundary displacement field of a two dimensional elastic structure containing an unknown crack, RBF interpolation was used, method that can generate response of the system corresponding to the different parameter sets not included in the initial selection $\mathbf{P}$. The matrix $\mathbf{P}$ stores the crack parameters sets $\mathbf{P}_i$ of all simulations, considered in our study as the crack's center position and the crack size. The amplitude matrix $\hat{\mathbf{A}}$ is defined as a multiplicative form of the function $\mathbf{G}$, defined as the matrix of interpolation parameters, and the matrix $\mathbf{B}$ containing the unknown coefficients:

$$\hat{\mathbf{A}} = \mathbf{B}\cdot\mathbf{G}. \tag{8}$$

The interpolation functions are expressed by (Buljak and Maier 2011; Ostrowski et al. 2008; Rogers et al. 2012), as follows:

$$\mathbf{g}_i = \mathbf{g}_i(|\mathbf{P}-\mathbf{P}_i|) = \frac{1}{\sqrt{|\mathbf{P}-\mathbf{P}_i|^2 + c^2}}, \tag{9}$$

$\mathbf{P}_i$ is the crack parameter corresponding to $\mathbf{U}_i$ ($i=1,2,\ldots,N$). The argument of the i-th RBF is the distance $|\mathbf{P}-\mathbf{P}_i|$, $\mathbf{P}$ and $\mathbf{P}_i$ being respectively current and reference parameters. c is the

RBF smoothing factor defined in the range from 0 to 1. If the knot points $\mathbf{P}_i$ or some of them are relatively close to each other, the matrix **G** could be singular, which can be circumvented by reducing the c value. After the evaluation of the coefficient matrix **B**, a low-dimensional model issued from (8) can be written in the following vector form:

$$\mathbf{a}(\mathbf{P}) = \mathbf{B}\cdot\mathbf{g}(\mathbf{P}), \tag{10}$$

Equation (7) can be expressed as an approximation of the snapshot **u** corresponding to a new parameter vector **P**:

$$\mathbf{u}(\mathbf{P}) = \hat{\mathbf{\Phi}}\cdot\mathbf{a}(\mathbf{P}), \tag{11}$$

This model will be referred as the trained POD-RBF network. It is capable of reproduce the unknown boundary displacement field of the structure that corresponds to any set of crack parameters **P**. It must be noted that extrapolation outside the range of **P** leads probably to poor precision of the model.

Increasing the value of the smoothing factor c leads to a better interpolation. But it can make the matrix **G** singular, depending on the closeness of knot points. In the present work, the parameter c was chosen to be constant for all functions, and equal to 0.6.

4 Implementation of the crack identification algorithm: POD-RBF coupled with GA

One of the main steps of the identification algorithm was the generation of the boundary displacements by using a reduced model instead of numerical simulations by FEM. The GA was coupled with the POD-RBF to inversely estimate the crack parameters.

GA is a general optimization method that belongs to the class of evolutionary algorithms. During the last decade, it was widely applied in various kinds of optimization problems (Abraham and Jain 2005). In a GA, feasible solutions – also called individuals – are randomly generated in the research domain. They evolve towards the better solution in an iterative process inspired in the natural evolution. Each of the possible solutions has a set of properties – or chromosomes – which are parameters generally represented in binary encoding. Individuals are allowed to reproduce and cross among themselves in order to obtain solutions with better fitness values. The highest probability of getting chosen as parent of new individuals is given to the best feasible solutions. The parent properties are combined by exchanging chromosome parts, producing new designs. Then, a possibility of mutation is imposed on the resulting individuals, which arbitrary changes digits inside a randomly selected chromosome. These basic operators are used as reference to the next iteration containing the next generation of the same size and with better fitness. This process is continued until stopping criterion is satisfied, commonly until a maximum number of generations is reached or a satisfactory fitness value has been achieved (Gen and Cheng 2000).

The vector of boundary displacement caused by the crack identity desired to be identified, was considered as a reference. It was compared with all boundary displacement vectors generated by the crack parameters proposed by the GA optimization method. With the aim of finding a vector displacement close to the reference, the minimization of the fitness function f (**P**) was done iteratively. This fitness function is defined as the error between **u**(**P**) and **u**($\mathbf{P}_0$) and calculated from the following equation:

$$\begin{cases} f(\mathbf{P}) = \dfrac{\|\mathbf{u}(\mathbf{P}_0) - \mathbf{u}(\mathbf{P})\|^2}{\|\mathbf{u}(\mathbf{P}_0)\|^2} \\ f\left(\mathbf{P}_{\text{optimal}}\right) = \min[f(\mathbf{P})] \end{cases}, \tag{12}$$

u($\mathbf{P}_0$) being the boundary displacements corresponding to the real crack.

These stages of the identification algorithm can be summarized as follows:

1. Creation of a starting population of N individuals, created in real encoding as a random generation. Each individual has three chromosomes corresponding to the crack parameter set **P**[x, y, s], where (x, y) represent the coordinates of the crack center and s is the length of the crack.
2. Evaluation of each individual by introducing the proposed parameters into the trained POD-RBF network that generates the corresponding boundary displacement vector **u**(**P**).
3. If the maximum number of generations or defined fitness level is reached, the algorithm is stopped, else it continues.
4. Storing the population according to their fitness value and then ranked. A proportion for breeding a new generation is selected. The top ranked populations are more likely to be selected.
5. Crossover of individuals to produce the population of the next generation.
6. Mutation of a specified percentage of the resulting population.
7. Replacement of the old population by new one and coming back to the step 2.

Through series of identification tests, the following genetic parameters were chosen based on the accuracy of results: population size = 1000, crossover rate = 0.8 i.e. 800 individuals were selected for crossover, the mutation rate = 0.01. The mutation was used to avoid the convergence of the solution toward local optimums by creating diversity.

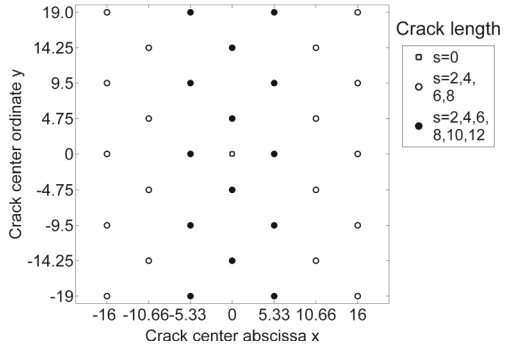

Fig. 2 Crack parameters sets considered in the case of the rectangular plate specimen

5 Identification of the size and position of the crack

This study is based on the y displacements of all boundary nodes, called snapshots. Crack parameter sets were defined in the ranges of −16 and 16 mm for the crack center abscissa x, −19 and 19 mm for the crack center ordinate y, and 0 and 12 mm in step of 2 mm for the crack size s. The considered crack parameter sets, corresponding to the 157 snapshots of the used matrix **U**, are depicted in Fig. 2.

The origin of the coordinate system was taken at the center of the plate (Fig. 1). We have used a reduced model containing 20 POD bases, chosen by applying the criteria that impose to the ratio between the eigenvalues of the neglected vectors and the largest of the retained ones to be less than 10^{-6}.

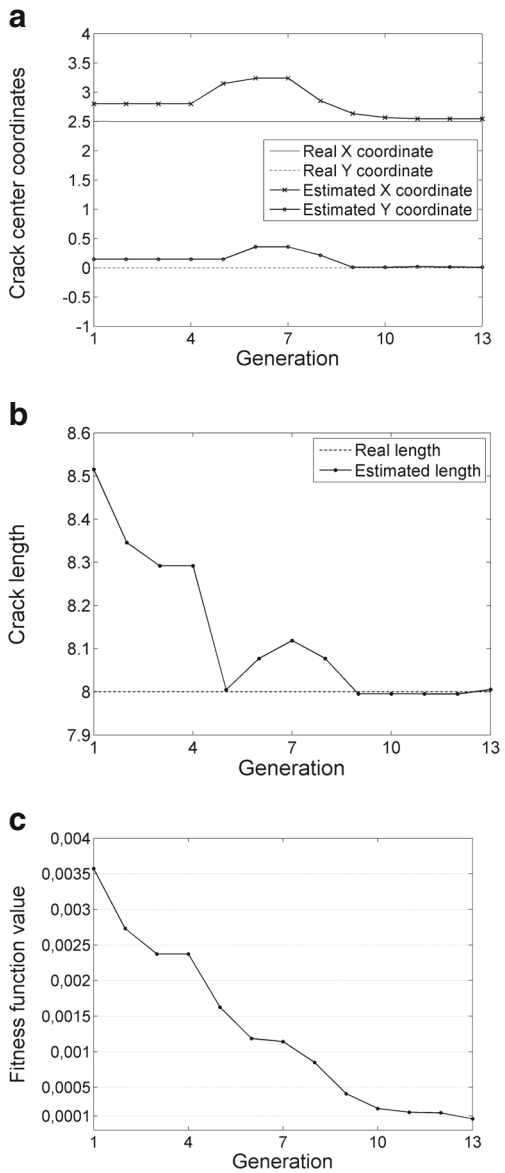

Fig. 3 Results for a crack with parameter $\mathbf{P}_1[2.5, 0, 8]$: (**a**) crack center coordinates, (**b**) crack length, and (**c**) fitness values, as a function of generation

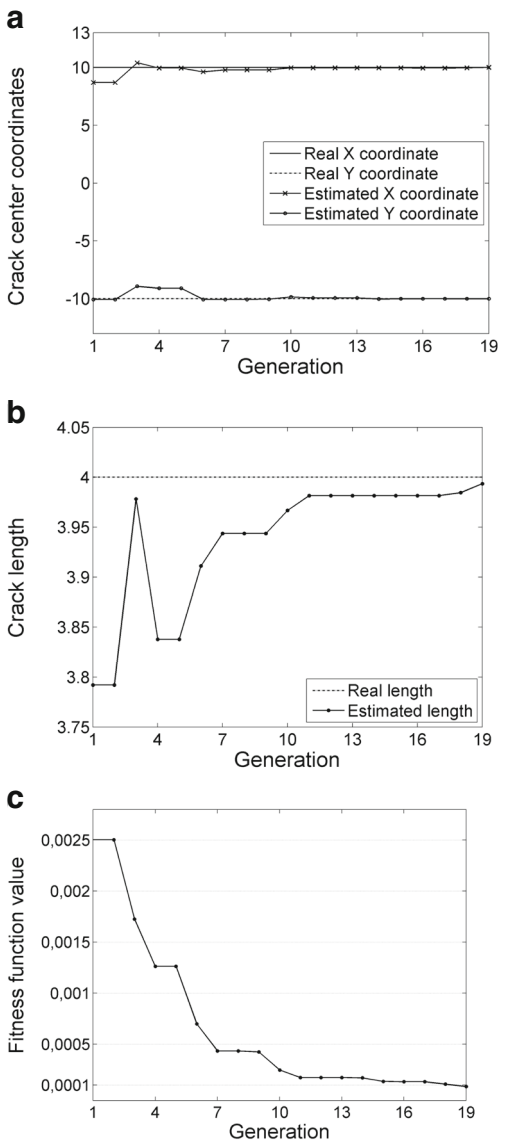

Fig. 4 Results for a crack with parameter $\mathbf{P}_2[10, -10, 4]$: (**a**) crack center coordinates, (**b**) crack length, and (**c**) fitness values, as a function of generation

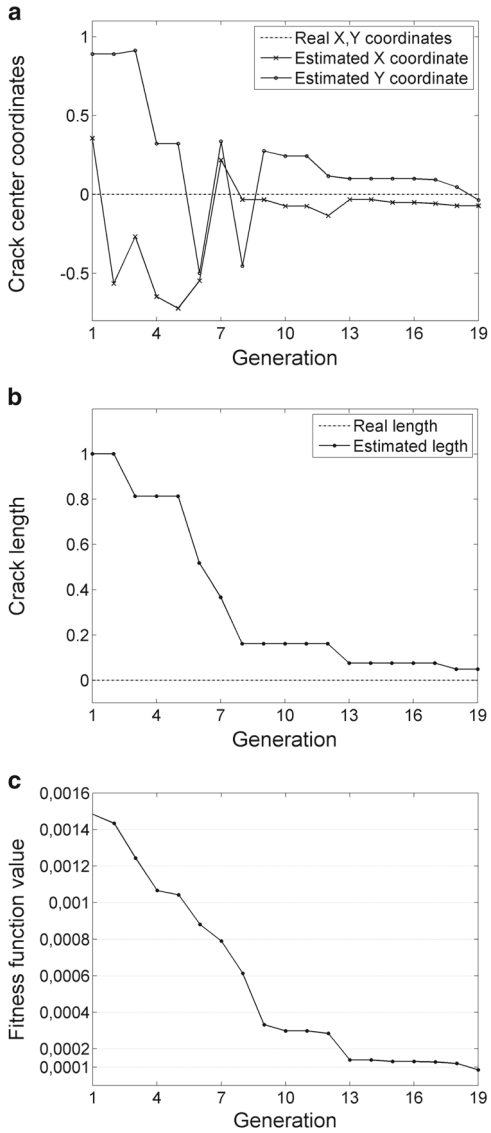

Fig. 5 Results for a crack with parameter $\mathbf{P}_3[0, 0, 0]$ (absence of crack): (**a**) crack center coordinates, (**b**) crack length, and (**c**) fitness values, as a function of generation

The combined POD-RBF-GA algorithm was used to identify three configurations of crack in the above described plate

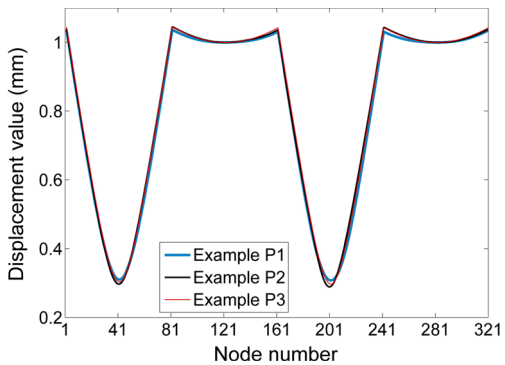

Fig. 6 Boundary displacement distribution for the three studied examples $\mathbf{P}_1[2.5, 0, 8]$, $\mathbf{P}_2[10, -10, 4]$ and $\mathbf{P}_3[0, 0, 0]$

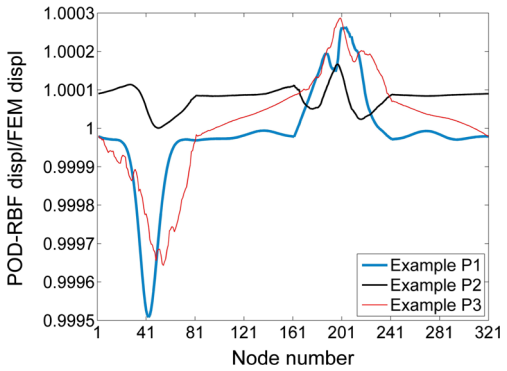

Fig. 7 Comparison between boundary displacements evaluated by POD-RBF technique for the approximated crack configurations, and FEM for the real crack configurations

(Fig. 1). Each configuration was defined by the parameter **P**[crack center abscissa, crack center ordinate, crack length]. The maximum number of iteration was set equal to 50. In addition, a stagnation criterion was imposed to stop the calculation if the fitness value does not improve after 10 iterations. Moreover, to avoid spending computational time once the result is satisfactory, fitness tolerance was introduced to be 10^{-4}. This identification method was implemented in MATLAB, on a PC with Intel Dual-Core Processor 3.0 GHz and 2 GB RAM.

In the first example, we tried to identify a crack of 8 mm length, located near the center of the specimen and represented by the parameter $\mathbf{P}_1[2.5, 0, 8]$. Results are illustrated in Fig. 3 showing the evolution of the crack center coordinates, the crack length and the fitness function value. Although the stopping criterion was reached at the 13th iteration (Fig. 3c), after 677 s, results showed that both crack position and crack length were determined with high precision after only 9 iterations (Fig. 3a, b). It can also be noted from Fig. 3a, b that the crack position was earlier predicted than its size, suggesting that the boundary displacements are more sensitive to the crack position than to the crack length.

In the second example, a crack of 4 mm length located at the lower left side of the specimen was considered. The parameter representing this crack is $\mathbf{P}_2[10, -10, 4]$. Similarly to the first example, results are shown in Fig. 4. These results highlighted that the algorithm leads to stable solutions in terms both of crack position and crack length, at the 7th iteration, as

Table 1 Effect of noise level on the identification of the size and the position of crack

Noise level	$s_{estimated}$	$x_{estimated}$	$y_{estimated}$	s_{error}	x_{error}	y_{error}
0 %	8.005	2.546	0.010	0.06 %	1.84 %	1.04 %
1 %	7.990	2.587	−0.008	0.12 %	3.48 %	0.86 %
5 %	8.030	2.644	0.204	0.37 %	5.76 %	20.41 %
10 %	8.063	3.068	0.363	0.79 %	22.72 %	36.36 %

Table 2 Effect of noise level on the identification of the crack size

Crack length (mm)	Without noise		Noise = 1 %		Noise = 5 %		Noise = 10 %	
	Estimated	Error	Estimated	Error	Estimated	Error	Estimated	Error
3	2.96	1.33 %	2.92	2.73 %	2.91	3.09 %	2.84	5.33 %
6	6.01	0.16 %	5.97	0.50 %	6.05	0.83 %	6.09	1.54 %
12	11.97	0.25 %	11.94	0.50 %	11.89	0.92 %	11.86	1.18 %

it can be observed in Fig. 4a, b. These figures also show that the precision of the identification procedure increases with increasing the iteration number and reaches better stable values at the 11th iteration. Figure 4c shows, in agreement with Fig. 4a, b, that the fitness function decreases sharply up to generation number 7. Then, it converges slowly, step by step, toward the solution to satisfy the stopping criterion at the 19th iteration, after 969 s.

In the third example, we examined the ability of the proposed identification approach to detect the absence of cracks. This configuration was represented by the parameter $\mathbf{P}_3$[0, 0, 0]. Results are depicted in Fig. 5 showing that the crack parameters converge to stable values after 9 iterations. This convergence is highly improved at the 13th generation and the fitness threshold is reached at the 19th iteration, after 970 s, leading to a crack length of 0.04 mm (Fig. 5b) that is 1000 times smaller than width of the specimen. So, it can be neglected. However, it should be noted that the precision of the numerical solution could be improved by decreasing the fitness stopping criterion. These results prove that the identification approach can also be used to check the safety of a structure.

Finally, Fig. 6 shows the displacement distribution on the boundary nodes (as labeled in Fig. 1), for the three studied examples. It can be clearly observed that the three curves are only slightly different, proving the sensitivity of the crack identification approach used in this study. Figure 7 presents the ratio "POD-RBF displacements/ FE displacements" at the boundary nodes, for the three configurations above investigated and represented respectively by the parameters $\mathbf{P}_1$[2.5, 0, 8], $\mathbf{P}_2$[10, −10, 4] and $\mathbf{P}_3$[0, 0, 0]. Here, POD-RBF displacements correspond to the estimated configurations. This figure shows that the maximal error is less than 5.10^{-4}, justifying the efficiency of the POD-RBF method in the calculation of the boundary displacements employed as input data in the crack identification approach. It must be noted that finite element-based inverse identification methods require much time to calculate such boundary displacements.

6 Stability of the method to measurement noise

In order to study the stability of the crack identification algorithm to measurement noise, the configuration $\mathbf{P}_1$[2.5, 0, 8] was reanalyzed but this time introducing three perturbation levels of 1, 5 and 10 % in the input deformation vector. The noise was determined by the white Gaussian law defining the standard deviation as the noise level. A number of 15 generations was fixed as a stopping criterion.

Table 1 shows results illustrating the algorithm performance. It appeared that 1 % of noise does not affect considerably the precision of both the predicted crack length and crack position. Even when the noise level is high, the algorithm still approximates the crack length with a good precision, unlike the crack position for which the approximation error increases with the noise level. The input data

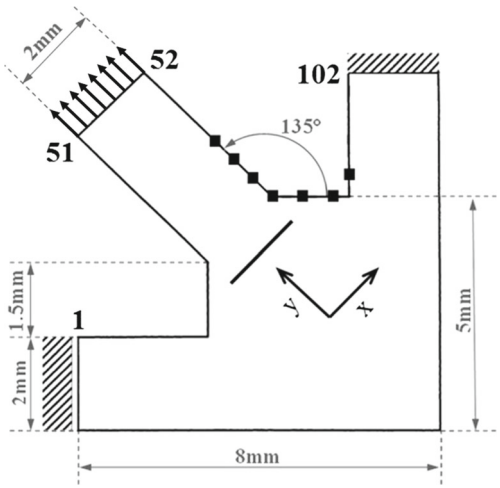

Fig. 8 Complicated cracked specimen

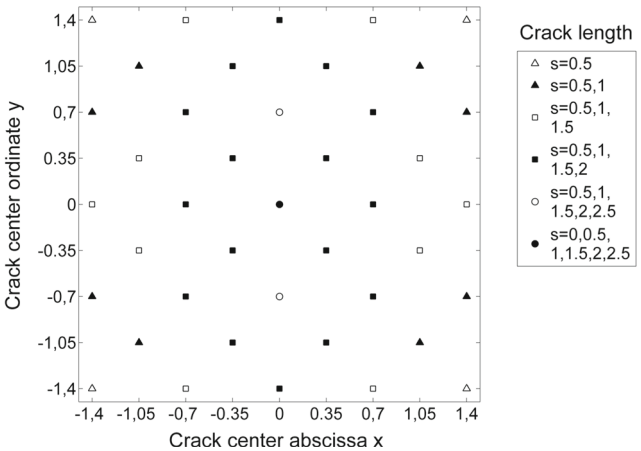

Fig. 9 Crack parameters sets considered in the case of the complicated cracked specimen

perturbation seems to have a more significant effect on the determination of the crack position than on the crack size.

Furthermore, the influence of noise level on the crack length prediction was deeply investigated by considering crack lengths of 3, 6 and 12 mm with a fixed position (x, y) =(2.5, 0). Table 2 shows the outcomes of the inverse identification for noise levels of 0, 1, 5 and 10 %. Results' precision decreases when the noise level increases, for the three investigated crack lengths. The highest errors occurred in the case of the 3 mm crack, and can be attributed to the difficulty of the algorithm to identify small cracks. Nevertheless, the prediction errors are still reasonable for the three investigated configurations, reflecting the high performance of the crack size identification approach.

7 Evaluation of the proposed approach as applied to a complicated crack model

To demonstrate the general applicability of the proposed approach, it was applied to a more complicated crack model (Fig. 8), already investigated by Burczynski and Beluch (2001) using a dual boundary element method combined with a linear hybrid algorithm. The specimen dimensions were arbitrary selected (Fig. 8) since they are not available in (Burczynski and Beluch 2001). Moreover, the same material properties of the rectangular plate analyzed above (Fig. 1), were adopted for this complicated specimen geometry. Both GA optimization and Particle Swarm Optimization (PSO) were used and their performances were compared.

PSO is a population-based optimization method inspired on the behaviour of bird flocks that is characterized by distinct social and psychological principles. Large attention has been paid to this method in few last decades. Its implementation requires a small number of parameters, which facilitates its application. The main idea of PSO is to consider the potential solution as a particle moving through the space, looking for the global optimum position. Initiated as a group of random particles, each particle is characterized by its position in the multidimensional space and by its movement speed. These particles cooperate each other to reach the solution, based both on their personal previous experience and the experience of other particles. More details about PSO method are given in (Eberhart and Kennedy 1995; Kennedy 2010).

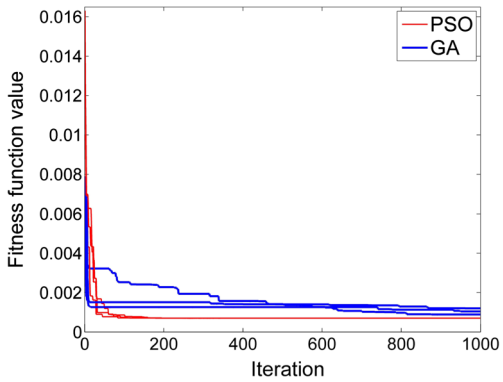

Fig. 10 History of fitness convergence of PSO and GA methods

Table 3 Crack identification results using 7 sensor points: comparison to literature results

Crack parameters	POD-RBF-PSO		Burczynski and Beluch (2001)	
	real	estimated	real	estimated
x_1	−1.1	−1.0905	0.05	0.0486
y_1	0	0.0488	−0.01	−0.0085
x_2	1.1	1.2225	0.01	0.0121
y_2	0	0.0488	−0.05	−0.0517

130 ABAQUS simulations were used to build the input data of the POD-RBF reduced model based on the first 8 POD. This input data consists of 102 boundary node displacements along y direction (Fig. 8). Crack parameters sets were defined in the −1.4 to +1.4 mm range for both the x and y coordinates of the crack center, and from 0 to 2.5 mm, in step of 0.5 mm, for the crack size. The considered crack parameter sets, corresponding to the 130 snapshots of the used matrix **U**, are depicted in Fig. 9.

The origin of the coordinate system was positioned at 5.5 and 2.5 mm respectively from the left side and the bottom side of the specimen. The same stopping criteria described in section 5 were adopted.

A crack of 2.2 mm length, represented by the parameter **P**[0, 0, 2.2], was considered as the unknown crack to be identified.

Based on POD-RBF model, both GA and PSO were used to minimize the fitness function using population of 100 individuals. In order to compare the performance of the two optimization algorithms, three applications were run with maximum iteration number of 1000. Their fitness convergence history is presented in Fig. 10 which highlights that PSO technique converges more quickly to the solution compared to GA

Table 4 Identification results of small and medium cracks using 7 sensor points

Real crack parameters			Estimated crack parameters (Error)			Iterations (time)	fitness
x	y	s	x	y	s		
1.3	1.3	0.6	1.367 (5.1 %)	1.318 (1.3 %)	0.600 (0 %)	500 (2351 s)	0.00083
−1.3	−0.2	1.4	−1.276 (1.8 %)	−0.215 (7.5 %)	1.422 (1.5 %)	440 (2030 s)	0.00005

technique. The best solution obtained by PSO and GA are respectively [−0.0086, 0.0334, 2.2001], corresponding to fitness value equal to 0.0007, and [−0.0225, 0.0401, 2.1695] corresponding to fitness value equal to 0.0008.

To analyze the influence of sensor point number on the precision of the proposed approach, the identification was achieved using only 7 sensor points represented in Fig. 8 by small full squares. The PSO method was used with population size of 100 individuals and maximum generation number of 500. The algorithm was stopped if no improvement happened in the fitness function value after 50 iterations.

A similar configuration was analysed by Burczynski and Beluch (2001) using a dual boundary element method combined with a linear hybrid algorithm. Theses authors identified the crack position by predicting the coordinates of the two crack tips, assuming that the crack length is known.

The proposed POD-RBF-PSO approach converged to the following solution: $x = 0.066$, $y = 0.048$ and $s = 2.313$, with the fitness value of 0.00023. These results and those obtained by Burczynski and Beluch (2001) are reported in Table 3 in terms of the coordinates of the two crack tips. This table demonstrates that the accuracy of our approach is equivalent to the one of the method used by Burczynski and Beluch (2001). However, 349,800 simulations were required to reach a solution by the dual boundary element method, using 406 iterations in 7188 s, while our approach needed 404 iterations and only 1863 s to converge toward a solution, allowing the reduction of computing time of ~74 %, without visible accuracy lost.

Furthermore, the proposed approach was applied, with reduced sensor points, in the identification of small and medium cracks located respectively at (1.3, 1.3) and (−1.3, −0.2) coordinates. The results are reported in Table 4 show that these cracks were accurately identified using only 7 sensor points.

8 Conclusion

In this paper, a crack identification approach was proposed. More precisely, based on several simulations of cracked plate under traction by FEM, a reduced model was built by combining POD and RBF methods, in order to get a solution with a good precision. The genetic algorithm was implemented for the inverse crack identification using boundary displacement as input data.

Results have clearly shown that the proposed algorithm is capable of accurately predicting the presence, the size and the position of crack in a structure, within few generations. It also presented a high stability against measurement uncertainty.

It proved that POD-RBF method could be applicable in crack identification, in both simple and complicated specimen geometries, based on boundary displacement, providing the advantage of the low computational cost.

The use of the population based algorithm in the optimization procedure helped to avoid typical limitations of the classical optimization approaches. It appeared that PSO method is more suitable than GA method for the problem investigated in this work.

Finally, it should be underlined that the approach presented in this paper could be applied using accessible experimental measurements, like strains or displacements as input data, using small number of sensor points.

As a perspective, it will be interesting to use the proposed approach to identify the crack orientation.

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