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Investment in the energy sector: An optimization model that contemplates several uncertain parameters

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ABSTRACT

Investments in the energy sector on the medium/long term are risky due to the uncertainties having in this sector: price volatility, unclear demands and indeterminate fossil reserve volumes, among others. Decision making tools plays an important role in order to attenuate the effect of uncertainties in the investment by including this aspect in the models. In this sense, mathematical programming models provide analytical tools to improve the decision making process. This paper presents a multi-period mathematical model for planning investments in the energy sector in a medium time horizon. The model considers several imprecise information of the energy market: uncertainty in the price of fossil resources, the trend in the growing demand and the variation in the availability of fossil reserves.

The main objective of this work is to formulate a decision making model in planning investments in the energy sector which can provide a unique strategic plan, robust enough to cover pessimistic and optimistic scenarios of the uncertainties. In particular, a fuzzy approach is used to formulate the problem, and is combined with sets of possibilistic techniques to transform the problem into a form that can be solved. In order to show the capabilities of the model, it is applied and solved for the Argentina's energy sector.

1. Introduction

Fossil fuels are currently the main source of energy to produce goods and services to cover the needs of human life. In the last decades, these resources have been strongly questioned for environmental and sustainability reasons; for those motives many renewable sources (wind, biomass, solar, etc.) has become alternatives to produce energy. Nevertheless, substituting fossils by renewables is not, in principle, economically viable, it requires new and efficient technologies, well oriented politics, and economic incentives to compete at the same level. With this situation in mind, an investment plan to produce energy is a complex task that requires a model for helping the decision making. It involves making choices in the medium and long terms about what to do with the financial assets. The length of time horizon implies dealing with imprecise information about future behavior of market conditions that can affect negatively the expected benefits. That is the case of the energy sector where it was a large fluctuation in the oil and gas prices, imprecision in the projected demand, and also vagueness in the availability of nonrenewable resources, all parameters that affect the profitability of investments. In this context, the model to make decisions for investment in the energy sector must include the uncertainties that can influence the selection of alternatives.

This paper proposes, a mathematical multiperiod model for planning the investments in renewable and conventional energy to satisfy the demands of different consuming markets (residential, commercial and industrial), considering the variability in: a) prices of oil and gas, b) energy demand and c) the availability of fossil resources. This work is based on a deterministic model presented by Flores et al. [1].The inclusion of uncertainty in the investment model gives more insight to the problem and brings more arguments for the analysis.

Several authors have dealt with uncertainty in the energy sector. Choi et al. [2] presents a modeling and optimization framework for temporal and spatial energy planning considering future advances in renewable energy technologies and uncertainty in demands and resource prices. Demand and price are predicted using a stochastic model and Monte-Carlo simulation; respect to the technological improvements in renewable energy the authors use a learning

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curve-based iterations approach. To solve the problem they decompose the original large MINLP formulation into two separate problems, a MILP for traditional energy resources with price forecasting and a non-linear programming (NLP) for renewable energy resources. The objective is to find an optimal energy planning policy that minimizes total cost for South Korea, which is divided into several regions. The optimal policy must also satisfy each region's demands for energy sources such as oil, natural gas and electricity under uncertain market prices of these resources. Nuclear and renewable energy sources are used only to generate electricity in this problem. The decision variables are the construction of new power plants including the number, locations, and types, allocation of energy and electricity production. Aquila et al. [3] proposes a framework for investment analysis of wind power generation under uncertainty using Monte Carlo simulation. For this purpose, the authors use a quantitative approach that allows the analysis of investments by simulating the net present value (NPV) for different scenarios. The approach uses the parameter Value at Risk (VaR) proposed by the same authors in a previous work, that considers the settlement of the differences, which depends on the behavior of the wind energy and the electricity prices in the Brazilian market. The authors aim to enhance decision-making for potential investors. The sensitivity analysis is made by changing the input variables by −10% to +10% interval regarding their base values from the deterministic case. Variables that impact more the NPV results are selected and represented in the stochastic analysis as random variables. Variables with smaller influence on NPVs are set in their base values. The authors conclude that the application of VaR contributed to analyze the worst scenario expected from the producer's point of view. Results show that the uncertain scenario reach much smaller values than those observed in the deterministic analysis, due to uncertainties present in the investment disbursement. Caralis et al. [4] analyze the profitability of wind farm investments in China by taking into consideration some relevant investment risks, such as wind potential, wind curtailment, access to the grid and macroeconomic parameters. They formulate a financial model in combination with Monte Carlo simulation to study the investment opportunity in wind power energy generation for four regions of China. The mathematical model is executed iteratively; each iteration is characterized by a randomly selected set of the examined uncertain parameters. In the conclusions the authors remarks that the results obtained shows that the regions of China are good places to install wind energy mills. The financial performance is satisfactory even in regions where wind potential is not high. The model proposed based on feed-in-tariffs is correct for the analysis performed. The work of Tarhan et al. [5] treats uncertainty in planning infrastructure for the exploitation offshore oil and gas fields. The authors analyze the most important uncertainties in the production of oil, focusing exclusively on exploitation and ensuring an acceptable level of production. Uncertainties are represented by discrete distribution. They propose a disjunctive/mixed-integer nonlinear programming model, which is converted into an MINLP model. Several scenarios are solved having a combination of discrete values of the uncertain parameters. The authors solve two examples; the results give the investment strategies for reducing uncertainties which represent nearly 10% and 22% better, respectively, than the solutions found by the expected value approach. The authors claim that these improvements came out because stochastic programming incorporates the uncertainty directly into the mathematical model and proposes decisions that hedge against possible outcomes of uncertain parameters. Cai et al. [6] developed a model of fuzzy-random interval optimization, to determine planning strategies and energy management under multiple uncertainties, which can be formulated as interval values, possibilistic and probabilistic distributions, as well as their combinations. The approach is applied to a region of three cities in a large time horizon, where multiple energy resources need to be allocated to multiple end-users sectors. Conventional and renewable energy resources (coal, refinery petroleum products natural gas, solar, wind, hydropower and nuclear) with limited availabilities are employed for meeting local demands. The example solved is very limited, just enough to show the model capabilities. In this case, the authors infer that the solutions obtained are useful to determine politic actions to address economic constraints, under system reliability. Although the authors say that the approach can be also applied to capacity expansion planning of energy-production, they do not show the way to do it. In another work, Fleten et al. [7] a method for evaluating investments in decentralized renewable power generation under price uncertainty. The model is limited to the study the investment in wind power generation for an office building. A stochastic formulation is used to model the electricity price uncertainty. They evaluate investment projects in wind turbines in Norway by maximizing the net present value. Al-Qahtani and Elkamel [8] propose a two-stage stochastic programming model for planning multiple sites of a network of oil refineries by including uncertainty in both, the prices of crude oil and the demand of products. For this purpose, they extend a deterministic model to include uncertainty and analyze the robustness of it. The above objective is to minimize the annualized cost that includes crude oil cost, operating cost, exchange piping cost among refineries, production system expansion cost, less the export revenue. The authors propose a two stage MILP stochastic problem. The uncertain parameters are formulated assuming a normal distribution form. In the conclusions, they point out that robustness of the proposed model, because is more stable respect to the crude oil price variability and also to the forecasted demand. Svensson et al. [9] present a mixed-binary programming model for long term planning investment in energy for industrial purposes, with the objective to maximize the net present value. The authors point out that the investments in energy for industrial purposes, in the long term, are difficult to evaluate because of the uncertainties present. They use stochastic programming for modeling uncertainties and assume that the probability distributions of the parameters are known. They use a two stage algorithm where decisions are made 'here-and-now' before uncertainties are solved. The approach work out simultaneously the immediate and later decisions. The objective function is the net present value where the investment cost on new facilities is included. The model is applied to a pulp mill which generates electricity; the uncertain parameters are the energy, lignin and bark prices. Although the model is applied to this example the authors claim that can be applied to other industrial processes where similar decisions must be taken. Fuss and Szolgayova [10] pointed out that renewable energy are still expensive but they can improve with technological changes, however its implementation strongly depends on the fossil fuel price volatility. Even more, the technological improvements are uncertain. To overcome this problem, the authors propose a stochastic model to analyze the impact of the investments in the electricity energy sector. The uncertain parameters are the fuel prices and the technical changes in renewable energy, which are modeled stochastically. The authors conclude that "Even the simultaneous inclusion of stochastic fossil fuel prices in the same model does not make renewable energy competitive compared to fossil-fuel-fired technology in the short run based on the data used. This implies that policymakers have to intervene if renewable energy is supposed to get diffused more quickly." Yoon and Ratti [11] analyze the effect of the energy price uncertainty in firm-level investment. The variance in the energy price is predicted by using a generalized autoregressive conditional heteroskedasticity (GARCH) model. The model used to analyze the investment is the estimating an error correction (ECM) model of capital stock adjustment. The model is applied to 2600 publicly traded U.S. manufacturing firms. The authors classify the firms in energy intensive and non-energy intensive. In the conclusions, the authors stated that “a rise in uncertainty about energy price plays a role in firm-level investment decisions by reducing the positive effect of sales growth on investment at firms. A rise in sales growth from 5% to 6% raises investment by firms by 2.70% in the absence of uncertainty, by 1.70% given uncertainty at the mean value, and by only 0.70% given uncertainty at double the mean value".

The previous works confirm the need to include uncertainty in those factors that affect the investments in energy in order to obtain more realistic and robust results. None of the previous approaches have dealt with a model like the one presented in this article where a possibilistic formulation gives a unique solution that is feasible over the wide range of values of the uncertain parameters involved. In this case, it is employed fuzzy set theory to deal with the uncertainties. This is an efficient resource to address inaccurate data that are based on estimates and trends for a medium-long term planning horizon [8,9]. In this sense the fuzzy model gives to the decision makers the possibility of jointly evaluate a whole range of variability of input data, improving the flow of information available to determine a robust investment plan. To solve the model under uncertainty within the fuzzy environments, it is necessary to find an equivalent representation that allows to simultaneously consider the variability of each unstable factor. The methodology for transforming a diffuse linear programming model depends on the parameters that have been included to handle the uncertainties. As a consequence, a large number of reformulations are made, some of which have been proposed by Baykasŏglu et al. [12], Diaz-Madroñero et al. [13] and Thakre et al. [14]. To show the capabilities of the model it is applied, as a case study, to the Argentinean's energy sector.

The paper is organized as follows: a problem description is presented in Section 2, then, Section 3, introduces the Fuzzy Model, Section 4 reports computational results and finally, section 5 presents the conclusions.

2. Problem definition

This paper proposes a diffuse model for the generation of an investments plan in energy that considers the use of renewable and nonrenewable sources to supply energy to different end-user sectors, in a context of extreme variability. As was expressed in the introduction, the model takes into account variability in prices and availability of fossil resources, and demand forecasts. The objective function is the minimization of the cost involving operating, investment and startup cost.

In the formulation, three levels of the supply chain that characterizes the energy sector are represented: the first level corresponds to different sources of primary energy (primary level), the second to the processing plants that converts the primary sources into a form of usable energy (secondary level) and the third one corresponds to the final consumers. The schema used is similar to the one presented in a previous work [8]. Primary energy is any form of energy available in nature before it is converted or transformed like fossil resources, wind, hydraulic, solar, geothermic, biomass, etc. The secondary level corresponds to plants/processes that transform primary energy into a form of consumable one. It includes refineries, bio-refineries, windmills, dams, solar heaters, nuclear plants, etc. The final consumers contemplate the transport, electricity and heating market, for industrial, commercial and residential uses.

The main decisions variables are: the investment in new energy sources, the capacity to be installed and the amount to invest, the expansion of the production level, the annual energy production of each source and the flow of secondary energy destined to cover each market. All of these decisions must be at least feasible in the wide ranges of uncertainty involved in the model.

Next sections describe in detail the objective function and the constraints that formulate the fuzzy model along with the transformations needed in order to solve it.

2.1. Objective function

The objective function of this model is the minimization of the cost along the time horizon (*Cost*). In eq. (1), the *Cost* is equal to the summation of costs incurred $CFS_{i,k,t}$ for each energy source *i* and the market *k* belonging to subset $Market_{i,k}$ and for each period *t*. *TI* is a parameter representing the cost update rate. The subset $Market_{i,k}$ relates primary energy sources that can meet the demands of destination ma.

Eq. (2) formulates the cost flow $CSF_{i,k,t}$over an annual balance; the term $\tilde{C}O_{i,k,t}\cdot x_{i,k,t}\cdot hr$represents the operating cost; $CI_{i,k,t}$the investment cost and $CS_{i,k,t}$ the start-up cost. The variable $x_{i,k,t}$ is the energy flow of primary energy of source *i* to market *k* in time period *t*, the parameter *hr* is the annual operating time, in hours. Meanwhile $x^{EP}_{i,k,t}$ and $x^{ES}_{i,k,t}$ are variables which stand for the flow of energy imports, for primary and secondary sources, respectively and $CIMP_{i,t}$.

$$Cost = \sum_{\substack{i\in Market_{i,k}\\ k\in Market_{i,k}\\ t}} \frac{CFS_{i,k,t}}{(1+TI)^{t-1}} \quad (1)$$

$$CFS_{i,k,t} = \tilde{C}O_{i,k,t}\cdot x_{i,k,t}\cdot hr + CI_{i,k,t} + CS_{i,k,t} + CIMP_{i,t}\cdot\left(x^{EP}_{i,k,t} + x^{ES}_{i,k,t}\right) \quad \forall t;\ \forall (i,k)\in Market_{i,k} \quad (2)$$

The main consequence of uncertainty in fossil resources price is the variation in the costs associated with the energy production ($\tilde{C}O_{i,k,t}$). This instability in the price of the primary energy turns out in a variation in the operating cost of secondary energy, affected by the uncertainty in prices of the oil or natural gas. The price variability is shown in Fig. 1 for oil price and in Fig. 2 for natural gas price.

The prices for oil and natural gas are represented with triangular fuzzy numbers as shown in Fig. 3.

The prices are defined by three critical or characteristic values for the coefficients $\tilde{P}r_{i,k,t} = (\mathrm{Pr}^{lo}_{i,k,t}, \mathrm{Pr}^{av}_{i,k,t}, \mathrm{Pr}^{hg}_{i,k,t}) = \{\mathrm{Pr}_{i,k,t}\in\mathbb{R}\big|\mathrm{Pr}^{lo}_{i,k,t}$

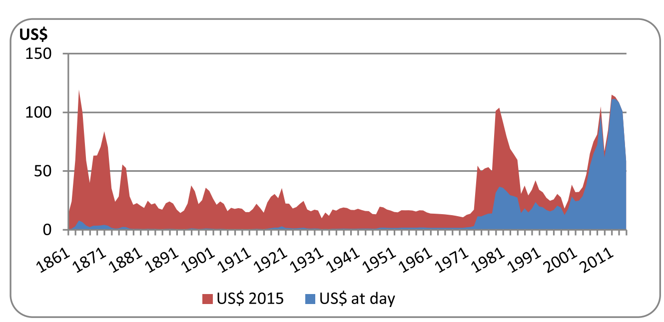

Fig. 1. Price of oil [15].

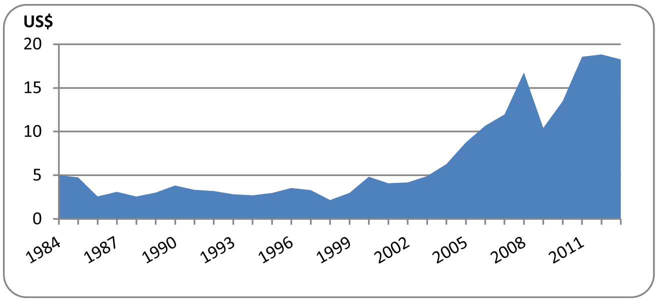

Fig. 2. Price for MBTU of natural gas [15].

$\leq \mathrm{Pr}_{i,k,t} \leq \mathrm{Pr}^{hg}_{i,k,t}\}$, where the superscript *lo*, *av*, and *hg* means low, average and high, respectively. The membership functions representing the shapes of the triangular numbers are shown below (3).

$$\Psi_{(\tilde{\mathrm{Pr}}_{i,k,t})}(x) = \begin{cases} \dfrac{1}{\mathrm{Pr}^{av}_{i,k,t} - \mathrm{Pr}^{lo}_{i,k,t}}\left(x - \mathrm{Pr}^{av}_{i,k,t}\right) + 1,\ if\ \mathrm{Pr}^{lo}_{i,k,t} \leq x \leq \mathrm{Pr}^{av}_{i,k,t} \\ \dfrac{1}{\mathrm{Pr}^{av}_{i,k,t} - \mathrm{Pr}^{hg}_{i,k,t}}\left(x - \mathrm{Pr}^{av}_{i,k,t}\right) + 1,\ if\ \mathrm{Pr}^{av}_{i,k,t} \leq x \leq \mathrm{Pr}^{hg}_{i,k,t} \\ 0 \quad ,\ if\ x \leq \mathrm{Pr}^{lo}_{i,k,t}\ or\ x \geq \mathrm{Pr}^{hg}_{i,k,t} \end{cases} \tag{3}$$

To model the price variability into the cost functions, the cost involved in eq. (2) is disaggregated into a series of equations that result in the first reformulation, which is based on the methodology developed by Zhang et al. [16] for trapezoidal coefficients, and later adapted by Mavrotas et al. [17] for the case of triangular numbers. This technique allows the transformation of a FMIL into a MILP multiobjective problem, being $Cost^{\mu}$ the function of cost evaluated at each critical values of the triangular number. The Appendix section contains more details about the transformations.

In this way, a new objective function is raised for each bound of the intervals, defined by the membership functions. The feasible region of the problem remains unchanged.

The new objective functions Eq. (4) determine the impact of the uncertainty in the price of fossil resources on downstream production costs.

$$Cost^{\mu} = \sum_{\substack{i \in Market_{i,k} \\ k \in Market_{i,k} \\ t}} \frac{CFS^{\mu}_{i,k,t} + CFS_{i,k,t}}{(+TI)^{t-1}} \qquad \forall \mu \tag{4}$$

Note that the index μ link different objective functions with the characteristic values that define the triangular numbers (lo, av, hg).

Eq. (5) represents the costs of oil and natural gas for the industrial market ($k = In$). Parameter $aCO_{i,k,t}$ characterizes a fraction of the operating costs related to its sales price. Therefore, the term $aCO_{i,k,t} \cdot \mathrm{Pr}^{\mu}_{i,k,t}$ represents the operative cost of the source i at time t in market k, where the value is assigned to the fuzzy price μ. The term $bCO_{i,k,t}$ represent the fixed cost per unit produced.

$$CFS^{\mu}_{i,k,t} = \left(aCO_{i,k,t} \cdot \mathrm{Pr}^{\mu}_{i,k,t} + bCO_{i,k,t}\right) \cdot x_{i,k,t} \cdot hr + CI_{i,k,t} + CS_{i,k,t} + CIMP_{i,t} \cdot \left(x^{EP}_{i,k,t} + x^{ES}_{i,k,t}\right) \quad \forall \mu;\ \ \forall t;\ \ i = P, GN;\ \ k = In \tag{5}$$

Eq. (6) corresponds to the production cost of petroleum derivatives ($Fraction_i$= gasoline, diesel, and fuel oil) where the cost fluctuation has a big impact. For this reason, the parameter $CO^{\mu}_{i,k,t}$ symbolizes the operating cost associated to price of raw material. In this case it takes the values $\mathrm{Pr}^{lo}_{oil,k,t}$, $\mathrm{Pr}^{av}_{oil,k,t}$ and $\mathrm{Pr}^{hg}_{oil,k,t}$.

$$CFS^{\mu}_{i,k,t} = CO^{\mu}_{i,k,t} \cdot x_{i,k,t} \cdot hr + CI_{i,k,t} + CS_{i,k,t} + CIMP_{i,t} \cdot \left(x^{EP}_{i,k,t} + x^{ES}_{i,k,t}\right) \quad \forall \mu;\ \forall t;\ \forall (i,k) \in Market_{i,k} \cap Fraction_i \tag{6}$$

Similarly, eq. (7) represents the costs for all industries that needs natural gas, where the price variability affects the cost $CO^{\mu}_{i,k,t}$.

$$CFS^{\mu}_{i,k,t} = CO^{\mu}_{i,k,t} \cdot x_{i,k,t} \cdot hr + CI_{i,k,t} + CS_{i,k,t} + CIMP_{i,t} \cdot \left(x^{EP}_{i,k,t} + x^{ES}_{i,k,t}\right) \quad \forall \mu; \forall t; \forall (i,k) \in Market_{i,k}; i = GN; k \neq In \tag{7}$$

Finally, eq. (8) expresses the costs for other sources in different markets.

$$CFS_{i,k,t} = CO_{i,k,t} \cdot x_{i,k,t} \cdot hr + CI_{i,k,t} + CS_{i,k,t} + CIMP_{i,t} \cdot \left(x^{EP}_{i,k,t} + x^{ES}_{i,k,t}\right) \quad \forall \mu; \forall t; \forall (i,k) \in Markets_{i,k}; i \neq Fraction_i, P, GN \tag{8}$$

Eq. (9) imposes a limit on the amount of money to invest in an energy source i for a market k in the period t through the parameter $CIup_{i,k,t}$.

$$CI_{i,k,t} \leq CIup_{i,k,t} \qquad \forall (i,k) \in Market_{i,k}, \forall t \tag{9}$$

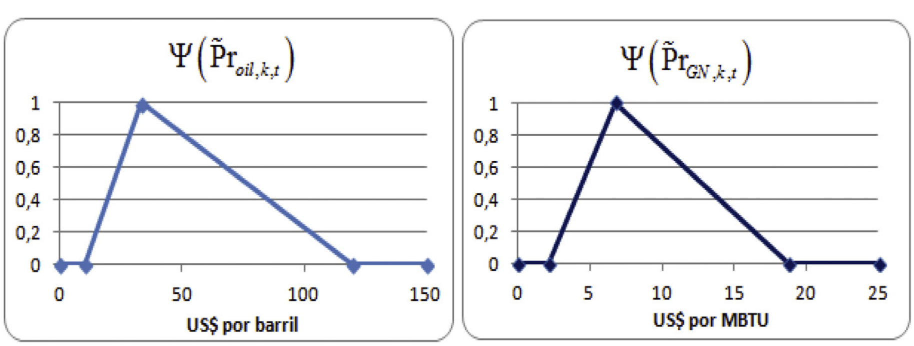

Fig. 3. Triangular fuzzy number for the price. (a) Oil price. (b) Natural Gas price.

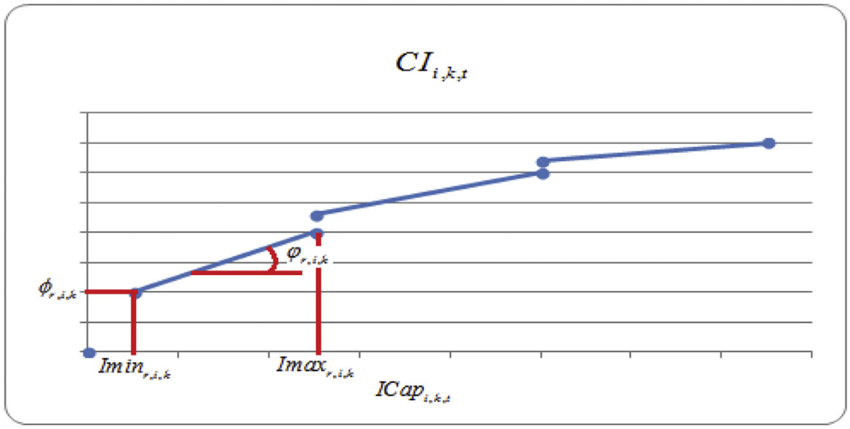

Fig. 4. Investment cost curve.

2.2. Investment decision

Investment and start-up costs are represented by discontinuous curves. In Fig. 4, it is illustrated the investment cost, where each segment is a straight line corresponding to an interval, represented by the parameters$(\varphi_{r,i,k}, \phi_{r,i,k})$. The start-up cost has a similar formulation where parameter $(\kappa_{r,i,k}, \omega_{r,i,k})$ are used. Those discontinuous costs are formulated by eqs. (15) and (16), respectively.

The investment in new energy sources involves discrete decisions which are formulated using equations (10–16). In eq. (10),$y_{r,i,k,t-T_{i,k}}$is a binary variable to handle the decision if a new investment is made $(y_{r,i,k,t-T_{i,k}}=1)$ or not $(y_{r,i,k,t-T_{i,k}}=0)$, in case of a positive value, then a capacity level $r$must be selected for source i, market $k$and period $t-T_{i,k}$. The variable $ICap_{i,k,t}$(Eq. (11)) denote the plant capacity, which is disaggregated in different levels r through the variable $xICap_{r,i,k,t}$. This variable is bounded by the parameters $Imin_{r,i,k}$and $Imax_{r,i,k}$(Eq. (12)) that characterizes the minimum and maximum values of capacity level, respectively. Similarly $CI_{i,k,t-T_{i,k}}$(Eq. (13)) is a variable that specifies the amount to invest, limited also by the corresponding disaggregated variable $xCI_{r,i,k,t-T_{i,k}}$(Eq. (15)). This variable is a function of the disaggregated capacity $(xICap_{r,i,k,t})$ and the decision about the investment $(y_{r,i,k,t-T_{i,k}})$. Finally, $CS_{i,k,t-1}$(14) is the start-up cost which is paid once the plant operation begins. It is disaggregated in a similar way than before (Eqs. (13) and (15)) thorough the capacity level and the investment decision.

$$\sum_{r\in R} y_{r,i,k,t} \le 1 \quad \forall t < \left(t^f - T_{i,k}\right);\ \forall (i,k)\in Market_{i,k} \tag{10}$$

$$ICap_{i,k,t} = \sum_{r\in R} xICap_{r,i,k,t}\ \forall t > T_{i,k};\quad \forall (i,k)\in Market_{i,k} \tag{11}$$

$$Imin_{r,i,k}\cdot y_{r,i,k,t-T_{i,k}} \le xICap_{r,i,k,t} < Imax_{r,i,k}\cdot y_{r,i,k,t-T_{i,k}}\ \forall r;\ \forall t > T_{i,k};\ \forall (i,k)\in Market_{i,k} \tag{12}$$

$$CI_{i,k,t-T_{i,k}} = \sum_{r\in R} xCI_{r,i,k,t-T_{i,k}}\ \forall t > T_{i,k};\ \forall (i,k)\in Market_{i,k} \tag{13}$$

$$CS_{i,k,t-1} = \sum_{r\in R} xCS_{r,i,k,t-1}\ \forall t > T_{i,k};\ \forall (i,k)\in Market_{i,k} \tag{14}$$

$$xCI_{r,i,k,t-T_{i,k}} = \varphi_{r,i,k}\cdot xICap_{i,k,t} + \phi_{r,i,k}\cdot y_{r,i,k,t-T_{i,k}}\ \forall r;\ \forall t > T_{i,k};\ \forall (i,k)\in Market_{i,k} \tag{15}$$

$$xCS_{r,i,k,t-1} = \kappa_{r,i,k}\cdot xICap_{r,i,k,t} + \omega_{r,i,k}\cdot y_{r,i,k,t-T_{i,k}}\ \forall r;\ \forall t > T_{i,k};\ \forall (i,k)\in Market_{i,k} \tag{16}$$

This group of equations defines the investment, the capacity and the start-up cost in the corresponding period t. The difference between the time when the investment is decided and the time to pay the start-up cost is given by the period required to build the facilities $(T_{i,k})$.

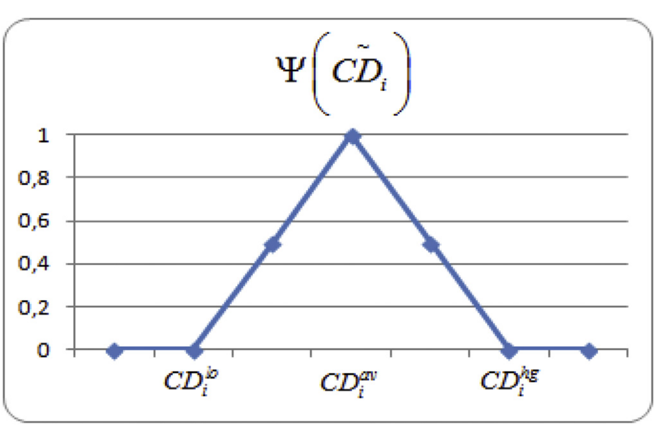

Fig. 5. Triangular fuzzy number for fossil reserves.

2.3. Operative restrictions

The annual energy production $(q_{i,t})$for source $i$period $t$is given by eq. (17.

$$q_{i,t} = \sum_{k\in Mercado_{i,k}} x_{i,k,t}\cdot hr \qquad \forall t;\ \forall i \tag{17}$$

Eq. (18) represents the energy production $(q_{i,t})$ due to fossil fuels i(naphtha, diesel, fuel oil) in period t.The parameter $Corte_i$represents the average fraction of fuel obtained from the crude oil amount processed $(q_{P,t})$.

$$q_{i,t} = Corte_i \cdot q_{P,t} \qquad \forall t; \ \ \forall i \in Fraction_i \tag{18}$$

Eq. (19) establishes that the energy flow $(x_{i,k,t})$is limited by the installed capacity of source i, market k in period t $Cap_{i,k,t}$. The parameter $f_{i,k}$ is a conversion factor of energy source $i$for market k. The conversion factors are needed to put the energy flow in the same measure units.

$$f_{i,k} \cdot x_{i,k,t} \le Cap_{i,k,t} \quad t; \forall (i,k) \in Market_{i,k} \tag{19}$$

2.4. Availability constraints

The amount of energy produced by fossil fuel sources must be less than or equal to the reserves available $(RD_{i,t})$of source $i$in period t, plus the imports in that period $(x^{EP}_{i,k,t})$. This situation is formulated in Eq. (20) for the non-renewable sources(NR).

$$q_{i,t} \le RD_{i,t} + \sum_k x^{EP}_{i,k,t} \qquad \forall i \in NR \tag{20}$$

Eq. (21) indicates that the reserves at the beginning of the time horizon(RD_{i,t^0}) are equal to the initial reserves $\tilde{C}D_i$.

$$RD_{i,t=t^0} = C^{\sim}D_i \qquad \forall i \in NR \tag{21}$$

The availability of fossil resources $\tilde{C}D_i = (CD^{lo}_i, CD^{av}_i, CD^{hg}_i)$is uncertain. This situation is depicted by a triangular number [18], in a similar way than the variability in prices. Fig. 5 presents the triangular fuzzy number defined for this parameter.

The membership functions for this parameter are defined in Eq. (22).

$$\Psi_{\left(\tilde{CD_i}\right)}(x) = \begin{cases} \dfrac{x - CD^{av}_i}{\left(CD^{av}_i - CD^{lo}_i\right)} + 1 & if \quad \left(CD^{lo}_i \le x \le CD^{av}_i\right) \\ \dfrac{x - CD^{av}_i}{\left(CD^{av}_i - CD^{hg}_i\right)} + 1 & if \quad \left(CD^{av}_i \le x \le CD^{hg}_i\right) \\ 0 & if \quad \left(x \le CD^{lo}_i, CD^{hg}_i \le x\right) \end{cases} \tag{22}$$

Then, by using the possibility operators of the fuzzy numbers, Eq. (21) can be rewritten as Eqs. (23) and (24) (see Appendix to find out about this transformation); where the variable α_t denotes the availability level of fossil resources that can be used.

$$RD_{i,t} \le \alpha_t \left(CD^{av}_i\right) + (1-\alpha_t)\left(CD^{hg}_i\right) \ \ \forall i \in NR, t = t^0 \tag{23}$$

$$RD_{i,t} \ge \alpha_t \left(CD^{av}_i\right) + (1-\alpha_t)\left(CD^{lo}_i\right) \ \ \forall i \in NR, t = t^0 \tag{24}$$

Eq. (25) evaluates reserves availability over the time horizon. It includes the uncertainty in discovering new non-renewable resources $(\tilde{N}ew_{i,t})$.

$$RD_{i,t} = RD_{i,t-1} - q_{i,t-1} + \tilde{N}ew_{i,t} + \sum_k x^{EP}_{i,k,t-1} \ \ t^f > t > t^0; \forall i \in NR \tag{25}$$

Eq. (25) can be rewritten similarly to Eq. (21). by replacing the parameter $\tilde{N}ew_{i,t}$ by its corresponding expression, resulting in Eqs. (26) and (27) (Transformation can be found in the Appendix).

$$RD_{i,t} \le RD_{i,t-1} - q_{i,t-1} + \alpha_t\left(New^{av}_{i,t}\right) + (1-\alpha_t)\left(New^{hg}_{i,t}\right) + \sum_k x^{EP}_{i,k,t-1} \ \ t^f > t > t^0; \ \ \forall i \in NR \tag{26}$$

$$RD_{i,t} \ge RD_{i,t-1} - q_{i,t-1} + \alpha_t\left(New^{av}_{i,t}\right) + (1-\alpha_t)\left(New^{lo}_{i,t}\right) + \sum_k x^{EP}_{i,k,t-1} \ \ t^f > t > t^0; \ \ \forall i \in NR \tag{27}$$

Eq. (28) defines a limit in the non-renewable reserves availability at the end of the time horizon by setting the value of ε_i.

$$RD_{i,t} \ge (1-\varepsilon_i) \cdot CD^{av}_i \qquad \forall i \in NR; t = t^f \tag{28}$$

Eq. (29) restricts that the capacity of a new installation $Cap_{i,k,t}$of a renewable energy plant of source i for all markets. It is constrained to the limit established for that source (CD_i).

$$\sum_{k \in Market_{i,k}} \frac{Cap_{i,k,t}}{f_{i,k}} \le CD_i \qquad \forall t; \ \ \forall i \notin NR \tag{29}$$

2.5. Demand constraints

Another cause of uncertainty is the demand $\tilde{D}_{k,t} = (D^{lo}_{k,t}, D^{hg}_{k,t})$represented by an L number, where $D^{lo}_{k,t}$and $D^{hg}_{k,t}$ are the

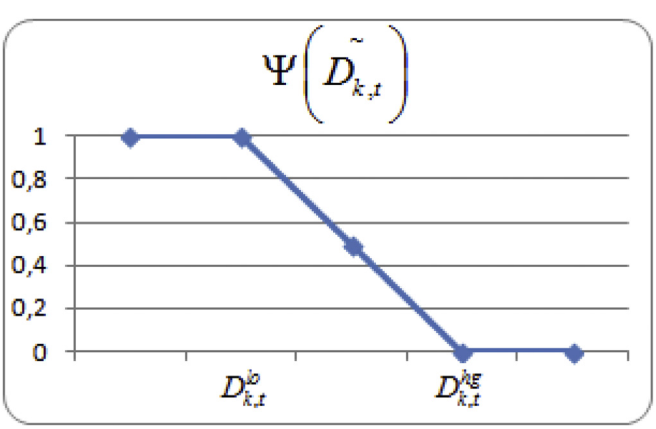

Fig. 6. Demand, L fuzzy number.

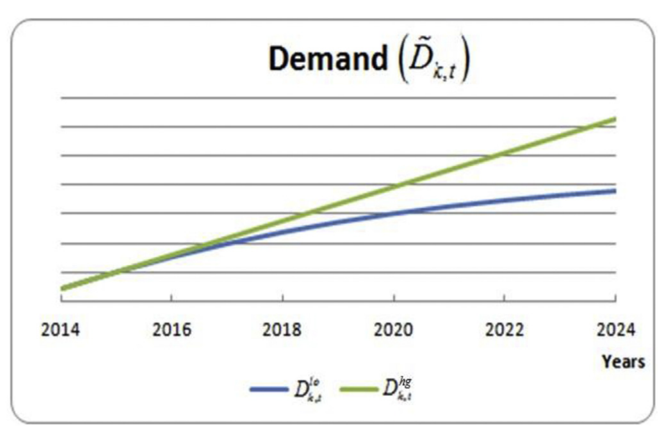

Fig. 7. Evolution of demand.

critical values defined for this case (see Fig. 6).

The membership functions for this fuzzy number are formulated in Eq. (30).

$$\Psi_{\left(\tilde{D}_{k,t}\right)}(x) = \begin{cases} 1 & \text{if} \quad D^{lo}_{k,t} \le x \\ \dfrac{D^{hg}_{k,t} - x}{\left(D^{hg}_{k,t} - D^{lo}_{k,t}\right)} & \text{if} \quad \left(D^{lo}_{k,t} \le x \le D^{hg}_{k,t}\right) \\ 0 & \text{if} \quad \left(x \ge D^{hg}_{k,t}\right) \end{cases} \tag{30}$$

For the demands, the uncertainty is represented by two curves, whose limits change across the time (Fig. 7). The differences between these curves are the increments along the study horizon. Note that each market has its corresponding demand represented by a pair of curves.

Eq. (31) establishes that energy production for a particular market k in each period $t$must be equal to the demand minus the contribution of imports of secondary energies.

$$\sum_{i \in Market_{i,k}} f_{i,k} \cdot x_{i,k,t} \cdot hr \ge \tilde{D}_{k,t} - \sum_{i \in Market_{i,k}} f_{i,k} \cdot x^{ES}_{i,k,t} \quad \forall t; \forall k \tag{31}$$

Eq. (31) is reformulated into Eq. (32) by replacing the demand uncertainty by its corresponding expression (See Appendix). The variable $\alpha_k{}'$ represents the membership value corresponding to the demand, very similar to the case of equations (23) and (24).

$$\begin{aligned} &\sum_{i \in Market_{i,k}} f_{i,k} \cdot x_{i,k,t} \cdot hr \ge \alpha_k{}' \cdot D^{lo}_{k,t} + (1 - \alpha_k{}') D^{hg}_{k,t} \\ &\quad - \sum_{i \in Market_{i,k}} f_{i,k} \cdot x^{ES}_{i,k,t} \quad \forall t; \forall k \end{aligned} \tag{32}$$

Eq. (33) restricts the relationship between gasoline $q_{Nf,t}$and bioethanol production from sugar cane $q_{BEC,t}$or corn $q_{BEM,t}$. This is set by the parameter$BioNF$which corresponds to the fraction of bioethanol admissible in gasoline.

$$q_{BEC,t} + q_{BEM,t} \le BioNF \cdot q_{Nf,t} \qquad \forall t \tag{33}$$

Eq. (34) keeps the production ratio between diesel $q_{GO,t}$and biodiesel $q_{BD,t}$. $BioD$is the fraction of biodiesel acceptable in diesel.

$$q_{BD,t} \le BioD \cdot q_{GO,t} \qquad \forall t \tag{34}$$

Up to this point, fuzzy model (eq. (1), (2), (9)–(21), (25), (28), (29), (31), (33) and (34)) is reformulated as an equivalent multi-objective model composed by eq. (4) as the objective functions and eqs. ((5)–(20), (23), (24), (26)–(29), (32)–(34) as constraints.

To find a solution of the multiobjective problem after this sequence of reformulations, it is necessary to apply a new transformation. In this case, is based on the max-min method. A membership function is defined for each objective function (Eq. (35)). The explanation about the transformation is also part of the Appendix.

$$\Omega_\mu = \begin{cases} 1 & Cost^\mu < Z^\mu_{lw} \\ 1 - \dfrac{Cost^\mu - Z^\mu_{lw}}{Z^\mu_{up} - Z^\mu_{lw}} & Z^\mu_{lw} \le Cost^\mu \le Z^\mu_{up} \ \forall \mu \\ 0 & Cost^\mu > Z^\mu_{up} \end{cases} \tag{35}$$

In Eq. (35), Z^μ_{lw} represents the most optimistic value of $Cost^\mu$, while the value Z^μ_{up} represents the pessimistic one.

Then the multiobjective problem is rewritten considering eq. (4) that defines the cumulative costs, and then converted into a conventional MILP problem as shown in Eq (36):

$$\begin{aligned} &Max\left(\alpha''\right) \\ &s.t. \\ &\alpha'' \le 1 - \frac{Cost^\mu - Z^\mu_{lw}}{Z^\mu_{up} - Z^\mu_{lw}} \forall \mu \\ &\alpha'' \le \alpha''_k \forall k \\ &\alpha'' \le \alpha \\ &Eq(4) - Eq(20), Eq(23), \\ &Eq(24), Eq(26) - Eq(29) \\ &Eq(32) - Eq(34) \end{aligned} \tag{36}$$

where α'' is now the membership value of the $Cost^\mu$ for the different prices, demands and fossil fuel reserves.

3. Case study: implementation and results

In order to show the capabilities of this model, it is applied to Argentina's energy sector. The energy matrix of this country is shown in Fig. 8 where can be observed that is highly dependent on fossil fuels (mainly gas and oil). It is composed of 46% natural gas, 36% oil, 5% hydro, nuclear 2.3%, 2% wood and 7% from other sources.

Conventional fossil resources are limited, if no new discoveries are made in the next years, reserves will be exhausted in the next two decades. Demands fluctuate according to the increase of Gross Domestic Product, the population and the energy prices. Fig. 9 shows the energy flow from the sources to the consumption markets. Since Argentina has comparative advantages for producing energy from renewable sources, some of them are included in the menu of investments possibilities: biofuels, windmills, solar and hydrokinetic turbines. Data collected to execute the model are: levels of reserves of fossil resources [20], market demands [19,21], capacity installed of different energy plants [22], extension of areas of cultivation [23], solar irradiation factor [24], wind power generation capacity, among others. These data can be found as Supplementary Material. In the case of fossil resource prices, market demands and availability of fossil reserves are predicted, for future years, according to the values collected in the past, analyzing its tendencies and distribution.

Note that deterministic solutions have membership value or possibility of occurrence equal to 1. In Fig. 10 it is also shown the solution achieved by the fuzzy model in color blue. It corresponds to an α'' value of 0.508. This is a unique and robust solution in the feasible range of the variables, obtained with the model presented in eq. (36). With that α'' value and the membership functions it is

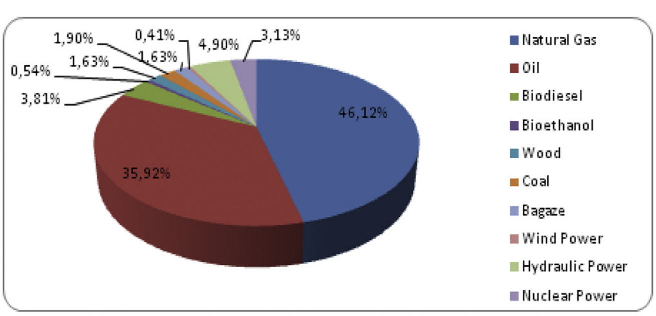

Fig. 8. Argentina's energy matrix 2014 [19].

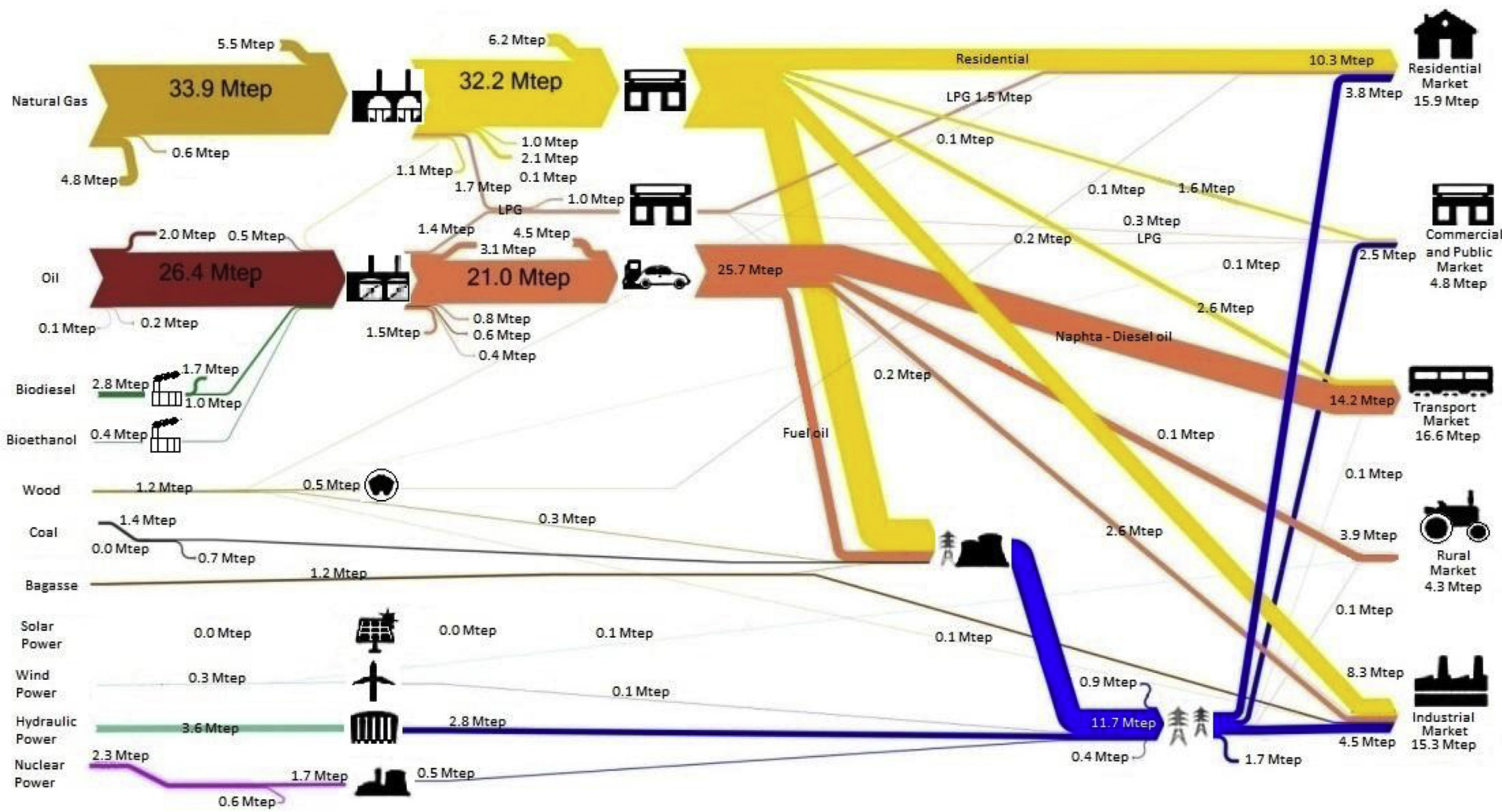

Fig. 9. Argentina's energy flow from sources to markets [19].

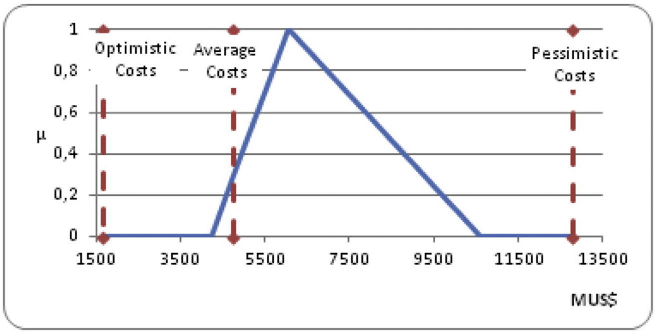

Fig. 10. Possibility of occurrence of costs.

possible to calculate the characteristic values (optimistic, average and pessimistic) of the fuzzy model, which are shown in Fig. 10 by the triangular shape. By comparing the deterministic against the fuzzy, it can be seen that the range of the latter is narrower, meaning that the financial risk is lower and also the cost. A minimum level of demand of 0.508 (αk) is guaranteed. This means that for each market it is guaranteed the satisfaction of the average demand of the estimated maximum.

The model in (36) is implemented in GAMS 23.7. The solver employed is Cplex 12.6.3.0 (Released Jul 11, 2016) for x86 64bit/MS Windows. The model statistics are detailed in Table 1, the model was run on an IBM PC with a processor Intel Core i7, 8 GB of RAM and Windows x64. The model has 4807 equations, 4316 continuous variables and 1311 discrete variables.

In order to understand and analyze the results, a comparison with a deterministic model is made. This model is composed of the same objective function and constraints of the fuzzy approach (eq. (1), (2), (9)–(21), (25), (28), (29), (31), (33) and (34)) with the particularity of not involving uncertainty in the data; so that the parameters $\tilde{C}O_{i,k,t}$, $\tilde{C}D_i$, $\tilde{N}ew_{i,t}$ and $\tilde{D}_{k,t}$ in equations ((2), (21), (25) and (31) are replaced by their corresponding fixed value. The deterministic model is evaluated in the three characteristic values of prices: low, average and high. The value of demands, availability of resources and discovery of new reserves are set in their average levels, while preserving flexibility in finding the period of the discovery. The deterministic model for the different characteristic values of the prices of fossil fuels is evaluated, obtaining for each case the cost values presented in Fig. 10, in red color. The red lines represent the optimistic, average and pessimistic cost values obtained in the objective function.

The value of $\alpha'' = 0{,}508$ are then represented in the triangular numbers of oil and gas prices represented in Fig. 11.

From Fig. 11 can be seen the fluctuation in the gas and oil price where the solution of the fuzzy model is feasible. The price range is shown by the red arrows. It is between 4.50 until 12.80 us$/MBTU for the gas, and between 21.5 until 76.5 us$/barrel for the oil price.

In a similar way, Fig. 12 shows the range on new resources availabilities involved by the solution obtained.

Fig. 13 shows an example of the energy demand that the model can cover a 100%. The upper value of the demand is represented by the red segment, all value included are guaranteed a 100%. Note that It can be drawn a graph for each market.

Table 1
Model statistics.

Blocks of Equations	49	Single Equations	4807
Blocks of Variables	26	Single Variables	4316
Non Zero Elements	14263	Discrete Variables	1311

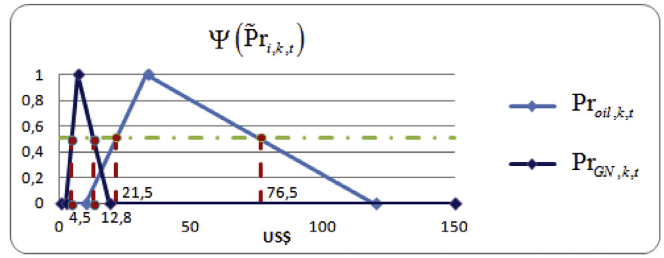

Fig. 11. Triangular numbers for oil and natural gas price.

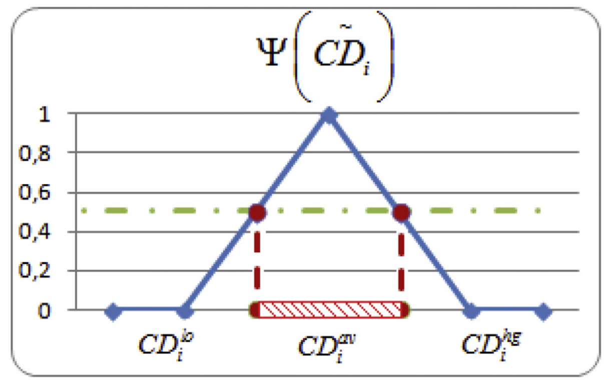

Fig. 12. Range of the resources availability covered by the fuzzy model.

Respect to the availability of resources, the solution defines the bounds where the solution reached is feasible. These bounds are between 50.8 MSm3 (mega standard cubic meters) and 83.9 MSm3 for oil; and from 50340 MSm3 to 83189 MSm3 of natural gas.

The cost range goes from MUS$ 4217.53 to MUS$ 10,597.40 for the fuzzy model. The lower cost, corresponding to the most optimistic scenario, is worse than the deterministic case (MUS$ 1680.52). However, it achieved a strong improvement for the worse case, where the pessimistic value reaches MUS$ 12,799.09. The most probable value of costs for the fuzzy model is MUS$ 6069.76, which is worse than the deterministic around MUS$ 5000.00.

These differences in the results from the diffuse and deterministic approaches respond to the fact that the diffuse formulation considers the variability in the uncertain factors while the deterministic does not. The diffuse model gives a robust solution that allow the variation in the data and the results remains feasible, while the solutions found in the deterministic only guarantee feasibility at the data point specified. The proposed investment

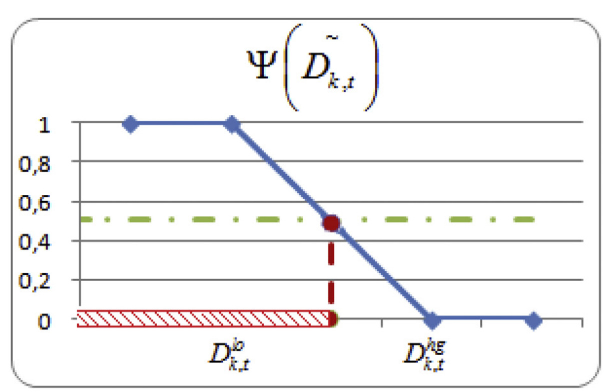

Fig. 13. Demand range where the energy supply is guaranteed a 100%.

Table 2
Schedule of installations. Fuzzy Model.

		2014		2015		2016		2017		2019	
		MUS$	TOE/hr	MUS$	TOE/hr	MUS$	TOE/hr	MUS$	TOE/hr	MUS$	TOE/hr
Biodiesel	Transport	144,34	135,94	144,34	135,94	114,56	64,74			56,58	26,64
Wind Power	Electrical	2224,1	373,14	2224,1	373,14	1883,99	259,02	1853,41	248,76		
Solar Power	Residential	410,84	662,43								
Hydraulic Power	Electrical	14,4	0,69	8,64	0,34						

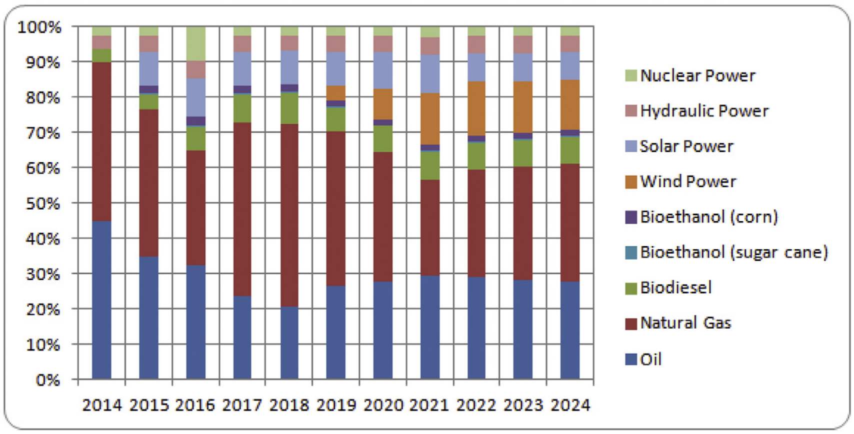

Fig. 14. Evolution of energy matrix for the fuzzy model.

Table 3
Schedule of installations. Deterministic Model (average value).

		2014		2016		2017		2018		2019	
		MUS$	TOE/hr	MUS$	TOE/hr	MUS$	TOE/hr	MUS$	TOE/hr	MUS$	TOE/hr
Bioethanol(sugar cane)	Transport	276,09	71,26								
Wind Power	Electrical	1384,76	170,14	341,58	38,21	341,58	38,21	341,58	38,21	341,58	38,21
Solar Power	Residential	410,84	662,43								
Hydraulic Power	Electrical	18	1,37								

planning in the diffuse model does not lose viability when uncertain data changes between the extremes proposed.

Table 2 list the investment in energy infrastructure for the fuzzy model. In the table can be seen the year, amount of money and capacity of the new facilities. These investments are needed in order to cover the cost range of the triangular curve of the fuzzy model, besides the demand of energy.

Fig. 14 shows the evolution in the use of resources for the fuzzy model. In this case, the results propose the installation of biodiesel plants in 2014, 2015, 2016 and 2019, being the latter much lower

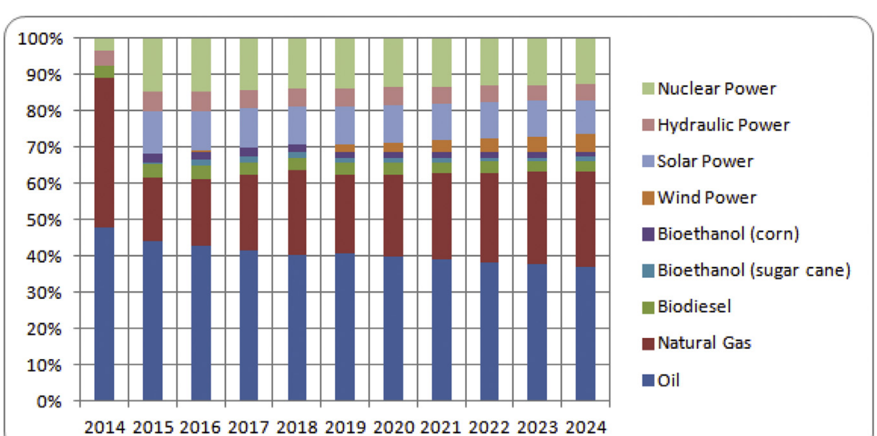

Fig. 15. Evolution of energy matrix for the deterministic model.

than the previous ones, these facilities have the purpose to adjust and satisfy the demand growth in the heavy transportation market and reduce the consumption of oil. Regarding the satisfaction of electric power needs, from Fig. 14 it can be seen that in 2014–2015, natural gas is used as primary source, in 2016 it increases the electric power produced by nuclear plants with the objective of reducing the gas consumption. From the beginning of the study horizon it decides to install four wind farms, these new facilities take five years to start the production, so this source begins in 2019 and continues growing its share in the energy matrix until 2021. Investments are decided from 2014 to 2017, with two facilities of high capacity, ending with two new investments of lower sizes. Hydroelectric turbines are installed to its maximum capacity from the beginning of the time horizon, even when they are economically viable, its contribution compared with other sources is marginal. A similar conclusion can be made with the use of solar power for heating, it is used from the beginning to its maximum capacity to replace the use of natural gas, but its contribution is limited.

In Table 3 can be found the investments resulting from the solution of the deterministic model when it is considered the average price in fossil resources. The differences in comparison with fuzzy model are important. In this case, investments in sugarcane bioethanol are proposed, and the amount and year of investment in wind energy are different. For the case of hydropower, changes are proposed regarding the installed amount. The only resource that remains constant is solar energy.

Fig. 15 shows the evolution in the energy use for the deterministic model. It is Interesting to observe how decisions regarding the use of fossil resources are modified; greater use of natural gas is seen in the fuzzy model, while in the deterministic model the most used resource is oil. Another big difference is reflected in the use of biodiesel; in the fuzzy model various facilities reaches a level of around 10% of the energy matrix, while in the case of the deterministic one the use is not important. The deterministic model installs bioethanol plants from sugarcane; this allows to reduce the use of natural gas. As far as the electric market is concerned, installations in wind energy are much smaller than those in the fuzzy case and use hydro-electric turbines as an alternative source. In the residential market it makes use of solar energy to its maximum capacity.

Comparing the fuzzy versus the deterministic approach, the first situation to observe is that the facilities installation proposed through the fuzzy approach are quantitatively larger, both in capacity and in “amounts of money” than those resulted by the deterministic approach. This is due to the presence of uncertainties in the levels of reserves and demands that force to sacrifice potential earnings. The fuzzy model has a foresight character to eventual changes in the demands and reserves, and for this reason it decides to preserve resources in stand by that allow to face decreases of the expected reserves and/or increases in the demands.

Another evident issue when comparing the results of both models is the lower use of oil in the diffuse approach. In this case, it is also regulated the use of natural gas to supply the demands of transportation and electric power. Alternative sources substitute the use of natural gas, it invests in wind mills to generate electricity to replace the use of that source. In contrast to the diffuse approach, the deterministic model predicts lower total costs as a result of not contemplating fluctuations in the parameters. In addition it makes intensive use of oil. Although wind farm facilities are installed to replace the gas needs, the proposed capacities are lower and also it makes and intensive use of the nuclear facilities.

In summary, the diffuse model proposes an investment plan that diversifies the energy matrix to a greater extent, by making a less intensive use of the fossil resources while the deterministic model focuses mainly on these.

4. Conclusions

This paper presents a model for planning investments in the energy sector that considers the impact of uncertainty regarding prices of fossil resources, demands and the availability of fossil reserves. With this purpose, a diffuse approach is proposed which brings together three highlighted characteristics: the improvement of the information capture and representativeness, the robust feasibility of its solution and an efficient computational performance. Each uncertain data is modeled different in order to properly represent the real behavior. The uncertainty in demand is formulated as an L-number to represent that the low demand must be satisfied completely while the upper bound is unlikely to cover. Prices and availability of fossil resources are denoted by a triangular number having a low, average and high value. These parameters are chosen because have a big impact on the investment decisions; however other sources of instability may be recognized in the sector such as the prices of secondary energies, the capacity and stability of renewable resources, among others.

By using fuzzy numbers it is possible to get a linear model, which is not always possible when considering other approaches for the uncertain parameters. However, this representation and the subsequent transformation process depend on the parameters affected by the uncertainty.

The results founded through the approach adopted are compared with those found from the deterministic one in order to underline the importance of improving the degree of realism in modeling. Other approaches can be used to introduce uncertainty, among them the most used are the stochastic. However, these forms of treatment are not necessarily comparable due the source of uncertainty that they deal are different in both cases. On the one hand stochastic approaches address uncertainty due to random events, while the diffuse ones deal with uncertainty regarding to concepts (or of linguistic nature). In this way, the methodologies chosen allow quantifying knowledge due to experience and captured by descriptions of the type “will be near”, “data tend”, “will be approximately”, among others. Moreover, this makes them very attractive alternatives to address uncertainties associated the construction of trends and predictions of future behavior.

In summary, with fuzzy model it is possible to find a unique solution where α takes the value of $\alpha = 0.508$. This value is replaced in the membership functions of the fuzzy parameters in order to obtain the ranges, for fossil reserves and fuel price, and also the demands, where the solution is feasible. The results are compared with a deterministic approach and the solutions obtained are quite different; the evolution in the investments and the energy matrix shows different patterns. It is mainly emphasized that the solution of the fuzzy model is robust in the feasible range of the variables, combining diverse resources and covering a wide range of the uncertainties values, giving the decision makers more elements to analyze. This synthesizes one of the major contributions of this paper: the proposed model is a tool to assist the decision maker in the results analysis to attenuate the effect of uncertainties in the investments in energy facilities.

Acknowledgements

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Nomenclature:

Sets

i	set for energy sources
k	set for markets
t	periods of time
r	capacities intervals
$Fraction_i$	subset of the oil products (gasoline, diesel, fuel oil)
$Markets_{i,k}$	subset that link the source i with market k
NR	subset of nonrenewable energy sources
μ	set of critical values (lo, av, hg)

Parameters

TI	interest rate
hr	annual operating hours
$\tilde{C}O_{i,k,t}$	operating cost of energy sources i, for the market k, in the period t, is a fuzzy parameter
$aCO_{i,k,t}$	fraction of the operating costs related to its sales price
$bCO_{i,k,t}$	fixed cost per unit produced
$CO^{\mu}_{i,k,t}$	operating cost
$T_{i,k}$	time required to build the facilities
$\tilde{D}_{k,t}$	fuzzy demand for market k in period t
$CIup_{i,k,t}$	upper limit for the amount of money to invest in an energy source i for a market k in the period t
$Imin_{r,i,k}$	lower limit r for the capacity
$Imax_{r,i,k}$	upper limit r for the capacity
$Cap0_{i,k}$	initially installed capacities for sources i and markets k
$f_{i,k}$	factor of performance for the conversion of the source i into the form required for the market k
$\tilde{C}D_i$	fuzzy initial reserves
$Corte_i$	represents the average fraction obtained by distilling the petroleum typical of Argentina
ε_i	minimum percentage of non-renewable reserves available at the end of the horizon time for source i
$BioNF$	relationship between gasoline and bioethanol production from sugar cane or corn
$BioD$	ratio of production between diesel and biodiesel

Variables

$Cost$	cost along the time horizon
$Cost^{\mu}$	cost evaluated at each critical values
$CFS_{i,k,t}$	costs incurred for each energy source i and the market k and for each period t
$x_{i,k,t}$	energy flow from source i to market k in period t
$x^{EP}_{i,k,t}$	: energy flow of imports, for primary sources
$x^{ES}_{i,k,t}$	energy flow ofimports, for secondary sources
$CIMP_{i,t}$	cost of imported energy
$CI_{i,k,t}$	investment cost
$xCI_{r,i,k,t}$	continuous variable disaggregated for $CI_{i,k,t}$
$CS_{i,k,t}$	start-up cost
$Cap_{i,k,t}$	Capacity available from the source i to market k in period t
$y_{r,i,k,t}$	binary variable to handle the decision if a new investment is made
$ICap_{i,k,t}$	increased capacity for new investments
$xICap_{r,i,k,t}$	continuous variable disaggregated for $ICap_{i,k,t}$
$q_{i,t}$	annual energy production for source $i$period t
$RD_{i,t}$	reserves available for source i in period t
$\tilde{N}ew_{i,t}$	fuzzy new reserves for non-renewable resources
Z^{μ}_{lw}	most optimistic value of $Cost^{\mu}$
Z^{μ}_{up}	pessimistic value of $Cost^{\mu}$
α_t	represents the belonging corresponding to the availability
α''_k	membership corresponding to the demand
α''	membership value of the $Cost^{\mu}$ for the different prices, demands and fossil fuel reserves

Appendix A. Fuzzy model formulation and transformation into a crisp one

A general fuzzy model can be represented as follows:

$$min\ \tilde{e}x + fy + gw$$

$$sa$$

$$A_1x = \widetilde{b_1}$$

$$A_2x \geq \widetilde{b_2}$$

$$By \geq c$$

$$Cw \geq d$$

$$x \geq 0, y \geq 0,\ w \in \{0,1\}$$

where $\widetilde{(\blacksquare)}$ represent fuzzy data. In particular are considered two kinds of fuzzy numbers (FNs), triangular fuzzy numbers (TFNs) and L-fuzzy numbers (LFNs). The first (Fig. 16) are defined by three critical or characteristic values, $\tilde{e} = (e^{lo}, e^{av}, e^{gr}) = \{e \in \mathbb{R} | e^{lo} \leq e \leq e^{gr}\}$ and $\widetilde{b_1} = (b_1^{lo}, b_1^{av}, b_1^{gr}) = \{b_1 \in \mathbb{R} | b_1^{lo} \leq b_1 \leq b_1^{gr}\}$, and the last one (Fig. 17) are defined as $\widetilde{b_2} = (b_2^{lo}, b_2^{gr}) = \{b_2 \in \mathbb{R} | b_2^{lo} \leq b_2 \leq b_2^{gr}\}$ with $b_2^{gr} = b_2^{lo} + \omega,\ \omega \geq 0$.An example of the definition and shape of each kind of membership function is shown below, being $\tilde{l} = \tilde{e}, \widetilde{b_1}$.

$$\Psi_{\tilde{l}}(x) = \begin{cases} \dfrac{x - l^{lo}}{l^{av} - l^{lo}} & , if\ l^{lo} \leq x < l^{av} \\ \dfrac{l^{gr} - x}{l^{gr} - l^{av}} & , if\ l^{av} \leq x \leq l^{gr} \\ 0 & , if\ x > l^{lo}\ or\ x > l^{gr} \end{cases}$$

$$\Psi_{\widetilde{b_2}}(x) = \begin{cases} 1 & , \quad if\ x < b_2^{lo} \\ \dfrac{x - b_2^{gr}}{b_2^{lo} - b_2^{gr}} & , if\ b_2^{lo} \leq x \leq b_2^{gr} \\ 0 & , \quad if\ \ x > b_2^{gr} \end{cases}$$

In order to solve this model, a number of reformulations are needed to convert this fuzzy linear model (FLM) into an equivalent crisp linear programming (CLP) model. The first transformation deals with the inaccurate coefficients in the objective function, so that the feasible region remains unchanged. This reformulation is based on [25] and was used by others authors as [26] and [27]. The technique consists in rewrite the objective function using the inherited properties from the triangular fuzzy numbers (TFN). Here, the critical values and the difference between them are used as shown in the following equations.

$$z^{lw} = \left(e^{av} - e^{lo}\right)x + hy + gw$$

$$z^{av} = e^{av}x + hy + gw$$

$$z^{up} = (e^{gr} - e^{av})x + hy + gw$$

In this way, is obtained a multi-objective fuzzy programming (MOFP) model that seeks to maximize the region I, increasing the possibility of obtaining better objective value, and the same time minimize the area of the possibility distribution, where the costs are upper (*region II*). This is seen graphically in Fig. 18.

$$max\ \ z^{lw}(region\ I)$$

$$min\ \ z^{\mu},\ \ \forall\,\mu = \ av, up(region\ II)$$

The second transformation is deduced of the methodology used by Ref. [28] and deals with uncertainty in the constraint sets. According [29] and [30] when $\widetilde{b_1}$and $\widetilde{b_2}$are fuzzy parameters and $A_1$and $A_2$are deterministic matrixes, the possibility operators are defined as:

$$Pos\left(A_1x = \widetilde{b_1}\right) = \Psi_{\widetilde{b_1}}(A_1x)$$

$$Pos\left(A_2x = \widetilde{b_2}\right) = \Psi_{\widetilde{b_2}}(A_2x)$$

This can also be deduced of the concepts of possibility theory developed by Ref. [31] and retaken by Refs. [32], [33], [34], among others. Specifically, the definitions used for obtain the previous equations sets that if $\tilde{\gamma}$and $\tilde{\beta}$ are fuzzy numbers then:

$$Pos\left(\tilde{\gamma} \le \tilde{\beta}\right) = sup\left\{min\left(\Psi_{\tilde{\gamma}}(u),\ \Psi_{\tilde{\beta}}(v)\right)u, v\in\mathbb{R}, u \le v\right\}$$

$$Pos\left(\tilde{\gamma} \ge \tilde{\beta}\right) = sup\left\{min\left(\Psi_{\tilde{\gamma}}(u),\ \Psi_{\tilde{\beta}}(v)\right)u, v\in\mathbb{R}, u \ge v\right\}$$

hence,

$$Pos\left(\tilde{\gamma} = \tilde{\beta}\right) = sup\left\{min\left(\Psi_{\tilde{\gamma}}(u),\ \Psi_{\tilde{\beta}}(u)\right)|u\in\mathbb{R}\right\}$$

Then, if $\tilde{\beta}$is crips:

$$\Psi_{\tilde{\beta}}(v) = \begin{cases} 1, & if\ v = \beta,\ \beta\in\mathbb{R} \\ 0, & otherwise \end{cases}$$

$$Pos(\tilde{\gamma} = \beta\ ) = sup\begin{cases} min(\Psi_{\tilde{\gamma}}(u),\ 1)|u = \beta,\ \beta\in R \\ min(\Psi_{\tilde{\gamma}}(u),\ 0)|u\ne\beta,\ \beta\in R \end{cases} = \begin{cases} sup(\Psi_{\tilde{\gamma}}(u)|u = \beta,\ \beta\in\mathbb{R}) \\ 0 \qquad |u\ne\beta,\ \beta\in R \end{cases} = \ \Psi_{\tilde{\gamma}}(\beta)$$

From the definition of this operator it is possible to adapt the techniques of stochastic chance-constraint in environment fuzzy, obtaining:

$$Pos\left\{A_1x = \widetilde{b_1}\right\} \ge \alpha$$

$$Pos\left\{A_2x \ge \widetilde{b_2}\right\} \ge \alpha''$$

Given that, $\widetilde{b_1}$is a TFN and $\widetilde{b_2}$is a LFN it follows:

$$\alpha \le Pos\left\{A_1x = \widetilde{b_1}\right\} = \Psi_{\widetilde{b_1}}(A_1x) = \begin{cases} \dfrac{A_1x - b_1^{lo}}{b_1^{av} - b_1^{lo}} &, if\ b_1^{lo} \le A_1x < b_1^{av} \\ \dfrac{b_1^{gr} - A_1x}{b_1^{gr} - b_1^{av}} &, if\ b_1^{av} \le A_1x \le b_1^{gr} \\ 0 &, if\ A_1x > b_1^{lo}\ or\ A_1x > b_1^{gr} \end{cases}$$

$$\alpha' \le Pos\left\{A_2x \ge \widetilde{b_2}\right\} = \Psi_{\widetilde{b_2}}(A_2x) = \begin{cases} 1, & if\ A_2x < b_2^{lo} \\ \dfrac{A_2x - b_2^{gr}}{b_2^{lo} - b_2^{gr}}, & if\ b_2^{lo} \le A_2x \le b_2^{gr} \\ 0, & if\ \ A_2x > b_2^{gr} \end{cases}$$

thus,

$$\alpha \le \frac{A_1x - b_1^{lo}}{b_1^{av} - b_1^{lo}} \Rightarrow \alpha*\left(b_1^{av} - b_1^{lo}\right) \le A_1x - b_1^{lo} \Rightarrow \alpha*b_1^{av} + (1-\alpha)*b_1^{lo} \le A_1x$$

$$\alpha \le \frac{b_1^{gr} - A_1x}{b_1^{gr} - b_1^{av}} \Rightarrow \alpha*(b_1^{gr} - b_1^{av}) \le b_1^{gr} - A_1x \Rightarrow A_1x \le (1-\alpha)*b_1^{gr} + \alpha*b_1^{av}$$

$$\alpha' \le \frac{A_2x - b_2^{gr}}{b_2^{lo} - b_2^{gr}} \Rightarrow \alpha'*\left(b_2^{lo} - b_2^{gr}\right) \le A_2x - b_2^{gr} \Rightarrow \alpha'*b_2^{lo} + \left(1 - \alpha'\right)*b_2^{gr} \le A_2x$$

Therefore, the two transformations performed up to this point give as a result a multi-objective crisp (MOC) model equivalent to the original FLM.

$$max \quad z^{lw}$$

$$min \quad z^{\mu} \quad , \ \forall\,\mu = \ av, up$$

$$sa$$

$$A_1x \ge \alpha*b_1^{av} + (1-\alpha)*b_1^{lo}$$

$$A_1x \le (1-\alpha)*b_1^{gr} + \alpha*b_1^{av}$$

$$A_2x \ge \alpha'*b_2^{lo} + \left(1 - \alpha'\right)*b_2^{gr}$$

$$By \le c$$

$$Cw \leq d$$

$$0 \leq \alpha \leq 1$$

$$0 \leq \alpha' \leq 1$$

$$x \geq 0, y \geq 0, \; w \in \{0, 1\}$$

Finally to solve the MOC model it used the max-min approach [35–37]. Thereby is determined a unique solution that maximizes the possibility of satisfaction of each constraints. For this, it defined a membership function for each objective function z^{μ} for $\mu = lw, \; av, up$ as:

$$\Psi_{z^{\mu}} = \begin{cases} 1, & if \; z^{\mu} < z^{\mu}_{lo} \\ \dfrac{z^{\mu}_{gr} - z^{\mu}}{z^{\mu}_{gr} - z^{\mu}_{lo}}, & if \; z^{\mu}_{lo} \leq z^{\mu} \leq z^{\mu}_{gr} \\ 0, & if \; z^{\mu} > z^{\mu}_{gr} \end{cases}$$

where the bounded are the positive ideal solutions (z^{μ}_{lo}) and negative ideal solutions (z^{μ}_{gr}) for each case:

$$z^{lw}_{lo} = \max_{x \in \mathbb{R}} z^{lw} x \;\; z^{lw}_{gr} = \min_{x \in \mathbb{R}} z^{lw} x$$

$$z^{av}_{lo} = \min_{x \in \mathbb{R}} z^{av} x \;\; z^{av}_{gr} = \max_{x \in \mathbb{R}} z^{av} x$$

$$z^{up}_{lo} = \min_{x \in \mathbb{R}} z^{up} x \;\; z^{up}_{gr} = \max_{x \in \mathbb{R}} z^{up} x$$

These definitions normalizes the objectives functions making them nominally comparables; so it can solve the three objectives resulting from the first reformulation together with the two feasibility conditions arising from the second transformation. For this reason, it formulates the follows expression:

$$max \left(\Psi_{z^{lo}} \wedge \Psi_{z^{av}} \wedge \Psi_{z^{gr}} \wedge \alpha \wedge \alpha'' \right) \equiv max \left(min \left(\Psi_{z^{lo}}, \; \Psi_{z^{av}}, \Psi_{z^{gr}}, \alpha, \alpha'' \right) \right)$$

which generates a compromise solution to maximize simultaneously the satisfaction of the objective function and each constraint that is linked to the fuzzy parameters. In this way, defining $\alpha'' = min\,(\Psi_{z^{lo}}, \; \Psi_{z^{av}}, \Psi_{z^{gr}}, \alpha, \alpha')$ it follows that the CLP model searched is:

$$max \; \alpha''$$

sa

$$\alpha'' \leq \frac{z^{\mu}_{gr} - z^{\mu}}{z^{\mu}_{gr} - z^{\mu}_{lo}} \quad , \forall \mu = lw, \; av, up$$

$$\alpha'' \leq \alpha$$

$$\alpha'' \leq \alpha''$$

$$A_1 x \geq \alpha * b^{av}_1 + (1 - \alpha) * b^{lo}_1$$

$$A_1 x \leq (1 - \alpha) * b^{gr}_1 + \alpha * b^{av}_1$$

$$A_2 x \geq \alpha' * b^{lo}_2 + \left(1 - \alpha'\right) * b^{gr}_2$$

$$By \leq c$$

$$Cw \leq d$$

$$0 \leq \alpha \leq 1$$

$$0 \leq \alpha'' \leq 1$$

$$0 \leq \alpha'' \leq 1$$

$$x \geq 0, y \geq 0, \; w \in \{0, 1\}$$

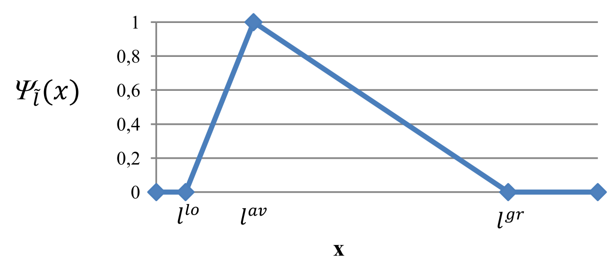

Fig. 16. Triangular fuzzy number.

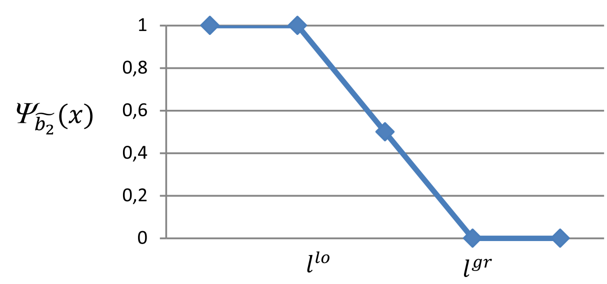

Fig. 17. L-fuzzy numbers.

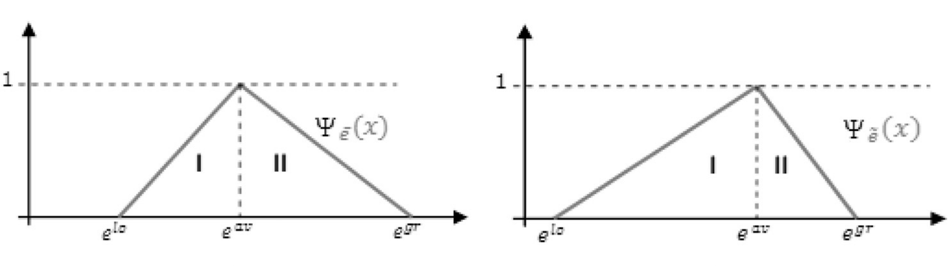

Fig. 18. Example of the methodology used for transformation the objective function.

Appendix B. Supplementary data

Supplementary data related to this article can be found at http://dx.doi.org/10.1016/j.energy.2017.07.103.

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