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FLEXIBILITY STUDY ON A DUAL MODE NATURAL GAS PLANT IN OPERATION

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This work addresses a flexibility study on a natural gas processing plant through the integration of a process simulator to a "worst case" flexibility strategy. The plant has a gas subcooled turboexpansion design, which is suitable for working in dual operation mode; i.e., in either ethane production or ethane rejection mode. The selected uncertain parameters (feed flowrate, condensable hydrocarbons content, carbon dioxide content, and ambient temperature) have great impact on process operating conditions. The use of the worse case algorithm with the KS overestimation function for inequality constraints has also been explored to improve computational time, and numerical results are compared for both solution strategies. Results show, in terms of both robustness and speed of computation, that this approach can be a useful tool to complement operational analysis of large processing units, commonly performed by simulation and "what if" studies.

Keywords: Optimization; Flexibility; Ethane extraction plant; Uncertain parameters

INTRODUCTION

Process flexibility is one important issue in the operability of chemical plants. At the academic level, it has been widely studied with mathematical programming techniques since the definition of the flexibility

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with low carbon dioxide content. The flexibility analysis explores a retrofit option in which the plant should operate with only two of three compression trains. The remaining train should be used in a new cryogenic plant. The uncertain parameters (carbon dioxide content, condensable components content, inlet gas volume flow and ambient temperature) have great impact on plant operating conditions. Numerical results show an important loss of flexibility in the plant with less compression horsepower for both operating modes, even though nominal optima are the same both with and without available horsepower limitations. This result highlights the need for a flexibility analysis within any retrofit study.

The flexibility problem has been solved with the original worst case algorithm (WC) and its extension that uses the KS function to overestimate the whole set of nonlinear constraints. A comparison of results shows that computational time can be reduced to a great extent by the use of the KS aggregation function, even though the problem must be solved several times to determine the appropriate ρ value.

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renders 77.75% propane recovery, which is an 18.3% decrease in propane production. These results show that plant flexibility, even in propane production mode, should be affected by the removal of a compressor.

Available horsepower and ethane purity are both active constraints in the nominal optimum and constitute the set of violated constraints in the first inner loop for ambient temperature in its upper bound. Convergence in this case has been very difficult to obtain because limited available horsepower sets the distillation column operation at a high pressure (less recompression requirement), but this fact worsens propane purity in bottoms product, which is also a violated constraint.

This problem has been solved with both the original WC algorithm and with the KS modification in three outer loops. Table IX shows computational time and permanently feasible point for both algorithms. In this case, there are only nonlinear constraints because there are fewer constraints that represent carbon dioxide precipitation conditions due to the warmer temperature profile in the distillation column. However, KSWC has been able to determine the permanently feasible optimum with an important reduction of computational time due to the application of the aggregated function that overestimates the set of nonlinear constraints. In ρ determination procedure, ρ succession has been 9, 11, 13, 15. Table IX shows that total CPU time, including ρ determination procedure, remains within 28% of the original WC time consumption in this large-scale problem.

CONCLUSIONS

We have integrated a rigorous process simulator to a flexibility optimization algorithm, taking into account uncertainty to determine the permanently feasible optimum in a gas subcooled turboexpansion plant design for two different operation modes: ethane production and ethane rejection. The analysis has been performed on a lean natural gas mixture

Table IX Comparison Between WC and KSWC Performance in Propane Production Mode (Case with Horsepower Limitations)

Variable	WC	KSWC
Propane production (kmol/h)	285.80	285.81
Adjusting parameter ρ	–	15
Number of outer loops	3	3
CPU time in inner loop (s) final ρ value	3481	102.75
CPU time in outer loop (s), final ρ value	93	155.72
Total CPU time including ρ determination procedure	3584	994.41

Table VII Nominal Optimum and Permanently Feasible Optimum for Propane Plant without Horsepower Limitation

Variable	Nominal optimum	Permanently feasible optimum
Tcold tank (K)	228.00	228.00
Pdem (bar)	18.00	18.00
Inlet gas compression work (kW)	18472.18	18472.18
Recompression work (kW)	18346.55	18515.75
Propane production (kmol/h)	672.20	645.74
Ethane recovery (%)	2.40	0.029
Propane recovery (%)	96.70	92.89

optimum (PFO) has been determined in two outer loop iterations of the KSWC algorithm, and it yields a propane recovery that is only 3.9% lower than the nominal optimum (92.89 against 96.7% propane recovery). Operating conditions are similar in both cases, and the constraint on available horsepower is not active in both optima.

As a second step, we have studied the plant flexibility when it operates with only two compression trains. Table VIII shows a comparison between objective function and operating variables at the nominal optimum and at the permanently feasible optimum. As in ethane production mode, propane recovery in the nominal optimum is slightly affected by the removal of a compression train: 96.70% propane recovery with the entire compression train and 95.20% propane recovery with only two of them. However, the permanently feasible optimum in this latter case

Table VIII Nominal Optimum and Permanently Feasible Optimum for Propane Production Mode with Compression Limitations

Variable	Nominal optimum	Permanently feasible optimum
Tcold tank (K)	228.00	235.55
Pdem (bar)	19.16	22.70
Bdem (kmol/h)	1122.70	966.88
$Tdem._{TOP}$ (K)	187.22	196.90
$Tdem._{BOT}$ (K)	347.39	361.96
TE work (kW)	8488.60	8093.13
Compression work (kW)	18472.18	18472.18
Recompression work (kW)	15952.31	10799.05
Subcooled fraction (%)	16.00	15.50
Ethane/propane ratio	0.06	0.02
Ethane recovery (%)	2.37	0.52
Propane recovery (%)	95.20	77.75

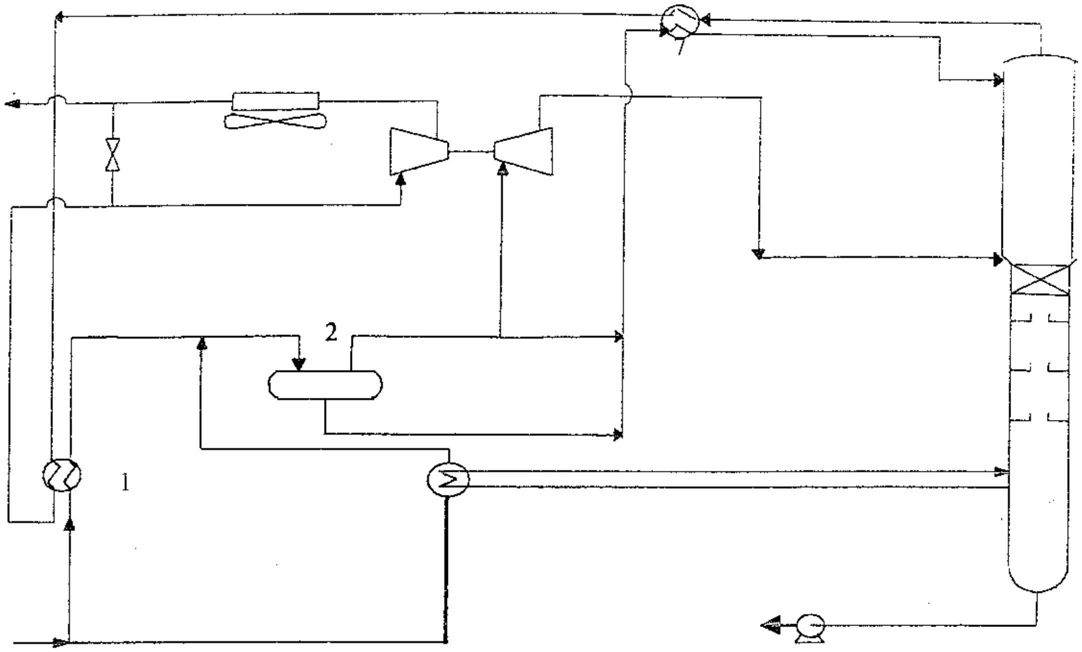

Figure 5. Gas subcooled process in ethane rejection mode. 1, gas-gas heat exchanger; 2, cold tank; 3, turboexpaneder; 4, demethanizer; 5, demethanizer side reboilder; 6, air coolers; 7, subcooler.

side reboiler can be integrated (see Figure 5). The cold tank also operates at warmer conditions. We have determined the permanent feasible optimum to maximize propane production with ethane reinjection to the pipeline for this plant for operation with horsepower limitations.

The optimization model is slightly different from the one for ethane production mode: the objective function is propane production and the second nonlinear inequality constraint corresponds to propane purity in bottoms product (ethane/propane ratio lower than 0.06). Uncertain parameters are the same as those in the previous case, and they vary within the bounds shown in Table II.

We have studied the flexibility of the plant in propane production mode for the plant operating with the original compression train. Numerical results are shown in Table VII. The permanently feasible

Table VI Computational Time in Outer and Inner Loops (Case: Horsepower Limitations)

		1	2	3
Outer loop	WC	107	156	261
	KSWC	53.39	121.61	173.90
Inner loop	WC	3343	2810	2535
	KSWC	38.22	35.86	8.96

Table V Permanently Feasible Optimum for Ethane Production in Gas Subcooled Design, with Original Worst Case Strategy (WC) and with KS Function (KSWC), for the Case with Horsepower Limitations

Variable	WC	KSWC
Ethane prod. (kmol/h)	556.77	556.77
Adjusting parameter ρ	–	13
Number of outer loops	3	3
CPU time in inner loop (s)	8688	83.04
Total CPU time (s), final ρ value	9212	431.44
Total CPU time (s), including ρ determination procedure	9212	1403.62

$$\lim_{\rho\to\infty} KS(x,\rho)_j = \max(g_j(x)).$$

However, too large ρ values may cause numerical overflow. A value between 5 and 10 has been suggested by Sobieszczanski-Sobieski (1992) and Sobieszczanski-Sobieski et al. (1987). In this work, slightly larger ρ values, between 9 and 15, have been required to obtain equivalence between WC and KSWC. Starting from an initial value $\rho = 9$, the KSWC has been applied to solve the NLP problem under uncertainty, increasing ρ value in two units until the same permanently feasible optimum (PFO) is obtained for two successive ρ values. In this case, the problem has been solved for $\rho = 9, 11, 13$; the last two values render the same PFO. The last row in Table V shows total CPU time including the problem solution with ρ determination procedure; it can be seen that CPU time remains within 15% of the original WC time consumption in this large-scale problem.

In the original algorithm, 28 maximization problems (one for each nonlinear constraint) are solved within each inner loop. In the KSWC algorithm, only the KS function (as an overestimator of the set of nonlinear constraints) is maximized in the inner loop. Moreover, the maximization of the KS function has shown a better performance in terms of robustness.

Plant Flexibility for the Maximization of Propane Production (with Ethane Rejection) with Two or Three Compressors

In gas subcooled designs, the plant can be operated at different operating modes: a) ethane recovery and b) ethane rejection. In this latter case, the process goal is the recovery of propane and heavier hydrocarbons; the distillation column has a much warmer profile (190–370 K) and no heat integration is performed between the bottom reboiler and inlet gas; only a the same (93% ethane recovery) for the plant operating either with three or with two compression trains. However, the permanently feasible optimum is quite different, about 90% ethane recovery in the original design and 66% ethane recovery in the proposed modification. The plant should have an important loss of flexibility to adapt its performance to uncertainties in operating parameters if there is a limitation in available horsepower.

The flexibility problem has been solved with both the original WC algorithm and the KSWC; they require three outer loops to converge to the permanently feasible optimum (66% ethane recovery). Figure 4 shows ethane production (objective function) and cold tank temperature values for each outer loop.

As regards computational times, there are important differences between both algorithms. Table V shows that outer loop computational time is on the same order for original WC and KSWC, but inner loop computational time with KSWC is 0.96% of that of WC. Table VI shows a detail of CPU time in outer and inner loops. As it has been proved in Raspanti et al. (2000), WC and KSWC feasible regions are equivalent when ρ tends to infinity. This means that the adjusting parameter ρ must be large enough to ensure that

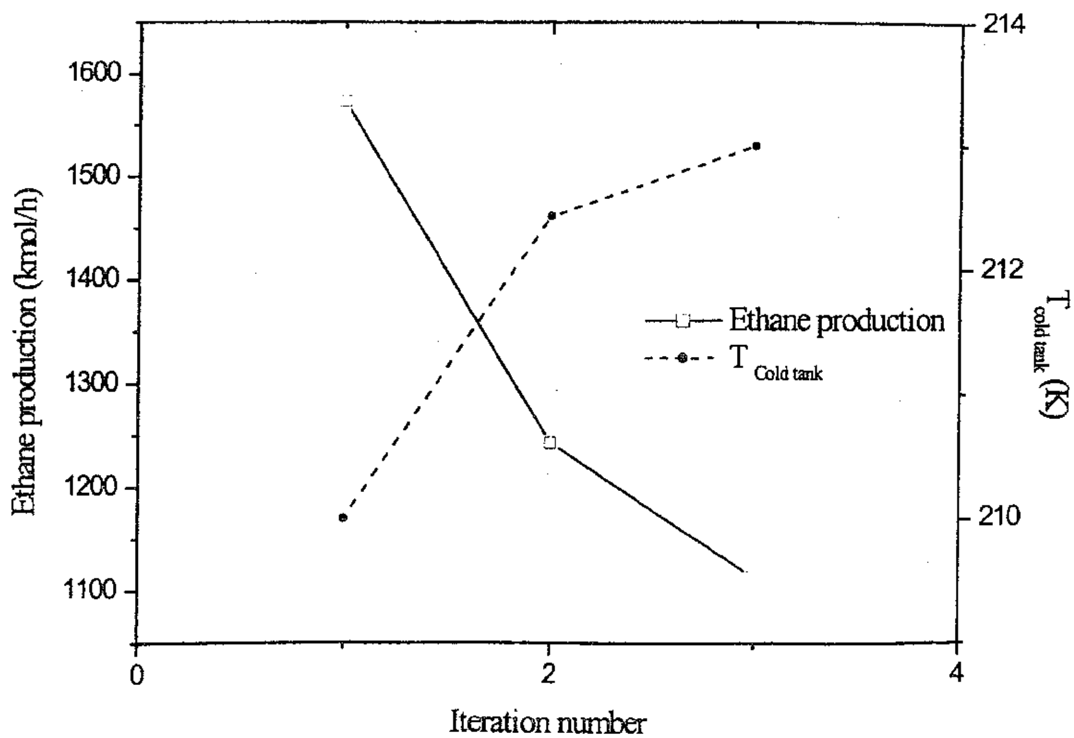

Figure 4. Outer loop iteration progress for gas subcooled process in ethane production mode.

After analyzing the plant flexibility with its original compression train, we have studied the plant performance and flexibility when there is a limitation on available horsepower: i.e., the plant operates with two of three compression trains. Variations in uncertain parameters (especially in feed flowrate, condensable hydrocarbons content and ambient temperature) clearly influence operating conditions. Table IV shows a comparison between objective function (ethane recovery) and main operating variables at the nominal nonlinear programming optimum and at the permanently feasible optimum (PFO), as determined through the worst case strategy. In the nominal optimum, both ethane purity and available horsepower in compression trains are active constraints. The set of violated constraints in the first inner loop is composed of available horsepower and temperature crosses in gas-gas heat exchangers for both the processed flowrate and ambient temperature in their upper bounds. In the second inner loop, the violated set is composed of ethane purity and heat duty in demethanizer bottom reboiler in its lower bound for ambient temperature of 294 K and the remaining uncertain parameters in their lower bounds.

Feed flowrate and ambient temperature are important uncertain parameters in this model because available horsepower is an active constraint in the nominal optimum due to the removal of one compression train.

Numerical results have highlighted the importance of the flexibility analysis in retrofit studies. In this plant, the nominal optimum is almost

Table IV Nominal Optimum and Permanent Feasible Optimum for Ethane Production (with Horsepower Limitations)

Variable	Nominal optimum	Permanently feasible optimum
Tcold tank (K)	210.00	213.00
Pdem (bar)	19.11	22.55
Bdem (kmol/h)	2836.6	2224.42
$Tdem._{TOP}$ (K)	167.22	177.23
$Tdem._{BOT}$ (K)	282.98	304.47
TE work (kW)	5039.10	4957.29
Compression work (kW)	18472.18	18472.18
Recompression work (kW)	15952.25	11130.30
Subcooled fraction (%)	22.00	22.00
Methane/ethane ratio	0.0035	1.E-5
Demethanizer bottom reboiler heat duty (kW)	4783.24	5373.14
Propane recovery (%)	99.60	99.06
Ethane recovery (%)	93.84	66.42

Table II Uncertain Parameters and Their Bounds

Uncertain parameter	Lower bound	Nominal value	Upper bound
CO_2 (%molar)	0.50	0.60	0.75
Ethane and heavier (%molar)	5.80	7.19	8.80
Inlet gas (MMscm/h)	21.50	22.00	22.50
Ambient temperature (K)	279.15	293.15	308.15

dependent factor; this coefficient is a linear function of ambient temperature, and is equal to 1 at 288.15 K and 0.88525 at 303.15 K (Gas Processors Suppliers Association, 1998);

6. CO_2 precipitation conditions in demethanizer; these constraints are written as:

   (CO_2 concentration in vapor phase) $\leq 0.90^*$ (CO_2 solubility in vapor phase)
   (CO_2 concentration in liquid phase) $\leq 0.90^*$ (CO_2 solubility in liquid phase)
   at each stage in the demethanizer column.

As a first step, we have determined the permanently feasible optimum (PFO) for the lean natural gas mixture in gas subcooled process (GSP) to maximize ethane production with the original available horsepower: i.e., with three compression trains. Table III shows numerical results for this case. The nominal optimum (93.86% ethane recovery) is slightly affected by variations in ambient temperature and feed flowrate and composition within the selected bounds: the PFO corresponds to 90.34% ethane recovery. The constraint on available horsepower is active neither in the nominal nor in the permanently feasible optimum.

Table III Comparison Between Nominal Optimum and Permanently Feasible Optimum for Plant in Ethane Production Mode and Original Compression Train

Variable	Nominal optimum	Permanently feasible optimum
Tcold tank (K)	213.67	212.14
Pdem (bar)	18.00	22.74
Inlet gas compression work (kW)	18472.18	18472.18
Recompression work (kW)	17693.78	10454.12
Ethane recovery (%)	93.86	90.34
Propane recovery (%)	99.61	99.32

compression trains (compressors + gas turbines) suggested the possibility of increasing plant capacity without the addition of compression horsepower. Lynch and Pitman (1999) have reported the retrofit of a natural gas processing plant to achieve 30% increase in plant feed handling without adding residue gas compression.

We have determined the permanently feasible optimum (PFO) for the plant with the original compression train and for the proposed modification and we have compared numerical results. Table I shows nominal feed gas composition. Uncertain parameters and their bounds are shown in Table II.

We have also compared the performance of the original worst case algorithm (WC) to the performance of its extension with the KS overestimator (KSWC) as regards computational time and solution accuracy. Nonlinear programming problems have been solved with a successive quadratic programming algorithm (Biegler and Cuthrell, 1985) and numerical results have been obtained in a Pentium II, 266 MHz.

Plant Flexibility for the Maximization of Ethane Production with Two or Three Compressors

In the maximization of ethane production under uncertainty, nonlinear constraints represent the following process specifications and operating bounds:

1. Reboiler heat duty in demethanizer column;
2. Ethane purity in demethanizer bottom flowrate, specified as methane/ethane molar flow ratio (lower than 0.04);
3. Carbon dioxide content in residual gas (lower than 0.02);
4. Temperature crosses in gas-gas heat exchangers and subcooler (minimum temperature difference: 10 K);
5. Available horsepower in each compression train. This constraint takes into account ambient temperature influence on the available energy from each turbine. Turbine efficiency is affected by a temperature-

Table I Natural Gas Composition

Component	% Molar
Nitrogen	1.80
CO_2	0.60
Methane	90.41
Ethane	4.31
Propane and heavier hydrocarbons	2.88

Inequality nonlinear constraints: They represent process specifications, purity requirements, available horsepower in compression trains, and carbon dioxide precipitation conditions.

Uncertain parameters: Typical input variations in this plant have been selected as uncertain parameters. Raw gas is produced from a large number of gas wells from different reservoirs and is also subject to fluctuations depending on factors such as maintenance of the wellhead facilities or abnormal operating conditions of upstream plants. The main variations in flowrate and composition are season dependent. In summer, the gas demand is reduced, and therefore only the richest composition wells are preferably kept in production; in winter, when the gas demand is maximum, additional wells with different and lighter compositions are put into production. Ambient temperature has been considered as an uncertain parameter because the available power output of gas turbines depends on the entering air temperature (the front end of these turbines is an axial air compressor). Consequently, the following uncertain parameters have been considered:

a) Carbon dioxide content in feed gas. This variable directly influences ethane recovery values because higher demethanizer pressures are required to process more acid mixtures with an important decrease in ethane production;
b) Condensable hydrocarbons content in the feed. This variable also affects ethane recovery because richer gases cannot produce the sufficient low temperatures required for the process, and mechanical refrigeration is needed to maintain acceptable levels for ethane recovery.
c) Total amount of feed gas.
d) Ambient temperature. Gas turbines horsepower changes with variations in inlet air density due to ambient temperature so this condition affects available horsepower in compressors.

DISCUSSION OF RESULTS

We have integrated the rigorous sequential process simulation routine described above to both alternative flexibility optimization algorithms, WC and KSWC. In this context, we have studied the flexibility of the gas subcooled turboexpansion plant for the maximization of production in two different operating modes: ethane production or propane production with ethane reinjection to the pipeline. As part of a retrofit study, we have analyzed the final flexibility of the original plant with three compressors for its operation with only two of them; the remaining train should be used as driver in a new cryogenic sector. Compression costs constitute the main capital costs in a revamp option, and a previous analysis of existing

PLANT FLEXIBILITY MODEL

In this paper, we study the flexibility of a dual-mode natural gas processing plant by solving the general nonlinear optimization problem under uncertainty (1) as follows.

Objective function: Maximization of ethane production or propane production with ethane rejection (depending on the operation mode).

Equality constraints: They represent the plant mathematical model; they are solved within a sequential rigorous simulator (De Beistegui et al., 1992). The most critical unit, with regard to proximity to the critical conditions, is the cold tank. The pressure of the cold tank is chosen below the predicted critical pressures, on the basis of Michelsen's phase envelope (1980) and critical point computations, using the SRK equation. A robust flash algorithm (Michelsen, 1982) gives a realistic description of the relative amounts of liquid and vapor and compositions. The simulation of process conditions following the above procedure has been confirmed by plant tests.

The simulation begins at the cold tank, where the entire refrigeration load is evaluated. The turboexpander, the subcooler, and the demethanizer are then simulated. A fraction of the feed gas is cooled by heat exchanging with side and bottom reboilers. If the remaining heat duty can be provided by the inlet gas-residual gas (cryogenic) heat exchangers, no external refrigeration is required. The demethanizer column is simulated using a modification of the Naphtali Sandholm procedure (1971) coupled with the Soave Redlich Kwong (SRK) equation of state for generating K-values and enthalpies (Christiansen et al., 1979). A rigorous prediction of CO_2 solubility is required at each stage of the demethanizer column because the separation of CO_2-methane mixtures is not always possible without going through the solid-vapor region, at pressures below methane critical pressure. Carbon dioxide solubility increases in mixtures with ethane, propane, and butanes. A predictive method, based on experimental CO_2 solid saturation pressures and computation of fugacity coefficients by using the SRK equation of state, has been used for the computation of CO_2 solubility in these multicomponent hydrocarbon mixtures (Fernández et al., 1991). Carbon dioxide solubility in both liquid and vapor phases is calculated at the hydrocarbon composition, pressure, and temperature at each stage of the demethanizer and it is compared with CO_2 current composition at each stage.

Optimization variables: From a previous sensitivity analysis and plant operating data, the following main continuous optimization variables have been selected: high pressure separator temperature (cold tank), Tct (K), demethanizer pressure, Pdem (bar), demethanizer bottom flowrate, Bdem (kmol/h), and the subcooled fraction in gas subcooled designs, Div (%).

For a sufficiently large ρ the KS function has the property of over-estimating the constraints (see Figure 3).

Based on this constraint aggregation strategy, the largest constraint violation at the inner level can be found solving the single NLP problem:

$$\underset{\boldsymbol{x},\ \boldsymbol{\theta}}{Max}\ KS(z^*, \boldsymbol{x}, \boldsymbol{\theta}) \tag{7}$$

s.t.

$$\boldsymbol{h}(z^*, \boldsymbol{x},\ \boldsymbol{\theta}) = 0$$
$$\forall \boldsymbol{\theta} \in \Gamma$$
$$\Gamma = \{\boldsymbol{\theta}/\theta^L \leq \boldsymbol{\theta} \leq \theta^U \wedge \boldsymbol{d}(\boldsymbol{\theta}) \leq 0\},$$

where

$$KS(z^*, \boldsymbol{x}, \boldsymbol{\theta}) = g_{\max} + (1/\rho)\ \ln[\Sigma \exp(\rho(g_j(z^*, \boldsymbol{x}, \boldsymbol{\theta}) - g_{\max}))]. \tag{8}$$

Problems (2) and (3) represent the original worst case algorithm (WC), and problems (2) and (4) are the KS-based modification or KSWC algorithm. Raspanti et al. (2000) have proved convexity properties of KS function under certain conditions.

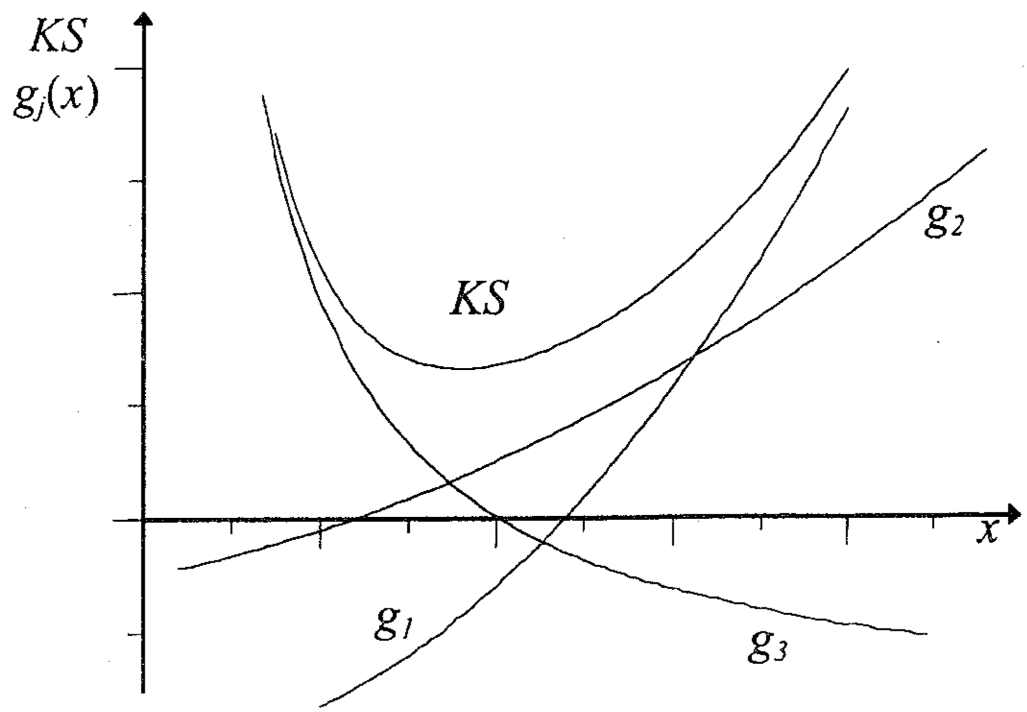

Figure 3. KS overestimation of a set of inequality constraints g_j.

x_v^k is the vector of state variables that verify the model equations for θ_v^k.

2. *Inner Level*: Fixing the optimization variables at z^*, the largest violation for each constraint is obtained by maximizing each single inequality constraint over the uncertain parameters. This means that the following J (number of inequality constraints) different nonlinear programming problems have to be solved:

$$\underset{\boldsymbol{x},\boldsymbol{\theta}}{Max}\; g_j(z^*, \boldsymbol{x}, \boldsymbol{\theta}) \qquad j = 1, \ldots, J \tag{3}$$

s.t.

$$\boldsymbol{h}(z^*, \boldsymbol{x}, \boldsymbol{\theta}) = 0$$
$$\forall \boldsymbol{\theta} \in \Gamma$$
$$\Gamma = \{\boldsymbol{\theta}/\theta^L \leq \boldsymbol{\theta} \leq \theta^U \wedge \boldsymbol{d}(\boldsymbol{\theta}) \leq 0\}.$$

The constraints that are violated at this level are added as new constraints at the outer level problem, and the sequence of outer and inner subproblems is repeated in an iterative way. The algorithm stops when no constraint violation is determined at the inner level, and the current solution z^* of the previous outer loop represents a point that remains feasible if the uncertain parameter realization lies inside the specified bounds.

The algorithm is robust but requires large computational time when a rigorous plant model is under study or when it involves a large number of inequality constraints. Raspanti et al. (1997) developed a new strategy, which proposed a modification of the inner level by the introduction of an aggregation function KS (Kreisselmeier and Steinhauser, 1983; Sobieszczanski-Sobieski, 1992). For the set of inequality constraints of problem (1), the KS function is:

$$KS(g) = (1/\rho)\ \ln[\Sigma \exp(\rho g_j)] \qquad j = 1, \ldots, J, \tag{4}$$

or

$$KS(g) = M + (1/\rho)\ \ln[\Sigma \exp(\rho(g_j - M))] \qquad j = 1, \ldots, J, \tag{5}$$

where ρ is an adjusting parameter defined by the user. Expression (5) is recommended if (4) generates very large values for the exponential term and M is a non-negative scalar, defined as

$$M \approx Max(g_j) \qquad \text{for } j = 1, \ldots, J. \tag{6}$$

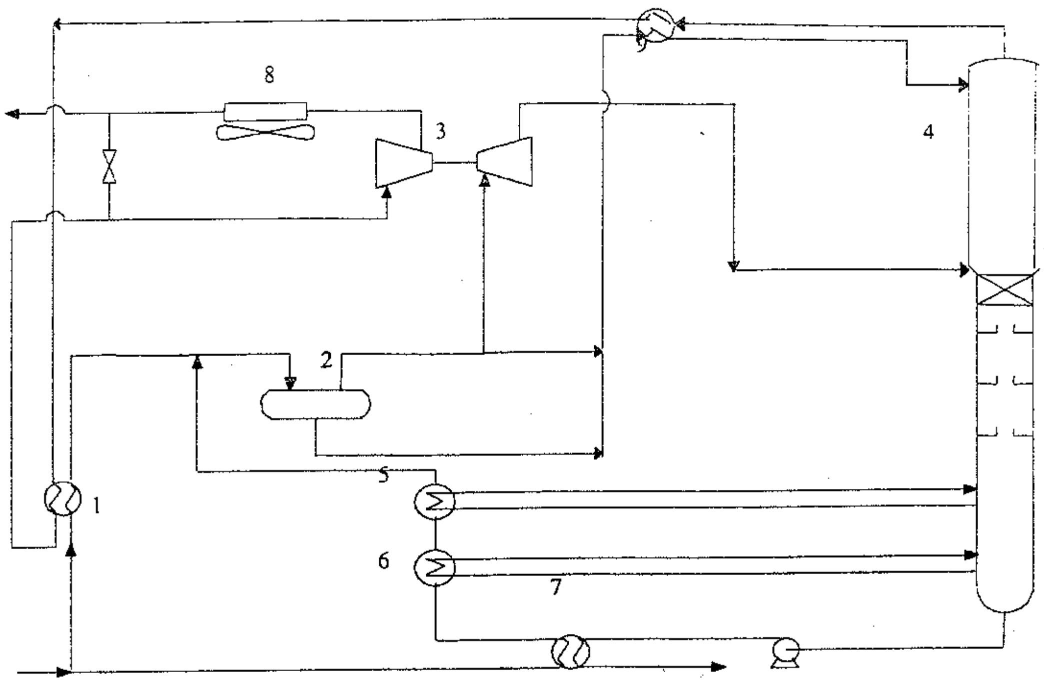

Figure 2. Gas subcooled process. 1, gas-gas heat exchanger; 2, cold tank; 3, turboexpander; 4, demethanizer; 5, demethanizer side reboilers; 6, bottom reboiler; 7, demethanized product exchanger; 8, air coolers; 9, subcooler.

parameters, $\boldsymbol{h}$ is the vector of equality constraints, and $\boldsymbol{g}$ is the vector of inequality constraints.

It is assumed that well-defined upper, lower, and nominal values for uncertain parameters are available. This strategy comprises two-level optimization problems:

1. *Outer Level*: For a fixed value of the uncertain parameters $\boldsymbol{\theta}$ (normally a nominal or average value, $\boldsymbol{\theta}^N$) the following problem (2) is solved to get an optimum z^*:

$$\begin{aligned}
&\underset{z,\boldsymbol{x},\boldsymbol{x}_v^k}{Min}\ \Phi(z,\boldsymbol{x},\boldsymbol{\theta}^N) \\
&\text{s.t.} \\
&\boldsymbol{h}(z,\boldsymbol{x},\boldsymbol{\theta}^N)=0 \\
&\boldsymbol{g}(z,\boldsymbol{x},\boldsymbol{\theta}^N)\le 0 \\
&\left.\begin{array}{l}\boldsymbol{h}(z,\boldsymbol{x}_v^k,\theta_v^k)=0 \\ g_v(z,\boldsymbol{x}_v^k,\theta_v^k)\le 0\end{array}\right\}\quad \begin{array}{l}k=1,\ldots,K \\ v\in V^k\end{array} \\
&z^L\le z\le z^U,
\end{aligned} \tag{2}$$

where k is the iteration index between both optimization levels, K is the total iteration number, V^k is the set containing the index of the violated constraints g_v at iteration k. Therefore θ_v^k is the value of θ that produces the largest violation of constraint v in iteration k, and

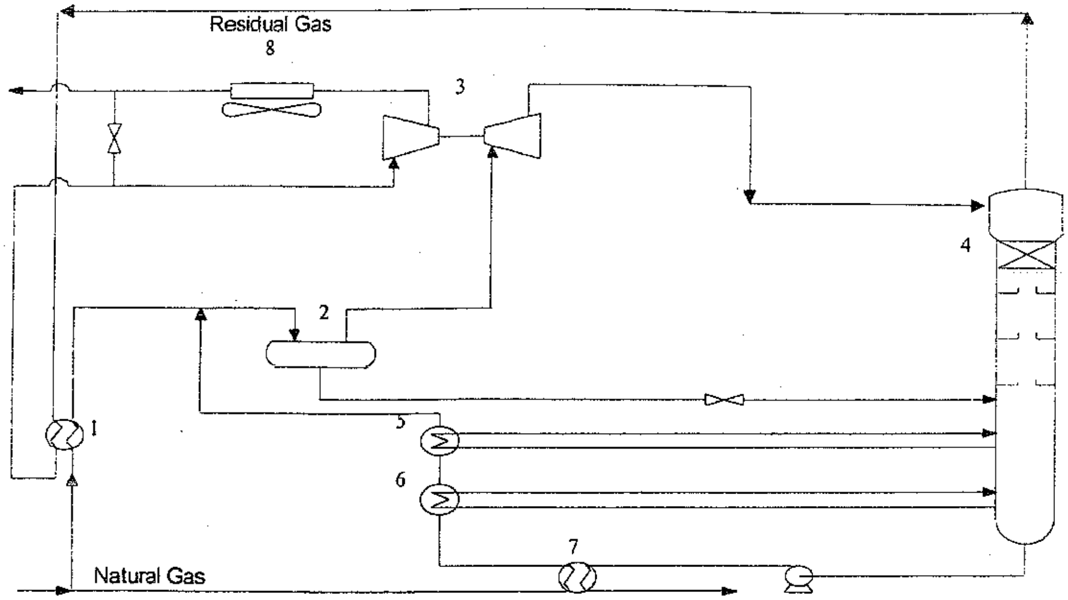

Figure 1. Cryogenic sector in a basic turboexpansion process. 1, gas-gas heat exchanger; 2, cold tank; 3, turboexpander; 4, demethanizer; 5, demethanizer side reboilers; 6, bottom reboiler; 7, demethanized product exchanger; 8, air coolers.

residual gas coming from the demethanizer. The remaining vapor is expanded through the turboexpander and fed to the middle of the column. The subcooled liquid is flashed and fed to the top of the demethanizer column, as shown in Figure 2.

FLEXIBILITY PROBLEM AND SOLUTION STRATEGY

In this work, we have studied the flexibility of a natural gas processing plant with the worst case (WC) methodology proposed by Bandoni et al. (1994). A brief description of the algorithm is given below. The general nonlinear programming problem under uncertainty can be formulated as:

$$\underset{z,\boldsymbol{x}}{Min}\,\Phi(z,\boldsymbol{x},\boldsymbol{\theta}^N) \tag{1}$$

s.t.

$$\boldsymbol{h}(z,\boldsymbol{x},\boldsymbol{\theta}) = 0$$
$$\boldsymbol{g}(z,\boldsymbol{x},\boldsymbol{\theta}) \leq 0$$
$$\forall\boldsymbol{\theta} \in \Gamma$$
$$\Gamma = \{\boldsymbol{\theta}/\theta^L \leq \boldsymbol{\theta} \leq \theta^U \wedge \boldsymbol{d}(\boldsymbol{\theta}) \leq 0\},$$

where Φ is the objective function, z is the vector of decision (optimization) variables, $\boldsymbol{x}$ is the vector of state variables, $\boldsymbol{\theta}$ is the vector of uncertain parameters, $\boldsymbol{\theta}^N$ is the vector of nominal values of the uncertain to be solved within the inner loop, with a consequent reduction of computational time. A comparison of computational times and results is presented.

TURBOEXPANSION PROCESSES

Turboexpansion designs have become increasingly popular in the natural gas processing industry since the first plant was built in 1963. Wilkinson and Hudson (1982, 1992) have proposed different turboexpansion plant designs to improve ethane recovery without inlet carbon dioxide removal, such as the gas subcooled process, the gas-liquid subcooled process, the liquid subcooled process, and the overhead recycle.

In a typical ethane extraction plant, inlet natural gas is filtered and compressed. It is then air cooled and dehydrated to avoid ice and hydrates formation. After this conditioning, the gas feed is divided into two equal streams, each of which is sent to a different cryogenic train for demethanization. The bottom products from the demethanizers are mixed and sent to a conventional separation process to obtain pure ethane, pure propane, butanes, and natural gasoline. After heat exchanging with the entering gases, the top product from the demethanizers, mainly methane, is recompressed to pipeline pressure and delivered as sales gas.

In a propane extraction plant based on turboexpansion (a dual-mode turboexpansion plant), the previously described demethanizer columns work as deethanizers and both methane and ethane are reinjected to the pipeline.

The cryogenic sector of a Basic Turboexpansion Process (BTEP) is shown in Figure 1. Natural gas is cooled by heat exchange with the residual gas and demethanizer side and bottom reboilers and, if necessary, with external refrigeration. The partially condensed gas feed is then sent to a high-pressure separator (cold tank). The vapor is expanded through the turboexpander to obtain the low temperatures required for high ethane recovery and is fed to the top of the demethanizer column. The liquid from the cold tank is directly flashed into the demethanizer at its lowest feed point. Methane and lighter components, such as nitrogen, constitute the top product and ethane and heavier hydrocarbons comprise the main components in the bottoms. Carbon dioxide, which is intermediate in volatility between methane and ethane, is distributed into top and bottom streams.

The top product, residual gas, cools the inlet gas and is then recompressed to pipeline pressure and delivered as sales gas. The demethanizer bottom product can be further fractionated to produce ethane, propane, butanes and natural gasoline.

In a gas subcooled turboexpansion process, a fraction of the vapor from the cold tank is condensed and subcooled by heat exchange with test (Grossmann et al., 1983) and the flexibility index (Swaney and Grossmann, 1985). Grossmann and Floudas (1987) formulated the flexibility problem as a mixed integer nonlinear programming case with the active set method, in which the inner optimization problem was replaced by algebraic equations that represented its Kuhn-Tucker conditions, and binary variables were introduced to model the complementarity conditions. Bandoni et al. (1994) proposed the worst case algorithm to study feasible operation in steady state as a two-level optimization method. More recently, Varvarezos et al. (1995) presented a sensitivity method to perform flexibility analysis in linear process systems and Hoch et al. (1995) studied the flexibility of rigorous distillation columns.

In this paper, we study the flexibility of a large-scale turboexpansion natural gas plant for different operation modes and optimization objectives: maximization of ethane recovery or maximization of propane recovery with ethane rejection. These plants are normally subject to variations in upstream conditions, such as pipeline pressure, ambient temperature and feed flowrate and composition, and must have the flexibility to achieve feasible operation over a wide range of uncertain conditions.

Through a detailed analysis of plant data, we have determined four measured uncertain parameters that have great impact on optimal plant operation. They are: a) carbon dioxide content in feed gas; b) heavy hydrocarbons content in the feed; c) total amount of feed gas; and d) ambient temperature.

The flexibility study has been performed using the "worst case" optimization strategy (Bandoni et al., 1994) for determining a permanent feasible operating point integrated to a sequential plant simulator (Diaz et al., 1997) for function evaluation. This strategy, which moves the nominal optimum into a permanent feasible region, is a two-level optimization algorithm. The outer loop gives the optimal operating conditions for a given set of nominal parameters, and the inner loop tests the feasibility of operating conditions for the corresponding outer loop and determines the combination of uncertain parameters that gives the worst violation of inequality constraints. The iterative procedure goes on until no constraint is violated in the inner loop.

In the case of a rigorous model of an existing plant, with a large number of inequality constraints, the same number of nonlinear optimization subproblems has to be solved within each inner loop, resulting in high computational times. Therefore, we have also studied the plant flexibility with an extension of the "worst case" (WC) algorithm that is based on an aggregation constraint approach (KSWC) that overestimates the set of nonlinear constraints (Raspanti et al., 1997). In this case, only one nonlinear subproblem has

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