STELLAR MOTIONS IN GALACTIC SATELLITES

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Abstract. The study of the motions of the stars that belong to a galactic satellite (i.e. a globular cluster or a dwarf galaxy orbiting a larger one) has some similarities, as well as significant differences, with that of the restricted three-body problem of celestial mechanics. The high percentage of chaotic orbits present in some models is of particular interest because it rises, on the one hand, the question of the origin of those chaotic motions and, on the other hand, the question of whether an equilibrium stellar system can be built when the bulk of the stars that make it up behave chaotically.

Key words: galactic satellites, stellar orbits, chaotic motion

1. Introduction

Many stellar systems are satellites of larger ones: the globular clusters, spheroidal systems, and the Magellanic clouds that orbit our own Galaxy, the Milky Way, are typical examples. It has long been recognized that the tidal field of the main galaxy affects the size of the satellite, imposing a limiting *tidal radius*, but aside from that effect, satellite models like the ones due to King, tend to ignore the tidal influence inside the body of the satellite (see, e.g. Binney and Tremaine, 1987). Nevertheless, Carpintero et al. (1999), Muzzio et al. (2000a,b) have shown that the motions of the stars that belong to the satellite are strongly affected, even those that pertain to the innermost regions of the satellite which are usually regarded as the ones best shielded from tidal effects.

The problem of the stellar motions inside a galactic satellite has some obvious similarities with the restricted three-body problem of celestial mechanics, as one can identify the main galaxy with the Sun, the satellite with a planet and the star with a minor body. A little thought shows, however, that there are also important differences. First, while the age of the Solar System is of the order of hundreds of millions of orbital periods, the age of a small stellar system is of the order of thousands of orbital periods only. Second, while the attractive force goes to infinity as one approaches the planet, that force goes to zero as one approaches the center of the galactic satellite. Third, while planets are usually taken as spherical (at least in the first approximation), galactic satellites are triaxially shaped by the tidal forces (and can have even more complicated forms, as the Magellanic clouds themselves show). Finally, planetary orbits do not depart much from circles, while

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galactic satellites tend to follow strongly elongated orbits (nevertheless, as in our previous investigations, here we will consider only the case of satellites on circular orbits). The similarities and differences between both problems were discussed by Muzzio et al. (2000b); here we want only to point out that this research may be of some interest to celestial mechanicists and to encourage them to undertake similar investigations.

Having that aim in mind, in the next section we explain the importance of stellar orbits for the construction of models of stellar systems and the problems posed by the presence of chaotic orbits. We then summarize the results of our previous investigations on this matter and present some new results.

2. The Importance of Stellar Orbits

Models of stellar systems must be *self–consistent*: the distribution of stars (i.e. mass) produces a gravitational potential; that potential determines the kind of orbits that the stars can follow in the system and the orbits, in turn, determine the distribution of the stars, which must be precisely the initial one to have self-consistency. The regions of space visited by the stars as they move on their orbits should, therefore, be consistent with the form of the stellar system we want to model: if we want to have a stellar system elongated in the, say, x direction, the bulk of the orbits has to be elongated in that direction too.

Now, since we usually want to build stationary models, all the orbits will obey the energy integral and they will be confined within the zero-velocity surface that corresponds to their energy. In three-dimensional space, chaotic orbits will obey, at most, one additional isolating integral, while regular orbits have two additional isolating integrals. Nevertheless, if the distribution function depends only on the energy, the stellar system must be spherical and, moreover, an equipotential surface tends to be more spherical than the equidensity surface through the same point (see, e.g. Binney and Tremaine, 1987). Therefore, chaotic orbits are not very good building blocks for stellar systems that depart much from sphericity (see Merritt, 1999 for further details).

3. Previous Results

Recognizing the importance of orbits to build a self-consistent stellar system, Carpintero et al. (1999) investigated what types of orbits might be present in a galactic satellite. They represented the satellite with a potential that was spherical at its center and increased its triaxiality outwards, just as it would happen with a satellite deformed by tidal effects. The satellite was placed on a circular orbit and stellar encounters among the satellite's stars were neglected. They reasoned that, if there were chaotic orbits under these conditions, even more could be expected

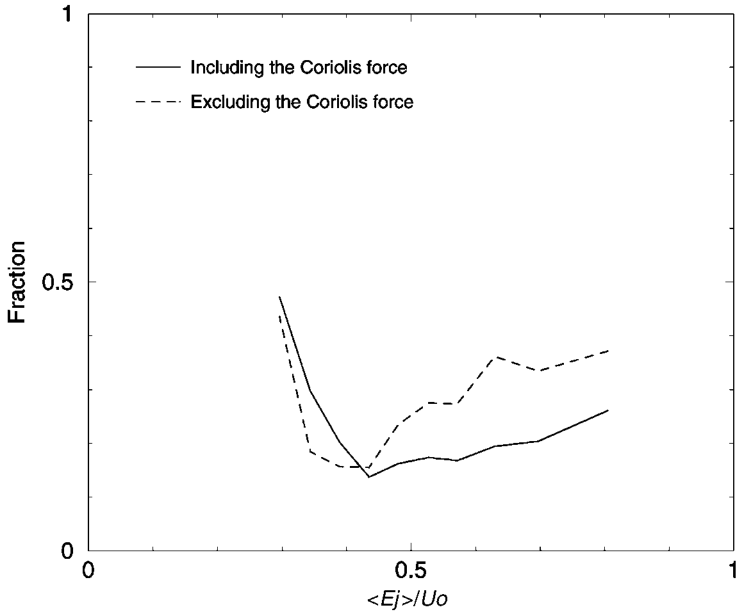

Figure 1. Fraction of chaotic orbits versus the value of the Jacobi integral normalized to the value of the central potential. We reproduce the values of the $b = 0.2290$ model of Muzzio et al. (2000b) and we add the new values obtained for the same model after eliminating the Coriolis force from the equations of motion.

under more realistic conditions (i.e. elongated orbit, stellar encounters, and so on). They investigated sets of initial conditions with fixed values of the Jacobi integral and starting, either with zero initial velocity, or from the main planes of symmetry with the velocity needed to yield the adopted value of the integral. The resulting orbits were classified using the Carpintero and Aguilar (1998) code of frequency analysis, and the Liapunov exponents were also obtained for a small number of orbits. The latter method was much slower than the former and it only allowed to distinguish chaotic from regular orbits, without providing a classification of the regular ones, but the results of both methods were very similar (see Figures 2 through 5 of Carpintero et al.), and the authors concluded that stellar orbits within galactic satellites were highly chaotic, affecting even the innermost regions of the satellite. Moreover, Liapunov times turned out to be surprisingly short, both in terms of orbital periods and satellite age.

Later on, Muzzio et al. (2000a) used the Carpintero and Aguilar (1998) code to classify the stellar orbits in satellites modelled according to the recipe of Heggie and Ramamani (1995). The satellites were again on circular orbits, but the big advantage offered by these models is that they are self-consistent (albeit involving certain approximations) and provide a distribution function. Muzzio et al., could then obtain the fractions of chaotic orbits, which turned out to be very high, indeed: almost one-fourth of the orbits of a satellite modeled after a dwarf galaxy

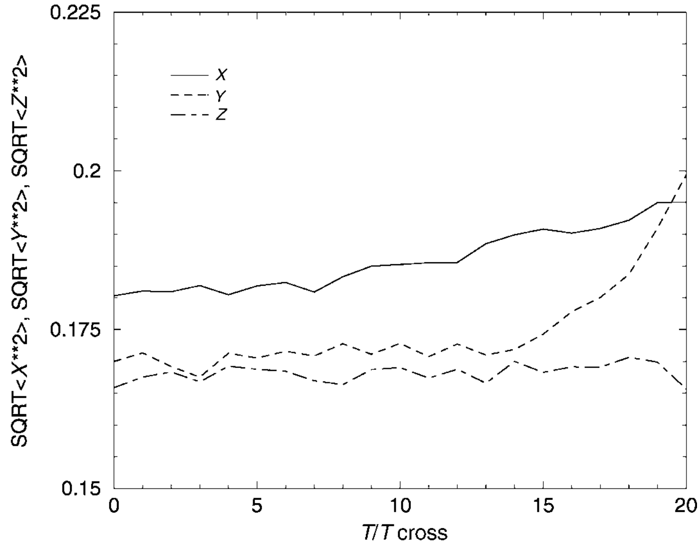

Figure 2. Evolution of the Heggie–Ramamani model with $Wo = 2.5$ of Muzzio et al. (2000a). The square roots of the mean square values of x, y and z are given as a function of the time (in units of crossing time). A total number of 6,944 bodies and a softening parameter of 0.0025 were used in the simulations.

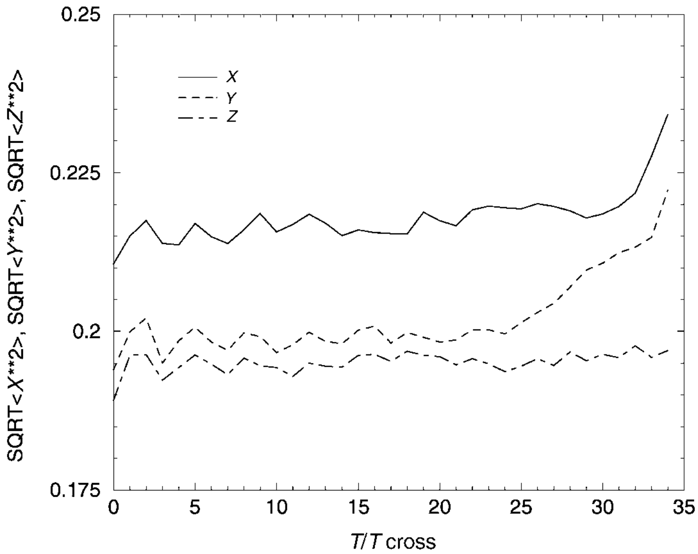

Figure 3. Same as Figure 2 for a Heggie–Ramamani model with $Wo = 0.5$. A total number of 20 000 bodies and a softening parameter of 0.0050 were used in the simulations.

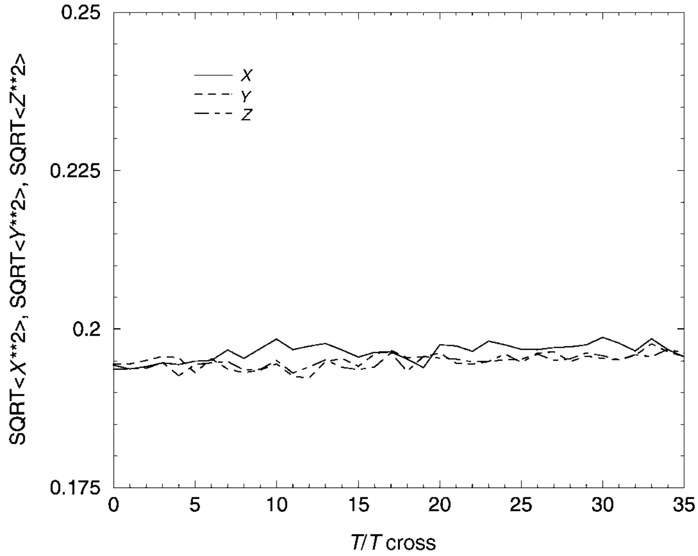

Figure 4. Same as Figure 2 for an isolated King model with $Wo = 0.5$. The x, y and z directions are purely arbitrary in this case. A total number of 20 000 bodies and a softening parameter of 0.0025 were used in the simulations.

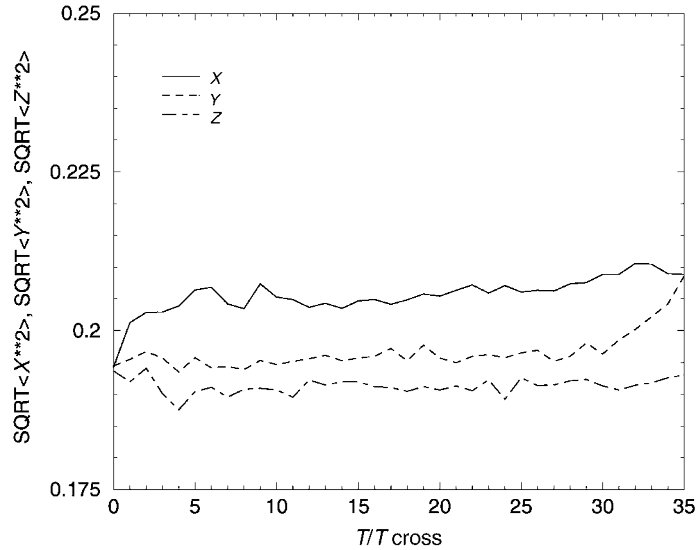

Figure 5. Same as Figure 2 for a King model with $Wo = 0.5$, placed on the same circular orbit as the Heggie–Ramamani models. A total number of 20 000 bodies and a softening parameter of 0.0050 were used in the simulations.

and between half and two-thirds of the orbits in models of globular clusters were found to be chaotic. They could also investigate the dependence of the fractions of chaotic orbits on the values of the Jacobi integral of the orbits, which they took as proof that the chaoticity arises from the interplay of the three forces that are present: the attraction from the cluster, the centripetal–centrifugal differential force and the Coriolis force. The role of those three forces was confirmed by Muzzio et al. (2000b) in their investigation of spherical satellites modeled according to the Schuster (or Plummer) law. About one-fourth of the stellar orbits in these models turned out to be chaotic. Summarizing all their results the authors concluded that, while mild triaxiality alone yielded very little chaos and a spherical satellite had a significant fraction of chaotic orbits, it was the combination of both triaxiality and tidal forces that yielded the highest fractions of chaotic orbits.

4. New Results

4.1. THE EFFECT OF THE CORIOLIS FORCE

Since the Coriolis force does not alter the values of the Jacobi integral, we repeated the analysis of the Schuster (or Plummer) models of Muzzio et al. (2000b) eliminating that force from the equations of motion. Figure 1 shows the fractions of chaotic orbits for the $b = 0.2290$ model. Clearly, the Coriolis force has an important effect on chaoticity, but it seems to be opposite to the one envisaged by Muzzio et al. (2000a): the presence of the Coriolis force marginally increases the chaoticity where that force is smallest (low $< Ej > /Uo$ values) and significantly decreases it where the Coriolis force is largest (large $< Ej > /Uo$ values). Curiously, however, for the $b = 0.0458$ model we got the opposite result. Therefore, while it is clear that the Coriolis force has an important role in the chaotization (or regularization) of the orbits of galactic satellites, that role seems to be much more complex than we had originally surmised and further studies are needed to decide which is its precise contribution.

4.2. STABILITY OF THE HEGGIE–RAMAMANI MODELS

Much of our previous work was based on the models of Heggie and Ramamani (1995), which have the great advantage of providing a known distribution function. Although these models are self-consistent, because they satisfy both the Boltzmann and Poisson equations, the solution is only a first order approximation. It is important, therefore, to check whether these are actually equilibrium models and, furthermore, whether they are stable in the long run. Heggie and Ramamani themselves did such a check, using an N-body simulation, but their results are far from convincing because their models only remained stable for a little less than two crossing times.

Two problems are immediately evident in the Heggie and Ramamani tests: they used only 1,000 bodies in their simulations, and their particle–particle interactions were purely Newtonian, with no softening included. The relaxation time of their models was then only about 15 crossing times, and even their isolated King model shows clear signs of evolution. Therefore, we decided to make our own N-body simulations, including larger numbers of bodies and using softened particle–particle forces. The NBODY2 code, kindly provided by S. Aarseth, was adopted for our experiments. We used the same constants and parameters as in our previous work, that is, gravitational constant and satellite mass equal to 1, orbit radius equal to 100 and angular velocity equal to 0.5.

Our first check was done with the $Wo = 2.5$ model of Muzzio et al. (2000a), with $N = 6{,}944$. We adopted softening parameters ranging between 0.0025 and 0.0050, which yielded relaxation times of the order of 100 crossing times (see, e.g. Huang et al., 1993). Figure 2 shows the evolution of the square root of the mean square values of the x, y and z coordinates (longest, intermediate and shortest axes, respectively, in the notation of Heggie and Ramamani) and it is clear that the model remains stable for at least 12 or 13 crossing times, that is, much longer than in the experiment of Heggie and Ramamani. Another $Wo = 2.5$ model, including 10 000 bodies, remained stable even longer, about 17 crossing times.

In order to test more triaxial cases, we also investigated models with $Wo = 0.5$. Simulations including 10 000 bodies remained stable for about 17 crossing times and this result encouraged us to perform another experiment, this time with 20 000 bodies. Figure 3 presents our results, in the same way as Figure 2, and it is clear that the model remains stable for at least 24 crossing times. Figure 4 provides the corresponding results for an isolated King model with $Wo = 0.5$, and Figure 5 those corresponding to that King model when it is subject to the same tidal field as the Heggie–Ramamani model of Figure 3. While the isolated model is very stable, indeed, displaying only a very mild expansion, the behavior of the King model suffering the action of the tidal forces resembles that of the Heggie–Ramamani model: it becomes unstable after 30 crossing times and the longer endurance can be attributed to the fact that, while the Heggie–Ramamani model fills in its Roche lobe from the start, the King model takes a while to reach a similar size in the x direction. The relaxation times for these models are very long, of the order of 300 crossing times.

Our experiments clearly show that increasing the number of bodies in the simulations extends the interval over which the Heggie–Ramamani models remain stable. It seems very likely that what is taking place in the experiments is completely analogous to what happens in a real globular cluster: particle–particle interactions increase the energy of some particles causing an expansion of the outermost regions and the tidal forces end up destabilizing the system. We feel that we can safely conclude that, in the continuous limit envisaged by the collisionless Boltzmann equation (i.e. for an infinite number of particles), the Heggie–Ramamani are indeed stable equilibrium models.

4.3. THE LACK OF CHAOTIC DIFUSSION

Merritt and Fridman (1996) have discussed the problems posed by chaotic orbits to the building of stationary stellar models. In their triaxial models, where orbits that fill in elongated regions of space (i.e. mainly boxes) are needed for self-consistency, chaotic orbits that tend to fill in all the region allowed to them by the energy integral are troublesome.

The high percentages of chaotic orbits found by Muzzio et al. (2000a) in the Heggie–Ramamani models seem, therefore, to pose a curious puzzle. A little reflection shows, however, that there is no problem at all, because the distribution function of those models depends only on the Jacobi integral and the stellar density results from integrating that function over all the velocities. Therefore, the isodensity surfaces coincide with the (effective) equipotential surfaces, so that chaotic orbits that fill in those equipotential (or zero-velocity) surfaces are perfect building blocks for the Heggie–Ramamani models.

As a check, we used a Heggie–Ramamani model with $Wo = 0.5$ (i.e. the most triaxial one of those investigated by us). We generated 10 000 initial conditions for that model and we followed the orbits in the (fixed) corresponding potential. In other words, now we are not using interacting particles, but letting the particles evolve in the Heggie–Ramamani potential with $Wo = 0.5$. Particle–particle interactions and self-consistency are thus suppressed, and the orbits can fill in all the space allowed to them by the isolating integrals that they obey. Orbit classification with the Carpintero and Aguilar (1998) code, as well as the Liapunov exponents computed for a small sample, showed that almost 40% of the orbits were chaotic.

Figure 6 presents our results; note that now the interval covered is 200 crossing times. The range of the Liapunov times for the chaotic orbits was approximately 15–75 crossing times, with a rough average of about 35 crossing times, so that the model remains perfectly stationary for several Liapunov times. To check whether the scatter was purely random, we also run 20 000 initial conditions for an interval of 20 crossing times. The dispersions computed for the 10 000 and 20 000 runs turned out to be proportional to the inverses of the square roots of the numbers involved, so that only noise seems to be present here.

5. Conclusion

Our previous work had shown that chaotic motions play an important role in galactic satellites: not only there are large fractions of chaotic orbits in the satellites, but the Liapunov times involved are short. The confirmation that the Coriolis force has a significant role on the (regular or chaotic) behavior of the stellar orbits inside the satellite emphasizes the need to take into account the tidal effects when building satellite models: clearly, those effects are not limited to imposing a tidal radius, but are present well inside that tidal radius too.

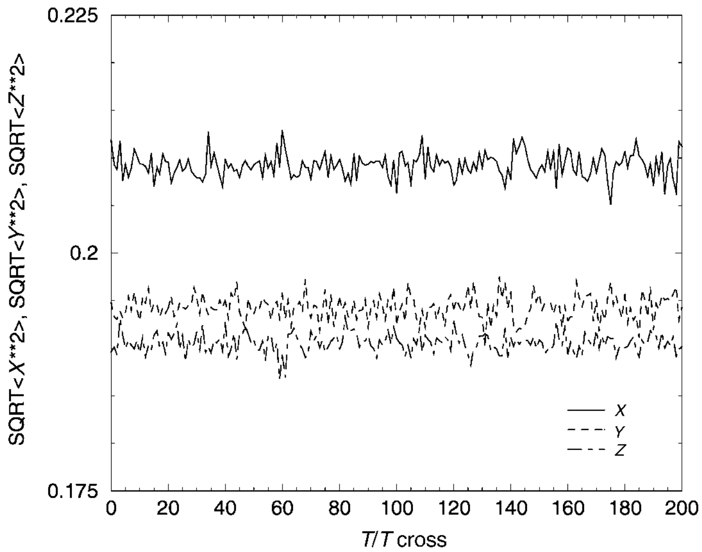

Figure 6. Same as Figure 2 for a Heggie–Ramamani model with $Wo = 0.5$. In this case the simulation is not self-consistent. Instead, 10 000 orbits whose initial conditions were randomly selected from the distribution function of that model were followed on the corresponding fixed potential of that same model.

We have also presented here a much needed confirmation that the Heggie–Ramamani models, on which much of our previous work was based, are truly stable. Not even the presence of large numbers of chaotic orbits prevents them from being stationary.

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