



# Equivalent linear stochastic seismic analysis of cylindrical base-isolated liquid storage tanks

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## ABSTRACT

Seismic performance of cylindrical liquid storage tanks base-isolated by bilinear bearings is investigated. The paper displays a stochastic parametrical study in which three design parameters, namely isolation period, yield strength and viscous damping ratio, characterizing the isolation system are taken into consideration. The earthquake excitation, modeled as a stationary random process, is characterized by a power spectral density function calculated via a compatible seismic design spectrum. The stochastic response of the base-isolated cylindrical tanks is obtained by the convolution between the frequency response function of the system and the input power spectrum. To determine effective damping and stiffness coefficients corresponding to the equivalent linear system a statistical linearization scheme was used. For the purpose of evaluating the seismic behavior under different conditions, two liquid levels (aspect ratios) and soil types (soft and stiff soil) were considered.

Thus, the study demonstrates the influence of each characteristic parameter of the isolation system and soil conditions on the response of cylindrical base-isolated tanks and principally allows visualizing the seismic performance that can be achieved through the selection of those parameters under certain soil conditions. Further, it is confirmed that soft soil conditions amplify the overall response of the system specially the base and sloshing displacements, as well as the normalized base shear to a lesser extent.

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## 1. Introduction

Storage tanks are very important components, principally in water supply, nuclear plants, refineries and petrochemical facilities. The importance goes beyond its economic cost because the effects of a failure are not limited to the risk of human lives and equipments in the proximity, but also can lead to serious consequences on the environment. Several tanks have been severely damaged and some failed with disastrous consequences revealing their vulnerability in almost every major earthquake (e.g., experiences from Valdivia, Chile 1960 [1], San Fernando, California 1971 and Whittier, California 1987 [2], San Juan, Argentina 1977 [3], Kocaeli, Turkey 1999 [4], Livermore, California 1980 [5], Coalinga, California 1983 [6]. Therefore, it is of critical interest to ensure operational reliability, since many of them are located in areas of high seismicity worldwide.

In order to improve the structural seismic performance and reduce the risk of damage or failure of liquid storage tanks, the base isolation technique extensively used in civil structures began to be implemented two decades ago and several experimental and theoretical studies have been carried out. Experimental tests on shake table were performed by Niwa and Clough [5] who investigated on the buckling of the cylindrical tanks. Similarly, Chalhoub and Kelly [7] demonstrated the effectiveness

of the isolation system in reducing the hydrodynamic forces. Kim and Lee [8] conducted pseudodynamic tests on cylindrical liquid storage tanks supported by elastomeric base isolators to predict the seismic loads exerted by the liquid on the tank wall. Park et al. [9] showed results and discussions on the seismic design of isolated pool-type tanks for the storage of nuclear spend fuel assemblies from experimental tests on scaled models. Malhotra [10] proposed an innovative isolation system by disconnecting the tank wall from the base plate and supporting it on a ring with horizontally flexible bearings. The author showed that this isolation system can reduce considerably the base shear, overturning moment, and axial compressive stress in the tank wall without significantly increasing the vertical displacements of the liquid surface. In recent years, analytical and numerical studies have been reported. Shrimali and Jangid [11] conducted parametric analyses to investigate the seismic performance of tanks isolated by different bearing types. By analytical studies on the seismic behavior of cylindrical liquid storage tanks isolated by linear elastomeric bearings under recorded earthquakes, Shrimali and Jangid [12] demonstrated that the response obtained by approximated methods was in good agreement with the exact response. Cho et al. [13] examined the seismic response of a base-isolated liquid storage tank on a half-space using a coupling method that combines finite elements and boundary elements. Assuming as excitation bilateral horizontal components of real earthquakes, Shrimali and Jangid [14,15] investigated, the response of liquid storage tanks isolated by sliding bearings and lead-rubber bearings (LRB),

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respectively. Similar studies based on the same isolation systems but under near-fault motions were conducted by Jadhav and Jangid [16]. Works concerning the efficiency of the friction pendulum system (FPS) for seismic isolation of liquid storage tanks were performed by Wang et al. [17]. Subsequently, Panchal and Jangid [18] proposed a more refined model for the FPS, which assumes the friction coefficient as function of isolator displacement. Abali and Uçkan [19] investigated the effects of the axial load variation on restoring force and damping of the FPS. In order to extend the displacement capacity and give more flexibility to the parametric optimization, Fenz and Constantinou [20–22] developed, tested and implemented double and triple friction pendulum systems. Recently by means of a parametrical study, Soni et al. [23] investigated the behavior of liquid storage tanks isolated by a double variable frequency pendulum isolator (DVFP) under bilateral excitation using a set of 20 far-field earthquake ground motions. Different to the traditional friction pendulum bearing, this device includes double friction elliptical surfaces which incorporate more versatility to achieve a required performance.

A satisfactory isolation system design should limit the overturning moment and base shear transmitted into the structure. This purpose is attained by the insertion of special bearings between the base and foundation of the tank which add lateral flexibility. The flexibility causes an increase in the base displacement that is controlled by incorporating damping elements in the bearings (cylindrical central lead core or high damping rubber) or supplemental dampers. However, excessive damping could reduce the efficiency of isolation system.

In view of this dilemma and in order to arrive at a compromise solution between the earthquake forces transmitted into the structure and the sloshing and base displacements, this paper presents a parametrical analysis of cylindrical liquid storage tanks isolated by means of lead-rubber bearings (LRB) under seismic loading. The study aims to investigate the influence of parameters characterizing the isolation system and soil conditions on the seismic performance of cylindrical isolated tanks. It is well known that the main contribution to the total uncertainty is due to the excitation, therefore to obtain general conclusions on the seismic response of liquid storage tanks, a Gaussian stationary random process characterized by its power spectral density function (PSDF) which is stochastically compatible with a given design spectrum is assumed. The stochastic response, in terms of root mean square value (rms) of the base shear and base and sloshing displacements, is obtained using the random vibration theory at full and half-full tank under stiff and soft soil conditions. The inelastic force-deformation behavior of the LRB isolator is represented by a bilinear hysteretic law which is the simplest model to capture the hysteretic energy dissipation of this device. This model causes some difficulties to estimate the response via Frequency Response Function (FRF) which is defined for linear systems. To overcome such difficulty, the effective damping and stiffness coefficients corresponding to an equivalent linear isolation system are estimated through the stochastic equivalent linearization technique.

## 2. Seismic ground excitation model

It is known that earthquake excitation is inherently random, however, if the evolution of the frequency content with time can be neglected, the input ground motion can be characterized by a power spectral density function (PSDF). In this study the earthquake excitation is assumed as a stationary Gaussian random process with zero mean characterized by means of a design spectrum compatible PSDF. Following the methodology developed by Venmarke [24] cited in the work conducted by Giaralis and Spanos [25], the design spectrum compatible PSDF of the input ground motion can be approximated by the following recursive equation:

$$S_g(\omega_j) = \frac{4\xi}{\omega_j \pi - 4\xi \omega_{j-1}} \left( \frac{S_a^2(\omega_j, \xi)}{\eta_j^2} - \Delta\omega \sum_{k=1}^{j-1} S_g(\omega_k) \right) \quad \omega_j > \omega_o \quad (1)$$

in which  $S_a(\omega_j, \xi)$  is the median pseudo-acceleration response spectrum, at a specific frequency  $\omega_j$  and  $\xi = 0.05$  is the assumed damping ratio;  $\Delta\omega$  is the frequency step in which the frequency range was discretized; the peak factor  $\eta_j$  is calculated by Eq. (2) and it represents the factor by which the rms value of the response of a SDOF oscillator must be multiplied to predict the level  $S_a$  below by which the peak response of the oscillator will remain, with probability  $p$ , throughout the duration of the input process  $T_s$ . Herein, the following approximated semi-empirical formula for the calculation of the peak factor is adopted, which is known to be reasonably reliable for earthquake engineering applications (Vermarke [24]):

$$\eta_j = \sqrt{2 \ln \left\{ 2v_j \left[ 1 - e^{(-q_j^{1.2} \sqrt{\pi \ln(2v_j)})} \right] \right\}} \quad (2)$$

in which

$$v_j = \frac{T_s}{2\pi} \omega_j (-\ln p)^{-1} \quad (3)$$

and

$$q_j = \sqrt{1 - \frac{1}{1 - \xi^2} \left( 1 - \frac{2}{\pi} \tan^{-1} \frac{\xi}{\sqrt{1 - \xi^2}} \right)} \quad (4)$$

being the Eqs. (3) and (4) close form expressions derived for a white noise PSDF which has to be *a priori* assumed without knowledge of  $S_g(\omega_j)$  when the peak factor is calculated by Eq. (2);  $T_s = 20$ s is the duration assumed for the underlying stationary process;  $p = 0.5$  is an appropriate probability assumed for the purposes of this study;  $\omega_o = 0.36$  rad/s denotes the lowest bound of the existence domain of Eq. (3) for a PSDF and  $T_s$  *a priori* assumed (Giaralis and Spanos [25]).

## 3. Structural model of the system

### 3.1. Model of the cylindrical base-isolated liquid storage tank

Scientific literature shows that, for an exhaustive analysis of liquid storage tanks regarding fluid–structure interactions, complex models incorporating Lagrangian–Eulerian approaches in Finite Element Method formulations should be used (Zienkiewicz and Bettles, [26], Cho et al. [27], Virella et al. [28], Livaoğlu and Doğangün [29]). However, in order to perform parametrical studies in complex problems, such as those that require non-linear and fluid–structure interaction analysis, and with the purpose to obtain general results from several simulations, it is convenient resorting to a simplified model. The first simple approach to represent the dynamical behavior of the liquid inside a cylindrical container was proposed in 1963 by Housner [30] and after extensively used by researchers and recommended by major seismic codes (Livaoğlu and Doğangün [31], Eurocode-8 [32]). In the Housner's two-mass model, the contained fluid assumed as incompressible, inviscid and irrotational, is replaced by two masses, one of which is rigidly attached to the tank wall and the other is connected to the wall by springs. According to the literature (Karamanos et al. [33], Sezen et al. [34]), although only one mass it is sufficient to represent the sloshing; additional higher-mode masses may also be included for the ground-supported tanks (Bauer [35]). Haroun and Housner [36] and Haroun [37] developed a three-mass model that takes the tank wall flexibility into account and showed that the hydrodynamic forces being exerted on the flexible wall consist of three components: 1) the convective (sloshing) component describing the portion of liquid mass localized at the top of the container, 2) the impulsive component representing the intermediate liquid mass vibrating along with the tank wall and 3) the rigid

component caused by the weight of the container and the portion of liquid mass localized at the bottom of the container, which rigidly move with the tank. It has been recognized in the scientific literature that, for practical seismic analysis, this model leads to simple, fast and sufficiently accurate analyses (Kim and Lee [8], Malhotra [10]). Therefore, in the present study, the dynamical behavior of the isolated liquid storage tank (Fig. 1a) is modeled by the mechanical model proposed by Haroun [37] with three degrees of freedom denoted by  $u_c$ ,  $u_i$ ,  $u_b$ , which correspond to the absolute displacements of the sloshing, impulsive and rigid masses respectively (see Fig. 1b).

The mechanical parameters of the model can be estimated according to the following formulations (Haroun [37]):

$$\begin{aligned} m &= \pi R^2 H \rho_w \\ m_c &= Y_c m \\ m_i &= Y_i m \\ m_r &= Y_r m \end{aligned} \quad (5)$$

in which,  $m$  and  $\rho_w$  are the mass and mass density of the contained liquid;  $R$  is the radius of the tank;  $H$  is the liquid height; the sloshing, impulsive and rigid lumped masses are denoted by  $m_c$ ,  $m_i$  and  $m_r$  respectively;  $Y_c$ ,  $Y_i$  and  $Y_r$  are the mass ratios which are function of average thickness of tank wall,  $t_h$ , and the aspect ratio of the tank defined as,  $S = H/R$ ;  $P$  is a dimensionless parameter expressed as a function of  $S$ .

For  $t_h/R = 0.004$ , the mass ratios and the dimensionless parameter  $P$ , are taken from Shriali and Jangid [12] which were derived from (Haroun [37]) and expressed in the matrix form as:

$$\begin{Bmatrix} Y_c \\ Y_i \\ Y_r \\ P \end{Bmatrix} = \begin{bmatrix} 1.01327 & -0.87578 & 0.35708 & 0.06692 & 0.00439 \\ -0.15467 & 1.21716 & -0.62839 & 0.14434 & -0.0125 \\ -0.01599 & 0.86356 & -0.39094 & 0.04083 & 0 \\ 0.037085 & 0.084302 & 0.05088 & 0.012523 & -0.0012 \end{bmatrix} \begin{Bmatrix} 1 \\ S \\ S^2 \\ S^3 \\ S^4 \end{Bmatrix} \quad (6)$$

The fundamental frequencies of impulsive mass,  $\omega_i$ , and of sloshing mass,  $\omega_c$ , are calculated by:

$$\omega_c = \sqrt{1.84 \left( \frac{g}{R} \right) \tanh(1.84S)} \quad (7)$$

$$\omega_i = \frac{P}{H} \sqrt{\left( \frac{E}{\rho_s} \right)} \quad (8)$$

in which  $E$  and  $\rho_s$  are elastic modulus and density of tank wall, respectively;  $g$  is the acceleration due to gravity. The sloshing and impulsive

masses are connected to the tank wall by means of equivalent springs and dampers having properties  $k_c$  and  $c_c$  for the sloshing mass and  $k_i$  and  $c_i$  for impulsive mass, obtained as:

$$\begin{aligned} k_c &= m_c \omega_c^2 \\ k_i &= m_i \omega_i^2 \\ c_c &= 2\xi_c m_c \omega_c \\ c_i &= 2\xi_i m_i \omega_i \end{aligned} \quad (9)$$

in which  $k_c, c_c$ , and  $k_i, c_i$  are the stiffness and damping coefficient of sloshing and impulsive masses, respectively and  $\xi_c, \xi_i$  are the critical damping ratio assumed by recommendations (Kim and Lee [8]).

### 3.2. Model of the isolation system with LRB

The LRB isolators consist of alternate layers of rubber and steel plates with sufficient vertical stiffness to support the total weight of the tank and a specified lateral flexibility and damping. Since experimental non-linear force–displacement behavior of this kind of bearing is almost bilinear (Fig. 2a) and as the distribution of isolators under the tank is always axial symmetric, for the full isolation system, the model illustrated in Fig. 2b was assumed in which  $f_b(x_b)$  denotes the bilinear restoring force of the full isolation system, and  $x_b$  is the displacement of the tank relative to the ground.

The mechanical properties of the full isolation system are perfectly defined by four specific parameters (see, Fig. 2), namely (i) typical yield displacement,  $x_y$ , (ii) total yield strength,  $F_y$ , (iii) post-yield and initial stiffness ratio,  $\alpha$ , and (iv) viscous damping coefficient,  $c_b$ , which lead to characteristic properties of the full isolation system determined as:

$$\begin{aligned} k_b &= \frac{F_y}{x_y} \\ T_b &= 2\pi \sqrt{\frac{M}{\alpha k_b}} \\ \xi_b &= \frac{c_b}{2M\omega_b} \\ F_o &= \frac{F_y}{Mg} \end{aligned} \quad (10)$$

in which  $k_b$  is the initial stiffness;  $M = m_c + m_i + m_r$  is the effective mass of the isolated liquid tank (the self-mass of the container is neglected since it is less than 5% of the effective mass);  $T_b$  is the post-yield isolation period;  $\xi_b$  is the viscous damping ratio;  $F_o$  is the total yield strength normalized with the effective weight of the tank,  $Mg$ ;  $\omega_b = \frac{2\pi}{T_b}$  is the post-yield circular frequency.

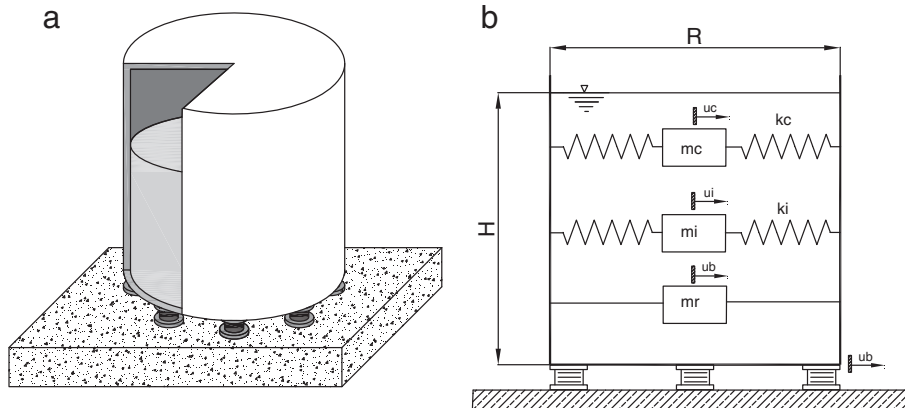


Fig. 1. a) Sketch of isolated liquid storage tank, b) tank model.

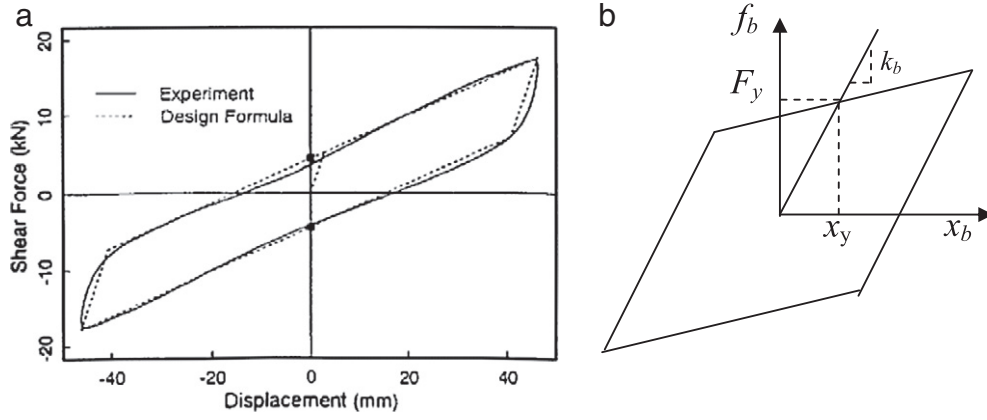


Fig. 2. a) Experimental test of LBR isolation system from Aiken et al. [38], b) bilinear model.

### 3.2.1. Stochastic equivalent linearization

As it was indicated before, to calculate the rms value of the response using stochastic vibration theory, it is necessary to estimate the effective damping and stiffness coefficients corresponding to an equivalent linear isolation system. In the linearization process, it is assumed that the behavior of the isolation-tank system is modeled as a single degree of freedom with equation of motion given as:

$$M\ddot{x}_b + c_b\dot{x}_b + f_b(x_b) = -M\ddot{u}_g \quad (11)$$

in which  $f_b(x_b)$  denotes the bilinear restoring force of the isolation system (see Fig. 2);  $\ddot{x}_b$ ,  $\dot{x}_b$ ,  $x_b$ , are the acceleration, velocity and displacement of the tank relative to the ground and  $\ddot{u}_g$  is the ground motion acceleration. Assuming that the response of the system can be approximated by an equivalent linear model, the equation of motion can be rewritten as:

$$M\ddot{x}_b + c_{eq}\dot{x}_b + k_{eq}x_b = -M\ddot{u}_g \quad (12)$$

in which  $k_{eq}$  and  $c_{eq}$  are the equivalent linear stiffness and equivalent viscous damping coefficient of the isolation system, respectively. Both equivalent parameters can be determined by different methods proposed in the literature (see, for example Robert and Spanos [39], Wen [40]) which are based on the minimization of the mean-square error between the responses obtained by Eqs. (11) and (12). In this paper, the values of  $k_{eq}$  and  $c_{eq}$  are computed from the bilinear model (Fig. 2) according to statistical equivalent linearization proposed by Caughey [41] as follows:

$$k_{eq} = k_b \left( 1 - \frac{8\mu}{\pi} \right) \int_1^\infty \left( (z\lambda)^{-1} + z^{-3} \right) (z-1)^{0.5} e^{-\frac{z^2}{\lambda}} dz \quad (13)$$

$$c_{eq} = c_b + \frac{2\mu}{\sqrt{\pi\lambda}} \frac{k_b}{\omega_{eq}} \left( 1 - \text{erf} \left( \frac{1}{\sqrt{\lambda}} \right) \right)$$

in which  $\mu = 1 - \alpha$ ;  $\lambda = 2 \left( \frac{\sigma_{x_b}}{d_v} \right)^2$ ;  $\omega_{eq} = \sqrt{\frac{k_{eq}}{M}}$ ;  $\text{erf}(\cdot)$  indicates the error function;  $\sigma_{x_b}^2$  is the mean-square displacement of the tank relative to the ground.

Once, the equivalent parameters  $k_{eq}$  and  $c_{eq}$  are calculated by Eq. (13), the mean-square displacement of the tank  $\sigma_{x_b}^2$ , can be evaluated by (Robert and Spanos [39]):

$$\sigma_{x_b}^2 = \int_{-\infty}^{\infty} |H_{eq}(\omega)|^2 S_g(\omega) d\omega \quad (14)$$

in which  $S_g(\omega)$  is calculated by Eq. (1) and the frequency response function (FRF) is expressed as:

$$H_{eq}(\omega) = \frac{1}{(k_{eq} - M\omega^2)^2 + (c_{eq}\omega)^2} \quad (15)$$

As the equivalent parameters in Eq. (13) depend on  $\sigma_{x_b}^2$ , the procedure is repeated until the calculated value  $\sigma_{x_b}^2$  in Eq. (14) coincides with the assumed value  $\sigma_{x_b}^2$  in Eq. (13) (only few cycles are necessary for the convergence).

### 4. Evaluation of the mean square response of the linearized system

The equations of motion of the three-degrees-of-freedom model of the isolated tank (see Section 3.1) subjected to earthquake excitation can be written in the matrix form as:

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = -[m]\{r\}\{\ddot{u}_g\} \quad (16)$$

in which  $\{x\} = \{x_c, x_i, x_b\}^T$  is the relative displacement vector;  $x_c = u_c - u_b$  is the displacement of the sloshing mass relative to the base of the tank;  $x_i = u_i - u_b$  is the displacement of the impulsive mass relative to the base of the tank;  $x_b = u_b - u_g$  is the displacement of the base of the tank relative to the ground;  $\{r\} = \{0 \ 0 \ 1\}^T$  is the influence vector and  $T$  denotes the transpose;  $[m]$ ,  $[c]$  and  $[k]$  are the mass, damping and stiffness matrices, respectively, expressed as:

$$[m] = \begin{bmatrix} m_c & 0 & m_c \\ 0 & m_i & m_i \\ m_c & m_i & M \end{bmatrix} \quad (17)$$

$$[c] = \begin{bmatrix} c_c & 0 & 0 \\ 0 & c_i & 0 \\ 0 & 0 & c_{eq} \end{bmatrix}$$

$$[k] = \begin{bmatrix} k_c & 0 & 0 \\ 0 & k_i & 0 \\ 0 & 0 & k_{eq} \end{bmatrix}$$

From random vibration theory, the root mean square response of each degree of freedom of equivalent linear system can be evaluated by (Roberts and Spanos [39]):

$$\{\sigma\} = \sqrt{\text{diag} \left( \int_{-\infty}^{\infty} [S_{xx}(\omega)] d\omega \right)} \quad (18)$$

in which  $\sigma = \{\sigma_{x_c} \ \sigma_{x_i} \ \sigma_{x_b}\}^T$  is the root mean square response vector in terms of the relative displacements of each degree of freedom and  $S_{xx}$  is the PSDF of the response computed from:

$$[S_{xx}] = [H(\omega)] [S_{gg}(\omega)] [H(\omega)]^* \quad (19)$$

in which  $(\cdot)^*$  denotes the complex conjugate, being  $[H(\omega)]$  the frequency response function (FRF) of the system defined as:

$$[H(\omega)] = ([k] - [m]\omega^2 + i[c]\omega)^{-1} \quad (20)$$



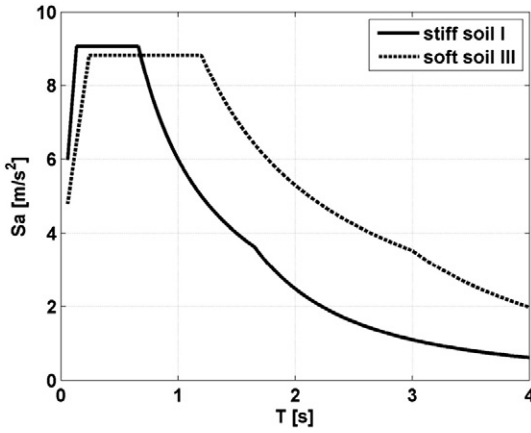


Fig. 3. Pseudo-acceleration response spectrum. Seismic zone IV, soil conditions I (stiff) and III (soft) (from INPRES CIRSOC 103 [42]).

in which  $(\cdot)^{-1}$  denotes the inverse matrix and  $[S_{gg}(\omega)]$  is the power spectral density function (PSDF) of the input  $\ddot{u}_g$  given by:

$$[S_{gg}(\omega)] = [m]\{r\}([m]\{r\})^T S_g(\omega) \quad (21)$$

Once the response of the base of the tank relative to the ground has been determined, the root mean square base shear normalized with the effective weight of the tank may be calculated by (Roberts and Spanos [39]):

$$\sigma_{F_s} = \frac{\sqrt{c_{eq}^2 \sigma_{\dot{x}_b}^2 + k_{eq}^2 \sigma_{x_b}^2}}{Mg} \quad (22)$$

in which  $\sigma_{x_b}^2$ ,  $\sigma_{\dot{x}_b}^2$  are mean square displacement and velocity of the base of the tank relative to the ground.

## 5. Numerical study

The parametrical study is conducted on an isolated cylindrical liquid storage tank built with steel plates with elastic constants  $E=200$  GPa,  $\nu=0.3$  and density  $\rho=7850$  kg/m<sup>3</sup> and filled with water (density equal to 1000 kg/m<sup>3</sup>). Two cases with different liquid levels were

assumed: (i) full tank with surface water height equal to  $H=11.3$  m and aspect ratio  $S=H/R$  1.85 (slender tank) which led to natural frequencies of the convective and impulsive masses  $f_c=0.2733$  Hz and  $f_i=5.963$  Hz, respectively, and (ii) half-full tank with the following properties,  $H=5.5$  m,  $S=0.9$  (broad tank),  $f_c=0.2638$  Hz and  $f_i=11.662$  Hz.

The damping ratios for the sloshing and impulsive masses are assumed, at both water levels, equal to  $\xi_c=0.005$  and  $\xi_i=0.02$ , respectively, from Kim and Lee [8].

To evaluate the influence of the parameters characterizing of the isolation system (see Section 3.2) on the system response, the parametrical analysis requires considering specific values of those parameters. In this study, the following properties of the isolation system, defined in Eq. (10), were assumed within a practical and useful range:

- 1) In the full-tank case, the characteristic properties of the isolation system were: post-yield isolation period,  $T_b=[1.5 \ 2.5 \ 4]$ s, viscous damping ratio,  $\xi_b=[0.025 \ 0.05 \ 0.075 \ 0.1 \ 0.125 \ 0.15 \ 0.175 \ 0.2]$  and, total yield strength normalized with the effective weight of the tank,  $F_o=[0.025 \ 0.05 \ 0.1]$ .
- 2) Maintaining the mechanical properties of the former case, the characteristic properties of the isolation system in the half-full tank case resulted in  $T_b=[0.83 \ 1.39 \ 2.22]$ s,  $\xi_b=[0.05 \ 0.12 \ 0.14 \ 0.18 \ 0.23 \ 0.27 \ 0.32 \ 0.36]$ ,  $F_o=[0.08 \ 0.16 \ 0.32]$ .

A typical yield displacement of the isolation system equal to  $x_y=0.015$  m was assumed in both cases.

The influence of the frequency content of the seismic excitation and soil conditions on the system response was evaluated assuming two pseudo-acceleration response spectra corresponding to stiff (I) and soft (III) soil conditions of the seismic zone IV (region with high seismicity, PGA=0.35 g) prescribed by the provisions of Argentine seismic code (INPRES CIRSOC 103 [42]) as illustrated in Fig. 3.

## 6. Analysis of the system response

From the design point of view, the most important response quantities of the system to be evaluated are: (i) base shear normalized with the effective weight of the tank,  $F_s$ , which is directly proportional to hydrodynamic forces (pressures) generated in the tank wall, (ii) displacement of the base of the tank relative to the ground,  $x_b$ , which is critical for the isolation system design, piping connections,

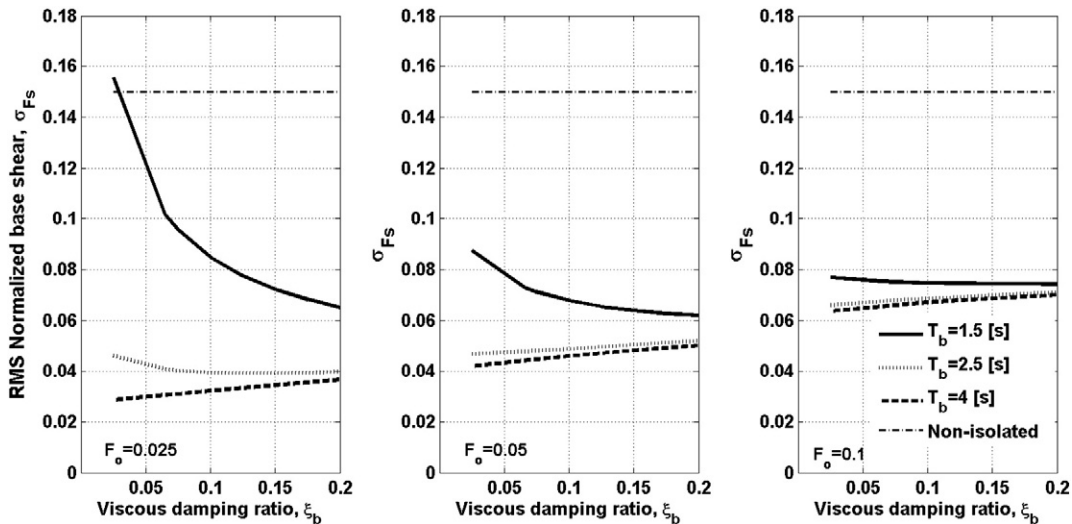


Fig. 4. Influence of post-yield isolation period  $T_b$ , normalized total yield strength  $F_o$  and viscous damping ratio  $\xi_b$ , on the normalized base shear,  $F_s$ ;  $S=1.85$ , soil type = I.

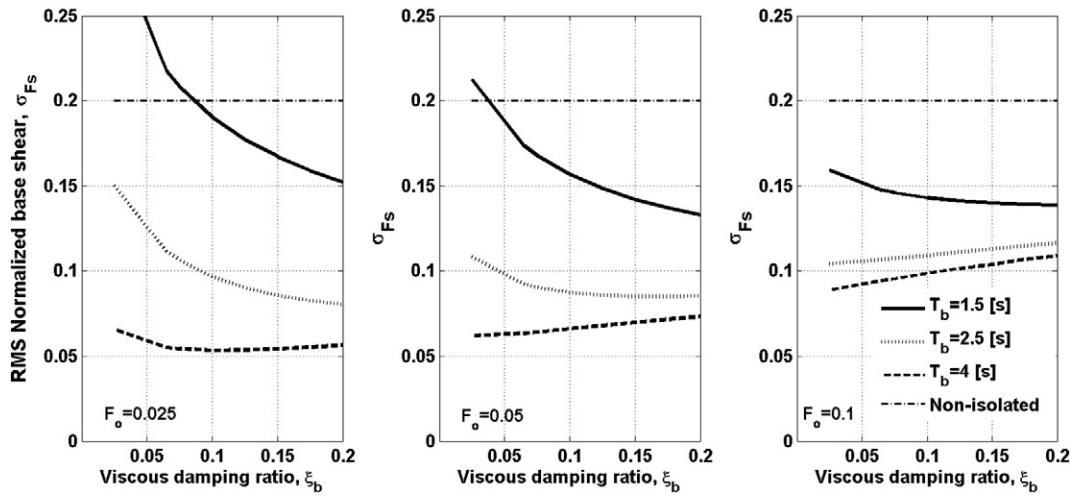


Fig. 5. Influence of post-yield isolation period  $T_b$ , normalized total yield strength  $F_o$  and viscous damping ratio  $\xi_b$ , on the normalized base shear,  $F_s$ ;  $S = 1.85$ , soil type = III.

dilatation joints and space requirements and (iii) displacement of sloshing mass relative to the base of the tank,  $x_c$ , which controls the vertical displacement of the liquid free-surface, hence, the freeboard requirements. The dependence of these response quantities on the characteristic parameters of the isolation system and soil conditions is evaluated at full and half-full tank in the next sections.

#### 6.1. Full tank (slender tank, $S = 1.85$ )

Figs. 4–9 show the effects of the characteristic parameters of the isolation system denoted by post-yield isolation period  $T_b$ , total yield strength normalized with the effective weight of the tank  $F_o$ , viscous damping ratio  $\xi_b$ , and soil conditions on the normalized base shear,  $F_s$ , base displacement,  $x_b$ , and relative displacement of the sloshing mass,  $x_c$ .

The effectiveness of the isolation system for reducing base shear is shown in Figs. 4 and 5 under stiff and soft soil conditions, respectively. While, in systems with post-yield isolation period  $T_b > 2.5$  s the effect of viscous damping on the base shear could be neglected, in systems with  $T_b < 2.5$  s and especially with total yield strength  $F_o < 0.05$  viscous damping causes an appreciable reduction in the transmitted shear.

From Figs. 4 and 5, in general, an important increase in the normalized base shear in soft soil conditions (soil type III) with respect to stiff soils (soil type I) is observed. As it was expected, the base shear also increases as the initial stiffness (or  $F_o$ ) of isolation system increases.

On the other hand, the flexibility provided by the isolation system causes an important base displacement which is clearly controlled by viscous damping and initial stiffness as shown in Figs. 6 and 7. Fig. 7 shows an important increase in the base displacement in soft soil conditions as compared to the response in stiff soils (Fig. 6). In general, negligible influence of the post-yield stiffness on the response, except in systems with total yield strength,  $F_o = 0.025$  is also observed.

From Figs. 4–7, it is inferred that in designing the isolation systems, there is a compromise solution between transmitted shear forces and base displacements. In order to obtain an adequate system response, the characteristic parameters should remain within the following ranges of values  $\xi_b > 0.1$ ,  $T_b > 2.5$  s,  $F_o \approx 0.05$ . Before making a decision, the designer should evaluate all the technical and economic factors involved in the particular project.

Figs. 8 and 9 indicate that, to reduce the sloshing displacement, it is necessary to design highly flexible isolation systems given that, in this case, the response has components of low frequency. The lowest

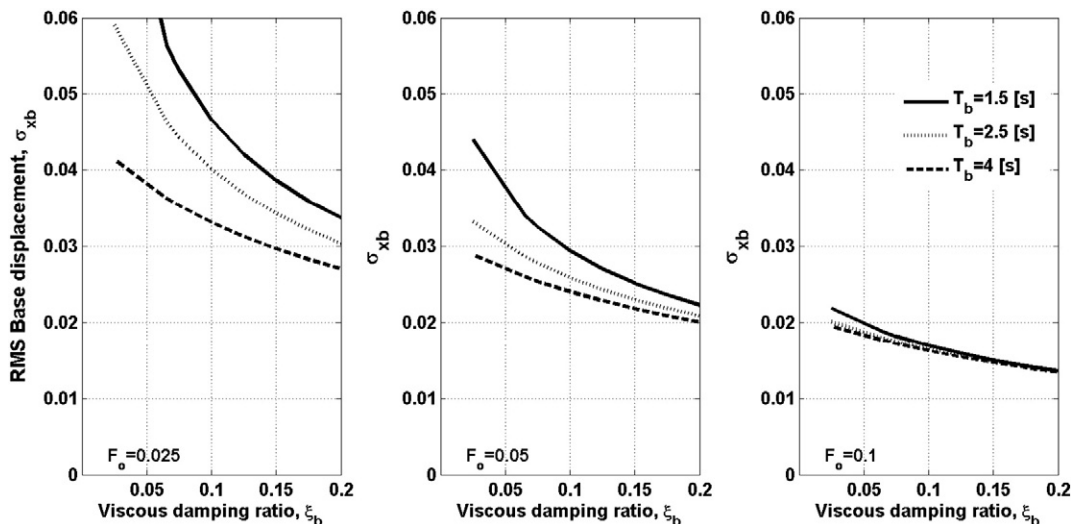


Fig. 6. Influence of post-yield isolation period  $T_b$ , normalized total yield strength  $F_o$  and viscous damping ratio  $\xi_b$ , on the base displacement,  $x_b$ ;  $S = 1.85$ , soil type = I.

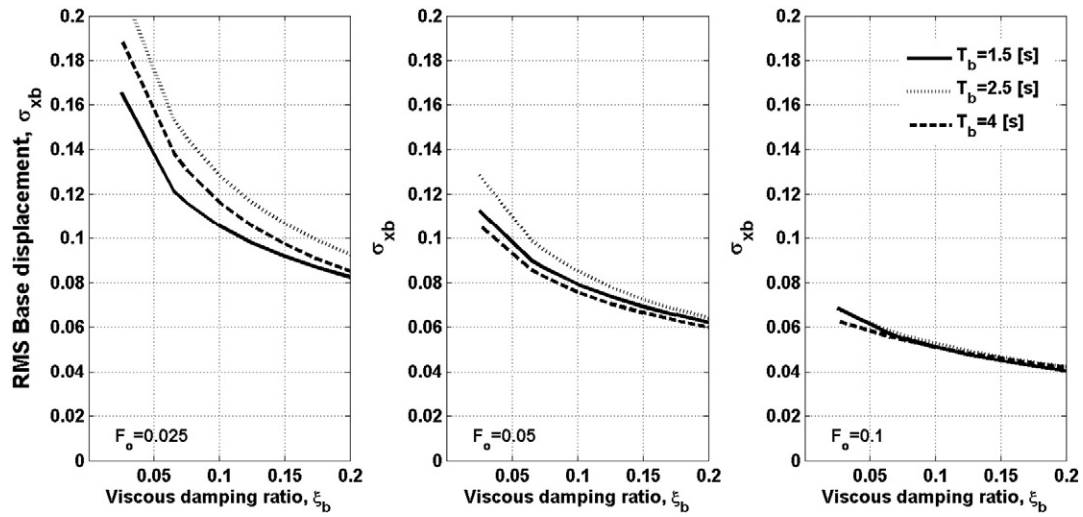


Fig. 7. Influence of post-yield isolation period  $T_b$ , normalized total yield strength  $F_o$  and viscous damping ratio  $\xi_b$ , on the base displacement,  $\sigma_{xb}$ ;  $S = 1.85$ , soil type = III.

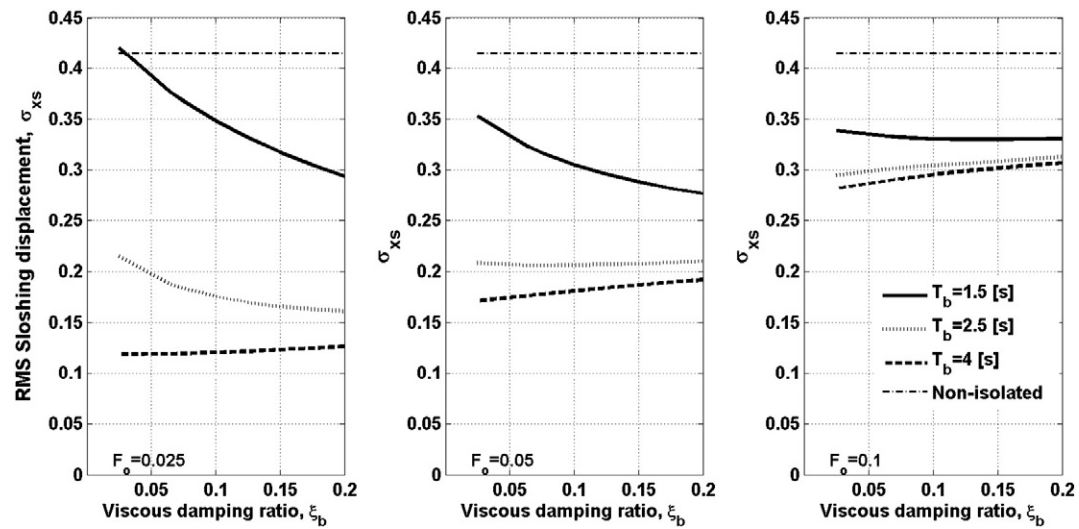


Fig. 8. Influence of post-yield isolation period  $T_b$ , normalized total yield strength  $F_o$  and viscous damping ratio  $\xi_b$ , on the sloshing mass displacement,  $\sigma_{xs}$ ;  $S = 1.85$ , soil type = I.

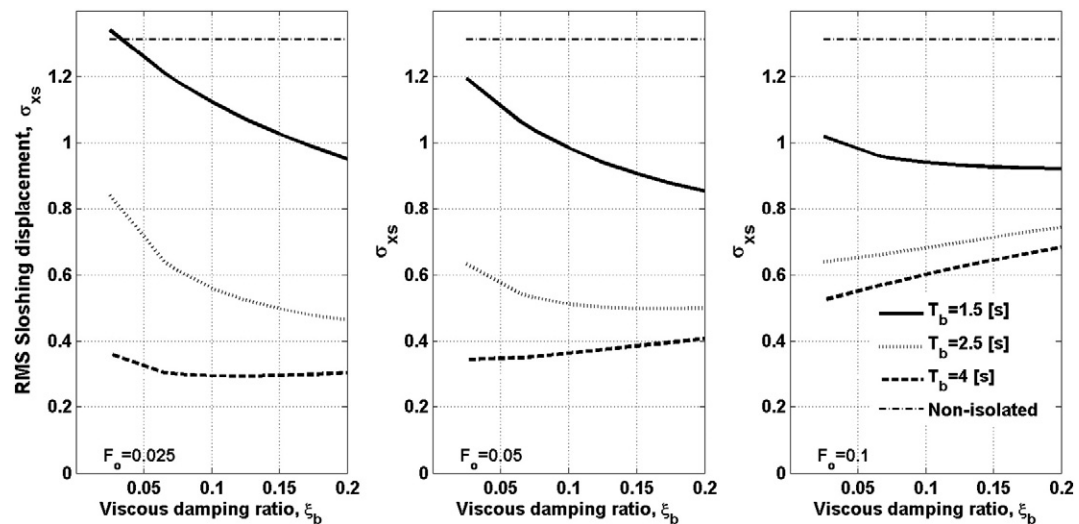


Fig. 9. Influence of post-yield isolation period  $T_b$ , normalized total yield strength  $F_o$  and viscous damping ratio  $\xi_b$ , on the sloshing mass displacement,  $\sigma_{xs}$ ;  $S = 1.85$ , soil type = III.

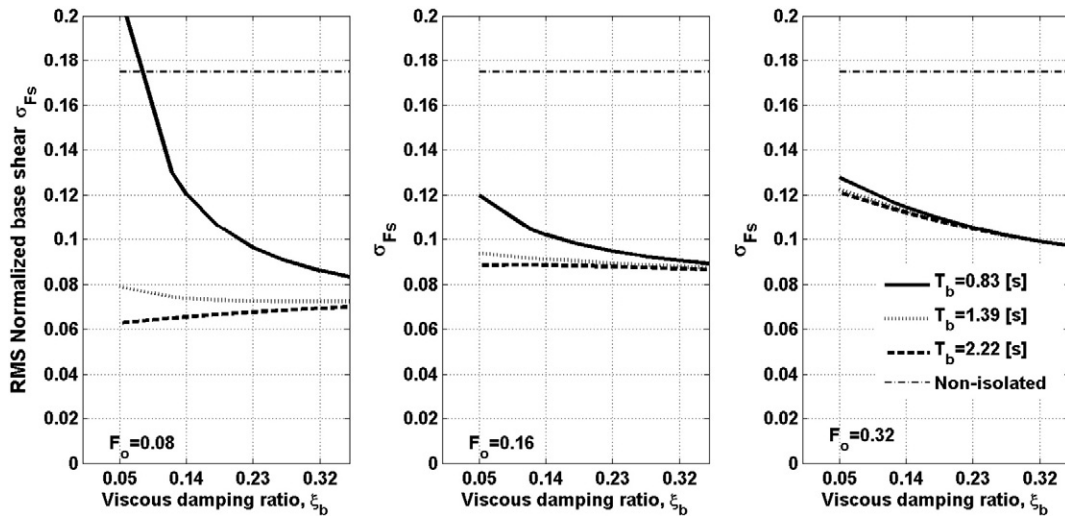


Fig. 10. Influence of post-yield isolation period  $T_b$ , normalized total yield strength  $F_o$  and viscous damping ratio  $\xi_b$ , on the normalized base shear,  $F_s$ ;  $S=0.9$ , soil type = I.

sloshing displacements are obtained in isolation systems with total yield strength  $F_o < 0.05$  and post-yield isolation period  $T_b > 2.5$  s in which the viscous damping has little influence. In stiff soils (Fig. 8), the systems with  $T_b = 1.5$  s exhibit an optimum total yield strength at  $F_o = 0.05$  in which the response displays minimum values. Similarly, in systems with  $T_b > 1.5$  s the sloshing displacement grows up with the increase of the initial stiffness. By comparing Figs. 8 and 9, it is also observed that in soft soil conditions, the sloshing displacement is appreciably larger than in stiff soils, because the system perceives higher excitation in low frequencies (see Fig. 3).

## 6.2. Half-full tank (broad tank, $S=0.9$ )

In this section, the seismic performance of the isolated half-full tank is analyzed and the response is displayed in the same terms as the former case.

Figs. 10 and 11 indicate that in systems with total yield strength  $F_o < 0.15$  the base shear depends on the post-yield period  $T_b$ , while in system with  $F_o > 0.2$ , the isolators practically remain in elastic range, thus, the response becomes insensitive to that parameter.

From Figs. 4, 5 and 10, 11, it is observed that the base shear transmissibility has increased because, at half-full tank, the system presents relatively larger total yield strength  $F_o$  (or initial stiffness).

Figs. 12 and 13 show that, the base displacement,  $x_b$ , in half-full tanks is notably less than in full tanks, as it was expected. Further, it is observed that the response is principally controlled by the initial stiffness and viscous damping, in both soil conditions. In the case of stiff soil, the post-yield stiffness has little influence in the base displacement,  $x_b$  (see Fig. 12).

Figs. 14 and 15 provide evidence that the sloshing displacement,  $x_c$ , is not influenced by the isolation system parameters when the initial stiffness is high enough ( $F_o > 0.2$ ). In any case, the isolation damping has little influence on the response.

Only with highly flexible systems (low initial and post stiffness,  $F_o < 0.1$  and  $T_b > 2$  s) that it is possible to achieve a moderated control of the surface liquid motion.

Similarly to the former case, there is an important amplification of the response in soft soil conditions. It is also observed that in both soil conditions, the sloshing displacement,  $x_c$ , in half-full tank is larger than in full tank (see Figs. 8, 9, 14 and 15).

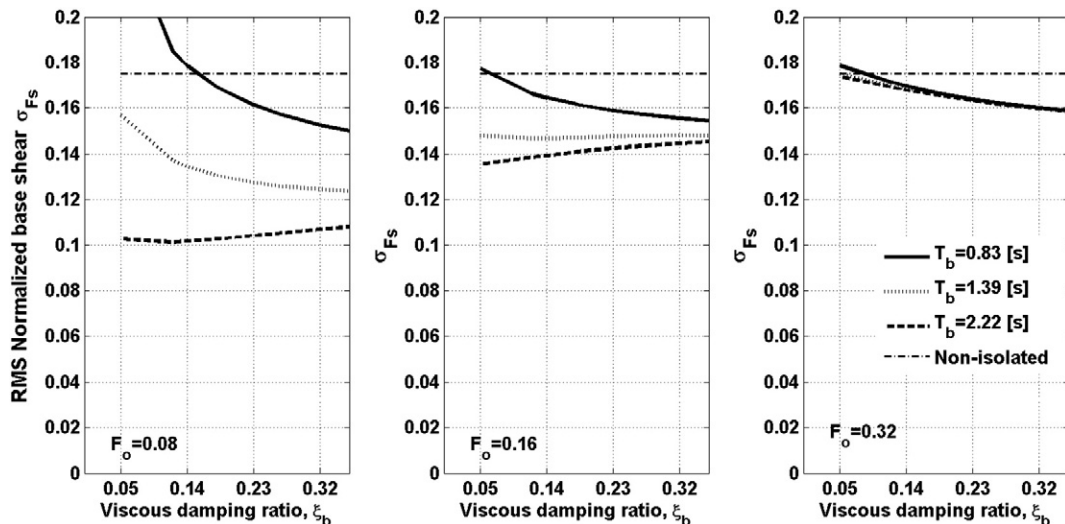


Fig. 11. Influence of post-yield isolation period  $T_b$ , normalized total yield strength  $F_o$  and viscous damping ratio  $\xi_b$ , on the normalized base shear,  $F_s$ ;  $S=0.9$ , soil type = III.



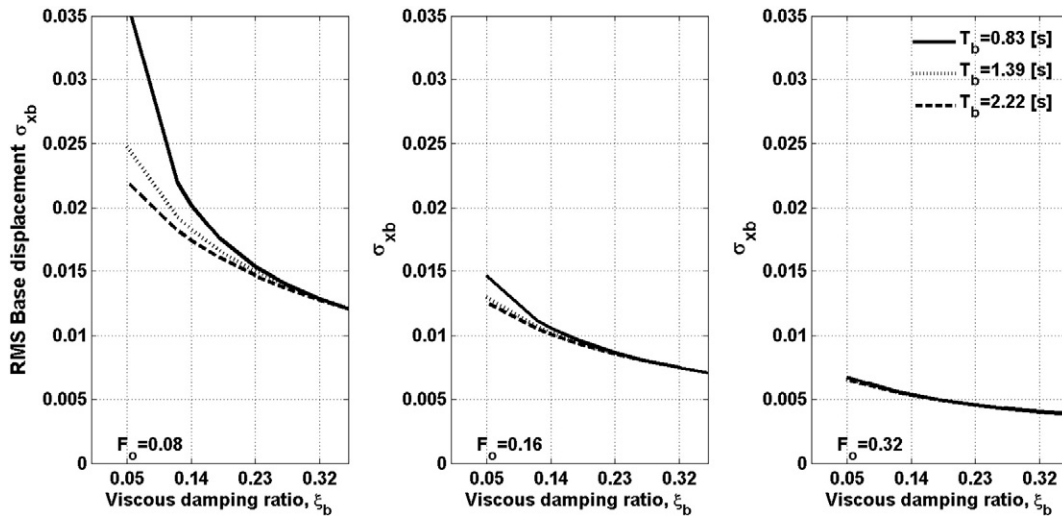


Fig. 12. Influence of post-yield isolation period  $T_b$ , normalized total yield strength  $F_o$  and viscous damping ratio  $\xi_b$ , on the base displacement,  $x_b$ ;  $S = 0.9$ , soil type = I.

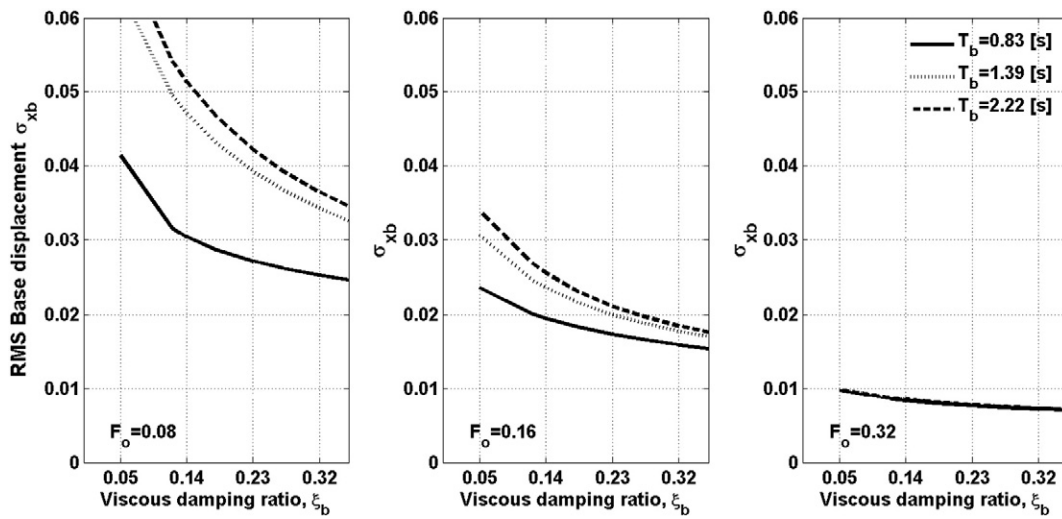


Fig. 13. Influence of post-yield isolation period  $T_b$ , normalized total yield strength  $F_o$  and viscous damping ratio  $\xi_b$ , on the base displacement,  $x_b$ ;  $S = 0.9$ , soil type = III.

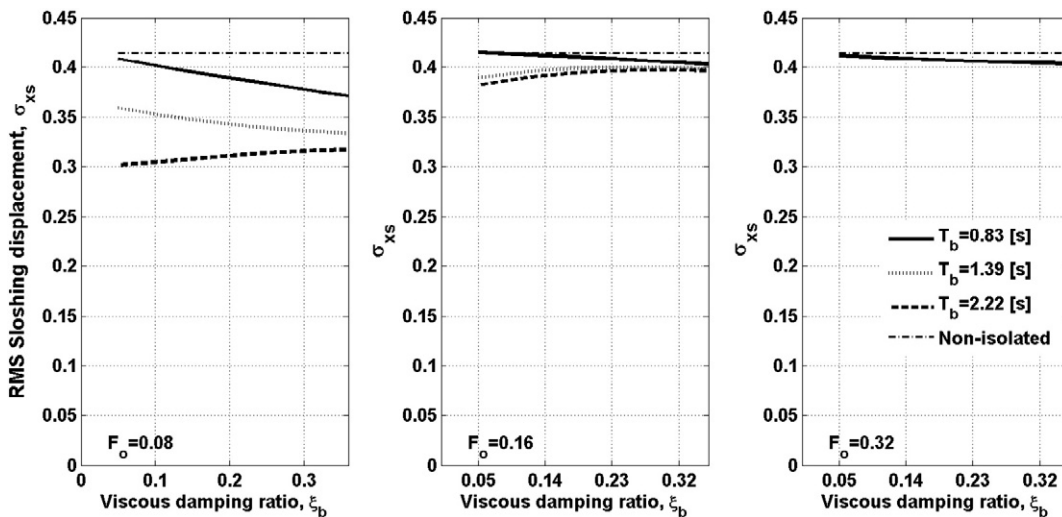


Fig. 14. Influence of post-yield isolation period  $T_b$ , normalized total yield strength  $F_o$  and viscous damping ratio  $\xi_b$ , on the sloshing mass displacement,  $x_s$ ;  $S = 0.9$ , soil type = I.

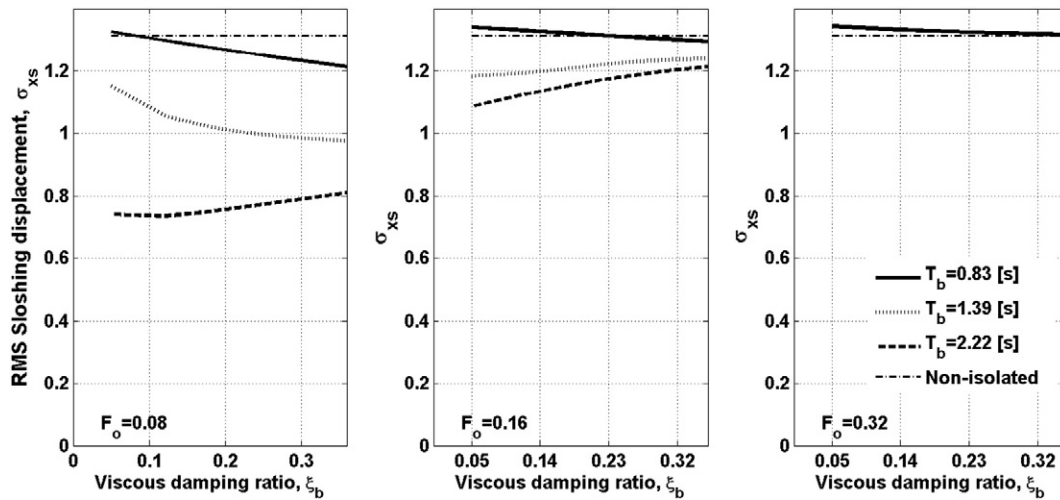


Fig. 15. Influence of post-yield isolation period  $T_b$ , normalized total yield strength  $F_o$  and viscous damping ratio  $\xi_b$ , on the sloshing mass displacement,  $\sigma_{xs}$ ;  $S = 0.9$ , soil type = III.

## 7. Conclusions

Seismic performance of cylindrical liquid storage tanks isolated by bilinear bearing was investigated by means of a parametrical study. Ranges of practical and useful values of post-yield isolation period, total yield strength and viscous damping ratio of the isolation system were taken into consideration. The earthquake excitation, assumed as a stationary random process, was characterized by a power spectral density function compatible with a given design spectrum. As the stochastic response in terms of root mean values was obtained in the frequency domain a statistical linearization scheme was used. To examine the seismic performance in different conditions, two liquid levels and soil types (soft and stiff soils) were considered.

From the results of the systems particularly analyzed, the following conclusions may be drawn:

1. Even though the flexibility provided by the isolation system (low values of total yield strength,  $F_o < 0.05$ ) leads to reduced base shear transmissibility (Figs. 4, 5 and 10, 11), it also causes important base displacement. Thus, the designer should adopt a compromise solution taking into consideration economic as well as technical aspects such as piping connections, dilatation joints, available space, etc.
2. While the base shear transmitted into structure is a little sensible to viscous damping of the bearings, especially in systems with enough flexibility ( $T_b > 2.5$  s and  $F_o < 0.1$ , Figs. 4, 5 and 10, 11), the damping as well as the initial stiffness of the isolation system are essential to control the base displacement (Figs. 6, 7 and 12, 13).
3. Soft soil conditions amplify significantly the overall response of the system; especially the base and sloshing displacements (Figs. 7–9 and 13–15) as well as the normalized base shear to a lesser extent.
4. The sloshing displacement is principally amplified by the post-yield stiffness (Figs. 8, 9) and, to a lesser extent, by the initial stiffness of the isolation system. In order to mitigate the surface liquid motion, it is necessary to design highly flexible isolation systems ( $F_o < 0.05$  and  $T_b > 2.5$  s).
5. In any case, the viscous damping of the isolation system has little influence on the sloshing displacement (Figs. 8, 9 and 14, 15).
6. Half-full tank displays relatively larger base shear than full tank (Figs. 4, 5 and 10, 11).
7. In order to obtain an adequate system response, the characteristic parameters should remain within the following ranges of values  $\xi_b > 0.1$ ,  $T_b > 2.5$  s,  $F_o \approx 0.05$ .

It is also worth noting that this work provides to the designers some guidelines for preliminary designs of cylindrical base-isolated tanks. In order to obtain a higher accuracy in the dynamical response

of the system, it is necessary to make use of sophisticated models like FEM that take into account the interaction of the fluid and soil with the structure as well as the non-linearities of the isolation system (Livaoglu and Dogangun [29]).

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