

On Two Different Lines of Argumentation in Kant's Theory of Schematism of Empirical and Mathematical Concepts

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In the chapter “On the Schematism of the Pure Concepts of the Understanding” of his *Critique of Pure Reason* (*KrV*), Kant devotes only two paragraphs to expound a theory of the schematism of empirical and mathematical concepts.¹ These passages have received less attention than those devoted to transcendental schematism (that is, the schematism of the categories or pure concepts of the understanding), and the interpretation of their meaning and scope is still a subject of great debate among Kant scholars. In these two paragraphs, Kant presents the schemata of empirical and mathematical concepts in ways that are not only quite different, but also clearly divergent, if not contradictory. On the one hand, Kant claims that a schema is a “method” (*Methode*), a “procedure” (*Verfahren*) or a “rule”

¹ See A140-2/B179-81. Following standard practice, references to the *Kritik der reinen Vernunft* (*Critique of Pure Reason*) are to the pagination of the first (A = 1781) and second (B = 1787) editions. Unless otherwise indicated, all quotations from this work are taken from P. Guyer and A. W. Wood's translation (Kant, 1787/1998). Apart from the references to *KrV*, all references to Kant's writings are to the academic edition of his works (Kant, 1900-). In this case, volume and page numbers are given, preceded by the title of the work and the abbreviation “AA” (for *Akademie-Ausgabe*). The *Jäsche-Logik* (*Jäsche Logic*) is quoted according to the translation by J. M. Young (Kant, 1992). In several instances, I have slightly modified these English translations in order to give a more literal rendering. All such modifications are indicated in the footnotes.

(*Regel*). On the other hand, he suggests that a schema is a “monogram” (*Monogramm*) or a “figure” (*Gestalt*) delineated in a general manner.

The two main contributions of this article are as follows. 1) I shall show that Kant’s characterisation of the schema both as a procedure and as a monogram reveals that there are two lines of argumentation of the schematism of empirical and mathematical concepts. I shall study in detail the significance and the limitations of these two lines of argumentation as well as the differences between them. 2) While examining Kant’s characterisation of schemata as procedures, I shall make a clear demarcation between concepts as rules and the thus-conceived schemata.

As I shall explain, the aforementioned two lines of argumentation are two different ways of approaching one and the same philosophical problem, namely, that of the applicability of empirical and mathematical concepts, which as such are always universal, to their corresponding intuitions, which as such are always singular. In this article I will limit myself to shedding light on the characteristics and limitations of these two lines of argumentation by means of a thorough textual analysis of the above-mentioned passages of the Schematism chapter. The purpose of this exegesis is to call attention on the complexities involved in Kant’s treatment of the aforementioned philosophical problem. I hold that my exegesis is a necessary first step in the reconstruction of Kant’s attempt to solve that problem.

However, it needs to be stated that the examination of the philosophical significance of the presence of these two lines of argumentation in the Schematism chapter lies beyond the scope of this article. There is not enough space here to discuss the problem whether Kant’s divergent characterisations of the schemata of empirical and mathematical concepts highlight a tension or contradiction in Kant’s thought, nor to determine which line of argumentation is the best Kantian

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solution to the philosophical problem of the applicability of empirical and mathematical concepts (in fact, as I will show, both lines of argumentation have difficulties and shortcomings).

I shall study these two lines of argumentation separately. After briefly introducing the problem of the Schematism chapter in the first section, I shall address Kant's account of the schema as a method or procedure (hereafter L1) in the second one. I shall also study the distinction between concepts as rules and schemata as procedures in this section. In the third section, I shall examine Kant's account of the schema as a monogram or figure delineated in a general manner (hereafter L2).

I. The Problem of the Schematism Chapter

In the Schematism chapter Kant asks the question: “Now how is the *subsumption* of the latter [i.e., spatiotemporal intuitions] under the former [i.e., pure concepts of the understanding], thus the *application* of the category to appearances² possible [...]?” (A137/B176). The answer to this question is a necessary task for transcendental philosophy. 1) The application of the categories to the empirical manifold is a condition of the possibility of our experience of empirical objects. 2) However, the application of a concept to intuitions is not possible if the concept and the intuitions are heterogeneous. 3) The categories and our spatiotemporal intuitions are completely heterogeneous. Therefore transcendental philosophy must show how, despite this heterogeneity, each category can be applied to empirical intuitions (see A137-8/B176-7).

The categories are heterogeneous from the empirical manifold because no content of any human spatiotemporal

² “Appearance” (*Erscheinung*) means here “[t]he undetermined object of an empirical intuition” (A20/B34).

intuition corresponds to the purely intellectual content of the categories. Hence, the subsumption of the empirical manifold under the categories cannot take place immediately, because that would be “contrary to logic” (*Letter to Tieftrunk*, AA:12, 224; December 11, 1797). It is thus necessary to introduce mediating elements, i.e., transcendental schemata. The function of these schemata is to produce homogeneity between the categories and the empirical manifold. Thus, the main contribution of the Schematism chapter is to specify the different sensible conditions (i.e., transcendental schemata) that produce homogeneity between each category and the corresponding aspect of the empirical manifold. This homogeneity makes the application of the former to the later possible.³

In the case of the schematism of empirical and mathematical concepts there is no heterogeneity between concept and intuition.⁴ Therefore, it is necessary to ask the question why there is in the Schematism chapter a theory of schematism of empirical and mathematical concepts. First, it is remarkable that the problems, which make necessary the doctrines of transcendental schematism and of schematism of empirical and mathematical concepts, are different ones (see Nolan, 1979, 123). According to Kant, the schemata of empirical and mathematical concepts are not needed to address a problem of heterogeneity, but the problem posed

³ A detailed discussion of the problem of transcendental schematism is beyond the scope of the present article. I have dealt with this problem in another place (see Arias Albusu, 2009a). However, it should be mentioned that “The Deduction of the Pure Concepts of the Understanding” only shows that: 1) the application of the categories is a necessary condition of the possibility of our objective experience; 2) this application must take place, in general, through the transcendental synthesis of imagination. In contrast, the main objective of the Schematism chapter is to specify the particular modes of this synthesis that make the application of each category possible.

⁴ The content of empirical concepts is formed from empirical intuitions, and the content of mathematical concepts can be exhibited in our pure intuition. For an account of the characteristics of these concepts, see section II, esp. II.1.

by the difference between the universality of these concepts and the singularity of their intuitions (see A140-2/B179-81).⁵ Second, it is important to note that, although these two problems are different in kind,⁶ they have a common source, that is, the heterogeneity between understanding and sensibility. 1) The categories and the empirical manifold are heterogeneous in content because the conceptual content of the categories arises solely from the understanding and the spatiotemporal character of the intuitive content of the empirical manifold stems from the forms of sensibility. 2) Universality, that is, the form of all concepts, is generated by the understanding (see *Jäsche Logic*, AA, 9:91-5) and the intuitions of sensibility are always singular (see *Jäsche Logic*, AA, 9:91; A320/B376-7).⁷

The fact that these problems have the same origin is probably the reason why Kant also presented in the Schematism chapter a theory of schematism of empirical and mathematical concepts. Furthermore, it may be said that Kant's intention was to introduce the new and special problem of heterogeneity between categories and empirical manifold by means of the common and well-known modern version of the problem of universals—a problem which, in his philosophy, has the same source as the first.

⁵ I quote this passage below.

⁶ And not just different in degree, as Pippin (1976, 163-4; 1982, 143-5) and Seel (1998, 229) seem to hold.

⁷ I have examined elsewhere the two different problems that the two different kinds of schematisms address (see Arias Albisu, 2009b). I have tried to show there that the gap between the universality of mathematical and empirical concepts and the singularity of their corresponding intuitions is best characterised as “difference” (*Unterschied*).

II. The Schemata of Empirical and Mathematical Concepts as Methods or Procedures (L1)

The following are the key passages from the Schematism chapter relevant for the study of the first line of argumentation of the schematism of empirical and mathematical concepts:

The schema is in itself always only a product of imagination; but since the synthesis of the latter has as its aim no individual intuition but rather only the unity in the determination of sensibility, the schema is to be distinguished from an image. Thus, if I place five points in a row, this is an image of the number five. On the contrary, if I only think a number in general, which could be five or hundred, this thinking is more the representation of a method for representing a multitude (e. g., a thousand) in accordance with a certain concept than the image itself, which in this case I could survey and compare with the concept only with difficulty. Now this representation of a general procedure of the imagination for providing a concept with its image is what I call the schema for this concept.

In fact it is not images of objects but schemata that ground our pure sensible concepts. No image of a triangle would ever be adequate to the concept of it. For it would not attain the generality of the concept, which makes this valid for all triangles, right- or oblique-angled, etc., but would always be limited to only one part of this sphere. The schema of the triangle can never exist any-where except in thought, and signifies a rule of the synthesis of the imagination with regard to pure figures in space. Even less does an

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object of experience or an image of it ever reach the empirical concept, rather the latter is always related immediately to the schema of the imagination, as a rule for the determination of our intuition in accordance with a certain general concept (A140-1/B179-80).⁸

For the moment, I understand the concept of image (*Bild*) as an intuitive, singular content that corresponds to a concept (cf. Mellin, 1797-1804, 1:706). In the passage quoted above, Kant claims that singular images do not reach the universality of mathematical and empirical concepts. Inasmuch as these concepts can be applied to the totality of their possible images, the former do not have the latter as their adequate correlates. For instance, an image of a particular dog necessarily belongs to a particular breed, whereas the concept of a dog is valid for all dogs of all breeds. Moreover, an image of a triangle is necessarily equilateral, isosceles, or scalene, whereas the concept of a triangle is applicable to these three classes of triangles. Thus, in order to mediate between concept and image, it seems necessary to introduce a third element: the schema. Concepts are grounded on schemata and they immediately refer to them. On the contrary, concepts are connected to images only in a mediated manner, that is, through the schemata.

The Schematism chapter deals in general with the problem of the conditions under which the application of concepts becomes possible. Thus, the fact that empirical and mathematical concepts are connected to their correlative intuitions only through schemata must be understood in the sense that schemata make possible the application of these concepts to their intuitions.⁹

⁸ (Translation modified).

⁹ Although the passage quoted above from the sixth and seventh paragraphs of the Schematism chapter (A140-1/B179-80) makes it clear that, according to Kant, a schematism of empirical and mathematical concepts is necessary, there is a passage in the second paragraph of that chapter (A138/B177) that

The following passage is crucial to support L1: “Now this representation of a general procedure of the imagination for providing a concept with its image is what I call the schema for this concept.” The expression “representation of a [...] procedure” can be understood as referring to the discursive description of a procedure. However, if the schemata for concepts were discursive descriptions of procedures, they would be themselves concepts, that is, concepts of procedures. This is an extremely problematic assertion. For, if it were the case that the application of a concept A can be made possible only by another concept B, Kant’s doctrine of the schematism would imply an infinite regress. In effect, to introduce a concept-schema would lead to positing another concept-schema as a way of explaining the adequation of the original concept-schema to its intuitions. This operation is clearly never-ending.¹⁰ It is possible, though, to avoid this conclusion by interpreting the term “representation” widely: that the schema is the representation of a procedure simply means that the word “schema” designates or refers to a procedure. In sum, I hold that the schema is the procedure itself.

Interpreted this way, the schemata of empirical and mathematical concepts appear to us as essentially dynamic. These schemata are procedures or ordered modes of action, that is, ordered sequences of certain kinds of actions that aim to accomplish a certain goal. Actually, “Verfahren” refers not to an act, but to a *way* of acting.

In the passage quoted above, Kant points out that the procedures-schemata are products of the imagination whose

seems to imply that only the schematism of the categories is necessary. Proper consideration of this problem lies beyond the scope of the present article. I have tried to show elsewhere that, despite appearances, the necessity of the schematism of empirical and mathematical concepts is in no way denied in the passage in question (see Arias, 2008).

¹⁰ F. J. Leavitt mentions this interpretation of the schema and raises a similar objection. See Leavitt (1991, 653-4).

purpose is to give a concept its corresponding image (*Bild*). How to interpret Kant's concept of image? J. Bennett has claimed that an image is nothing but a "mental picture" (Bennett, 1966, 141ff), and G. J. Warnock makes reference to an "imagined model" (Warnock, 1949, 81-2). In both interpretations, images are fundamentally different than objects in that they are mental facts. There is certainly textual support for Bennett and Warnock's interpretation: for instance, in the passage quoted above, Kant refers to the image *of* an object; in another, he defines imagination as the faculty for representing an object even when it is not present in intuition (see B151). These passages suggest that an image is a private mental representation that the subject may have even when there is no correlative object.

It is clear that an explanation of the concept of image (*Bild*) presupposes an exposition of the concept of imagination (*Einbildungskraft*). For the purpose of this article it suffices to consider briefly two problems regarding Kant's statements on imagination: 1) the different theories of imagination of the first (A = 1781) and second (B = 1787) editions of *KrV*; 2) the relation between the already mentioned definition of imagination from B151 and the theory of imagination of B.¹¹

¹¹ I follow closely the interpretation of Kant's concept of imagination given by M. Caimi (2007, 81-8; 2008). Kant distinguishes between productive and reproductive imagination (see B139-40, B141-2, B152). Reproductive imagination performs a synthesis subject to the empirical laws of association. These laws of association make reference to the principle according to which the empirical representations that have repeatedly followed one another create a habit in the mind such that when one empirical representation is present to the mind, the associated one also arises (see *Anthropology from a Pragmatic Point of View*, AA, 7:176). This synthesis is purely subjective and amounts to a fact in the mental history of a subject. On the contrary, productive imagination performs a synthesis governed by concepts of the understanding. This synthesis is a condition of the constitution of objects of knowledge. My explanation of the concept of imagination applies only to productive imagination.

As regards the first problem, it is worth noting that Kant has not devoted any section of *KrV* to the imagination and vacillated about how to characterise it. In A, the imagination is one of the three autonomous and independent faculties of the mind. These faculties are sense (i.e., sensibility), imagination and apperception (i.e., understanding) (see A94, A115). In B, the imagination is no more an autonomous faculty and turns into a function of the understanding, which, as a consequence of this shift in Kant's thinking, remains as the only spontaneous faculty. The transcendental synthesis of imagination is "an effect of the understanding on sensibility and its first application [...] to objects of the intuition that is possible for us" (B152) and the transcendental action of imagination is a "synthetic influence of the understanding on the inner sense" (B154. See also B129-30, B150, B162 fn. and Kant's handwritten correction to his own copy of the first edition of *KrV*—A 78—: AA, 23:45). According to Kant's mature theory of imagination, this faculty is nothing more than a mode of the understanding, in which it does not obey exclusively to its own laws of logic, but also accepts and takes into account the heterogeneous laws of the sensible. The task of determining sensibility requires a specification of the faculty of synthesis of the understanding. This specification of the understanding is imagination. Such a specification is necessary because sensibility presents its own fundamental conditions for the sensible manifold that the understanding must synthesise, that is, extension or the form of spatial dispersion and succession or the form of temporal dispersion. As M. Caimi points out, this conception of imagination follows a tradition initiated by Descartes. According to Descartes, the power of imagining is the power of understanding inasmuch as the latter is directed towards bodies or *res extensa* (see

Descartes, 1641¹, 1642/1904, 71-3). So it may be said that for Descartes imagination is the corporeal intellect.¹²

In regard to the second problem, I have already mentioned that in B151 Kant defines imagination as “the faculty for representing an object even [*auch*] *without its presence* in intuition”. Thus, it is necessary to explain the relation between this definition and the above-described theory of imagination of B. I will limit myself to briefly presenting three key observations on this point. Firstly, as M. Caimi indicates, the definition of imagination from B151 (i.e., as a faculty for representing objects without their presence in intuition) can be found in Christian Wolff (see 1720¹, 1751/1983; 130). Wolff could not accept Descartes’ theory of imagination because for him there was no independent sensibility to which the activity of imagination as a specification of the understanding could be applied. Following Leibniz, Wolff thought that sensibility is not an independent faculty and that the sensible is nothing but a confused representation of the intellectual.¹³ But what prevents Wolff from adopting Descartes’ theory of imagination is not a valid reason for Kant to reject it (see Caimi, 2008, 44-5). As it is well known, Kant thinks that sensibility is independent from the understanding and that only from the unification of these two different faculties can cognition arise (see A51-2/B75-6). According to Kant, sensibility provides a sensible manifold of intuition, which is not a confused intellectual representation (see A43-4/B60-2, A267-8/B323-4, A270-1/B326-7, A275-6/B331-2). Thus, it is perfectly possible for Kant to hold the theory of

¹² For a discussion of this conception, see A. Ferrarin (1995b, 78-81). According to Ferrarin, the mind needs the faculty of imagination in order to know bodies (see 79). Caimi holds that, in Spinoza’s view, imagination has a similar function (2008, 42-3).

¹³ However, according to Caimi, there is at least one passage where Wolff seems to admit that the laws of imagination are specifically different from those of the understanding (see Caimi, 2008, 44, fn. 12; Wolff, 1720¹, 1751/1983; 133).

imagination that, as we have seen, he offers in B. Secondly, in B151 Kant states that imagination is the faculty for representing an object *even* or *also* (*auch*) without its presence in intuition. As G. S. A. Mellin already said, the German word “auch” can be understood in the sense that for Kant imagination is the faculty for representing in intuition an object with its presence and *also* without it (see Mellin, 1797-1804, 2:219).¹⁴ So what is decisive in Kant’s conception of imagination is that its synthesising activity is an application of the spontaneity of the understanding to the manifold of sensible intuitions, regardless the presence or absence of the object of intuition. Thirdly, there is abundant textual support in *KrV* for the thesis that imagination synthesises something present in sensibility. I shall give only one telling example. The synthesis of apprehension attributed to imagination in A is exercised “immediately upon perceptions” (A120).¹⁵ This example makes it clear that imagination can operate on something that is given in sensibility, i.e., perceptions “in the mind” (*ibid.*).

In conclusion, imagination is the spontaneity of understanding as oriented to sensibility and its fundamental operation is the synthesis of the sensible intuitive manifold.¹⁶ This synthesis is governed by a representative concept¹⁷ and it

¹⁴ I owe this reference to Caimi (2008, 47).

¹⁵ I quote this passage at length below. See also B160.

¹⁶ W. Metz offers a similar interpretation of Kant’s concept of imagination. Metz claims that a “main result” of his research is “that the transcendental imagination neither is nor can be a third aprioristic faculty of cognition, but that it designates the transcendental synthesis *in concreto*, i.e., in its universal reference to perceptions as a whole” (Metz, 1991, 141). Thus, productive imagination is the pure understanding *in concreto*, and pure understanding is only the productive imagination that has not yet led itself to concretion (86). It is remarkable that the synthesis of imagination has been understood as an interpretation of the sensible manifold (see Young, 1988, 143, 145, 153; Allison, 2004, 188-9).

¹⁷ By “representative concept” I mean those concepts of the understanding which may have a corresponding object in intuition. Such concepts are the categories and the empirical and mathematical concepts. Concepts such as the ideas of reason are not designated by that expression.

can be performed regardless the presence or absence of that which is intuited. Thus, imagination plays a decisive role in the constitution of objects of knowledge. In fact, imagination is responsible for the synthesis of the manifold that constitutes this manifold as an object. In the context of an exposition of the necessary connection between the understanding and all possible appearances (Transcendental Deduction—A—), Kant writes:

But since every appearance contains a manifold, thus different perceptions by themselves are encountered dispersed and separate in the mind, a combination of them, which they cannot have in sense itself, is therefore necessary. There is thus an active faculty of the synthesis of this manifold in us, which we call imagination, and whose action exercised immediately upon perceptions I call apprehension. For the imagination is to bring the manifold of intuition into an *image*; it must therefore antecedently take up the impressions into its activity, i.e., apprehend them (A120).

An image is a representation that is produced by our imagination when, through an action of synthesis, it unifies a sensible manifold. For that reason, “image” is not to be taken, in the Schematism chapter, as a synonym for “private mental image”. In fact, an image can also be understood as the result of a synthesis of the empirical manifold received in sensibility when the latter becomes affected by an entity that is independent of the subject. Such a synthesis is a necessary condition for the constitution of the empirical manifold as an object. In conclusion, “image” means both an imagined entity (for example, an imagined empirical object or the representation of a geometrical figure in pure intuition) and, inasmuch as it is a correlate of the synthesis of the imagi-

nation, an actual empirical object. In both cases, the image is an intuitive content structured and unified according to what is thought in a concept.¹⁸

I will now interpret the definition of the schema as a procedure for providing a concept with its image taking into account the previous explanation of the concept of image. The schema of an empirical concept is simply the procedure to imagine an object corresponding to the concept, or to constitute an empirical manifold as an object by interpreting it as a particular case of the concept.¹⁹ The schema of a geometrical concept consists in the procedure undertaken by the imagination to draw, in the pure intuition of space, the corresponding geometrical figure. The schema of an arithmetical concept is the procedure through which we successively enumerate a collection of formal intuitions of space (for instance, dot and lines) taken as unities.²⁰

¹⁸ La Rocca and Chipman deny that “image” should be restrictively understood as “private mental image” (see Chipman, 1972, 47; La Rocca, 1989, 136-9). It may be said that an image, inasmuch as it is an intuitive content structured by a concept, is one of the possible sensible aspects of that concept (see Heidegger, 1929/1991, 92-7).

¹⁹ While the application of the categories constitutes empirical intuitions in general as empirical objects in general, the application of empirical and mathematical concepts constitutes sensible intuitions as objects of a certain kind. (On the problem of the status of mathematical objects, see fn. 20). The application of empirical concepts presupposes the application of all the categories, and the application of mathematical concepts presupposes primarily the application of the categories of quantity. An examination of these issues is beyond the scope of this article.

²⁰ The objects of mathematical concepts are not objects in the strict sense of the word (that is, empirical ones), but pure or formal intuitions produced by an action of synthesis (see B137-8, B160-1 fn. This action of synthesis is called the construction of the mathematical concept. See A713/B741). Mathematics proceeds by determining pure intuition and thus provides *a priori* cognitions of nothing but the intuitive *form* of empirical objects (see B147). Despite this fact, Kant makes reference to the “object” (*Objekt*, *Gegenstand*) of mathematical concepts (see A713/B741. For a discussion of this topic, see Kim—2006, 144-8—and Parsons—1984, 109-11—). According to Kant, geometrical concepts govern a synthesis that consists in tracing or drawing (*ziehen*, *zeichnen*, *verzeichnen*) from part to part a figure in pure intuition (see A162-5/B203-5). The figure so drawn is the object of the geometrical concept and can be regarded as a formal intuition. In contrast,

Therefore, a schema is a universal procedure generated by imagination to objectively synthesise sensible manifolds according to a particular concept. This concept is applied to the sensible manifold through the synthesis of the procedure-schema. In this way, the sensible manifold is constituted as an object corresponding to the thus-applied empirical or mathematical concept.²¹ Whereas empirical and math-

arithmetical concepts can govern two different kinds of actions of synthesis. First, such an action can be the enumerating (*zählen*) of a discrete collection of formal intuitions (such as points or strokes) or of empirical objects (such as the fingers or the beads of an abacus) (see B15, A78/B104, A140/B179, A240/B299. For a fuller development of this conception of arithmetical concepts, see Young—1982; 1984—). These formal intuitions or empirical objects are taken as homogeneous unities (i.e., they are considered merely as individual or unitary objects or intuitions of the same kind, and abstraction is made of their differentiating particular qualities or properties). Empirical objects are used just in order to pay attention to strictly homogeneous regions of pure space taken as unities (see Sutherland, 2006, 556-7). Thus, the primary objects of arithmetical concepts are still pure or formal intuitions, and the empirical objects can be considered as objects of these concepts only in a secondary sense. The same applies to the relation between geometrical concepts and their objects (see *On a Discovery*, AA, 8:191-2 fn.). Second, the construction of an arithmetical concept can also occur through the calculation of the size of a formal intuition or an empirical object in space. This calculation consists in choosing a unit of measurement and determining how many times it is contained in the intuition or the object (see A242/B300. On the problem of measuring and enumerating—*Messen und zählen*—, see also *Reflection* No. 5727—AA, 18:338; 1780-1789—). As regards algebra, I will limit myself to presenting the principal conclusions on this topic reached by L. Shabel. Shabel states that in early modern mathematics “algebra was not conceived as a separate mathematical discipline with its own object of investigation. Rather, algebra was conceived as a method of reasoning about the objects of arithmetic and geometry, and was thus used as a tool for solving arithmetic and geometric problems” (1998, 590). This conception of algebra was present “in the popular textbooks by Christian Wolff that Kant used to teach algebra” (ibid. See also 608). The same conception of algebra can be found in Kant: “[S]ymbolic constructions’ [that is, algebraic constructions (see A717/B745)] are not *kinds* of constructions, that is, constructions of or out of symbols or characters. Rather, they are that which symbolize ostensive or geometrical constructions [i.e., the constructions of arithmetical and geometrical concepts]” (615. See also 597). Since algebraic concepts do not have any kind of specific objects associated with them, these concepts will not be considered in this article.

²¹ “An *object* [Object], however, is that in the concept of which the manifold of a given intuition is *united*” (B137). “Hence we say that we cognize the

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ematical concepts serve as rules of synthesis, the schemata are procedures of synthesis performed by imagination according to those rules.²²

An objection that might be raised against my interpretation that the schemata are procedures governed by conceptual rules is that there are two passages where Kant apparently interprets the schemata of empirical and mathematical concepts as *rules*. On A141/B180, he holds that the schema of a triangle “signifies a rule of the synthesis of the imagination with regard to pure figures in space.” And he adds that an empirical concept “is always related immediately to the schema of the imagination, as a rule for the determination of our intuition in accordance with a certain general concept” (ibid.). I claim that these passages must be interpreted in the following way: the schema of the concept of triangle is a procedure of synthesis of the imagination governed by the concept; the schema of an empirical concept is a procedure through which we determine our intuition according to the concept that serves as a rule. The schema is by no means a rule different from the conceptual rule. On the contrary, the schema is the procedure that is performed according to the conceptual rule.

A brief explanation of the role played by concepts as rules of synthesis will provide us with a better understanding of the difference between concepts as rules and schemata as

object [*Gegenstand*] if we have effected synthetic unity in the manifold of intuition” (A105).

²² In the same paragraph in which he equates schema with procedure, Kant employs the concept of method (*Methode*) (see the passage from A140/B179 quoted above). It is clear in the context that also the latter concept refers to the schema. So it may be objected that my choice of the term “procedure” is arbitrary. However, in A855/B883 we find the following sentence: “If something is to be called a method, it must be a procedure in accordance with *principles*.” This statement provides support for my interpretation of the schema as a procedure, for I conceive of the schema as a procedure governed by a certain principle, that is, the concept that serves as a rule. There is, then, no contradiction between my interpretation of the schema and the characterisation of the schema as a method.

procedures. In the Transcendental Deduction (A), Kant states: “as far as its form is concerned the latter [i.e., the concept] is always something universal, and that serves as a rule” (A106); “the representation of a universal condition in accordance with which a certain manifold (therefore in the same way) *can* be posited is called a *rule*” (A113).²³ We have already seen that concepts serve as rules in the sense that they govern the synthesis that constitutes the intuitive manifold as an object. According to Kant, the concept, as regards its form,²⁴ is “something universal” (*etwas Allgemeines*) that serves as a rule, and a rule is “the representation of a universal condition [*Vorstellung einer allgemeinen Bedingung*] in accordance with which” the manifold “can be posited”, we may say, always “in the same way” (see Kant, 1787/2009, CI, fn. 386). It seems that concepts can serve as rules only because they represent universal conditions (that is, there is “something” universal in them) according to which the manifold can be unified or synthesised always in the same way.

The question that must be posed is: what is this universal “something” or “condition” that enables concepts to serve as rules of synthesis? The answer to this question can be found in the *Jäsche Logic*. There Kant writes that the concept “is a universal representation or a representation of what is common to several objects, hence a representation *insofar as it can be contained in various ones*” (AA, 9:91).²⁵ A concept is universal because it represents properties that are common

²³ (Translation modified). While the fact that Kant’s characterisation of concepts as rules is given in the Transcendental Deduction (A) may suggest that he is only interested in explaining the role played by the categories as rules of synthesis, the examples of the empirical concept of body (A106) and of the mathematical concept of triangle (A105) given in the same context prove that Kant’s theory of concepts as rules applies to all kinds of representative concepts.

²⁴ The form of all concepts is universality (see fn. 35 and *Jäsche Logic*, AA, 9:91).

²⁵ (Translation modified).

to all the objects that fall under the concept. The representations of a common property or common properties contained in a concept are themselves concepts or, as Kant says, “partial concepts”. These partial concepts (*Teilbegriffe*) are also called by Kant marks (*Merkmale*) (see AA, 9:58-9). The mark or marks contained in a concept are what is thought in it, that is, its content (*Inhalt*).²⁶ For example, it may be said that the concept of body contains, among others, the marks of extension, impenetrability and figure (see B12).

I claim that the content of the concept, i.e., a set of discursive marks, is that universal “something” or “condition” that enables the concept to serve as a rule of synthesis. These sets of marks serve as rules of synthesis insofar as they determine the unity of elements that the schemata as universal procedures of synthesis must effect in the sensible manifold.²⁷ Thus, the content of a representative

²⁶ Although there is passage where Kant makes a distinction between marks and partial concepts (see AA, 9:95), I agree with H. J. Paton when he states that, for the sake of simplicity, it is convenient to speak of concepts as made up of marks, ignoring the distinction between marks and partial concepts made in that passage (see Paton, 1936/1970, 1:195-6. Kant himself ignores this distinction in other passages. See AA, 9:59-62; A43/B60).

²⁷ It should be mentioned that Kant makes a distinction between discursive and intuitive marks. In regard to this point, H. Smit has drawn attention to the *Reflection* No. 2286: “A mark is a partial representation (which), as such (is a ground of cognition). It is either intuitive (a synthetic part): a part of intuition, or discursive: a part of a concept, which is an analytic ground of cognition” (AA, 16:299-300; 1780-1789. See also the *First Introduction to the Third Critique*—AA, 20:226-7 fn.—and the first lines of *Reflection* No. 2282—AA, 16:298—). According to Smit, a mark is a part or partial representation of an intuition insofar as it can be understood as an element of the content of that which is intuited. For example, the property “rectangular” is included, as an intuitive content, in the intuition of this page you are now reading. In this context, “rectangular” means this particular rectangular shape of a certain size and colour. In contrast, the concepts “rectangle” and “body” are examples of discursive marks; that is to say, they are partial and universal representations, which may be applied to several rectangular bodies (see Smit, 2000, 254-5). To subsume an intuition under a certain concept means “to recognize an intuitive mark as a singular instance of a discursive mark”. The forms of our sensibility (space and time) render a partial representation

concept prescribes the mode in which a schema must “posit” the manifold always “in the same way”.²⁸

I shall try to explain this point further. The sets of marks thought in representative concepts determine the aim that must be achieved by the acts of synthesis of the imagination. These concepts represent the union or necessary mutual belonging of the elements that the synthesis of the imagination has to unify. It is in this sense that representative concepts serve as rules of synthesis. In other terms, concepts rule the action of synthesis inasmuch as they determine the unity that it must generate out of the sensible manifold.²⁹ For instance, the discursive mark “biped”, which is thought in the empirical concept of man, demands the imagination to combine two legs, not one more nor one less, in the objective representation of every particular man. As for mathematical concepts, the situation is analogous. For example, the mathematical concept of triangle demands the imagination to combine, in all triangular objective representation, three and only three sides. Hence the content of a representative concept, that is, a set of discursive marks, requires the imagination to carry out a certain and determinate synthesis of sensible elements.³⁰ The schema of this concept is noth-

an intuitive mark, while the logical form of concepts (i.e., universality) renders a partial representation a discursive mark (259). Therefore, it may be said that the unity of sensible elements made necessary by the set of discursive marks thought in a concept is a unity of intuitive marks.

²⁸ In other words, the property (or set of properties) contained in a representative concept of the understanding demands the imagination to find in empirical intuitions (in the case of empirical concepts) or draw in pure intuition (in the case of mathematical concepts) the particular instance (or set of particular instances) of that property (or set of properties). The fulfilment of this demand is a necessary condition of the subsumption of the intuitive elements under the concept.

²⁹ Kant expresses this idea when he equates concepts with the consciousness of the synthetic unity of the manifold (see A103ff). For an interpretation of this function of concepts, see Ginsborg (1997, 51-2).

³⁰ R. P. Wolff maintains that, for Kant, concepts “cease to be *things* (mental contents, objects of consciousness) and become *ways of doing things* (rules, forms of mental activity)” (1963, 70). My analysis has shown that Wolff’s

ing but the procedure of the imagination through which that certain and determinate synthesis is achieved.³¹

If “rule” is interpreted as the ordered way in which an action of synthesis is performed, then schemata are rules governed by conceptual principles. However, this discussion is entirely semantical. I hold that my terminology is the correct one for, as Kant himself claimed, we cannot explain the subsumption of sensible elements under a concept-rule by resorting to other rules (this would entail entering into an infinite regress). This is why schemata, as conditions for the application of conceptual rules, cannot be taken themselves as rules (see A133/B172; *Critique of the Power of Judgment*, AA, 5:169; *On the Common Saying*, AA, 8:275).³²

On the other hand, the studies on schematism that interpret schemata of empirical and mathematical concepts as rules present serious difficulties. One can analyse these interpretations by dividing them into two groups according to how they evaluate the distinction between concepts and schemata. In what follows I present and criticise these groups of interpretations.

position on this point is one-sided. Concepts can serve as rules of synthesis because their representative content determines the aim of such a synthesis. R. Aquila and S. M. Bayne also reject the view that concepts are just rules of synthesis. According to Aquila, for Kant concepts are not only rules, but also possible mental contents (see 1974, 9, 13, 15-6). For his part, Bayne states that “the development of the concepts as rules theory” is “the completion of the concepts as marks theory” (2011, 135).

³¹ L. J. Stern’s interpretation of the sense in which concepts are rules is compatible with my own reading (see 1974, 160-1). Concepts are rules inasmuch as they “determine the constituents of something” (161). Furthermore, according to Stern, the function of empirical concepts “is discriminatory; they mark off the contents of a particular synthesis” (164). However, Stern holds that the form of concepts performs the function that, according to my view, is performed by procedure-schemata (see 163).

³² C. La Rocca has stated that schemata must not be understood as rules, but as procedures (see La Rocca, 1989, 134-5, 152). However, La Rocca does not develop his thesis in detail.

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First, consider the interpretation that holds that both concepts and schemata are rules that differ only in character. S. Feldman, for instance, claims that while concepts are “meaning rules” that allow us to produce valid analytic judgments, schemata are “reference rules” that make valid synthetic judgments possible (see Feldman, 1989). This interpretation finds inspiration in the work of S. Körner. Körner distinguishes between the “logical grammar” of concepts, that is, the “non-referential rules” that govern our use of concepts when they relate to other concepts, and the “referential rules” (schemata) that relate concepts with their corresponding perceived objects (see Körner, 1955, 71). This first group of interpretations does not take into account that concepts themselves serve as rules of synthesis of a sensible manifold. When Kant identifies concept and rule he does not have in mind meaning or intradiscursive rules, but rules for the constitution of objects.

Second, consider the view of N. Kemp Smith (1918, 338-9), R. P. Wolff (1963, 211-2)³³ and P. Guyer (1987, 164-5), who claim that empirical and/or mathematical concepts are identical to their schemata, that is, that they both are the same rule, method or procedure. This second group of interpretations directly contradicts the main lines of Kant’s argument in the Schematism chapter. In this chapter, Kant holds that the application of a concept is possible only if a non-conceptual element, that is, the schema, intervenes.³⁴

³³ In this case the identification between schema and concept is only suggested.

³⁴ See A139-42/B178-81. I will now consider a group of interpretations that also rejects Kant’s position, but in a different way. These interpretations do not hold that concepts are identical to their schemata, but that schemata are unnecessary or superfluous. G. J. Warnock claims that the Schematism chapter is based on an illegitimate distinction between having and applying a concept. Warnock equates the application of concepts with a capacity for certain verbal behaviour. He thus states that “to ask how I can apply a concept that I have, is to ask how I can use a word that I know how to use”. Warnock concludes that having a concept implies the ability to apply it, and that, for that reason, schemata are unnecessary (see Warnock, 1949, 80). J.

Contrary to these two groups of interpretations, mine shows that there is actually a difference between empirical and mathematical concepts and their corresponding schemata, even when this difference cannot be determined according to a distinction of different kinds of rules but according to the distinction between rule and procedure.

Bennett, too, considers that the possession of concepts implies the ability to apply them, but he adds that this is the case if and only if the concept has instances and the subject has no sensory disabilities. Contrary to Warnock, Bennett equates the application of concepts with a capacity for object recognition (see Bennett, 1966, 146). L. Chipman criticises both Warnock and Bennett. Chipman tries to show that a subject without sensory disabilities may possess a concept that has instances and yet be unable to apply it. He offers the example of the concept of “tadpole”. Chipman considers that we could know some marks of this concept without being able to recognise its corresponding objects in experience, i.e., we could know where tadpoles are to be found, that they are very small, that the tadpole stage is part of the life cycle of a frog, etc., and yet not be able to recognise them when confronted with a jar of water containing tadpoles. This is because we could lack the capacity to synthesise a sensible manifold in accordance with the concept “tadpole” (that is, we could lack the corresponding schema). Chipman claims that his argument shows that the possession of a concept does not necessarily imply the capacity to apply it. He then concludes that this fact proves that there is a difference between a concept and its schema (see Chipman, 1972, 44-5). Although Chipman’s argument is valuable, it does not address the problem of concept application from a Kantian point of view. I think that the incapacity to apply the concept of a tadpole, as Chipman presents it, is due to the fact that we do not possess the relevant discursive marks, such as “round head”, “long and restless tail”, etc. I hold, with Chipman, that the schema is different from the concept and makes its application possible, but only inasmuch as it is the procedure of synthesis produced by imagination in accordance with the set of marks that constitutes the content of the concept. The synthesis of a sensible manifold is always governed by that content. Chipman considers instead that the schema, independently of the concept, is the only responsible for the synthesis (see 46). D. O. Dahlstrom presents and discusses the above-mentioned interpretations (see Dahlstrom, 1981, 210-4).

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II.I. On the Distinction between Empirical and Mathematical Concepts

Now I will consider more closely the major differences between empirical and mathematical concepts in order to make a clearer demarcation between concepts as rules and schemata as procedures. I begin by briefly presenting two characteristics of mathematical concepts. Firstly, mathematical concepts are arbitrary, because their content is deliberately made up by the subject; that is to say, the content is not given through the nature of our understanding, like in the case of the categories, nor through experience, like in the case of empirical concepts (see A729/B757; *Logic Jäsche*, AA, 9:93).³⁵ The subject generates a mathematical concept by choosing some marks and uniting them to form the content of that concept. For example, we may combine the marks “closed plane figure”, “three” and “line segment” to form the content of the geometrical concept of a triangle. Secondly, mathematical concepts can be constructed *a priori* (see A713-4/B741-2, A729-30/B757-8). This means that the content of a mathematical concept can serve as rule for producing, by means of a procedure of synthesis, a pure or formal intuition that corresponds to the former. For instance, our imagination considers the marks of the content of the concept of a triangle and generates a universal procedure to draw, in the pure intuition of space, complex intuitions of particular triangles. In sum, the characteristics of mathematical concepts that are relevant for my present purpose are: 1) the subject may become aware of the whole content of the concept, because the latter is deliberately made up through a definition (see A729-30/B757-8; *Logic Jäsche*, AA, 9:141-4); 2)

³⁵ Note that all representative concepts are “made” (*gemacht*) as regards their form, that is, universality (see *Jäsche Logic*, AA, 9:91, 93).

the imagination must generate a universal procedure to determine *a priori* the manifold of pure intuition in accordance with the content of the concept, i.e., that faculty does not have to consider the special features of the empirical manifold given *a posteriori*.³⁶

In order to illustrate the difference between mathematical concepts and their schemata, I will now present the example of the geometrical concept of circle and its corresponding schema. The concept of circle is that of a closed, plane curve, all of whose points are equidistant from a point within it called the centre. In order to actually draw circular figures, imagination must be able to interpret this characterisation by producing a procedure whose form can be stated as follows: 1) draw two points on a plane, P and Q; 2) draw a line segment r between P and Q; 3) take the extreme P of segment r and rotate it around Q. In this way, one builds a circle of radius r and centre Q.

As regards empirical concepts, it is necessary to briefly examine two problems: 1) the genesis of these concepts out of empirical intuitions; 2) the uncompletable character of their content. In the *Logic Jäsche* Kant presents the logical acts of the understanding (that is, comparison, reflection and abstraction) through which concepts are made with regard to their form (i.e., universality), and offers the famous example

³⁶ Although mathematical concepts can be constructed through mere imagination in pure intuition (*pure or schematic construction*) as well as by means of instruments on some empirical material (*empirical or technical construction*), the second kind of construction, which is “improperly so-called”, is based and depends upon the first (*On a Discovery*, AA, 8:191-2 fn.; see also fn. 20; Shabel, 2006, 109). Both kinds of construction are carried out in an *a priori* manner. We may, for example, draw on paper an empirical figure corresponding to the geometrical concept of a triangle. However, this construction is carried out in a completely *a priori* manner, because the figure is not drawn according to a pattern borrowed from experience, but according to the content of the corresponding mathematical concept. Therefore, even in the case of empirical construction, our imagination does not have to consider primarily the specific features of the empirical manifold, but the content of the *a priori* concept (see A713-4/B741-2).

of the formation of the concept of a tree. I see a spruce, a willow and a linden. “By first *comparing* these objects with one another I note that they are different from one another in regard to the trunk, the branches, the leaves, etc.; but next I *reflect* only on that which they have in common among themselves, the trunk, the branches, and the leaves themselves, and I *abstract* from the quantity, the figure, etc., of these; thus I acquire a concept of a tree.” (*Logic Jäsche*, AA, 9:94-5).³⁷ As B. Longuenesse suggested, Kant’s description of these operations does not intend to establish a chronological order between them. For example, the effort to reflect on what these intuitions have in common presupposes an activity of comparison with regard to their differences and an attempt to abstract from them. Hence, it seems that each one of these operations “depends on the others and all proceed simultaneously” (1993/1998, 116). In addition, I think that the three operations are exercised on intuitive marks given in sensibility, and that, according to Kant, the reflection on what these intuitive marks—despite their differences—have in common renders the relevant intuitive marks discursive or universal ones.

As R. B. Pippin pointed out, Kant’s account of the genesis of the concept of a tree seems to presuppose conceptual activity. For example, the recognition of those three different but associable objects (i.e., the spruce, the willow and the linden) appears to presuppose the application of the concept of a three. Furthermore, the reflection on the branches, the trunk and the leaves that renders these intuitive marks constituent discursive marks of the concept of a tree seems to presuppose the application of the concepts of a branch, a trunk and a leaf (see Pippin, 1979, 10ff.; 1982, 112ff.).³⁸

³⁷ (Translation modified. The emphasis is mine).

³⁸ Similar critiques have been advanced by H. Ginsborg and U. Eco (see Ginsborg, 1997, 53; 2006, 39; Eco, 1997/1999, 88). It is worth mentioning

In this article I will limit myself to summarising a work that offers a plausible and consistent reconstruction of a Kantian theory of the formation of empirical concepts. In this work, C. La Rocca addresses the problem of Kant's analyses of the cognitive processes that give rise to empirical knowledge. According to La Rocca, these analyses can be found especially in the *Reflections* and *Lessons* on logic (see 2004, 270). La Rocca states that the application of categories makes possible to identify and re-identify empirical objects on the basis of their spatiotemporal permanence and mutability. In this way it is possible to have an experience of them, not certainly as empirical objects of a particular kind, but as empirical objects in general. Such an experience involves the capacity to intuit these objects as sets of intuitive marks and, for example, the ability to "cut out" the former from their background (see 276-7, 285-6). However, although the application of the categories makes possible the experience of empirical objects even without the corresponding empirical concepts, it is not a sufficient condition for the formation of these concepts (see 277-8).

The first step in the formation of empirical concepts consists in making pertinent some intuitive marks of an empirical object (for example, the colour and scent of a rose) for the identification of the latter (see 274-7). This operation does not yet need any empirical concept. Such concepts can be formed only by the generalisation of a pertinent intuitive mark or a group of them (see 282). This generalisation presupposes, among others, an activity of comparison of several intuitions and several concepts (see 284). The sets of discursive marks that are generated in this manner have originally a partially arbitrary character (see 278). Despite this arbitrariness, the so-formed empirical

that this problem has led G. Schrader to "regard the suggestion that empirical concepts are abstracted from empirical intuitions as a non-critical doctrine" (1958, 270. See also 266, 273, 277).

concept can already perform its cognitive functions, that is, the designation and classification of empirical objects. But only through a long process of revision consisting in provisional inductions and analogies can the empirical concept deploy its full cognitive force (see 279, 284, 290). The aim of this partially unconscious process is the production of “an always incomplete generality” (292-3). Therefore, the content of empirical concepts is uncompletable.³⁹

It is not necessary to study here in detail La Rocca’s interpretation. For the purpose of this article it suffices to have shown that it is possible to give a consistent Kantian account of the formation of empirical concepts. Now I will briefly consider a feature of empirical concepts, namely, the uncompletable character of their content. This feature is due to the peculiarities of the genesis of these concepts. As we have already seen, the universality of empirical concepts is not true or strict, but assumed and comparative, because it depends on generalisations of what we have yet perceived (see B3-4). Thus, the aggregate of discursive marks that constitutes the content of an empirical concept can never be completed, “but rather resembles a straight line *without limits*” (*Jäsche Logic*, AA, 9:59). An empirical concept cannot be defined, because it is never certain “whether by the word that designates the same object one does not think at one time more, at another time fewer marks of it [...]. One makes use of certain marks only as long as they are sufficient for making distinctions; new observations, however, take some away and add some” (A727-8/B755-6).⁴⁰

³⁹ However, see pp. 279-80. It is noteworthy that also H. Ginsborg offered an interesting reconstruction of a Kantian theory of the formation of empirical concepts. Ginsborg’s account focuses on the notion of the synthesis of imagination as “exemplary” of the rules or empirical concepts which govern it (see Ginsborg, 1997).

⁴⁰ (Translation modified).

Hence, the situation becomes complicated when it comes to schemata of empirical concepts. As we have already shown, the subject may become aware of the whole content of a mathematical concept and then produce a mathematical object in accordance with the former. This production is carried out in an *a priori* manner; that is, our imagination does not have to consider the particular features of the empirical manifold (see fn. 36). In contrast, the content of empirical concepts is always provisional and incomplete. Furthermore, the application of an empirical concept presupposes the interpretation of the specific features of the empirical manifold as an object corresponding to that concept.⁴¹ An empirical object can be given to sensibility in multiple ways. The ordered sequence of acts of synthesis of the empirical manifold varies according to the perspective through which the object is given. Moreover, manifolds that can be subsumed under the same concept can have radically different aspects. Thus, it becomes quite difficult, if not just impossible, for example, to conceive of a universal procedure of the imagination to synthesise, according to the provisional set of marks thought in the concept of a dog, the totality of its corresponding possible empirical manifolds. Perhaps the general form of a procedure of synthesis corresponding to the concept of a dog can be stated as follows: first, look for an animal with a furry coat; second, look for four legs; third, look for four paws; fourth, look for a tail; fifth, look for a head with a snout, a nose, two furry ears and two eyes, etc. It seems clear that this procedure would not work if, for example, someone perceives the head of the dog prior to its legs or the dog has only three legs by virtue of an accident. How could imagination, then, generate

⁴¹ Of course, this is not the case when an empirical concept is applied through the production of an imagined object corresponding to the concept.

a procedure of synthesis that works in all possible cases?⁴² I just do not have an answer to this question.⁴³ One might criticise Kant for dealing in the Schematism chapter with empirical and mathematical concepts as if they were essentially the same kind of concepts.⁴⁴

III. The Schemata of Empirical and Mathematical Concepts as Monograms or General Figures (L2)

In the second section of this article, I addressed the first group of characterisations of the schemata of empirical and mathematical concepts, namely, the characterisations that conceive of the schemata as methods or procedures (L1). In this section, I will study the second group of characterisations (L2). According to it, schemata of empirical and mathematical concepts are monograms or figures delineated in a general manner. I will show that, when schemata are characterised this way, a schema is thought of as a universal image whose aspect resembles that of all the particular cases of a certain concept.

The function of the schemata of mathematical and empirical concepts is to establish a connection between the universality of a concept and the singularity of an intuition. According to L1, schemata mediate between these terms

⁴² Perhaps the procedure-schema of an empirical concept should be thought of as what may be called an *open procedure*. The order of the steps of this procedure would depend on the order in which intuitive marks are given to sensibility. However, this kind of procedure would not be in line with L1. According to L1, as we have seen, schemata are *universal ordered ways* of performing actions of synthesis. In addition, there is no textual support in Kant's texts for the interpretation of schemata as open procedures.

⁴³ It is even more difficult to conceive of universal procedures to apply higher concepts such as "intelligent", "friendly" (see Bayne, 2004, 11-2) "evolution" and "entropy" (see Warnock, 1949, 81-2).

⁴⁴ The necessity of distinguishing between the schemata of empirical concepts and the schemata of mathematical concepts was pointed out by A. Ferrarin (see 1995a, 148-9).

inasmuch as they are universal procedures to constitute singular intuitions as cases of their corresponding universal concepts. When interpreted as general figures or monograms (L2), schemata also mediate between those terms. However, in this case, schemata are not procedures, but representations. This does not mean that, according to L2, a schema is a type of representation different from both concepts and intuitions but, as I will show, an intuitive, universal characteristic that is produced in conformity with the content of a concept. I quote now the key passages that support my claim:

The concept of a dog signifies a rule in accordance with which my imagination can delineate the figure of a four-footed animal in a general manner, without being restricted to any single particular figure that experience offers me or any possible image that I can exhibit *in concreto*. This schematism of our understanding with regard to appearances and their mere form is a hidden art in the depths of the human soul [...]. We can say only this much: the *image* is a product of the empirical faculty of productive imagination, the *schema* of sensible concepts (such as of figures in space) is a product and as it were a monogram of pure *a priori* imagination, through which and in accordance with which the images first become possible, but which must be connected with the concept, to which they are in them-selves never fully congruent, always only by means of the schema that they designate (A141-2/B180-1).⁴⁵

In the previous section I have argued that the concept of image refers to an intuitive, singular content corresponding to a concept. In the passage I have just quoted, Kant claims

⁴⁵ (Translation modified).

that the concept of a dog signifies a rule according to which our imagination can trace, in a universal manner, the figure of a four-legged animal, without being restricted to any particular image produced by imagination or found in actual experience. According to this claim, the empirical concept of a dog is objectively meaningful insofar as imagination can delineate, in agreement with the conceptual rule, a figure (*Gestalt*) that intuitively expresses this rule in its universality. The sentence that immediately follows the passage starts this way: “This schematism of our understanding”. When examining transcendental schemata, Kant defines the “*schematism* of the pure understanding” as “the procedure of the understanding with these schemata” (A140/B179). Thus, the term “schematism” refers to the procedure of the understanding with the schemata. In the case of the schemata of mathematical and empirical concepts, the procedure with the schemata refers both to the production of schemata and the production of singular images through schemata.⁴⁶ On the one hand, then, Kant claims that the figure traced in a universal manner is not a singular image; on the other hand, he describes the action of tracing that figure as a “schematism.” This suggests that the universal figure is the schema of the concept of a dog. A concept would then be a rule according to which imagination traces or delineates a schema, whereas a schema would make singular images possible. It is worth noting that both L1 and L2 conceive of the schemata and the images as products of the imagination.

In the passage quoted above, Kant refers to “sensible concepts”. These concepts are the pure sensible concepts, that is, mathematical concepts. Kant claims, then, that the schema of a mathematical concept is a product and a mono-

⁴⁶ Therefore, the term “procedure” (*Verfahren*) does not have in the passage quoted above the meaning that I have explained in the second section. In the passage quoted above, “procedure” refers to that which in general does the understanding with the schemata: on the one hand, it generates them through imagination; on the other hand, it produces images through them.

gram (*Monogramm*) of the pure *a priori* imagination, through which and in accordance with which images are possible. This characterisation is analogous to the one he uses to explain the schema of the empirical concept of a dog. In fact, a mathematical concept serves as a rule for imagination to trace a monogram or schema. This monogram would be the model according to which images are produced. In A833/B861 Kant writes: “Umriß (*monogramma*)”. A monogram would then be an outline. And in A570/B598 he relates a monogram to a silhouette (*Schattenbild*) and to a sketch (*Zeichnung*) that, more than constituting a determinate image, hovers in the middle of various experiences. In the same context, Kant alludes to an unattainable model (*Muster*) for possible empirical intuitions.

According to Kant’s example, we can link the concept of a dog to its corresponding intuitions only by imagining, in accordance with the content of the concept, something like the figure of a dog in general. Such a figure would be the schema of the concept of a dog, and it would consist in the intuitive exhibition of the concept in its universality (see Pippin, 1976, 168; 1982, 147-8). If this interpretation were correct, then a schema would not be—as in L1—the procedure performed by imagination each time a conceptual rule is applied, but a product of an original application of the conceptual rule. This application is original inasmuch as it does not generate a singular image, but a schema according to which singular images are produced. The procedure-schema is an ordered way of performing an action of synthesis that is common to all applications of a conceptual rule. L1 conceives of the schema as making possible an infinite number of equivalent applications of the rule. In this line of argumentation, each application of the rule produces a singular image. By contrast, according to L2, the application of the rule is original, because its product is a

schema or monogram that exhibits the content of the concept in its universality. Singular images can only be produced through and in conformity with this monogram or product-schema.⁴⁷

It would seem that Kant's thought is that, if we are to subsume intuitions under a concept, we should possess something like a universal image that corresponds to the concept. In this way of conceiving of schematism, the German term "Schema" is understood as a model and a sketch. The schema is presented as a model because it is an intuitive characteristic through which singular images are produced. The schema is also presented as a sketch because the intuitive characteristic must be indeterminate enough as to encompass the totality of cases covered by the concept. This is in line with Kant's distinction between a monogram and a determinate image.⁴⁸

⁴⁷ According to G. Seel, the conception of the schema as a procedure (that is, L1) is only valid for mathematical concepts, whereas the conception of the schema as a monogram (i.e., L2) is only valid for empirical concepts (see 1998, 232-5). However, there are two reasons why this interpretation is not very plausible. First, after defining the schema as a procedure, Kant describes in a similar manner the relations between mathematical and empirical concepts and their schemata (see A141/B180, AA, 3:136:6-13). Second, as we have seen, the characterisation of the schema of the empirical concept of a dog as a general figure is analogous to the characterisation of schemata of mathematical concepts as monograms.

⁴⁸ G. Seel argues that Kant's description of the schema of the concept of a dog can be interpreted in two ways. According to the first interpretation, imagination delineates the figure of a dog in general. Such a figure is the schema of the concept of a dog and must be understood as an outline and a pattern. According to the second interpretation, imagination can always trace various concrete and particular figures of dogs, without being restricted to any determinate figure (1998, 234). There are two reasons, in my view, why the first interpretation is the correct one. First, Kant's characterisation of the schema as a monogram is in agreement with the first interpretation mentioned by Seel. Second, Kant states that imagination "delineate[s] the figure of a four-footed animal in a general manner". The fact that the schema of the concept of a dog depicts "a *four-footed animal* in a *general manner*" indicates that the former is a general and indeterminate figure (the emphasis is mine. See below).

The intuitive characteristic that constitutes the schema would be present in all singular images whose production it makes possible. As I have already stressed, the schema is a mediating representation between the universality of the concept and the singularity of intuition. Actually, it is an intuitive characteristic that expresses the concept in its universality and is included in the aspect of every singular image.⁴⁹

I have previously called the schema a “universal image”. The image is universal because it consists in an intuitive exhibition of the concept in its universality. The schema is not, then, a singular and determinate image that, as such, would not express the universality of the concept. I stress here that, in the expression “universal image”, the term “image” has a narrower meaning than it has in Kant. Both “Gestalt” and “Monogramm” have a strongly visual connotation. According to this way of interpreting schematism, there would be innumerable empirical concepts—those that cannot be expressed in figures—without any corresponding schema. Examples of such concepts are those of “scent of a rose” and “bark of a dog” (see Noske, 2001, 325-6).⁵⁰ L1 is free from this problem. According to L1, a schema is a universal procedure of the imagination to synthesise empirical manifolds, independent of the sense organ where they originate.⁵¹

⁴⁹ For a fuller development of the interpretation of the schema as an undetermined sketch, see Düsing (1995, 50-6).

⁵⁰ Although according to L2 the schema has a strongly visual connotation, it is worth mentioning that one can also recognise a figure, for example, with one’s hands (see *Anthropology from a Pragmatic Point of View*, 7:154-5). Thus, L2 is valid for concepts whose objects can be perceived through both senses of sight and touch (see Noske, 2001, 326).

⁵¹ In the above-quoted passage from A120 Kant claims that an image (*Bild*) is formed through the apprehension of the “manifold of intuition” (*Mannigfaltige der Anschauung*), “perceptions” (*Wahrnehmungen*) or “impressions” (*Eindrücke*). Thus, it may be said that images are generated through the synthesis of the manifold of sensible intuitions which do not

L2 faces a second major difficulty. As it is well known, Kant conceives of representative concepts as rules of synthesis whose application constitutes sensible manifolds as objects. However, L2 may suggest that the application of concepts should not be thought of as constitutive of objects.⁵² In other words, the claim that the schema is a universal figure suggests that an intuition is subsumed under a concept when the aspect of the intuition coincides with that of the figure that, as a schema, corresponds to that concept. In this case, there would not be a unifying constitution of the sensible manifold, but the mere act of comparing the aspect of the intuition with that of the schema. Therefore, it is possible to interpret that L2 is not entirely in line with the very core of Kant's theory of representative concepts, namely, the *constituting* character of their application.

However, Kant claims that the product-schema makes the singular images possible. Should not this thesis oblige us to try to reconcile the two lines of argumentation we have examined? In fact, the hypothesis that the product-schema is the model through which the procedure-schema synthesises sensible manifolds could reconcile L1 and L2. Nonetheless, this hypothesis must be rejected because it seems unnecessary and superfluous to introduce a principle of synthesis that is different from what is thought in the concept.⁵³

I have explained above that the schemata of empirical and mathematical concepts as universal figures or monograms can be understood as indeterminate images. A third difficulty I want to mention before concluding this section is that it seems impossible for us to form so indeterminate images as L2 requires. Of course, one may claim that we can form

necessarily belong to the sense of sight. See A525-6/B553-4 and *On a Discovery*, AA (8:205).

⁵² See fn. 19.

⁵³ Neither is it possible to hold that the procedure-schema generates the product-schema, because the former produces singular images and the latter is a universal figure.

images that are indeterminate only to a certain extent, such as the image of a human being in which the colour of his or her eyes is not represented. However, it seems clear that we cannot form images that are indeterminate enough as to encompass, through a common intuitive characteristic, all the intuitions corresponding to a concept. We cannot imagine, for instance, a “man in general” without imagining it as being young or old. We cannot either imagine a “triangle in general” that is not equilateral, isosceles, or scalene.⁵⁴

That it is not possible for us to conceive of images that represent all the instances of a concept becomes evident in Kant’s own example. It seems that Kant, trying to remain faithful to the idea that schemata must be to a great extent indeterminate, holds that the schema of the concept of a dog is the figure of a four-legged animal. However, this schema of the concept of a dog is so indeterminate that it could also work as the schema of the concepts of a cat or a horse. This schema could not be a criterion for deciding on the subsumption of a given intuition under the concept of a dog. On the other hand, our representation of an animal necessarily entails the representation of qualities other than that of being a four-legged animal.⁵⁵

⁵⁴ Although I cannot develop this point here, it is important to note that this objection is inspired by George Berkeley’s criticism of the doctrine of abstract and general ideas (see 1710¹, 1734/1949; 27-30). In effect, Berkeley seems to hold that images as such are perfectly determinate in all respects. Consequently, he seems to reject abstract general ideas because these ideas, were they in existence, should be indeterminate images (see Pitcher, 1977, 69ff; Baxter, 1997, 307-9). It is true that Berkeley’s criticism can be avoided by adopting the so-called theory of selective attention. According to this theory, to have a general idea only means to pay selective attention to the features of a singular idea in which it resembles others (cf. Berkeley, 1710¹, 1734/1949; 31ff.). However, it seems that the aforementioned theory cannot be attributed to Kant. In fact, Kant, on the one side, makes reference to the action of tracing a figure *in a general manner* and, on the other, points out that the monogram makes the singular images possible.

⁵⁵ R. E. Butts mentions the example of the schema of the concept of a dog given by Kant and remarks that “[u]nless the notion is developed with great

It would seem, then, that it is impossible for us to form images that are indeterminate to the extent that L2 requires.

IV. Conclusion

In Kant's texts, there is textual support for two irreconcilable interpretations of the schemata of mathematical and empirical concepts. According to L1, schemata of mathematical and empirical concepts are procedures to constitute objects of these concepts. This constitution is a synthesis of particular, sensible manifolds. To each kind of concept—empirical, arithmetical, and geometrical—corresponds a particular mode of synthesis. This line of argumentation is in agreement with Kant's characterisation of concepts as rules of synthesis. In fact, in this case a schema is the procedure required by a concept that serves as a rule. The content of a concept, i.e., that which is thought in it, requires a certain and determinate synthesis of the sensible manifold, and our imagination responds to this exigency by generating a universal procedure of synthesis (schema). Nonetheless, we have seen that it is quite difficult to conceive of those universal procedures in the case of empirical concepts.

According to L2, the schemata of empirical and mathematical concepts are universal images or figures. These universal images are generated by imagination in accordance with the content of concepts. Such images can be understood as intuitive characteristics that express the content of concepts in their universality and whose aspect resembles that of all the particular cases of the concepts in question. These particular cases are produced through and in

care (a task that Kant did not himself undertake to carry out) we can become quickly lost in the notorious problem of the general image that represents all individuals of a certain type" (Butts, 1969, 296).

conformity with the universal images that, as schemata, correspond to their universal concepts. However, considered in isolation, this way of interpreting the schemata may give support to the idea that the application of concepts does not constitute objectivity. In this case, a schema would work as a model to identify particular cases of the concept on the basis of a comparison between their aspect and that of the model. Furthermore, if it is true that a schema of this kind represents all the particular cases of a concept, then this schema should be thought of as a radically indeterminate image. So conceived, the notion of a schema would be vulnerable to an objection inspired by Berkeley's criticism of abstract ideas, a criticism that seems successful when ideas are understood as strongly indeterminate images.

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