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# Estimation of notch sensitivity and size effect on fatigue resistance

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**Abstract**

This paper addresses the problem of high cycle fatigue resistance associated to notches and the role of short crack propagation in the fatigue notch sensitivity quantified by the notch factor $k_f$. An integrated fracture mechanics approach is proposed to estimate the fatigue notch sensitivity, by including the effect of both blunt and sharp notches.
Whether fatigue strength at a given life is controlled by crack initiation (very blunt notches, $k_f = k_t$), by microstructuraly short cracks (blunt notches, $k_f < k_t$), or by mechanically short crack propagation (sharp notches, $k_f << k_t$), depends on the stress concentration $k_t$, the notch length D and the material threshold to crack initiation $\Delta\sigma_{eR}$, to short crack propagation $\Delta K_{th}$ and to long crack propagation $\Delta K_{thR}$.
The approach includes the prediction of the fatigue crack propagation threshold for short cracks, previously developed to analyze the short crack behavior in metallic materials with or without blunt notches, and is integrated adding the influence of sharp notches and accounting for the controlling parameters. It estimates the fatigue resistance of the component by comparing the threshold for fatigue crack propagation as a function of crack length, $\Delta K_{th}$, with the applied $\Delta K$ for the given configuration. Estimations for results reported in published bibliography are presented.
The proposed fracture mechanics approach allows accounting for the effects of notch acuity, notch size and intrinsic material fatigue properties on fatigue notch sensitivity. It opens the door to a new simple method for predicting fatigue notch sensitivity and fatigue strength of components with geometric concentrators by using parameters that can be easily measured or estimated, without the necessity of any fitting parameter.



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## 1. Introduction

When components containing notches or holes of a given elastic stress concentration factor $k_t$ are tested under cyclic loading and their fatigue strengths measured, two observations are generally made [1-7]. One is that the fatigue notch factor, $k_f$, which is given as the ratio of the un-notched specimen fatigue strength to that of the notched specimen fatigue strength, is generally less that $k_t$. This phenomenon has been called fatigue notch sensitivity. The other observation is that the smaller the size of the notch or hole the smaller is the value of $k_f$. This phenomenon has been associated to the notch size effect.

The fatigue notch factor $k_f$ depends on notch geometry and material and many interpretations of the $k_f < k_t$ effect has been given. The most popular and traditional interpretation of the $k_f < k_t$ effect is based on the concept of critical distance or process zone volume [8-10]. The material is not sensitive to the Peak stress on the surface but rather to an average stress over some finite volume of material that would be involved for the fatigue damage process to develop. A collection of the most commonly used methods are presented in Ref. [11]. They usually include material parameters to account for the notch sensitivity, but no physical stress at a particular point, located at a material-constant distance from the notch meaning is given for them. Besides, all formulas show considerable scatter when applied to different materials. The following formulae proposed by Peterson [12] is an example:

$$k_f = 1 + \frac{k_t - 1}{1 + \frac{A}{\rho}} \tag{1}$$

where ρ is the notch root and $A$ is a material parameter. The Peterson´s approaches [12] give the simplest formulas and they use the local stress at a particular point, located at a material-constant distance from the notch root. The critical distance concept is motivated by experimental observations on the notch radius influence; the smaller the notch radius, the larger the stress gradient and the smaller the average stress or the local stresses at a given distance from the notch root.

The major failing of the above historical approach is that failure is associated with bulk stress or strain parameters and not to the behavior of a fatigue crack and its local associated stress field. Such bulk parameters do not take into account the various regimes of fatigue process.
More recently, some fracture mechanics based approaches have been proposed by using the same concept, as the critical distance/line/volume methods proposed by Taylor [16]. For instance, in the case of the critical distance method, the material parameter is estimated from the plain fatigue limit (defined, for instance, as the endurance limit for $10^7$ cycles), and the threshold for fatigue propagation for long cracks. However, this concept has limitations to explain the effect of specimen size, even though it is said that it can also be accounted for by the same argument [17]. Besides, they do not take into account explicitly the physical mechanisms of crack initiation and short crack propagation.

Classical LEFM analyses of crack propagation conditions [18-22] have pointed out that the presence of cracks can be a major cause of the $k_f < k_t$ effect. An important advance in understanding why $k_f$ is less than $k_t$ and quantifying the fatigue notch sensitivity was made by El Haddad et al. [4], Tanaka et al [2,21], McEvily and Minakawa [20], Taylor et al [17], Chapetti et al [23-24], among others [3,18] based upon the growth behavior of small fatigue cracks and by using different models.

In the present paper we deal with this matter again, using an integrated fracture mechanics approach and accounting for both the fatigue notch sensitivity and the notch size effect. The main purpose of this paper is to propose a simple approach for dealing with the fatigue notch sensitivity and the notch size effect, based on fundamental understanding of the factors responsible for these phenomena.

## 2. Threshold conditions and non-propagating cracks

As shown by a Frost diagram (Fig.1, diagram that shows fatigue limit as a function of stress concentration factor $k_t$) [25], there is a critical $k_t$ beyond which fatigue limit is controlled by crack propagation [15] (sharp notch regime). The fatigue notch factor $k_f$ is lower than $k_t$ because the crack propagation stress (90 MPa; full symbols in Fig.1) becomes higher than the crack initiation stress (260 MPa/$k_t$; open symbols in Fig.1, where 260 MPa is the plain fatigue limit for the material analyzed in [25]). The Frost diagram shows a region of non-propagating cracks for sharp notches. These cracks have initiated but the driving force is not high enough to make them to propagate till fracture. The crack propagation threshold condition can be then estimated from the long crack propagation threshold $\Delta K_{th}$.

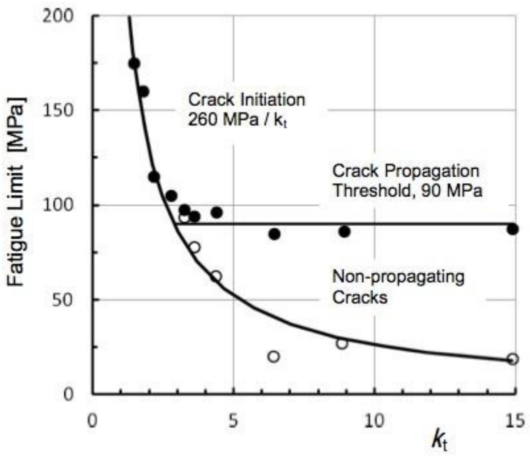


**Fig.1** The Frost diagram of fatigue limit versus stress concentration factor $k_t$. Mild steel specimens having V-notches 1.3 mm deep and root radii varying from 0.005 mm to 0.3 mm [25].

However, it is well known now that even in the blunt notch region the fatigue limit could be given by a threshold condition associated to microstructurally-short non-propagating cracks [1,19,23], and this matter is analyzed in detail in next sections. Here we can then summarize that in "sharp" notches (high stress concentration factor, $k_t$), mechanically-short non-propagating cracks exist at the fatigue limit of the notched component, whereas "blunt" notches (small $k_t$), exhibit microstructurally-short non-propagating cracks. In the case of blunt notches the driving force that is sufficient to initiate a crack at the notch root and overcome the strongest microstrucural barrier, is also sufficient to cause continuous propagation of the crack to failure and the fatigue strength is given by a microstructural threshold determined by a $\Delta\sigma$ criterion. On the other hand, in the case of sharp notches the fatigue strength is given by a mechanical threshold defined by a $\Delta K$ criterion (applied stress intensity factor range), and the development of mechanical non-propagating cracks is allowed by the existence of a stress gradient high-enough and the development of the crack closure effect. In this case the fatigue strength becomes independent of the stress concentration factor $k_t$ and is governed mainly by the notch depth $D$ and the fatigue threshold $\Delta\sigma_{th}$ for physically small or long cracks (the stress range associated to the threshold for fatigue crack propagation as a function of crack length, see Figure 2.a. below) [4,5,20]. So there is a clear transition in the mechanism that defines the fatigue limit of the notch and the associated controlled parameters.

This transition is not accounted for by using the traditional approaches mentioned in the previous section. Those models need material parameters that should be experimentally obtained for a given material and are not independent of the notch geometry. They can only explain the notch sensitivity for sharp notches for a given notch depth $D$. If the geometrical parameter $D$ changes, the material parameter of the model should be also changed. It is clear that those models have empirical bases and that they cannot predict fatigue resistances properly.

### 3. The fatigue blunt notch sensitivity

In a previous work [23,24], a model for the notch size effect on the basis of the experimental evidence that both, the plain- and the blunt-notched fatigue limit represents the threshold stress for the propagation of the nucleated short cracks, was derived. The derived relationship characterizes the fatigue notch sensitivity by means of the parameter $k_{td}$ defined as the stress concentration introduced by the notch at a distance $d$ from the notch root surface equal to the distance between microstructural barriers (e.g. grain boundaries), as follows:

$$k_{td} = \frac{k_t}{\sqrt{1+\frac{4.5\,d}{\rho}}} \tag{2}$$

where ρ is the notch radius.

Defining $d_i$ as the mean distance between microstructural barriers $i$, and $\Delta\sigma_{edi}$ as the fatigue limit associated to a given barrier $i$, the fatigue limit $\Delta\sigma_e$ of the notched component at a given $k_t$ would be given by the greatest $\Delta\sigma_{edi}$ at that $k_t$, as follows:

$$\Delta\sigma_e\Big|_{k_t} = \max\ \Delta\sigma_{ed_i}\Big|_{k_t} = \max\left[\frac{\Delta\sigma_{eRd_i}\sqrt{1+4.5\frac{d}{\rho}}}{k_t}\right]_{k_t} \tag{3}$$

and

$$\Delta\sigma_{ed_i} = \frac{\Delta\sigma_{eRd_i}}{k_{td_i}} \tag{4}$$

where $\Delta\sigma_{eRdi}$ is the effective resistance of the barrier $i$ and $k_{tdi}$ is the stress concentration introduced by the notch at a depth $x = d_i$, for the stress ratio $R$. The concept is shown schematically in Fig 2(b) by considering three consecutive microstructural barriers placed at distances $d_1$, $d_2$ and $d_3$ from the surface ($d_1 < d_2 < d_3$), with their effective resistance $\Delta\sigma_{eRd1}$, $\Delta\sigma_{eRd2}$ and $\Delta\sigma_{eRd3}$, respectively, as we can see in Fig 2(a). From $k_t = 1$ to $k_{t1}$ the fatigue limit of the notch component is given by $\Delta\sigma_e = \Delta\sigma_{eRd1}/k_{td1}$, from $k_{t1}$ to $k_{t2}$ by $\Delta\sigma_e = \Delta\sigma_{eRd2}/k_{td2}$, and so on.

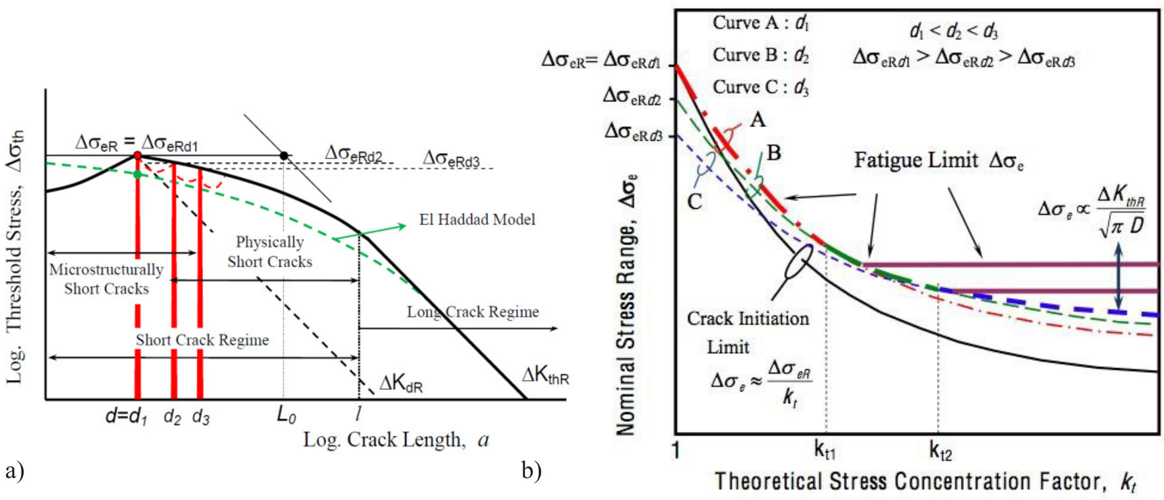


**Fig. 2** **(a)** Kitagawa-Takahashi-type diagram showing the threshold between propagation and non-propagating cracks; after [26]. **(b)** The fatigue limit $\Delta\sigma_e$ of blunt notches defined as the greatest fatigue limit associated with the effective resistance $\Delta\sigma_{eRdi}$ and the position from the notch-root surface $d_i$ of the microstructural barriers $i$, see Eq. (3) [23]. El Haddad model is also schematically shown in (a) [3].

In references [23,30] this concept was used to analyze the influence of the position and the effective resistance of the microstructural barriers on the fatigue notch sensitivity of several steels. It was shown experimentally that the plain fatigue resistance is mainly given by the first (and the strongest) microstructural barrier. As the stress concentration effect increases, deeper barriers with lower effective resistance start to define the fatigue resistance.

The simplest assumption is to consider $d_1$ and $\Delta\sigma_{eR}$ as its resistance, and $k_f$ equal to the $k_{td}$ associated to this strongest barrier, which is conservative. The smaller is the value of $d$, the closer is the fatigue limit to that given by $\Delta\sigma_{eR}/k_t$ for blunt notches.

The Kitagawa diagram schematized in Fig. 2(a) also shows the point stress model concept applied to the results (black point), in which the intrinsic $L_0$ crack length is obtained with the plain fatigue limit and the threshold for long cracks. It is easy to understand that in this approach the non-propagating crack length associated to the fatigue limit of any blunt notch (and so for any $k_t$) is constant and equal to $L_0$. This result is a consequence of that the threshold condition is given by the threshold for long crack, $\Delta K_{thR}$ (a constant value for a given stress ratio $R$), for any configuration. As a result, the fatigue limit is associated to the same intrinsic crack length for any $k_t$ value for blunt notches.

It can then be addressed that the point stress analysis gives accepted results but it cannot catch the real mechanism associated to the definition of the fatigue limit. Besides, it is easy to see and demonstrate that it gives underestimations, in some cases significant.

## 4. A fracture mechanics approach for thresholds estimation

Taking in consideration all the introduced concepts, in the case of blunt notches, for which the short crack behavior is important, we can use a fracture mechanics approach proposed previously to estimate a continuous crack propagation threshold curve integrating the short and long crack regime [26], starting from a depth given by the first and strongest microstructural barier (associated to the plain fatigue limit, or the microstructural threshold for fatigue crack propagation as was defined by Miller [1], see Figure 2a). In that study an expression to estimate the threshold for fatigue crack propagation as a function of crack length was obtained by using only the plain fatigue limit, $\Delta\sigma_{eR}$, the threshold for long crack, $\Delta K_{thR}$, and the microstructural characteristics dimension, $d$ (e.g., grain size). The expression was defined from a depth given by the position d of the strongest microstructural barrier that defines the smooth fatigue limit (e.g. first grain boundary). A microstructural threshold for crack propagation, $\Delta K_{dR}$, is defined by the plain fatigue limit $\Delta\sigma_{eR}$ and the position $d$ of the strongest microstructural barrier (see Figure 2a).

A total extrinsic threshold to crack propagation, $\Delta K_{CR}$, is then defined by the difference between the crack propagation threshold for long cracks, $\Delta K_{thR}$, and the microstructural threshold, $\Delta K_{dR}$. The development of the extrinsic component is considered to be exponential and a development parameter $k$ is estimated as a function of the same microstructural and mechanical parameter used to define the material threshold for crack propagation. The material threshold for crack propagation as a function of the crack length, $\Delta K_{th}$, is then defined as [26]:

$$\Delta K_{th} = \Delta K_{dR} + \left( \Delta K_{thR} - \Delta K_{dR} \right) \left[ 1 - e^{-k(a-d)} \right] = Y \, \Delta\sigma_{th} \sqrt{\pi \, a} \qquad a \geq d \tag{5}$$

where $\Delta K_{dR}$ and $k$ are given by:

$$\Delta K_{dR} = Y \, \Delta\sigma_{eR} \sqrt{\pi \, d} \tag{6}$$

$$k = \frac{1}{4d} \left( \frac{\Delta K_{dR}}{\Delta K_{thR} - \Delta K_{dR}} \right) \tag{7}$$

Figure 3 shows schematically the definition of the threshold curve given by expression (5) in terms of the stress intensity factor threshold range $\Delta K_{th}$, respectively, as a function of crack length. For a crack length $a = d$, $\Delta K_{th} = \Delta K_{dR}$, and $\Delta K_{th}$ tends to $\Delta K_{thR}$ for long cracks. The expression allows a definition of a crack initiation period as the number of load cycles necessary to initiate a crack of depth *d* (micro-crack initiation), from which the crack propagation behavior can be analyzed ($a \geq d$). This applies to materials free of cracks or crack-like defects. In the case of a component with defects the crack initiation period is usually minimized, and the initial crack length for the crack propagation period will be given by the maximum crack-like defect.

Once the threshold as a function of crack length is known, the fatigue crack propagation behavior or the threshold condition associated to a given component can be estimated by accounting for the difference between the total applied driving force, defined by the applied stress intensity factor range for the given geometrical and loading configuration ($\Delta K$), and the threshold for crack propagation ($\Delta K_{th}$), as follows:

$$\frac{da}{dN} = C\left(\Delta K - \Delta K_{th}\right)^{m} \tag{8}$$

where *C* and *m* are material parameters [6,7] and the threshold for crack propagation defines the effective driving force applied to the crack. Both parameters, $\Delta K$ and $\Delta K_{th}$, should be known as a function of crack length.

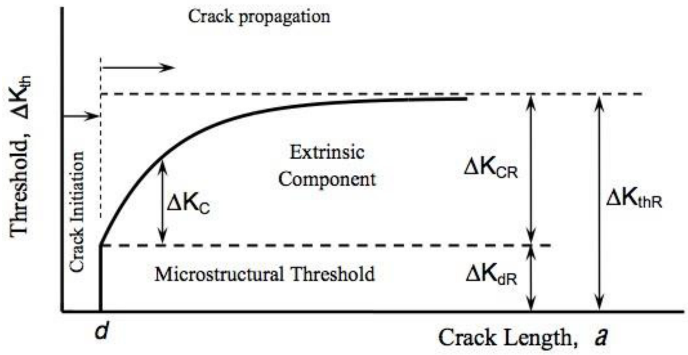


**Fig. 3** Defined fatigue crack propagation threshold as a function of crack length, given by expression (5).

The fatigue limit for a given component configuration and the associated non-propagation crack can then be easily estimated without knowing the fatigue crack propagation data (*C* and *m*). Figure 4 shows schematically the resistance curve concept that use the threshold for fatigue crack propagation $\Delta K_{th}$ and the applied driving force $\Delta K$ (both as a function of crack length). The value of $\Delta K$ should be greater than $\Delta K_{th}$ for any crack length (between the initial crack length $a_i$ and the final crack length $a_f$). The fatigue limit $\Delta\sigma_e$ will be given by the nominal stress $\Delta\sigma$ for which the applied $\Delta K$ becomes equal to $\Delta K_{th}$ for a given crack length (see Figure 4). Then the tangent condition defines the associated non-propagating crack length associated to the component configuration. The maximum defect size allowable for that configuration can also be known, so that the approach can also analyze the defect size sensitivity of the analyzed configuration.

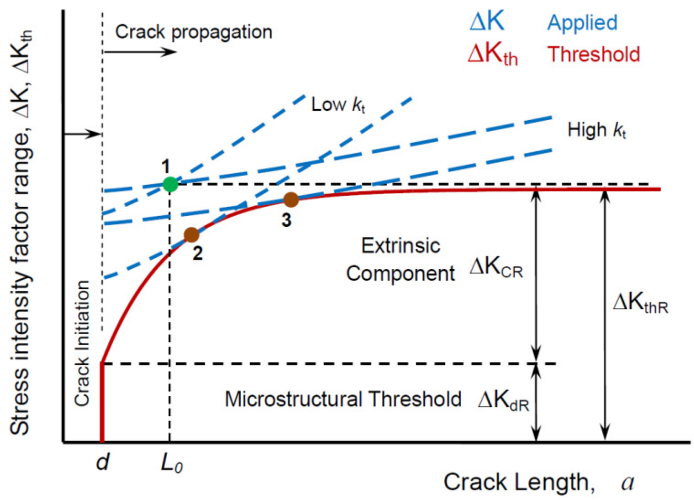


**Fig. 4** Schematical explanation of the condition for the threshold associated to the fatigue limit of a given joint configuration, in terms of stress intensity factor range.

In Figure 4 the differences with the point stress approach can also be clearly understood. In this case the threshold condition is given for any configuration by the threshold for long crack, $\Delta K_{thR}$ (a constant value for a given stress ratio *R*). Besides, the fatigue limit so defined is associated to the intrinsic crack length for any $k_t$ value for blunt notches. This is the base of the point stress model, for which the point 1 in Figure 4 (a kind of fixed barrier that should be overcome by the applied driving force) gives all the configurations that define the fatigue limit as a function of $k_t$ for blunt notches. See also the green point in Figure 2(a). Applications of the approach can be found in references [23-30].

By using the threshold curve approach, the threshold condition depends on the relative position of the applied driving force ΔK and the threshold for fatigue crack propagation, both as a function of crack length. As a result, the non-propagating crack associated to the fatigue limit (a threshold condition for fatigue crack propagation) depends on both curves (ΔK and $\Delta K_{th}$). In this case, if the stress concentration factor increases, the threshold condition moves from point 2 to point 3, as it is shown schematically in Figure 4.

Here a further step should be done, in order to account for the transition between blunt and short notches and the introduction of the notch size effect in the definition of the fatigue resistance.

## 5. The proponed Fracture Mechanics Approach for notches

In the case of blunt notches, and considering the concepts introduced in section 3, the following expression to estimate the crack driving force in the case of a blunt notch, $\Delta K_{BN}$, can be used to account for a short crack in a notch stress field [6,7]:

$$\Delta K_{BN} = Y\left(k_{td}\,\Delta\sigma_n\right)\sqrt{\pi a} = \frac{k_t}{\sqrt{1+4.5\dfrac{a}{\rho}}}\,Y\,\Delta\sigma_n\sqrt{\pi a} \tag{9}$$

For the case of sharp notches, for which the total effective crack length (*a*+*D*) should be considered, the following expression can be used [6,7]:

$$\Delta K_{SN} = Y\Delta\sigma_n\sqrt{\pi\left(a+D\right)} \tag{10}$$

The actual applied ΔK should transit gradually from expression (9) to expression (10), as it is shown schematically in Figure 5.

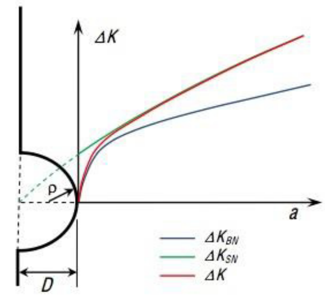


**Fig. 5** Schematical explanation for the transition of the actual crack driving force *ΔK* from expression (9) to expression (10).

In order to get a unique expression for the crack driving force ΔK for any $k_t$, we propose the following:

$$\Delta K = \Delta K_{BN} + \left(\Delta K_{SN} - \Delta K_{BN}\right)\left[1 - e^{-g(a-d)}\right] \tag{11}$$

Where $\Delta K_{BN}$ and $\Delta K_{SN}$ are given by expressions (9) and (10), respectively.

According to the proposed expression (11), the transition is given by the function *g*, estimated as:

$$g = \frac{f}{\sqrt{D\rho}} \tag{12}$$

where *D* is the notch depth, ρ is the notch radius, and *f* is a numerical parameter. This expression was chosen after making a deep analysis of the different combinations of the main parameters controlling the transition (*D* and ρ). Figure 6 shows some results of that analysis for the chosen expression and for a given hypothetical material.

Besides, there are previous indications of the expression $(D\rho)^{1/2}$ as the one controlling the transitions [6,7]. However, the physical mechanism of the transition in the definition of the fatigue resistance could not be clarified.

In order to obtain a model to carry out actual estimations, the parameter *f* should not depend of the material or the notch geometry. In order to check this matter, a parametric analysis has been done using several *f* values. Results have shown that this parameter can be fixed properly as equal to 2, for which very good transition for all studied cases could be obtained. Figure 7 shows an example of the analysis carried out to define the parameter *f*.

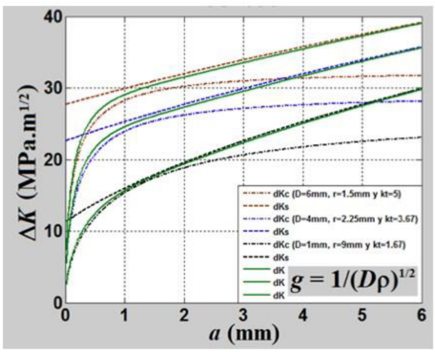


**Fig. 6** Some results of the analysis of the different combinations of the main parameters controlling the transition (*D* and ρ, through expression 12).

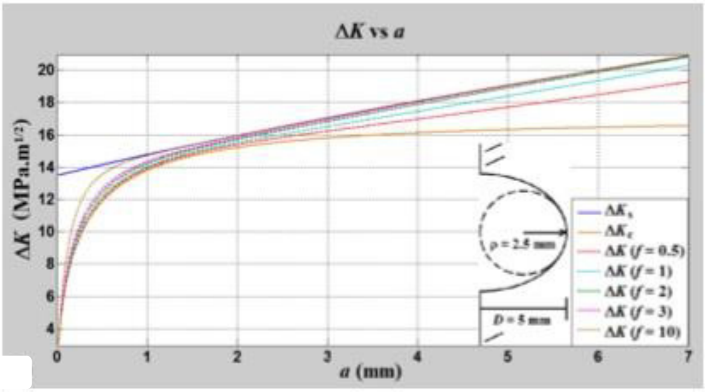


**Fig. 7** Some results of the analysis carried out to define the parameter *f* of expression (12).

The fatigue threshold condition that defines the fatigue resistance of the configuration for any notch geometry and sizes can then be estimated comparing the applied ΔK given by expression (11) and the threshold for fatigue crack propagation as a function of crack length estimated by using expression (5).

### 6. Applications

In order to show the ability of the approach to estimate the fatigue notch sensitivity and size effect in fatigue, two sets of experimental results taken from references [31] and [32] are analyzed. Table 1 shows data used for estimations. Experimental results and estimations made by using the proposed model are shown in Figures 8 and 9, respectively. Full circles indicate fracture, hollow circles indicate no fracture and without non-propagating cracks, and crossings indicate no fracture with non-propagating cracks in specimens. In all figures three estimations curves are shown, corresponding to $k_t$, $k_{td}$ (first barrier, at *d*), and $k_f$.

It can be seen very good predictions, not only in the estimation of the level of the fatigue resistance, but also in the estimation of the transition from blunt to sharp notches. It is important to remark that the results are really estimated, since the procedure does not require any fitting parameter. Further applications, analysis and results will be published soon.

**Table 1.** Data for estimations. Experimental results from references [31], and [32] are shown in Figures 8 and 9 together with estimations.

| Ref | Material | R | $\Delta\sigma_{eR}$ [MPa] | $\Delta K_{thR}$ [MPa m$^{1/2}$] | *d* [mm] | *D* [mm] | $\rho$ [mm] | $k_t$ |
|---|---|---|---|---|---|---|---|---|
| [31] | 0.22C Steel | -1 | 395 | 13 | 0.068 | 5.08 | 1.27 | 2.2 |
| | | | | | | | 0.5 | 4.47 |
| | | | | | | | 0.25 | 6.71 |
| | | | | | | | 0.1 | 9.23 |
| [32] | SM41B Steel | -1 | 326 | 12.36 | 0.064 | 3 | 3 | 2.63 |
| | | | | | | | 0.83 | 4.23 |
| | | | | | | | 0.39 | 5.72 |
| | | | | | | | 0.16 | 8.48 |

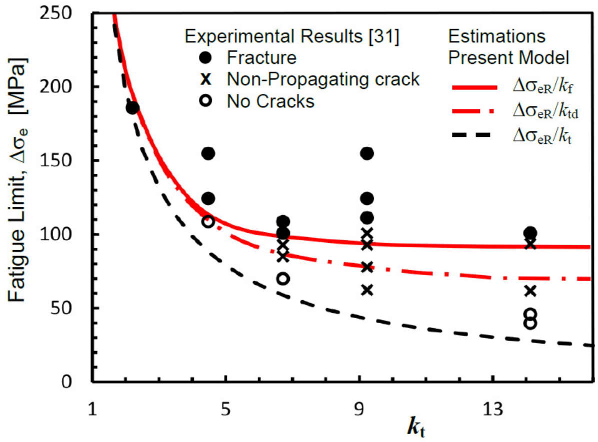


**Fig. 8** Experimental data from [31] for 0.22C steel, and estimations by using the proposed model.

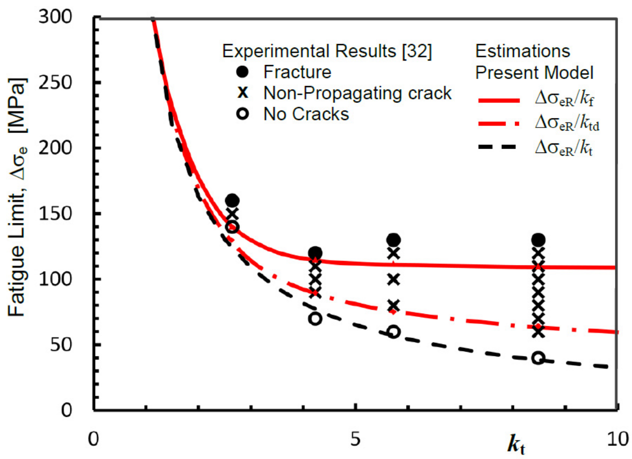


**Fig. 9** Experimental data from [32] for SM41B steel, and estimations by using the proposed model.

## 7. Concluding remarks

In this paper an integrated fracture mechanics approach is proposed to account for both the fatigue notch sensitivity and the notch size effect. The main purpose was to propose a simple approach for dealing with the fatigue notch sensitivity and the notch size effect, based on the fundamental understanding of the factors responsible for these phenomena. The model considers the effects of notch acuity, notch size and material properties to account for the fatigue notch factor $k_f$ for both, blunt and sharp notches. Applications, estimations and results showed very nice agreements with experimental results. The model is simple to apply, accounts for the geometrical mechanical and material parameters that define the fatigue notch sensitivity and notch size effect, and allow acceptable predictions.

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