NUMERICAL SIMULATION OF CONDUCTION-ADVECTION PROBLEMS WITH PHASE CHANGE

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Abstract

This paper introduces a finite element temperature-based model to represent conduction-advection dominated heat transfer when the material is undergoing phase-change. A fixed domain method is applied, so that the energy balance through the interface does not need to be explicitly accounted for in the weak formulation of the problem, obtained either by the standard Galerkin or Streamline Upwind Petrov-Galerkin (SUPG) procedures. The model is applied to analyze a continuous casting process.

1. Introduction

Phase-change plays a prevailing role in several technological applications. Metal solidification is a typical example. During phase-change a considerable amount of (latent) heat is absorbed or released by the material, usually causing a strong non-linearity in the analysis. In a previous work (Fachinotti *et al.*, 1999) we have developed an efficient temperature-based model to deal with severe non-linearity in conduction phase-change problems. The aim in this paper is to extend this methodology in order to include advection effects.

The finite element method (FEM) was used in the formulation. Since the energy balance conditions at the interface become hidden in the weak statement of the problem, we do not need to consider moving boundary conditions and therefore, a fix mesh can be used. One of the features distinguishing our model is the discontinuous integration scheme (Steven, 1982): the integrals over those elements undergoing change of phase are computed as the sum of the contributions from each phase. Moreover, using linearly interpolated triangles or tetrahedra, the interface becomes straight or plane, enabling exact analytical integration. Therefore, no regularization of the latent heat effects is involved and the FEM version of the governing transport equation holds exactly.

While heat transport is essentially diffusive, the standard Galerkin FEM performs satisfactorily well. This is no longer valid for convection-dominated transfer. In such case, Galerkin solutions are not able to represent steep thermal variations downstream, giving raise to node-to-node oscillations. Such behavior is prevented using upwind schemes. In this work, we employ the technique formulated by Brooks and Hughes (1982), known as streamline upwind Petrov-Galerkin (SUPG) method. In this way, an artificial diffusion is added, acting only in the flow direction.

Regarding the severe singularity associated with the interface, other crucial item is the solution of the resulting system of equations. To this end a Newton-Raphson incremental-iterative algorithm is implemented, assuring good convergence properties.

The application of this thermal model offers a first insight into the complex thermo-mechanics of casting processes. The temperature field allows an introductory knowledge of microstructure and shrinkage based on simple criteria (Bellet *et al.*, 1996). At the same time, it constitutes the data needed to update the mechanical temperature-dependent properties of the material during the analysis of stresses and strains.

In the continuous casting process, liquid metal is poured into a water-cooled open mould. After that, the strand arrives at the secondary cooling zone that consists of a set of water sprays. Similar analyses have been described by Boehmer *et al.* (1993) and Funk *et al.* (1993), except that the strand section here is circular. Actually, the problem will be considered axisymmetric. Further, as we suppose that the caster has already reached its operating conditions, a steady state analysis is performed.

2. Problem Definition

Under the assumptions of incompressibility and negligible viscous dissipation and assuming the heat flux described by Fourier's law, the energy balance equation at a material point x inside the closure Ω at the instant t may be written in terms of the enthalpy $\mathcal{H}$ (per unit volume) as follows

$$\frac{d\mathcal{H}}{dt} - \nabla\cdot(\boldsymbol{\kappa}\cdot\nabla T) = Q, \tag{1}$$

where T is the temperature, $\boldsymbol{\kappa}$ the thermal conductivity tensor and Q the heat source (per unit volume). If there were no phase-change, this initial and boundary value problem is completed by the following conditions

$$\begin{aligned} &T = T_0 && \text{at } t = 0, \\ &T = T_w && \text{at } \delta\Omega_T, \\ &(\boldsymbol{\kappa}\cdot\nabla T)\cdot\mathbf{n} = \bar{q} && \text{at } \delta\Omega_q, \\ &(\boldsymbol{\kappa}\cdot\nabla T)\cdot\mathbf{n} = h(T_\infty - T) && \text{at } \delta\Omega_c, \end{aligned} \tag{2}$$

being $\delta\Omega_T$, $\delta\Omega_q$ and $\delta\Omega_c$ non-overlapped portions of the boundary $\delta\Omega$, where temperature, heat flux and convection to an external fluid, respectively, are prescribed.

In a multiphase problem, Eqn. 1 must hold for each single-phase closure Ω_i. Since temperature at the interface Γ has a constant value T_{pc} and the jump energy balance must be satisfied, two additional boundary conditions arise

$$\begin{aligned} &T = T_{pc} && \text{at } \Gamma, \\ &\langle \mathcal{H}(\mathbf{V}\cdot\xi - u(\xi)) - (\boldsymbol{\kappa}\cdot\nabla T)\cdot\xi \rangle = 0 && \text{at } \Gamma, \end{aligned} \tag{3}$$

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