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LETTER TO THE EDITOR
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Comments and replies on the paper 'Prediction of short fatigue crack growth of Ti–6Al–4 V', FFEMS, 2014, 37, 1075–1086, by K. Wang, F. Wang, W. Cui, T. Hayat and B. Ahmad

COMMENTS

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In this article,[1] the authors claim to have proposed a modified model for short fatigue crack growth rate *da/dN* based on both Cui and Wang's model and Chapetti's model. We think that the publication has several shortcomings and that some statements should be properly clarified.

When estimating the threshold for short-crack propagation, the authors used the model proposed by Chapetti,[2] changing the name of some parameters and claiming to propose a new model (refer to expressions 13, 14 and 15 of their publication[1]). On Fig. 1 in Ref. [1], proposed by Chapetti,[2] the fatigue limit $\Delta\sigma_{eR}$ and the microstructural dimension d define the minimal intrinsic threshold for short fatigue crack growth, ΔK_{dR}, as

$$\Delta K_{dR} = 0.65 \quad \Delta\sigma_{eR} \quad \sqrt{\pi d} \tag{1}$$

The threshold for fatigue crack growth as a function of crack length is then defined as[2]

$$\Delta K_{th} = \Delta K_{dR} + (\Delta K_{thR} - \Delta K_{dR})\left[1 - e^{-k(a-d)}\right] \quad \text{with } k = \frac{1}{4d}\frac{\Delta K_{dR}}{(\Delta K_{thR} - \Delta K_{dR})} \tag{2}$$

where ΔK_{thR} is the threshold for long fatigue crack growth under the stress ratio R.

However, the material threshold for crack propagation is redefined in Ref. [1] as (refer to expressions 13 and 14 in Ref. [1])

$$\Delta K_{th} = \Delta K_{th-s} + \Delta K_{th-cl} \tag{3}$$

$$\Delta K_{th-cl} = (\Delta K_{thR} - \Delta K_{th-s})\left[1 - e^{-k(a-d)}\right] \tag{4}$$

where it is said that ΔK_{th-s} is the material threshold of short fatigue crack (minimal intrinsic threshold for short fatigue crack growth), and ΔK_{th-cl} is the component of the stress intensity factor range corresponding to crack closure. The values for the k factor (a material constant that reflects the rate of the extrinsic component development with crack advance, according to nomenclature) in the main equation (2) are not provided anywhere in the paper. This should be clearly reported.

On the other hand, if the parameter d is the one they estimate as expression (12) on their publication[1] seems to read

$$d = \left(\frac{\Delta K_{th-s}}{Y\Delta\sigma_R}\right)^2 \frac{1}{\pi} \tag{5}$$

they should calculate ΔK_{th-s}. There is an issue in the way the model was applied in the fact that it is not shown how this was done. This topic should be properly clarified. For all applications in the publication, the authors should show the values for the independent parameters that they use and how they calculate the ones that are needed for the estimations.

When the authors carried out the comparison in Fig. 2 of their publication[1], they reported a wrong value for the parameter d (the microstructural dimension, needed for the estimation) when using Chapetti's model. The authors should report if they used a value of 0.2 mm for the grain size of the Ti–4Al–6 V alloy analyzed, instead of 0.02 mm, one order of magnitude smaller,[2,3] or it is only a typing mistake.

There was no numerical value or expression for the calculation of the geometrical factor $Y(a)$ used in expression (12) of Ref. [1]. Again, the values used for the estimations should be reported.

Moreover, the authors claim that their model can predict propagation for both short and long cracks including initiation phase (refer to conclusions in Ref. [1]: '…The model can be applied from the short crack to the long crack, including the crack initiation and threshold region…'). The authors seem to have performed some algebra on the definition for the crack propagation threshold for short cracks under a certain stress ratio, defining an 'intrinsic crack length', d, from this threshold (refer to expression 5 in Ref. [1]). Again, as we have mentioned before, the authors should clarify how they measure or estimate the parameter ΔK_{th-s} to calculate the intrinsic parameter d. Here arises another issue that should be clarified, because the value of the intrinsic parameter d (that seems to be estimated from expression 5) could be smaller than the microstructural dimension (grain size for the analyzed Ti alloy). This drives to the authors to say that they can estimate fatigue crack propagation for crack length smaller than grain size. They should clearly propose a valid hypothesis for this statement, accounting for the different fatigue crack mechanisms acting during the creation of a crack with a length similar to the microstructural dimension (microcrack initiation). Chapetti's model is clearly defined from d onwards and allows defining a transition crack length between initiation and propagation stages, given by the microstructural dimension d.

To conclude, we think that to understand the proposal, it becomes imperative to define all parameters and expressions properly, as well as all the hypotheses and simplifications that they use. Extra nomenclature should be defined in some cases. For instance, d is used for two different parameters: the grain size and the intrinsic short crack parameter. The same happens with ΔK_{th-s}, which should be properly defined. Without detailed and proper descriptions of the model and the proposed expressions, and definitions of the assumption or the measurement of the parameters that are needed, it is impossible to continue with the analysis of the estimations that the authors presented in Figs. 5 to 9 of the paper under discussion.

REPLIES

By K. Wang[1,2], F. Wang[3] & W.C. Cui[3]
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We would like first to express our great appreciation to Prof. Chapetti and Dr. Carr for the valuable comments and questions about this publication, which are very important for us to more deeply understand the issue. We have carefully studied the questions they raised, and really found that some problems existed in our publication.

The research background of this paper is that short cracks may exist in the deep-sea manned cabin during manufacturing process. So prediction of short crack growth is very important for ensuring the safety of the manned cabin. The authors' previous research work has mainly focussed on long crack growth rate model and fatigue life estimation. In considering the problem of short cracks, Chapetti's model is applied.

In terms of their threshold question, in our previous work, we proposed a model for long fatigue crack growth, in which the elastic–plastic behaviour in calculating the stress intensity factor range is considered by increasing the actual crack length by adding one-half of the plastic zone size and expressed in expressions (10)–(12) in Wang et al.[1] as follows:

$$a_{\mathrm{mod}} = aF, \quad F = \frac{1}{2}\left[\sec\left(\frac{\pi}{2}\frac{\sigma_{\max}}{\sigma_{fl}}\right) + 1\right], \quad \Delta K = Y(a)\sqrt{aF}\Delta\sigma \tag{1}$$

where the value of F is larger than 1.0, and the definition of ΔK is different from that in the Chapetti's model.[2] In terms of the parameter k, it is found through preliminary analysis that the modified model could not be used to predict the crack growth rate in the vicinity of fatigue limit. So the constant value 4 of their original model is replaced by a variable parameter, λ. A simple comparative analysis of crack growth rate with the variation of λ is carried out here for illustrating the issue shown in Fig. 1. When determining the value of λ, the fatigue crack propagation rates under fatigue limit level will be first determined by the expression (15) in Wang et al.[1] for different values of λ, and the actual parameter λ used in the equation is defined as its maximum value under crack arrest.

In terms of parameter d, we accepted that the physical meaning of the parameter d was indeed ambiguous in our paper. We suggest to use r_e to represent the inherent defect of the material whilst d is retained as the microstructual dimension in Chapetti's model. As a matter of fact, r_e has been introduced in our former model to represent a material constant of the equivalent inherent flaw length (a minimum crack size for engineering metals), and the parameter can be defined as follows.

$$r_e = \left(\frac{\Delta K_{th-s}}{Y\Delta\sigma_R}\right)^2 \frac{1}{\pi F} \tag{2}$$

Obviously, the parameter r_e is smaller than grain size because of the value of ΔK_{th-s} and the parameter F. r_e is determined by the value of ΔK_{th-s} whilst the actual value

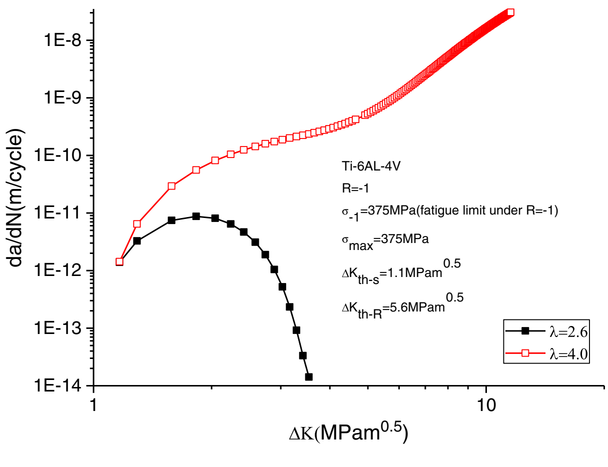

Fig. 1 Comparison of crack growth rates under different values of λ.

of ΔK_{th-s} should be determined either from test or some prediction models. However, to be honest, up to now, we have not established the test method on how to actually determine the value of ΔK_{th-s} but just proposed a possible estimation method using existing measurements or theoretical models. With the development of the detecting instrument, the ΔK_{th-s} can be determined from the test. The fatigue crack length with micrometer scale can be a natural initiation by cyclic fatigue load, and inspected in a light microscope at any time.[4,5] On the other hand, the value of ΔK_{th-s} can be determined by the interpolation method from the threshold test data for larger crack length. For example, the ΔK_{th-s} was approximately determined by the EI Haddad model,[6,8] $\Delta K_{th} = \Delta K_{th0}/\sqrt{1+a_0/a}$,where ΔK_{th0} is the crack-size independent threshold stress intensity range for long cracks, a_0 is an intrinsic crack length defined as $a_0 = {}^{1}/_{\pi}(\Delta K_{th0}/\Delta\sigma_{th0})^2$ and $\Delta\sigma_{th0}$ is the fatigue limit for smooth specimens.[6] So the ΔK_{th-s} can be approximately determined from the relation, and compared with the test data of threshold stress intensity range as shown in Fig. 2.

In terms of the value of d in Fig. 2[1], we checked and found that it was a typing mistake. In terms of the geometrical factor $Y(a)$ used in expression (12),[1] a constant value of 0.65 is used for the semicircular surface crack propagating.

As we know, when the crack length was smaller than the grain size, the short fatigue crack could propagate, and ΔK was affected by the strain of the crack surface.[3] So, we extrapolate Chapetti's model a bit by introducing our previous concept of equivalent inherent flaw length r_e. However, this extrapolation may not be extended too far away from the microstructural dimension d. When the driving force ΔK was greater than the ΔK_{th-s}, the crack propagates. The ΔK_{th-s} increased with the crack length and was affected by the crack closure level. When

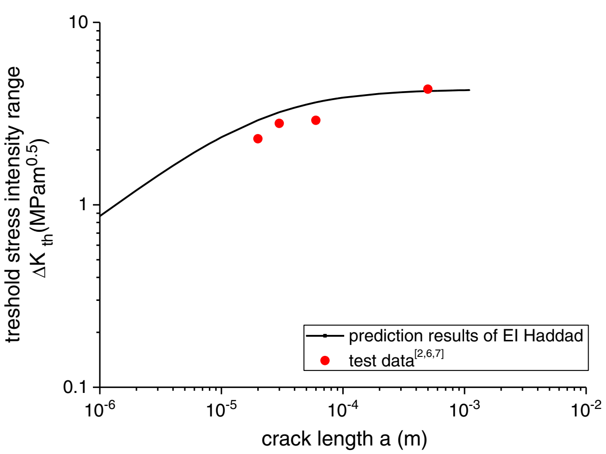

Fig. 2 Comparison of the test data and prediction results according to the EI Haddad model.

the ΔK_{th-s} reaches the level of the ΔK_{th-R} onwards, it will become a constant.

We totally agree with Prof. Chapetti and Dr. Carr that we should define all parameters and expressions properly, as well as all the hypothesis and simplifications. So, we made the following correction. The parameter d in its positions to express intrinsic short crack should be replaced by r_e and accordingly, the related equations (12)–(15) in the present publication[1] should be modified to

$$r_e = \left(\frac{\Delta K_{th-s}}{Y\Delta\sigma_R}\right)^2 \frac{1}{\pi F} \tag{8}$$

$$\Delta K_{th} = \Delta K_{th-s} + \Delta K_{th-cl} \tag{9}$$

$$\Delta K_{th-cl} = (\Delta K_{thR} - \Delta K_{th-s})\left[1 - e^{(-k(a-r_e))}\right] \tag{10}$$

$$da/dN = A\frac{\left[\Delta K - (\Delta K_{thR} - \Delta K_{th-s})\left(1 - e^{(-k(a-r_e))}\right) - \Delta K_{th-s}\right]^m}{1 - (K_{\max}/K_{cf})^n} \tag{11}$$

where the sequence numbers of the equations are those used in publication.[1]

FINAL COMMENTS

by Mirco D. Chapetti and Gustavo E. Carr

At first, we would like to express our thanks to the authors for the effort to answer our questions and to clarify the referenced publication. Finally, we want to express our final remarks.

We understand how the authors propose to modify the driving force applied to the crack to consider elastic–plastic behaviour and the unstable crack growth for high-applied stress levels. However, our concern is mainly related with the attempt to modify the Chapetti model in order to estimate the threshold for short crack propagation:

$$\Delta K_{th} = \Delta K_{dR} + (\Delta K_{thR} - \Delta K_{dR})\left[1 - e^{-k(a-d)}\right]$$

$a \geq d$ Chapetti estimation

$$\Delta K_{th} = \Delta K_{th-s} + (\Delta K_{thR} - \Delta K_{th-s})\left[1 - e^{-k(a-r_e)}\right]$$

$a \geq r_e$ Wang *et al* estimation

The authors change the initial crack length for crack propagation, and instead of d (grain size), they define a value r_e (intrinsic crack length). Besides, they change the minimum threshold for short crack propagation and instead of

$$\Delta K_{dR} = 0.65\Delta\sigma_{eR}\sqrt{\pi d} \text{ Chapetti´s model}$$

they use

$$\Delta K_{th-s} = \frac{\Delta K_{th0}}{\sqrt{1+\frac{a_0}{r_e}}} (\text{El Haddad, with } a = r_e), \text{ with}$$

$$r_e = \left(\frac{\Delta K_{th-s}}{Y\Delta\sigma_R}\right)^2 \frac{1}{\pi F} \text{ Wang } et\ al \text{ model}$$

Because they need ΔK_{th-s} to estimate r_e, it seems from Fig. 2 that they prefer to make r_e equal to 1 μm and calculate ΔK_{th-s} by using the El Haddad model. In this case, this value is much smaller than the grain size of the material analyzed for comparisons (0.02 mm), in contraposition to the limitation the authors expressed, that the extrapolation might not be extended too far away from the microstructural dimension. The authors include in the crack propagation estimation some part of the stage of the crack nucleation within the grain. We think that the estimation of the short crack propagation between r_e and d cannot be properly estimated with the proposed model, and we hope that the authors can explain carefully and improve this matter in future publications.

Besides, they estimate the parameter k for the development of the component $(\Delta K_{thR} - \Delta K_{th-s})$ by using the expression proposed by Chapetti for the development of the component $(\Delta K_{thR} - \Delta K_{dR})$

$$k = \frac{1}{4d}\frac{\Delta K_{dR}}{(\Delta K_{thR} - \Delta K_{dR})} \text{ Chapetti model}$$

and replacing the number 4 for the parameter λ, d for r_e and ΔK_{dR} for ΔK_{th-s}, as follows:

$$k = \frac{1}{\lambda r_e}\frac{\Delta K_{th-s}}{(\Delta K_{thR} - \Delta K_{th-s})} \text{ Wang } et\ al \text{ model}$$

They estimate λ for each application by calculating its maximum value under crack arrest. We think that this procedure is not suitable for real fatigue short crack growth estimations. Finally, it is important to emphasize that the expression proposed by Chapetti to estimate the parameter k is associated to the fatigue limit and the position of the related strongest barrier (d), and that the threshold stress for crack propagation decreases for crack lengths $a > d$. With changes made by the authors, that is no longer satisfied.

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