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A different method to update monthly median h_mF2 values

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Abstract

The height, h_mF2, and the electron density, N_mF2, of the F2 peak are key model parameters to characterize the actual state of the ionosphere. These parameters, or alternatively the propagation factor, *M3000F2*, and the critical frequency, f_oF2, of the F2 peak, which are related to h_mF2 and N_mF2, are used to anchor the electron density vertical profile computed with different models such as the International Reference Ionosphere (Bilitza, 2002), as well as for radio propagation forecast purposes. Long time series of these parameters only exist in an inhomogeneous distribution of points over the surface of Earth, where dedicated instruments (typically ionosondes) have been working for many years. A commonly used procedure for representing median values of the aforementioned parameters all over the globe is the one recommended by the ITU-R (ITU-R, 1997). This procedure, known as the Jones and Gallet mapping technique, was based on ionosondes measurements gathered from 1954 to 1958 by a global network of around 150 ionospheric stations (Jones and Gallet, 1962; Jones and Obitts, 1970). Even though several decades have passed since the development of that innovative work, only few efforts have been dedicated to establish a new mapping technique for computing h_mF2 and N_mF2 median values at global scale or to improve the old method using the increased observational database. Therefore, in this work three different procedures to describe the daily and global behavior of the height of the F2 peak are presented. All of them represent a different and simplified method to estimate h_mF2 and are based on different mathematical expressions. The advantages and disadvantages of these three techniques are analyzed, leading to the conclusion that the recommended procedure to represent h_mF2 is best characterized by a Spherical Harmonics expansion of degree and order equal to 15, since the differences between the h_mF2 values obtained with the Jones and Gallet technique and those obtained using the abovementioned procedure are of only 1%.

Keywords: h_mF2; Electron density profile; ITU-R database; Ionosphere

1. Introduction

It is well known that the ionosphere's climatology shows geographic and diurnal variations, as well as long term variations related to the seasons and the 11 year solar cycle. Several ionospheric models have been developed to predict climatological values of the electron density (ED) vertical profile for every location and any particular moment of the day and the year and solar activity conditions. The determination of such profiles is a very difficult task since the ionosphere represents a highly dynamic and very complex system with a huge variety of parameters that need to be estimated if a reliable and accurate representation of the actual state of the ionosphere is desired. Two of these parameters, and perhaps the most important ones, are the ED and height of the F2 layer peak of the ionosphere, N_mF2 and h_mF2, respectively. In most ionospheric models, these two parameters govern the behavior and shape of the whole ED profile and determine its principal features. Therefore their accurate estimation is crucial to render a good and precise description of the ED distribution of the ionosphere. For this purpose, several efforts have been

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conducted during the last decades in order to obtain a good spatial and temporal representation of these two parameters and hence, a precise description of the main characteristics of the ionosphere (e.g. Jones and Gallet, 1962; Jones and Gallet, 1965; Jones and Obitts, 1970; Bilitza et al., 1979).

An important number of ionospheric models, such as the International Reference Ionosphere (IRI; Bilitza, 2001), the NeQuick (Nava et al., 2008) or the La Plata Ionospheric Model (LPIM; Brunini et al., 2012) use particular functions to reconstruct the ED profile of the ionosphere. Specifically, IRI employs International Radio Consultative Committee (CCIR) models with some modifications for the bottom side and Epstein functions for the topside of the ionosphere; NeQuick implements semi Epstein functions for both, the bottom and the topside; and LPIM uses Chapman functions for the bottom side and a vary-Chapman representation for the topside (Reinisch et al., 2007). All these types of functions depend on several ionospheric parameters, like the critical frequencies (related to the ED through a law of proportionality) of the E, F1, and F2 layers, and the corresponding peak heights and scale heights of these ionospheric layers. Nevertheless, for every one of the aforementioned models, the shape of the entire ED profile is dominated by the critical frequency and the height of the F2 peak.

The first way to tackle the problem of accurately determining N_mF2 and h_mF2 is to directly measure these parameters using ionosondes and, to lesser extent, incoherent scatter radars. Ionosondes are capable of measuring the critical frequency of the F2 peak, f_oF2, and then using the simple relation of proportionality (Bilitza, 2002)

$$N_mF2 = 1.24 \cdot 10^{10} \cdot (f_oF2)^2, \quad (1)$$

the electron density of the peak is determined. In Eq. (1), f_oF2 is measured in MHz and N_mF2 in m^{-3}. Conversely, the determination of the height of the F2 peak (h_mF2) from ionosondes is a more complex problem since a trained eye for manually scaling the ionograms and an appropriate inversion technique to invert virtual into true height are required to get accurate values of this parameter. However, thanks to decades of dedicated measurement campaigns, high quality h_mF2 datasets have been obtained.

In other words, nowadays ionosondes can provide precise and reliable values of f_oF2 and h_mF2, but the global distribution of these instruments is rather inhomogeneous and some kind of modeling has to be introduced in order to describe the spatial and temporal behavior of the critical frequency and the altitude of the F2 peak in those regions of the world poorly covered by observations. Perhaps the first scientists to do this were William B. Jones and Roger M. Gallet, who in the year 1962 presented an insightful and still used technique to represent the geographical and diurnal variations of the monthly medians of f_oF2 and the 3000 km maximum usable frequency factor, *M3000F2* (Jones and Gallet, 1962; Jones and Obitts, 1970). This last parameter (related to the height of the F2 peak through several expressions which will be discussed later) can also be deduced from ionograms and represents the maximum usable frequency at which the ordinary component of an electromagnetic signal emitted from the ground can travel to a point located also on the ground but at a distance of 3000 km, normalized to the critical frequency of the F2 peak (Bilitza, 2002)

$$M3000F2 = \frac{MUF}{f_oF2}, \quad (2)$$

where the acronym MUF stands for maximum usable frequency.

The mapping technique presented by Jones and Gallet was so successful that the CCIR (afterwards, the Radio Communication Sector of the International Telecommunication Union, ITU-R) established it as the recommended one to describe the spatial and temporal behavior of f_oF2 and *M3000F2* (ITU-R, 1997).

This mapping technique represents the diurnal variations of f_oF2 and *M3000F2* through the Fourier series expansion

$$\Omega = a_0 + \sum_{j=1}^{J}[a_j \cdot \cos(jt) + b_j \cdot \sin(jt)], \quad (3)$$

where Ω is the parameter to be mapped; t is the Universal Time (UT); and J is the maximum number of harmonics for mapping the diurnal variation ($J = 6$ for f_oF2 and $J = 4$ for *M3000F2*). The geographical variation of these parameters is taken into account in the Fourier coefficients through the following expressions:

$$a_j = \sum_{k=0}^{K} U_{2j,k} \cdot G_k,\ j \geqslant 0 \text{ and } b_j = \sum_{k=0}^{K} U_{2j-1,k} \cdot G_k,\ j \geqslant 1. \quad (4)$$

Where $K = 75$ for f_oF2 and $K = 49$ for *M3000F2*; U are the numerical coefficients of the expansion (988 coefficients for f_oF2 and 441 coefficients for *M3000F2*); and G are special functions whose explicit forms depend on the k index. For example

$$G_{54} = \sin^8(\mu) \cdot \cos^2(\varphi) \cdot \cos(2\lambda), \quad (5)$$

where φ and λ are the geographic latitude and longitude; and μ is the so-called modip latitude, defined as

$$\mu = arc\tan\left(\frac{I}{\sqrt{\cos(\varphi)}}\right), \quad (6)$$

I being the magnetic inclination at an altitude of 350 km above the surface of the Earth (Rawer, 1984).

The numerical coefficients (U in Eq. (4)) were estimated by means of the Least Squares method applied to the observations provided by a global network of around 150 ionospheric sounders. The quantity of unknowns to be estimated was increased until the RMS error of the adjustment did not further reduce. This procedure led to establish the maximum number of terms for mapping f_oF2 and *M3000F2* (*J* and *K* in Eqs. (3) and (4)). The computational limitations of those times forced Jones and his co-authors to construct a special basis of mathematical functions that fulfilled the condition of orthogonality with respect to the geographical coordinates of the available network of ionospheric stations (Jones and Gallet, 1962; Jones and Obitts, 1970). This procedure avoids the inversion of the normal matrix and other numerical problems that may arise from an ill conditioned system of normal equations. Inspired by the classical Spherical Harmonics functions they developed a mathematical basis comprising the *G* functions above mentioned.

The mathematical basis developed by Jones and co-authors proved to be very well suited to represent the behavior of f_oF2: with no more than 988 coefficients, the technique was able to map very well the sharp peaks and the deep valley between them that f_oF2 exhibits in response to the Appleton anomaly. Besides, the use of the modip latitude helped to cope with the distortion introduced in this complex ionospheric structure by the terrestrial magnetic field. On the other hand, the same mapping technique was also used to describe the daily and global behavior of *M3000F2*, providing an acceptable representation for radio communications purposes.

Nevertheless, the availability of vast ionospheric datasets provided by modern observational techniques, such us ground and space-borne GPS determinations and satellite altimetry missions, presents a unique opportunity to update the ITU-R f_oF2 and h_mF2 global maps database. Particularly, this research is focused on the analysis of three mathematical methods capable of estimating the height of the F2 layer peak but that also present the convenient advantage of being more appropriate for assimilation studies of h_mF2.

Present ionospheric datasets offer today a much better global coverage, a fact that suggests that an assimilation procedure could be performed for a different set of geographical locations (than the one Jones and co-authors implemented) and without the necessity of implementing any *M3000F2*–h_mF2 relating formula (to be discussed in the following section). It has to be clear that Jones and his co-authors developed their f_oF2 and *M3000F2* mapping technique based on the available set of ionospheric stations and computational capabilities of those times, and on the fact that the mentioned parameters were very important for radio communications purposes. However, since the objective of this research is to represent the daily and global behavior of the height of the F2 peak in a simpler manner but with the same degree of accuracy given by the Jones and co-authors technique, it is of importance understanding that there is no need for the implementation of any formula relating *M3000F2* with h_mF2. In other words, it is equally valid to represent the daily and global behavior of h_mF2 using a different numerical procedure that presents the advantage of accurately reproducing the results obtained with the Jones and co-authors technique, but that is based on a simpler mathematical formulation. This is the main objective of the work that will be described throughout the following sections.

In summary, the mapping technique developed by Jones and his co-authors represented a great advance in a time when computer facilities were extremely limited compared to the present ones. Although more than 40 years have passed since those days, the CCIR model is still at the core of well credited ionospheric models like the IRI. Hence, it seems opportune to revisit the work by Jones and co-authors to explore the possibility of updating their mapping technique, taking advantage of the fact that standard computers offer today much more possibilities than the one Jones et al. had. Particularly, in this paper we explore the possibility of simplifying the mathematical representation of the h_mF2 parameter starting from the *M3000F2* parameter determined by Jones and co-authors.

Consequently, three different procedures to estimate values of the height of the F2 layer peak, characterized for introducing a significant simplification with respect to the standard numerical technique commonly used to determine h_mF2 will be presented. Each one of these procedures applies a different mathematical formulation. The first one implements the basis of functions developed by Jones and co-authors, while the remaining two are based on Spherical Harmonics expansions of degree and order equal to 15, but with different time dependences through the corresponding coefficients. The three techniques were conceived to reproduce as accurate as possible the h_mF2 global maps obtained using the ITU-R database, but through the implementation of a simpler and more straightforward numerical procedure. Hence, it is important to remark that the main objective of our research does not consist on an improvement of the Jones and co-authors technique, but on a simplification of the manner in which the height of the F2 layer peak can be determined.

2. The M3000F2–h_mF2 relation

The decision of developing an empirical expression for *M3000F2* instead of h_mF2 was based on the suitability of *M3000F2* for radio communications studies and on the fact that *M3000F2* could be easily scaled from ionograms. Nevertheless, since the middle of the 20th century, many studies have been conducted in order to obtain a method to relate the *M3000F2* parameter to the height of the F2 peak. Some examples worth to mention are Shimazaki (1955), Bilitza et al. (1979), Obrou et al. (2003) and Rawer and Eyfrig (2004). In these papers, several relating expressions were proposed and their advantages and disadvantages analyzed.

The original relation due to Shimazaki (1955) assumes an approximately parabolic layer and is based on the fact that a strong anti correlation exists between h_mF2 and *M3000F2*. This results in the following relation between *M3000F2* and h_mF2

$$h_mF2 = \frac{1490}{M3000F2} - 176, \quad (7)$$

where h_mF2 is measured in km.

Wright and Mcduffie (1960) reported some discrepancies between different measured datasets and the corresponding values calculated with the rather simple Shimazaki (1955) formula. Later on, Bradley and Dudeney (1973) found that the discrepancies were a consequence of ignoring that the refraction in the lower ionospheric layers, particularly the E layer, deforms the layer profile away from a parabolic shape. To account for these effects, they included a correction factor, CF, dependent on the $\frac{f_oF2}{f_oE}$ ratio, f_oE being the critical frequency of the ionosphere's E layer,

$$h_mF2 = \frac{1490}{M3000F2 + CF} - 176, \quad (8.a)$$

with

$$CF = \frac{0.253}{\frac{f_oF2}{f_oE} - 1.215} - 0.012. \quad (8.b)$$

Lately, Eyfrig (1973) found that the 11 year solar cycle also influences the relation between *M3000F2* and h_mF2; so a new expression for CF was proposed, depending on the $\frac{f_oF2}{f_oE}$ ratio and the sunspot number, R. But even with the dependence on the solar activity incorporated, the Bradley and Dudeney (1973) formula still showed some problems. Besides, the correction factor given by Eq. (8.b) led to a non realistic pole. To overcome this problem, Dudeney (1975) presented a new formula based upon inverted profiles from two high latitude ionospheric stations. This expression is nowadays known as the "Dudeney formula"

$$h_mF2 = \frac{1490 \cdot M3000F2 \cdot \sqrt{\frac{0.0196 \cdot M3000F2^2 + 1}{1.2967 \cdot M3000F2^2 - 1}}}{M3000F2 + CF} - 176, \quad (9.a)$$

where CF is given by

$$CF = \begin{cases} \dfrac{0.253}{\frac{f_oF2}{f_oE} \cdot \frac{\exp\left[20\left(\frac{f_oF2}{f_oE} - 1.75\right)\right] + 1.75}{\exp\left[20\left(\frac{f_oF2}{f_oE} - 1.75\right)\right] + 1} - 1.215} - 0.012 & \text{, if the E layer is present} \\ -0.012, \text{if the E layer is not present.} \end{cases} \quad (9.b)$$

Finally, Bilitza et al. (1979) checked the different formulas and based on F2 peak altitudes obtained from incoherent scatter radar measurements, proposed an even more involved expression for CF in which, apart from depending on the $\frac{f_oF2}{f_oE}$ ratio and on the solar activity (through the sunspot number, R), a dependence on the geomagnetic latitude was included

$$h_mF2 = \frac{1490}{M3000F2 + CF} - 176, \quad (10.a)$$

with

$$CF = \frac{F_1(R) \cdot F_4(R, \vartheta)}{\frac{f_oF2}{f_oE} - F_2(R)} + F_3(R), \quad (10.b)$$

and

$$F_1(R) = 0.00232 \cdot R + 0.222, \quad (10.c)$$

$$F_2(R) = 1.2 - 0.0116 \cdot \exp(0.0239 \cdot R), \quad (10.d)$$

$$F_3(R) = 0.096 \cdot \frac{(R - 25)}{150}, \quad (10.e)$$

$$F_4(R, \vartheta) = 1 - \frac{R}{150} \exp\left(-\frac{\vartheta^2}{1600}\right), \quad (10.f)$$

ϑ being the geomagnetic latitude.

For the purposes of this work, only the Bilitza formula (Eqs. (10.a)–(10.f)) was taken into consideration.

3. A different approach to determine the height of the F2 peak

The so-called ITU-R database (ITU-R, 1997) was established using the Jones and Gallet mapping technique and observations collected from 1954 to 1958 by a network of approximately 150 ionospheric sounders unevenly distributed around the world. This database consists of two sets of U coefficients (see Eq. (4)), one for low solar activity ($R_{12} = 0$; where R_{12} is the 12 month running mean value of the monthly mean sunspot number) and another for high solar activity ($R_{12} = 100$), for every month of the year. Each set comprises the coefficients needed for the spatial representation of the median values of both parameters, f_oF2 and *M3000F2*, that is $988 + 441 = 1429$ coefficients. For a given month and R_{12} value, the U coefficients must be linearly interpolated from the tabulated values. Then, the Dudeney formula or the Bilitza one has to be used in order to convert the *M3000F2* values computed from this database into h_mF2 values.

Inspired on the suggestion made by Jones and Gallet (1962) that their mapping technique could also be used to describe the spatial and temporal behavior of the height of the F2 peak, we proposed a Fourier series expansion to directly represent the diurnal and geographic variations of h_mF2. Specifically, this means that

$$h_mF2 = a'_0 + \sum_{j=1}^{J} \left[a'_j \cdot \cos(jt) + b'_j \cdot \sin(jt)\right], \quad (11)$$

where the geographic variation is accounted for by the Fourier coefficients in the same way as it is done for *M3000F2* and f_oF2, that is, using the special G functions

$$a'_j = \sum_{k=0}^{K} U'_{2j,k} \cdot G_k, \; j \geqslant 0 \text{ and } b'_j = \sum_{k=0}^{K} U'_{2j-1,k} \cdot G_k, \; j \geqslant 1. \quad (12)$$

This idea is indorsed by the fact that the height of the F2 peak also exhibits clear and distinctive geographical and diurnal patterns, which can easily be seen when plotting measured or calculated (using Eqs. (9) or (10)) h_mF2 values against a map of the geographic coordinates or against the time of the day (UT). Besides, it is of importance to mention that once the coefficients involved in Eq. (12) are obtained, the usage of Eq. (11) to compute new h_mF2 values represents a significant simplification with respect to the standard procedure used to estimate h_mF2, since no $M3000F2$–h_mF2 relating formula will be necessary at this point.

In order to determine the best representation of the height of the F2 peak daily behavior, two maximum numbers of harmonics were proposed: $J = 4$ and $J = 6$. The analysis was performed for both high solar activity ($R_{12} = 100$) and low solar activity ($R_{12} = 0$).

The values of the h_mF2 parameter were calculated using the ITU-R database. In more detail, a geographical grid of 73 by 73 points in latitude and longitude was setup. Therefore, for each point of this grid, for every month of the year, for every hour of the day, and for both $R_{12} = 100$ and $R_{12} = 0$, f_oF2 and $M3000F2$ median values were computed using the Jones and Gallet mapping technique in connection to the tabulated coefficients from the ITU-R database. After that, the $M3000F2$ values were converted into h_mF2 values using the Bilitza formula. This means that for each level of solar activity and for every month of the year, one set of h_mF2 values (each one comprising $73 \times 73 \times 24 = 127896$ values) was determined.

Once the h_mF2 values were calculated, the corresponding system of equations was determined

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{h}, \tag{13}$$

where $\mathbf{x}$ is the vector comprising the unknowns of the problem, i.e.: the U' coefficients for the geographical representation of the height of the F2 peak; $\mathbf{h}$ is the vector that contains the calculated h_mF2 values mentioned in the last paragraph; and $\mathbf{A}$ is the design matrix, whose rows contain the corresponding special G functions multiplied by the cosines and sines of the Fourier series expansion for each point of the grid and each hour of the day. The size of this matrix depends on the maximum number of harmonics (J in Eq. (3)): $\mathbf{A}$ is a 127896×441 matrix for $J = 4$, and a 127896×988 matrix for $J = 6$. These two systems of equations were determined for every month of the year and for both high and low solar activities, meaning that a total amount of 48 systems of equations was constructed.

Finally, these systems of equations were solved using the Least Square method. The results obtained consist of four sets of U' coefficients for every month of the year

$$\mathbf{x}_{N,M,SA} = (U'_1, U'_2, \ldots, U'_N)^T_{M,SA}, \tag{14}$$

where $N = 441$ and 988, $M = 1, 2, \ldots, 12$ and $SA =$ high and low for high solar activity and low solar activity, respectively. Once the U' coefficients for the geographical representation of h_mF2 have been estimated, new $h_m\hat{F}2$ values can be calculated using Eqs. (11) and (12), and be compared with those used as inputs in the $\mathbf{h}$ vectors.

The comparisons mentioned in the last paragraph are of significant importance since they will allow us to evaluate the potential benefit of increasing the maximum number of harmonics from $J = 4$ to $J = 6$. Consequently, the following differences were computed

$$\Delta h(\varphi_u, \lambda_v, t_w)_{N,M,SA} = h_m\hat{F}2(\varphi_u, \lambda_v, t_w)_{N,M,SA} - h_mF2(\varphi_u, \lambda_v, t_w)_{N,M,SA}, \tag{15}$$

with $u = 1, 2, \ldots, 73$ and $v = 1, 2, \ldots, 73$, and $w = 1, 2, \ldots, 24$, describing the different grid points. To analyze these differences, hourly global maps of $h_m\hat{F}2(\varphi_u, \lambda_v, t_w)_{N,M,SA}$ and $\Delta h(\varphi_u, \lambda_v, t_w)_{N,M,SA}$ (for $J = 4, 6$) were represented. It is worth mentioning that in order to make the reading simple and straightforward, only the results for a particular month (January) and representative hours of the average day (UT = 6, 12) for that month are presented and analyzed. However, very similar results were obtained for the remaining eleven months. Fig. 1 shows

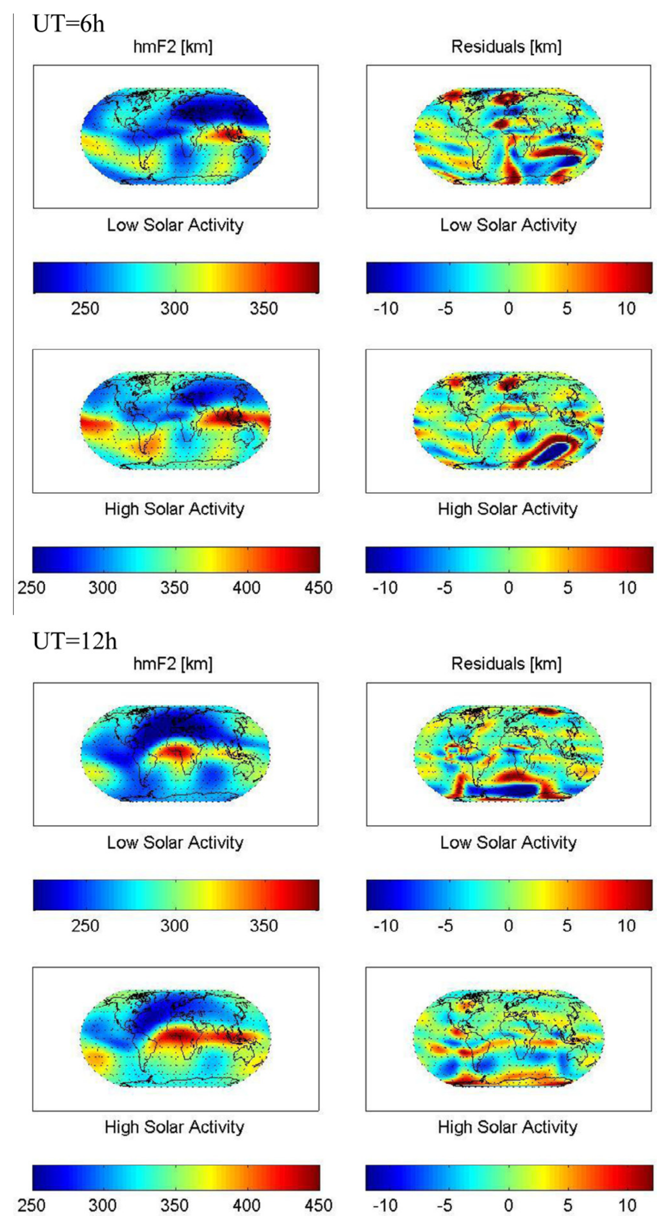

Fig. 1. Series of global maps of $h_m\hat{F}2^J$ and Δh^J(residuals) for $J = 4$; for some representative hours of the day (UT = 6, 12).

eight maps corresponding to the $J = 4$ representation, while Fig. 2 presents analogous maps to those shown in Fig. 1 but for $J = 6$.

After analyzing both figures, it would seem that the h_mF2 representation for $J = 6$ (Fig. 2) works better, basically because the corresponding height differences for both levels of solar activity maintain hourly average values 2 km lower than the average values obtained from the $J = 4$ representation. For both levels of solar activity, the maps of height differences in Fig. 2 present a smoother global distribution, although some particular (non physical) structures can be observed, specifically over the Indian and South Atlantic Oceans and over Northern Europe and Greenland. These structures are not only present in Fig. 1, but they also show an enhanced behavior.

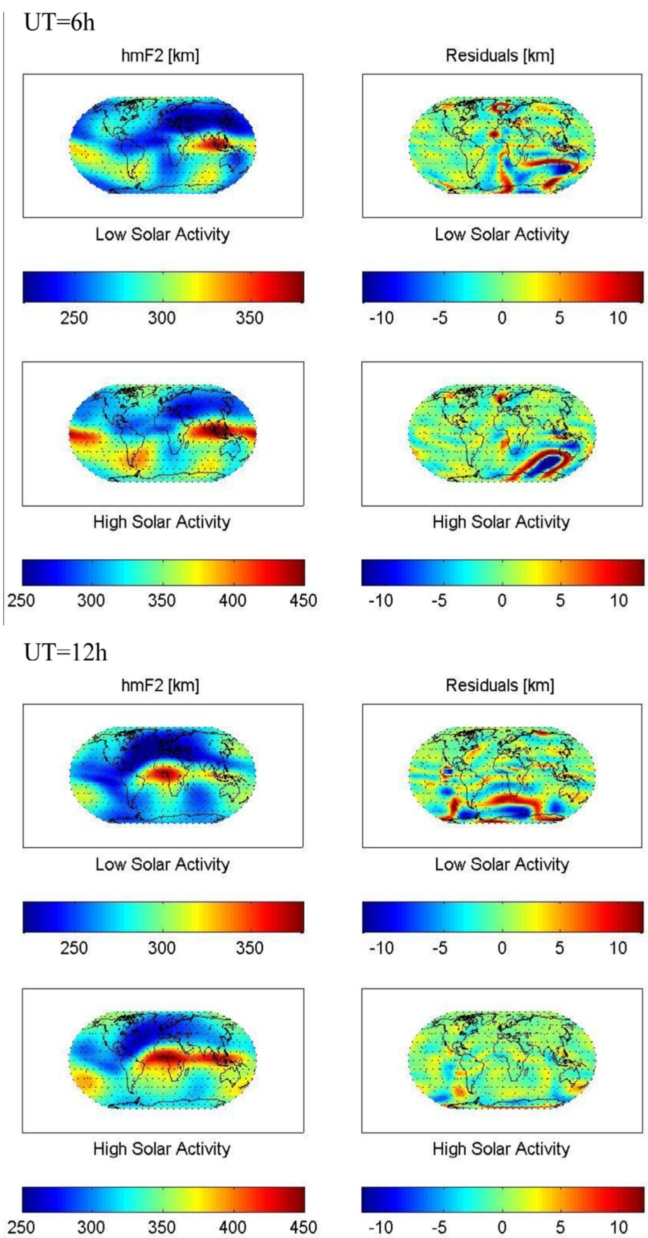

Fig. 2. Series of global maps of $h_m\hat{F}2^J$ and Δh^J(residuals) for $J = 6$; for some representative hours of the day (UT = 6, 12).

Furthermore, in Fig. 1 the aforementioned structures last longer, almost all day for the Indian and South Atlantic Oceans one, than in Fig. 2 where this last structure disappears around UT = 12 h. Even though some other regions of high values can be observed, for example one over North America around UT = 6 h, the most persisting and noticeable is this large loop shaped structure that extends over the Indian Ocean and part of the South Atlantic Ocean. One possible explanation for the differences represented by these structures could be that our technique fails to exactly reproduce the h_mF2 values rendered by the Jones and co-authors technique, probably because once the corresponding coefficients have been estimated, our method directly calculates h_mF2 values without using any *M3000F2–*h_mF2 relating formula, meaning that any structure (actual or fictitious) introduced at this stage by any of these relating formulas will not be mapped by our method. This important difference between the two mathematical techniques compared in this work could explain the presence of the Indian Ocean and South Atlantic structure as well as the other mentioned structures. In simple terms, the observed (non physical) structures might be a consequence of the simplification introduced by our technique. Lastly, it is important mentioning that the words "(non physical) structures" were implemented to emphasize the fact that they refer to regions where the differences between the compared techniques are most significant, rather than to actual ionospheric physical structures.

Based on the explanation given in the last two paragraphs, the $J = 6$ representation seems more appropriate to describe the diurnal and geographic behavior of the height of the F2 peak because once the Sun has moved away from the aforementioned oceanic regions, the loop shaped structure vanishes, while it persists for the $J = 4$ representation.

Now, with respect to the h_mF2 hourly maps, the $J = 6$ representation also seems to be the appropriate one. Again, this can be deduced from the analysis of Figs. 1 and 2. Particularly, the maps in Fig. 2 show a more extended F2 peak than those corresponding to Fig. 1, with a particular tail that could be interpreted as the sunset F2 peak observed in daily plots of measured h_mF2 values against the time of the day (Obrou et al., 2003). In fact, the plot portrayed in Fig. 3 supports this last statement since it clearly illustrates that h_mF2 values calculated with the $J = 6$ representation better depict the recently mentioned second peak, a feature that is not so well mapped with the $J = 4$ representation (compare the continuous line curves against the dashed ones to appreciate the difference). However, it has to be clear that our technique represents the sunset F2 peak with the same accuracy (or deficiency) as the Jones and co-authors one, since both techniques rely on the ITU-R database.

Finally, and regarding the magnitude of the height of the F2 peak, the only important fact to highlight is that for both high and low solar activities no appreciable differences between the corresponding maps of Figs. 1 and 2 can

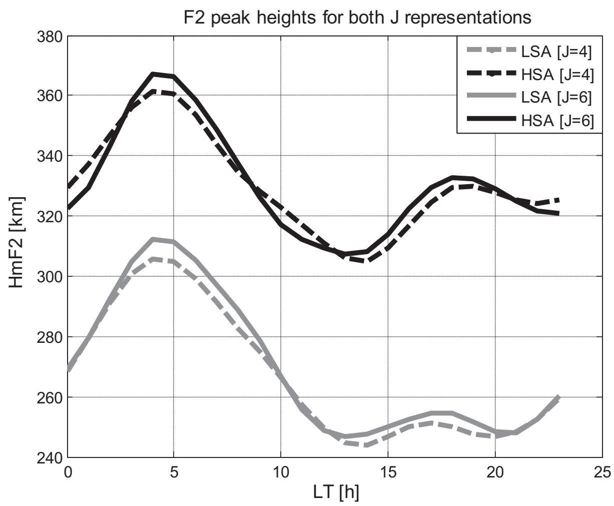

Fig. 3. Height of the F2 peak against local time for a particular location of the globe. The acronym LSA means low solar activity while HSA stands for high solar activity. For both levels of solar activity, the corresponding J value is given in brackets.

be observed. For high solar activity, the average height value is around 335 km, while for low solar activity the average height is of approximately 275 km. Moreover, Fig. 4 reinforces this last statement since it only shows small differences between the $J=6$ and $J=4$ representations with respect to the magnitude of h_mF2 for any given value of modip latitude (similar plots were obtained for different moments of the day and values of geographical longitude). As expected, it can clearly be seen in both Figs. 1 and 2 that the height of the F2 peak follows the daily apparent movement of the Sun and the terrestrial magnetic field isolines.

4. Comparison with Spherical Harmonics

Even though the $J=6$ representation proved to be a simpler and reliable way to determine h_mF2 values, the

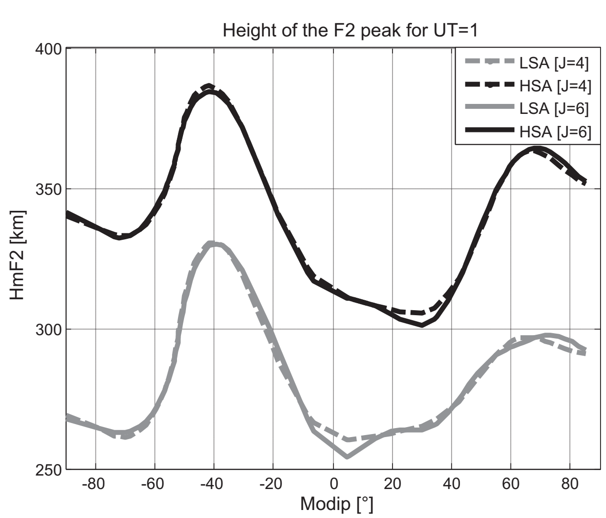

Fig. 4. Height of the F2 peak against modified dip latitude (modip) for UT = 1 h, and for a location with longitude $\lambda = 10.9484$.

structures observed in the corresponding Δh (residuals) plots of Fig. 2 suggest the implementation of some other mathematical basis to describe the behavior of the height of the F2 layer peak. Since a global analysis has been performed in this research, the simplest option would be to map the h_mF2 parameter using a Spherical Harmonics expansion.

The Spherical Harmonics are mathematical functions widely used to represent the behavior of several atmospheric parameters at a global scale. For example, the LPIM implements a Spherical Harmonics expansion of degree and order equal to 15, to hourly map the global characteristics of the vertical Total Electron Content (vTEC) of the ionosphere (Brunini et al., 2004; Azpilicueta et al., 2006).

Based on this comments, two different Spherical Harmonics representations were considered to globally describe the height of the F2 peak. The first one consists on the Spherical Harmonics expansion of degree and order equal to 15, that is

$$h_mF2 = \sum_{l=0}^{L}\sum_{m=0}^{l}\left(a_{lm}(t)\cos\left(2\pi\frac{mh}{24}\right) + b_{lm}(t)sin\left(2\pi\frac{mh}{24}\right)\right) \times P_{lm}(sin(\mu)), \quad (16)$$

where t is the Universal Time; h is the hour angle ($h = t + \lambda - 12$); λ and μ are the geographical longitude and modip latitude for the corresponding points of the grid previously established (see the preceding section); $a_{lm}(t)$ and $b_{lm}(t)$ are the time dependent coefficients (mathematically represented by a stepwise function with a refreshing interval of 1 h); L is the maximum degree of the expansion (15 for this work); and P_{lm} are the associated Legendre functions.

Although strictly speaking the aforementioned coefficients depend on the Universal Time, the expansion described by Eq. (16) will be referred to as the "independent" one, since its coefficients are considered constant values for each UT period of one hour. Again, both levels of solar activity were taken into consideration, and the values of h_mF2 utilized for the fitting process were those obtained using the ITU-R database. Consequently, 24 systems of equations analogous to those described in the preceding section but with a different design matrix, were solved using the Least Square method in order to obtain the Spherical Harmonics expansion coefficients needed to compute new h_mF2 values, h_mF2'. These new values of the height of the F2 peak were compared with those obtained using the ITU-R database (Jones and co-authors technique), for every month of the year and both levels of solar activity. The results for both high and low solar activity levels are presented in Fig. 5, which is divided in two blocks of four plots each; the first one showing the global behavior of h_mF2' and the residuals for UT = 12 h, and the second one describing the characteristics of the same quantities but for UT = 18 h. For simplicity reasons, only the results for a particular month (January) and representative hours

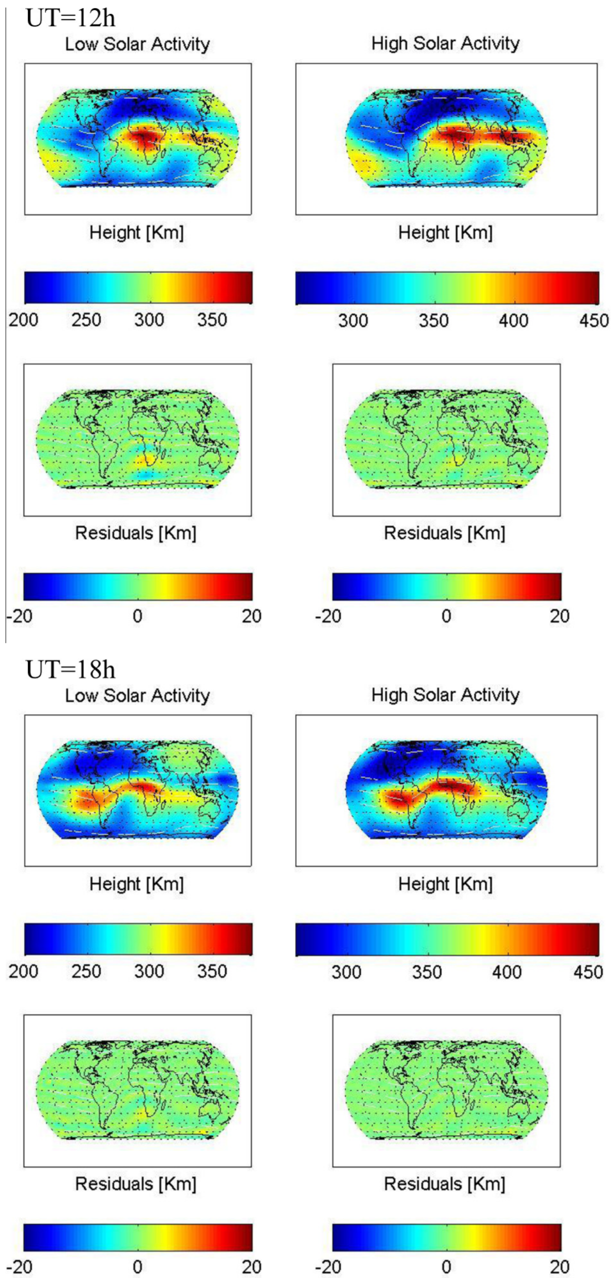

Fig. 5. Hourly maps of the height of the F2 peak computed using the "independent" Spherical Harmonics expansion and corresponding height differences between the h_mF2 values calculated using the ITU-R database and those obtained using the mentioned Spherical Harmonics expansion (residuals) for representative hours of the day.

of the day (UT = 12, 18) are showed; although very similar results were obtained for the remaining hours of the day and months of the year.

Analyzing the mentioned figure it can clearly be deduced that there are no considerable differences between the Jones and co-authors technique and the "independent" Spherical Harmonics expansion, since the residuals present absolute values not higher than 5 km, which represent a difference of only 1%. In other words, the description of the maximum values of the height of the F2 layer peak given by the "independent" Spherical Harmonics expansion does not differ from that provided by the Jones and co-authors technique, being a proof of this the fact that both techniques show similar extended and pronounced distributions of the highest values of h_mF2. The same behavior is observed for both levels of solar activity with the exception that for low solar activity periods the maximum h_mF2 values are about 100 km smaller than those corresponding to periods of high solar activity.

The second Spherical Harmonics expansion considered in this research is similar to the first one, but exhibits an important difference: the coefficients of the expansion depend on the Universal Time through a Fourier series expansion of degree 6

$$h_mF2 = \sum_{l=0}^{L}\sum_{m=0}^{l}\left(a'_{lm}(t)\cos\left(2\pi\frac{mh}{24}\right) + b'_{lm}(t)sin\left(2\pi\frac{mh}{24}\right)\right) \times P_{lm}(sin(\mu)), \quad (17.a)$$

with

$$a'_{lm}(t) = c_0 + \sum_{j=1}^{6}[c_j \cdot \cos(jt) + d_j \cdot \sin(jt)], \quad (17.b)$$

$$b'_{lm}(t) = e_0 + \sum_{j=1}^{6}[f_j \cdot \cos(jt) + g_j \cdot \sin(jt)], \quad (17.c)$$

where t represents the Universal Time; h is the hour angle; μ is the modip latitude; and L is once more equal to 15.

Again, the geographical grid of points was the same as in the previous experiments, the h_mF2 values were computed using the ITU-R database; and both high and low solar activity periods were considered. The corresponding 24 systems of equations were solved using the Least Square method, providing a solution of 24 data packages comprising the corresponding c_0, e_0, and c_j, d_j, e_j, f_j and g_j (for $j = 1, 2, \ldots, 6$) coefficients; which are essential to compute the new values of the height of the F2 peak for every hour of the day and month of the year. These new h_mF2 values were compared with those obtained with the Jones and co-authors technique. The results for high solar activity and low solar activity are presented in Fig. 6. Once more, the referred figure is divided in two blocks of four maps each, four of them describing the global behavior of the height of the F2 peak, and the other four depicting the residuals.

From a thoroughly inspection of Fig. 6, it can be concluded that this second Spherical Harmonics expansion produces different results than those obtained using the "independent" Spherical Harmonics expansion. Specifically, this second technique fails to represent with the same degree of definition showed by the Jones and co-authors procedure, the distribution of the maximum values of the h_mF2 parameter; although a similar structure following the daily apparent movement of the Sun and the modip isolines can be appreciated, particularly for the highest values of h_mF2. Besides, the already mentioned tail shaped distribution of the maximum h_mF2 values is not as pronounced

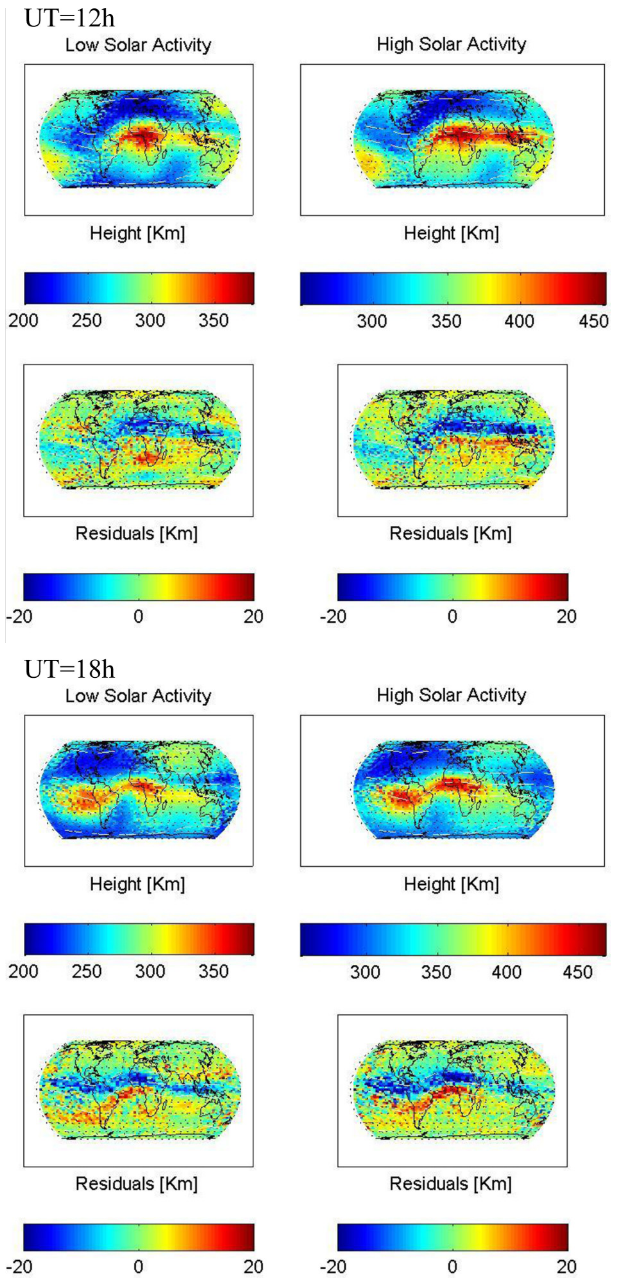

Fig. 6. Hourly maps of the height of the F2 peak computed using the "Fourier time dependent" Spherical Harmonics expansion and corresponding height differences between the h_mF2 values calculated using the ITU-R database and those obtained using the mentioned Spherical Harmonics expansion (residuals) for representative hours of the day.

and extended as it is for the "independent" Spherical Harmonics expansion. Furthermore, Fig. 6 shows important differences (ranging from −20 km up to 20 km) between the Jones and co-authors technique and this second Spherical Harmonics expansion for low modip latitude locations, which correspond to the region where the aforementioned tail shaped structure is observed. In summary, this "Fourier time dependent" Spherical Harmonics expansion would not seem the appropriate one to map the global behavior of the height of the F2 peak, perhaps because the coefficients dependence on time through the Fourier expansion could be introducing structures that do not exist in the real ionosphere.

Finally, it is worth mentioning that the characteristics described in the last paragraph can be observed for both high and low solar activity levels with the exception of a difference of around 100 km in the magnitude of the h_mF2 parameter.

5. Conclusions and further work

The first method described in this paper to represent the diurnal and geographic behavior of the h_mF2 ionospheric parameter shows a good daily behavior for both high and low solar activities, particularly when the maximum number of harmonics is $J = 6$. For the indicated maximum number of harmonics our technique presents a better agreement with those h_mF2 quantities obtained using the ITU-R database. Specifically, the obtained height differences for $J = 6$ behave better than those corresponding to $J = 4$, since the first ones show a more even global distribution. Besides, the $J = 6$ representation of the height of the F2 peak shows a more pronounced and extended distribution of the maximum values of the height of the F2 peak (in accordance with the position of Sun and the terrestrial magnetic field isolines) than the $J = 4$ representation.

In addition, a comparison between the Jones and co-authors technique and two different Spherical Harmonics expansions (with degree and order equal to 15), one comprising constant coefficients and the other characterized by time dependent coefficients through a Fourier series expansion of degree 6, was performed. The fitting of these two different Spherical Harmonics expansions gave similar results to those obtained using the Jones and co-authors functions, with the most noticing difference being that the so-called "independent" Spherical Harmonics expansion showed a better agreement than the "Fourier time dependent" method, a fact that can easily be deduced from the maps depicting the residuals, in which differences of around 5 km correspond to the "independent" Spherical Harmonics expansion, but the differences between the Jones and co-authors technique and the "Fourier time dependent" Spherical Harmonics expansion range from −20 km up to 20 km.

Based on these comments and the analyses performed in the preceding sections, we conclude that the "independent" Spherical Harmonics expansion represents the best way to describe the diurnal and geographic behavior of the height of the F2 peak. No important differences were observed between the values calculated with our techniques and those obtained using the ITU-R database (these differences reached maximum absolute values of 12 km for the $J = 6$ representation and of 5 km for the "independent" Spherical Harmonics expansion, which for an average height between 275 and 335 km only represent discrepancies of around 4% and 1%, respectively), for which it can readily be deduced that both techniques are appropriate to represent the spatial and temporal variations of the height of the F2 peak, although the "independent" Spherical Harmonics expansion (given by Eq. (16)) is the recommended one, since it accurately reproduces the h_mF2 global maps obtained using the ITU-R database. Furthermore, the technique described by Eq. (16) might perform better in researches in which a correction of the Jones and Gallet mapping technique's coefficients using actual data is required. In Brunini et al. (2011) a method to correct the ITU-R database using GPS actual data is described. However, numerical instabilities associated to the Dudeney formula (particularly to the derivates of the Dudeney formula with respect to *M3000F2*) are reported in that paper. This means that since the techniques presented in this paper compute new h_mF2 values without using any *M3000F2*–h_mF2 relating formula, they could be better suited for procedures in which corrections to the Fourier series expansion coefficients based on actual data are desired. But, in order to fully understand that the technique given by Eq. (16) is better suited for assimilation studies of the height of the F2 layer peak and hence present a well justified conclusion, it is important to further the discussion.

Some ionospheric assimilation algorithms developed during the last years (Galkin et al., 2012; Nava et al., 2011) might benefit with a simplification of the mathematical representation of the F2 peak parameters. A significant point in the development of these assimilation (or ingestion) techniques is the determination of the design matrix (if a Least Squares type of solution is desired) or the transition matrix (if a Kalman filter type of solution is desired), which strongly depend on the ionospheric model behind the data assimilation algorithm. Electron density (N_e) models like the IRI or NeQuick can be described (in a simplified manner) as a function that primarily depends on the F2 layer peak parameters, f_oF2 and *M3000F2*, which in turn depend upon the ITU-R coefficients ($X_1, \ldots, X_n$ in the case of f_oF2, and $Y_1, \ldots, Y_m$ in the case of *M3000F2*). Symbolically:

$$N_e(X_1,\ldots,X_n;Y_1,\ldots,Y_m) = F(f_oF2(X_1,\ldots,X_n), \times M3000F2(Y_1,\ldots,Y_m)). \quad (18)$$

Under these conditions, the ith row of the design/transition matrix reads:

$$\mathbf{a}_i = \left[\frac{\partial N_e}{\partial f_oF2}\frac{\partial f_oF2}{\partial X_1},\ldots,\frac{\partial N_e}{\partial f_oF2}\frac{\partial f_oF2}{\partial X_n}, \times\frac{\partial N_e}{\partial M3000F2}\frac{\partial M3000F2}{\partial Y_1},\ldots,\frac{\partial N_e}{\partial M3000F2}\frac{\partial M3000F2}{\partial Y_m}\right]_i, \quad (19)$$

whose determination implies the usage of the special G functions to compute the derivatives of *M3000F2* with respect to the ITU-R coefficients.

Assuming that the model formulation is modified in the following way

$$N_e(X_1,\ldots,X_n;Y_1,\ldots,Y_m) = F'(f_oF2(X_1,\ldots,X_n), \times h_mF2(Y'_1,\ldots,Y'_m)), \quad (20)$$

and that the "independent" Spherical Harmonics basis (instead of the one defined by the special G functions) is implemented to represent the geographical variability of h_mF2, the corresponding row of the design/transition matrix reads

$$\mathbf{a}'_i = \left[\frac{\partial N_e}{\partial f_oF2}\frac{\partial f_oF2}{\partial X_1},\ldots,\frac{\partial N_e}{\partial f_oF2}\frac{\partial f_oF2}{\partial X_n}, \times\frac{\partial N_e}{\partial h_mF2}\frac{\partial h_mF2}{\partial Y'_1},\ldots,\frac{\partial N_e}{\partial h_mF2}\frac{\partial h_mF2}{\partial Y'_m}\right]_i. \quad (21)$$

The benefit of using the modified formulation can be assessed by comparing the conditioning number of the design/transition matrix of both formulations. To accomplish this purpose, both matrixes were computed over a global grid with equally distributed nodes every 5° in longitude and 2.5° in latitude (73 by 73 points). Then, the conditioning numbers, which are given by the ratio between the maximum and minimum eigenvalues of the corresponding matrixes (Dahlquist and Bjorck, 1973), were determined. This simple experiment showed a dramatically reduction of the conditioning number from 10^{22} to 10^9, and served as a demonstration that the technique given by Eq. (16) is the recommended one for assimilation studies of the height of the F2 layer peak.

To sum up, even though the h_mF2 mapping techniques presented in this paper (given by Eq. (11) for $J = 6$, and Eq. (16)) represent different and reliable ways to describe the spatial and temporal variations of the height of the F2 peak, future works comprising comparisons with measured h_mF2 values are necessary to finally establish the ultimate accuracy and reliability of our research. On the other hand, since the real time IRI group has already begun to assimilate measured f_oF2 values to update the ITU-R database (Reinisch et al., 2012), the technique characterized by Eq. (16) presents a convenient way for the assimilation of measured h_mF2 data to update the monthly median h_mF2 coefficients for the calculation of the h_mF2 maps.

The coefficients sets for the twelve months of the year, for both high and low solar activity obtained with the $J = 6$ representation and the "independent" Spherical Harmonics expansion, plus the corresponding subroutines (in Fortran 90) that determine the values of h_mF2 are available at http://www.fcaglp.unlp.edu.ar/~fconte.

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