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Ion tori around $f(R)$-Kerr black holes

E.A. Saavedra[1], F.L. Vieyro[1,2] & D. Perez[2]

[1] *Facultad de Ciencias Astronómicas y Geofísicas, UNLP, Argentina*
[2] *Instituto Argentino de Radioastronomía, CONICET-CIPCPBA–UNLP, Argentina*

Contact / saavedraenz@gmail.com

Resumen / Los agujeros negros ofrecen uno de los mejores escenarios para explorar las desviaciones de la Relatividad General en el régimen de campo fuerte. Las teorías alternativas de la gravedad, en particular las teorías $f(R)$, que han sido ampliamente analizadas en contextos cosmológicos, pueden ser puestas a prueba mediante modelos astrofísicos, en particular, procesos de acreción alrededor de los agujeros negros. En este trabajo, exploramos las manifestaciones astrofísicas de un toro de iones ópticamente delgado entorno a un agujero negro de$f(R)$-Kerr. Primero, calculamos la geometría del toro mediante el análisis del movimiento de partículas en el espacio-tiempo $f(R)$-Kerr. Suponiendo un plasma de dos temperaturas, caracterizamos las cantidades termodinámicas del toro y comparamos nuestros resultados con los correspondientes a la Relatividad General. Analizamos el impacto de la variación del escalar de Ricci sobre la emisión sincrotrón asociada a un agujero negro supermasivo.

Abstract / Black holes offer one of the best scenarios to explore deviations from General Relativity in the strong field regime. Alternative theories of gravity, in particular $f(R)$ theories, that have been extensively analyzed in cosmological contexts, could be tested through astrophysical models of accretion process around black holes. In this work we model a low accretion process onto an $f(R)$-Kerr black hole by an optically thin ion torus. We first compute the torus geometry by analyzing the particle motion in $f(R)$-Kerr spacetime. Assuming a two temperature plasma, we characterize the thermodynamics quantities of the torus and compare our results with those corresponding to General Relativity. We analyze the impact of the variation of the Ricci scalar on the synchrotron emission associated with a supermassive black hole.

Keywords / black holes — accretion, accretion disks — gravitation

1. Introduction

General Relativity (GR) is a theory of space, time and gravitation formulated by Albert Einstein in 1915. Though GR has been extensively validated through experiments and observations (Will, 2006), it is not free of problems.

When GR is applied to scales of galaxies and galaxies clusters, it requires the introduction of a new type of substance, the so-called dark matter to explain the rotation curves and galaxy velocities. At much larger scales, an unknown dark energy field is postulated within the theory in order to account for the observed accelerated expansion of the universe.

Instead of introducing unknown entities into the world, an alternative approach consists of modifying the theory of gravitation. These new theories should reproduce GR predictions on the scales of the Solar System, but they might differ in some not well-explored domains, such as large scales and in the strong gravity regime.

One way to develop an alternative theory of gravitation is by modifying the relativistic action. In $f(R)$ theories, the Ricci scalar R in the action is replaced by a general function $f(R)$ that allows for additional degree of freedom and new gravitational phenomena (de La Cruz-Dombriz & et al., 2009; De Felice & et al., 2010). One of the main motivations for developing $f(R)$-gravity was that it could predict the effects of the accelerated expansion of the universe without invoking a dark energy field.

Though $f(R)$-theories have been extensively tested at large scales, the strong field regime of the theory deserves much further exploration. In particular, the astrophysical phenomena that occur very close to black holes could provide specific signatures that might unveil deviations from GR. Various methods can be employed to investigate these theories in the strong field regime, for instance, by Pulsar Timing and Laser-Interferometer Gravitational-Wave Detectors (Shao et al., 2017).

Pérez et al. (2013) were the first to study relativistic thin accretion disks around black holes in $f(R)$-gravity. They were able to constrain the values of a free parameter of the theory, denoted, R_0, through comparisons of current observational data of Cygnus X-1.

Relativistic geometrically thin and optically thick accretion disk models are good approximations of the flow around black holes when the accretion rate is significant. However, when the accretion rate is low, the density drops, and the Coulombian interaction between ions and electrons is not efficient. This gives rise to a two temperature plasma that radiates inefficiently. Since a fraction of the energy is kept in the gas, the disk inflates and becomes geometrically thick.

The best models available that describe hot accretion disks are Advection Dominated Accretion Flow (ADAF) (see for e.g., Narayan & et al., 2008; Gutiérrez et al., 2021). Abramowicz (1978) developed a similar model that has the advantage of being analytical; these are known in the literature as polish doughnuts or ion tori. It has been applied, in the framework of GR, to model the emission of Sgr A* (Straub et al., 2012).

In this short article, we extend the ion tori model to $f(R)$-gravity. In particular, we compute the geometrical structure of an ion torus around an $f(R)$-Kerr black hole and its emission spectra. In particular, We analyze the impact of the variation of the Ricci scalar on the synchrotron emission associated with a supermassive black hole. We compare the results obtained with those corresponding to GR and analyze the main differences.

2. $f(R)$ theory

In $f(R)$ gravity, the Lagrangian of the Hilbert-Einstein action is generalized to:

$$S[g] = \frac{c^3}{16\pi G}\int (R + f(R))\sqrt{-g}d^4x, \tag{1}$$

where g is the determinant of the metric tensor, and $f(R)$ is an arbitrary function of the Ricci scalar.

The geometry that describes a black hole with mass, electric charge, and angular momentum, which is axisymmetric, stationary, and has a constant Ricci scalar, was used by Cembranos et al. (2011) to study black holes in the context of $f(R)$ gravity, and is given by

$$\begin{aligned} ds^2 &= -\frac{c^2}{r^2\Xi^2}(\Delta_r - a^2)dt^2 \\ &- \frac{2ac}{r^2\Xi^2}(r^2 + a^2 - \Delta_r)dtd\varphi \\ &+ \frac{d\varphi^2}{r^2\Xi^2}[(r^2+a^2)^2 - \Delta_r a^2], \qquad (2) \\ \Delta_r &\equiv (r^2+a^2)(1 - \frac{R_0}{12}r^2) - \frac{2GMr}{c^2} \\ \Delta_\theta &\equiv 1 + \frac{R_0}{12}a^2\cos^2\theta \\ \rho^2 &\equiv r^2 + a^2\cos^2\theta\,, \qquad \Xi \equiv 1 + \frac{R_0}{12}a^2. \end{aligned}$$

In the limit $R_0 \to 0$, the line element (2) becomes Kerr's in General Relativity.

3. Ion Tori model

We consider a flow that consists of a perfect fluid torus orbiting a Kerr-$f(R)$ black hole (see Straub et al. 2012 for an application of this model in the context of GR to Sgr. A*). The 4-velocity of timelike geodesics has the form $u^\mu = (u^t, 0, 0, u^\phi)$, which results in circular orbits. The geodesic motion has two constants, one for stationarity and the other for axisymmetry

$$\begin{aligned} E &= -u_t = -u_t(g_{tt} + \Omega g_{t\varphi}) \qquad (3) \\ L &= u_\varphi = u_{t(g t\varphi} + \Omega g_{\varphi\varphi}), \qquad (4) \end{aligned}$$

where $\Omega = u_\varphi/u_t$ is the angular velocity with respect to a distant observer. The specific angular momentum is

$$\ell = \frac{L}{E} = -\frac{u_\varphi}{u_t}. \tag{5}$$

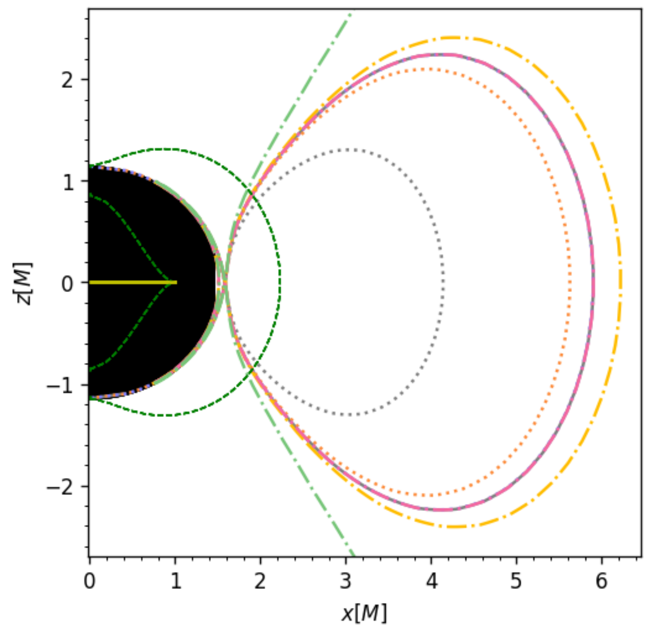

Figure 1: Meridional cut associated with different values of R_0. For $R_0 > 0$, the values are represented by dashdot lines, and for $R_0 < 0$, the values are represented by dotted lines. The R_0 values are shown in different colors: gray for -10^{-3}, orange for -10^{-4}, green for 10^{-3}, and yellow for 10^{-4}. The values close to 0, such as -10^{-6} and 10^{-6}, do not show significant differences compared to the Kerr case. The dashed green line represents the outer and inner ergosphere for a Kerr black hole. The solid yellow line represents the singularity.

The equation for conservation of energy and momentum can be solved to yield an expression for the gravitational potential function:

$$W(r,\theta) = \frac{1}{2}\ln\frac{-g_{tt} + 2\Omega g_{t\phi} + g_{\phi\phi}}{(g_{tt} + \Omega g_{t\phi})^2}. \tag{6}$$

The angular velocity of the torus, Ω, and the metric components, $g_{\alpha\beta}$, determine the equipotential surface that defines the torus boundaries. This surface crosses itself in a cusp, allowing accretion onto the black hole without the need for viscosity and the resulting loss of angular momentum (it is a Roche lobe overflow-like accretion). We can obtain analytical expressions for the temperatures of both gas species by assuming a two-temperature gas and using a polytropic equation of state:

$$T_e = [(1-\omega)\mathcal{M} + \omega\mathcal{M}_\xi]\mu_e(1-\beta)m_u P/k_B\epsilon \tag{7}$$

$$T_i = [(\mu_e/\mu_i)\mathcal{M} + \omega(\mathcal{M} - \mathcal{M}_\xi)]\,\mu_i(1-\beta)m_u P/k_B\epsilon \tag{8}$$

The dimensionless potential ω is calculated by dividing the magnetic pressure P_{mag} by the total pressure P, resulting in the fraction $\beta = P_{mag}/P$. This fraction is then multiplied by the mass density ϵ, the mean molecular weights for each species $\mu_{e,i}$, the atomic mass unit μ, the Boltzmann constant k_B, and the quantities $\mathcal{M} = \frac{\mu_i}{\mu_e + \mu_i}$ and $\mathcal{M}_\xi = \frac{\mu_i\xi}{\mu_e + \mu_i\xi}$.

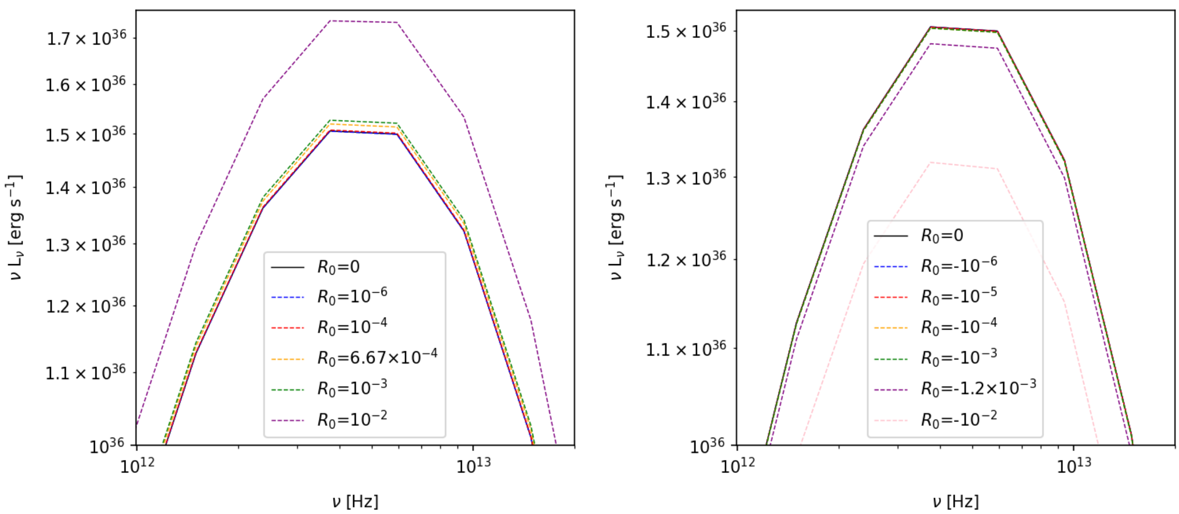

Figure 2: Zoom of the calculated Synchrotron emission for different values of R_0

Parameter	λ	$T_{e,c}$	ξ	ϵ_c	β	n	a
Value	0.7	10^{11}	10^{-2}	10^{-17}	0.1	3.0	0.99

Table 1: Free parameters used for modeling the Ion Tori. $T_{e,c}$ is in K units and ϵ_c is in g cm^{-3} units.

We compute numerically the thermal spectrum of the torus. We assume that electrons produce synchrotron radiation and free-free emission through interactions with both electrons and ions.

4. Results

In the model of the accretion disk in the $f(R)$-Kerr metric, a set of free parameters is considered, including the dimensionless specific angular momentum of the torus λ, the central electron temperature $T_{e,c}$, the ratio of the electron temperature to the ion temperature at the torus center ξ, the central mass density ϵ_c, the magnetic pressure fraction with respect to the total pressure $\beta = P_{mag}/P$, the polytropic index n, the mass M, specific black hole spin a, and the Ricci scalar R_0. The free parameters that we used to model the torus can be found in Table 1.

We study the ion tori structure for R_0 varying from $R_0 \in [-1.2 \times 10^{-3}; 6.67 \times 10^{-4}]$ for Kerr-$f(R)$ black holes, values given by Pérez et al. (2013) for a standard accretion disk. These should be compatible with the ion tori structure. In Fig. 1, we show the variations in the normalized potential of the Ion Tori. When $R_0 < 0$, the potential increases compared to Kerr (i.e. when $R_0 = 0$), causing the torus to become larger. When $R_0 > 0$, the potential decreases compared to Kerr, causing the torus to become smaller. This can be observed by comparing the values of the normalized potential for different values of R_0.

We calculate the synchrotron radiation for different values of R_0. In Fig. 2 it can be observed that for $R_0 > 0$, as the values of R_0 increase, the synchrotron luminosity increases. On the other hand, for $R_0 < 0$, as they become smaller, the synchrotron luminosity decreases. We note that exactly the opposite occurs compared to the standard disk cases studied by Pérez et al. (2013), this is mainly due to the normalized potential.

We choose a = 0.99, given the observational evidence that points out the existence of a class of highly spinning supermassive black holes (Reynolds, 2013; Aschenbach, 2010). However, our goal is to conduct a more thorough investigation in a forthcoming paper and delve deeper into the parameter space. Specifically, we plan to perform a fit analysis on the observational Sgr A* data, examining how both spin, Ricci scalar, and inclination affects the results.

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References

Abramowicz M., 1978, A&A, 63, 221
Aschenbach B., 2010, MmSAI, 81, 319
Cembranos J.A.R., de la Cruz-Dombriz A., Jimeno Romero P., 2011, arXiv e-prints, arXiv:1109.4519
De Felice, et al., 2010, Living Reviews in Relativity, 13, 3
de La Cruz-Dombriz A., et al., 2009, PhRvD, 80, 124011
Gutiérrez E.M., Vieyro F., Romero G., 2021, A&A, 649, A87
Narayan R., et al., 2008, NewAR, 51, 733
Pérez D., Romero G.E., Perez Bergliaffa S.E., 2013, A&A, 551, A4
Reynolds C.S., 2013, Classical and Quantum Gravity, 30, 244004
Shao L., et al., 2017, Physical Review X, 7, 041025
Straub O., et al., 2012, A&A, 543, A83
Will C.M., 2006, Living Reviews in Relativity, 9, 3