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Abstract

In this paper a k-nearest neighbor type estimator of the marginal density function for a random field which evolves with time is considered. Considering dependence, the consistency and asymptotic distribution are studied for the stationary and nonstationary cases. In particular, the parametric rate of convergence $\sqrt{T}$ is proven when the random field is stationary. The performance of the estimator is shown by applying our procedure to a real data example.

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Density estimation for spatial-temporal models

Liliana Forzani · Ricardo Fraiman · Pamela Llop

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Abstract In this paper a k-nearest neighbor type estimator of the marginal density function for a random field which evolves with time is considered. Considering dependence, the consistency and asymptotic distribution are studied for the stationary and nonstationary cases. In particular, the parametric rate of convergence $\sqrt{T}$ is proven when the random field is stationary. The performance of the estimator is

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Keywords Spatio-temporal data · Density estimation · Local times

Mathematics Subject Classification 91B72 · 62G07 · 60J55

1 Introduction

In the last decade there has been a significant growth on research of functional data as well as spatial-temporal data (see Tang et al. 2008). However, not all the theory developed for curves has been extended to random fields as is the case of nonparametric marginal density estimation which so far has only been performed for $\mathbb{R}^N$-valued random fields. For instance, for mixing stationary random fields, Tran and Yakowitz (1993) proved the asymptotic normality of the k-nearest neighbor estimator. Tran

L. Forzani · P. Llop (✉)
Facultad de Ingeniería Química and Instituto de Matemática Aplicada del Litoral, UNL-CONICET, Santa Fe, Argentina
e-mail: lloppamela@gmail.com

R. Fraiman
Departamento de Matemática and Ciencias, Universidad de San Andrés, Buenos Aires, Argentina

R. Fraiman
CMAT, Universidad de la República, Montevideo, Uruguay

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(1990) obtained the asymptotic normality of kernel type estimator, while Carbon et al. (1996) studied its L_1 convergence. The uniform consistency of this kind of estimator was shown by Carbon et al. (1997) and further extensions were studied by Hallin et al. (2001, 2004).

For functional data, the problem of nonparametric marginal density estimation has been considered by several authors in different setups. The case when a single sample path is observed over an increasing interval $[0, T]$ as T grows to infinity has been studied by Rosenblatt (1970), Nguyen (1979), and Castellana and Leadbetter (1986). In particular, the latter showed that for continuous time processes a parametric speed of convergence is attained by kernel type estimators. More recently, some extensions have been obtained by Blanke and Bosq (1997), Blanke (2004), Kutoyants (2004) among others. In particular, Labrador (2008) proposed a k-nearest neighbor type estimator using local time ideas. Later, Llop et al. (2011) redefined the k-nearest neighbor estimator for the case when an independent sample is observed, obtaining parametric rates of convergence and its asymptotic normality.

This work addresses the problem of nonparametric marginal density estimation for random fields which evolve in time. More precisely, given the following random field:

$$\mathcal{X}(\mathbf{s}) = \mu(\mathbf{s}) + e(\mathbf{s}), \quad \mathbf{s} \in \mathbf{S} \subset \mathbb{R}^d, \tag{1}$$

we estimate its marginal density function using a k-nearest neighbor type estimator defined via local time when a dependent sample of identically distributed random fields $\mathcal{X}_1, \ldots, \mathcal{X}_T$ is given. For this estimator we study its asymptotic properties. The functional nature of the data (a random surface) plays a fundamental role which allows us to obtain parametric rates of convergence of this density estimator in the stationary case, contrary to what generally happens in nonparametric problems. This kind of data appears naturally when analyzing the evolution of some measurements in a geographical area (such as the Amazon), or when recording responses from the brain during a time interval, among other interesting practical problems.

This paper is organized as follows: in Sect. 2 we recall some well-known dependence notions and give a new one which will be used in this work. Section 3 is dedicated to theoretical results. More precisely, in Sect. 3.2 we introduce the estimator for the stationary case, prove its consistency obtaining strong rates of convergence, and show its asymptotic normality. In Sect. 3.3, we extend the definition given in Sect. 3.2 to nonstationary random fields, and also obtain its rates of convergence. Section 4 is devoted to numerically show the performance of our estimation methods. In Sect. 4.1 a simulated example is presented for $d = 2$, and in Sect. 4.2 a real data example of fMRI images corresponding to the brain in the *resting state* is considered. Main and auxiliary proofs are given in Appendices B and C, respectively.

2 Dependence notions

In this section we will review some known dependence notions and introduce a new one which will be used in this work to find convergence rates of our density estimators. We will start with the classical α-mixing condition, introduced by Rosenblatt (1956) whose definition is the following:

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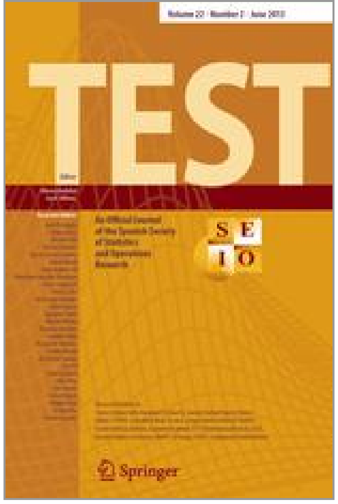

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Authors

- Liliana Forzani [1]
- Ricardo Fraiman [2] [3]
- Pamela Llop [1]

Author Affiliations

- 1. Facultad de Ingeniería Química and Instituto de Matemática Aplicada del Litoral, UNL-CONICET, Santa Fe, Argentina
- 2. Departamento de Matemática and Ciencias, Universidad de San Andrés, Buenos Aires, Argentina
- 3. CMAT, Universidad de la República, Montevideo, Uruguay

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