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Mode I stress intensity factor for cracked thin-walled open beams

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A general analytical method to determine the mode I stress intensity factor for thin-walled beams is presented. This method is based on the concept of crack surface widening energy release rate, which is expressed in terms of the G^* integral and the thin-walled beam theory. A distinctive aspect of this technique is the incorporation of the warping effect, which is a common feature in thin-walled beams that significantly influences in the stress distribution. This characteristic gives generality to the method, allowing the analysis of crack scenarios that have not been yet considered by other authors. The results show a good agreement with shell finite element solutions and other results available in the literature.

1. Introduction

Thin-walled beams are widely employed in modern engineering structures. For this reason, the study of crack behavior in these structural components represents a topic of crucial importance. The mode I stress intensity factor is a very significant parameter in the integrity evaluation and risk analysis of structures. The determination of an exact solution for the stress intensity factor is usually a difficult enterprise. In thin-walled beams, the presence of sectional warping constitutes an additional problem. Although some approaches have recently been proposed in this direction [1–3], no one have regarded flexural–torsional loads, which activate the warping effect.

The purpose of this article is to present a technique to determine the mode I stress intensity factor for cracked thin-walled beams. This technique is based on the G^* integral concept and the thin-walled beam theory. G^* integral [4] is derived from the conservation law and the concept of crack mouth widening energy release rate. It has shown to be easy to employ in the determination of closed forms for the stress intensity factors in several crack problems [1,2,5]. Thin-walled open beam theory [6–8] considers the warping effect derived from the natural flexural–torsional coupling of this kind of structures. In the determination of mode I stress intensity factor, warping is taken into account by considering the energetic contribution of the bimomental force.

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Nomenclature

a	crack depth (also semi-major axis of the elliptic crack)
$\tilde{a}$	boundary of the elliptic crack
A	cross-sectional area
b	dimension of a flange
B	bimomental beam force (also point B, origin of the system B: x, s, n)
c	semi-minor axis of the elliptic crack
C	center of gravity of the uncracked cross section
C_{ij}	element of order (i, j) of the inverse of the constitutive matrix
C_w	warping constant
G^*	crack mouth widening energy release rate
$\mathbf{G_c}$	vector containing elements of the inverse of the constitutive matrix
h	dimension of the web
$\mathbf{I_i}$	identity matrix of size i
I_y, I_z	second moments of area
I_{yz}	product moment of area
$I_{y\omega}$, $I_{z\omega}$	product of warping
$\mathbf{J}$	constitutive matrix
K_I	mode I stress intensity factor
L	length of the beam
n	coordinate normal to the cross-section middle line
N	axial beam force
M_y, M_z	bending moments
$\mathbf{Q}$	vector of generalized forces
$\mathbf{Q_c}$	vector containing squares and products of the generalized forces
r	radial coordinate
s	circumferential coordinate
S	cross-sectional perimeter
S_y, S_z	first moments of area
S_ω	first moment of warping
t	beam thickness
$\mathbf{T}$	stress vector
T_x, T_s	elements of the stress vector
u	axial displacement
$\mathbf{u}$	displacement vector
U	strain energy
U_0	strain energy density
v	circumferential displacement
x, y, z	Cartesian coordinates
Y, Z	coordinates of a point located in the middle line of the cross-section
α_i	coefficients of the axial stress in cracked cross-section
γ	vector of shape functions associated to cracked cross-section
Γ	integration path
Δ	vector of generalized strains
η_x, η_s	components of the unit outward normal vector
θ	angular coordinate
θ_x	warping variable
θ_y, θ_z	bending twists
λ	auxiliary integration variable
ν	Poisson's ratio
ξ	crack location (axial coordinate)
Π	total potential energy
σ_{xx}	axial stress
ω_p	primary warping function
$(\cdot)^{(0)}$	superscript associated to the uncracked cross-section
$(\cdot)^{(c)}$	superscript associated to the cracked cross-section
$(\cdot)^{(R)}$	superscript associated to the cracked flange or web of the beam

2. G* integral and mode I stress intensity factor

From the conservation law, the two-dimensional G^* integral can be defined as

$$G^* = t\int_\Gamma \left(U_0\eta_x - \boldsymbol{T}\frac{\partial \boldsymbol{u}}{\partial x}\right)d\Gamma, \tag{1}$$

where U_0 is the strain energy density, $\boldsymbol{\eta} = \{\eta_x, \eta_s\}$ the unit outward normal, $\boldsymbol{T} = \{T_x, T_s\}$ the stress vector applied on the outer side of the path Γ and t the thickness of the beam. The vector $\boldsymbol{u} = \{u, v\}$ contains the displacements from the Irwin–Westergaard field [9,10]. Let the crack in Fig. 1b. be a two-dimensional simplification of the three-dimensional edge-crack in Fig. 1a. For the path Γ_{dfg}, G^* represents the energy release rate due to the moving crack boundary dfg in the x direction. As the crack mouth widens, G^* can be regarded as the crack mouth widening energy release rate [4].

Considering plane strain and employing elementary concepts from Theory of Elasticity, the integral in expression (1) can be solved as [1,4,5]

$$G^* = \frac{K_I^2(1-\nu^2)t}{\pi E}, \tag{2}$$

where K_I is the mode I stress intensity factor, E is the Young modulus and ν the Poisson ratio.

3. Energy release rate for cracked thin-walled beams

3.1. A cracked thin-walled beam

The points of the beam are referred to a Cartesian coordinate system (x, y, z), which origin C is located at the centroid of the uncracked cross-section. A circumferential coordinate s and a normal coordinate n are also defined in the middle line of the cross-section. A point lying on this middle line has coordinates Y and Z.

A crack with depth a and location $x = \xi$ is regarded as an elliptical hole under the condition of $c \to 0$ [1,4,5]. Thus, the crack boundary can be expressed as

$$\tilde{a}(x) = a\sqrt{1 - \frac{(x-\xi)^2}{c^2}}. \tag{3}$$

3.2. Constitutive expression for mode I loading

The constitutive equation associated to a thin-walled beam in mode I loading can be written as [8]

$$\mathbf{Q} = \boldsymbol{J}\boldsymbol{\Delta}, \tag{4}$$

where $\mathbf{Q}$ is the vector of generalized beam forces, $\boldsymbol{J}$ the constitutive matrix and $\boldsymbol{\Delta}$ the vector of generalized strains. Their expressions are

$$\mathbf{Q} = \{N, M_y, M_z, B\}^T, \tag{5}$$

$$\boldsymbol{\Delta} = \left\{\frac{\partial u}{\partial x}, -\frac{\partial \theta_y}{\partial x}, -\frac{\partial \theta_z}{\partial x}, -\frac{\partial \theta_x}{\partial x}\right\}^T, \tag{6}$$

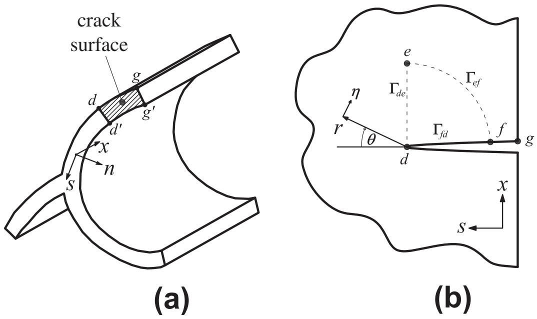

Fig. 1. (a) Three-dimensional edge-crack in a generic beam and (b) its corresponding two-dimensional simplification.

$$\boldsymbol{J} = E\begin{bmatrix} A & S_y & S_z & S_\omega \\ & I_y & I_{yz} & I_{y\omega} \\ & & I_z & I_{z\omega} \\ & & & C_w \end{bmatrix}. \tag{7}$$

The generalized beam forces have been defined in Eq. (5): N as the axial force, M_y and M_z as the bending moments and B as the bimoment. The cross-sectional constants are

$$\begin{gathered} A = t\int_S ds, \quad S_y = t\int_S Zds, \quad S_z = t\int_S Yds, \quad S_\omega = t\int_S \omega_p ds, \\ I_y = t\int_S Z^2 ds, \quad I_z = t\int_S Y^2 ds, \quad I_{yz} = t\int_S YZds, \\ I_{y\omega} = t\int_S Z\omega_p ds, \quad I_{z\omega} = t\int_S Y\omega_p ds, \quad C_w = t\int_S \omega_p^2 ds, \end{gathered} \tag{8}$$

where S denotes the contour perimeter of the cross-section and ω_p is the primary warping function [8]. The generalized strains in vector $\boldsymbol{\Delta}$ are defined in terms of the generalized displacements: u as the axial displacement, θ_y and θ_z as the bending rotations and θ_x as the warping variable.

3.3. Strain energy

The axial stress distribution on the cracked cross-section is assumed to be of the same mathematical form as in the case of an uncracked thin-walled beam. The following expression is then proposed for the axial stress acting on the cracked cross-section [11]

$$\sigma_{xx}^{(c)} = \alpha_0 + \alpha_Z Y + \alpha_Y Z + \alpha_\omega \omega_p^{(0)}, \tag{9}$$

where $\omega_p^{(0)}$ corresponds to the warping function of the uncracked cross-section. As an approximation, $\omega_p^{(0)}$ is taken to be valid also in the cracked zone. Considering that static equilibrium must be preserved, coefficients α_0, α_Y, α_Z and α_ω are obtained by solving the following linear system.

$$\begin{cases} N|_{x=\xi} = t\int_{S^{(c)}} \sigma_{xx}^{(c)} ds \\ M_z|_{x=\xi} = t\int_{S^{(c)}} Y\sigma_{xx}^{(c)} ds \\ M_y|_{x=\xi} = t\int_{S^{(c)}} Z\sigma_{xx}^{(c)} ds \\ B|_{x=\xi} = t\int_{S^{(c)}} \omega_p^{(0)}\sigma_{xx}^{(c)} ds \end{cases}, \tag{10}$$

where $S^{(c)}$ is the contour perimeter of the cracked cross-section, which depends on the x coordinate, since it depends on $\tilde{a}(x)$. With axial stress fully defined, the associated strain energy of the beam in Fig. 2. can be expressed as

$$U = \frac{1}{2}\left[\int_0^{\xi-c} \mathbf{Q}^T\left(\boldsymbol{J}^{(0)}\right)^{-1}\mathbf{Q}dx + \frac{t}{E}\int_{S^{(c)}}\int_{\xi-c}^{\xi+c}\left(\sigma_{xx}^{(c)}\right)^2 dxds + \int_{\xi+c}^{L}\mathbf{Q}^T\left(\boldsymbol{J}^{(0)}\right)^{-1}\mathbf{Q}dx\right], \tag{11}$$

where $\boldsymbol{J}^{(0)}$ is the constitutive matrix associated to the uncracked cross-section. Considering (9) and (10), the expression (11) is reformulated as

$$U = \frac{1}{2}\int_0^{\xi-c}\mathbf{Q}^T\left(\boldsymbol{J}^{(0)}\right)^{-1}\mathbf{Q}dx + c\mathbf{Q_c}|_{x=\xi}\boldsymbol{I}_{10}\int_0^1 \gamma d\lambda + \frac{1}{2}\int_{\xi+c}^{L}\mathbf{Q}^T\left(\boldsymbol{J}^{(0)}\right)^{-1}\mathbf{Q}dx, \tag{12}$$

where $\boldsymbol{I_{10}}$ is the order 10 identity matrix, $\boldsymbol{\gamma}$ is a vector of shape functions expressed in terms of the cracked cross-sectional constants (see Appendix A for details) and

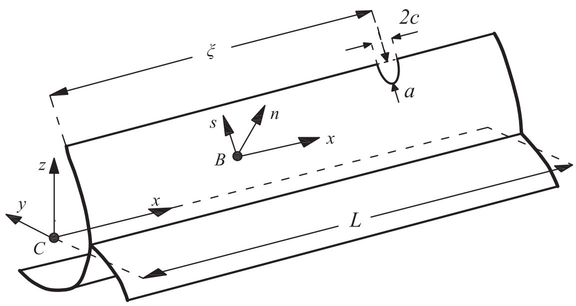

Fig. 2. Generic cracked thin-walled beam. Crack regarded as an elliptical hole ($c \to 0$).

$$\boldsymbol{Q_c} = \left\{N^2, NM_y, M_y^2, NM_z, M_yM_z, M_z^2, NB, M_yB, M_zB, B^2\right\}. \tag{13}$$

The additional integration variable $\lambda = (x - \xi)/c$ has been defined in Eq. (12).

3.4. Energy release rate

From Clapeyron's theorem, the work of external loads is $V = 2\,U$. The potential energy is given by $\Pi = U - V$. Then the crack surface widening energy release rate can be expressed as

$$G^* = \lim_{c\to 0}\frac{\partial U}{\partial c} = \frac{1}{2}\frac{\partial}{\partial c}\left(\int_0^{\xi-c}\boldsymbol{Q}^T\left(\boldsymbol{J}^{(0)}\right)^{-1}\boldsymbol{Q}dx + \frac{1}{2}\int_{\xi+c}^{L}\boldsymbol{Q}^T\left(\boldsymbol{J}^{(0)}\right)^{-1}\boldsymbol{Q}dx\right) + \boldsymbol{Q_c}|_{x=\xi}\boldsymbol{I}_{10}\int_0^1\gamma d\lambda. \tag{14}$$

This expression can be rearranged by applying the fundamental theorem of calculus to give

$$G^* = \boldsymbol{Q_c}|_{x=\xi}\left(\boldsymbol{I}_{10}\int_0^1\gamma d\lambda - \boldsymbol{G_c}^{(0)}\right), \tag{15}$$

where

$$\boldsymbol{G_c}^{(0)} = \{C_{11}, 2C_{12}, C_{22}, 2C_{13}, 2C_{23}, C_{33}, 2C_{14}, 2C_{24}, 2C_{34}, C_{44}\}, \tag{16}$$

and the coefficients C_{ij} are the elements of the matriz $(\boldsymbol{J}_{(0)})^{-1}$.

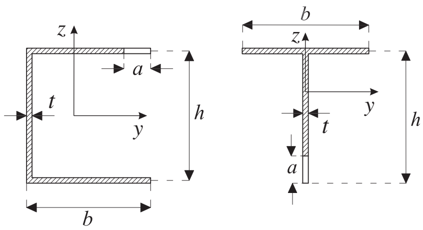

Fig. 3. Cross-sectional shapes used and its corresponding crack dispositions.

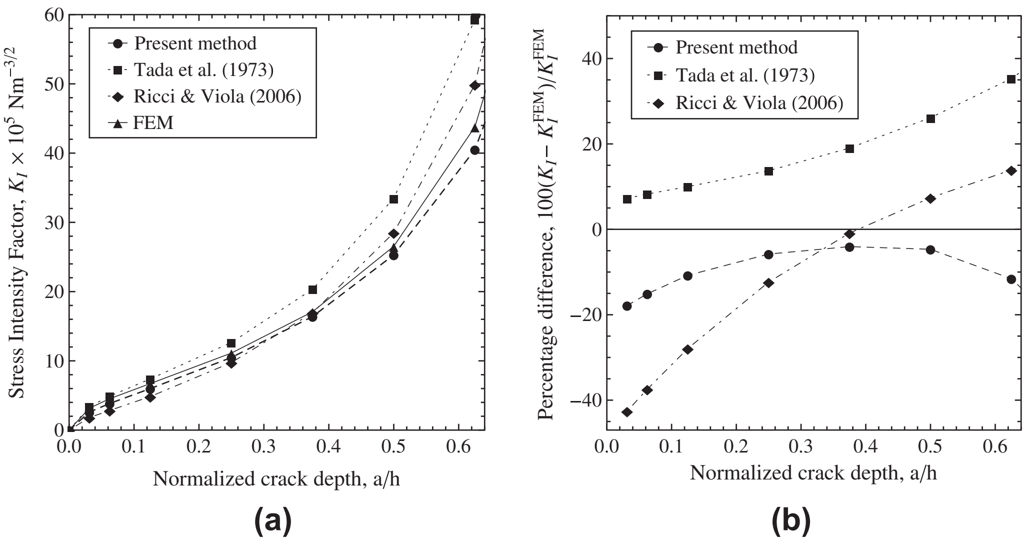

Fig. 4. (a) Stress intensity factor for a cracked thin-walled T beam under axial loading (N = 6 kN, no warping). (b) Percentage difference with respect to FEM results.

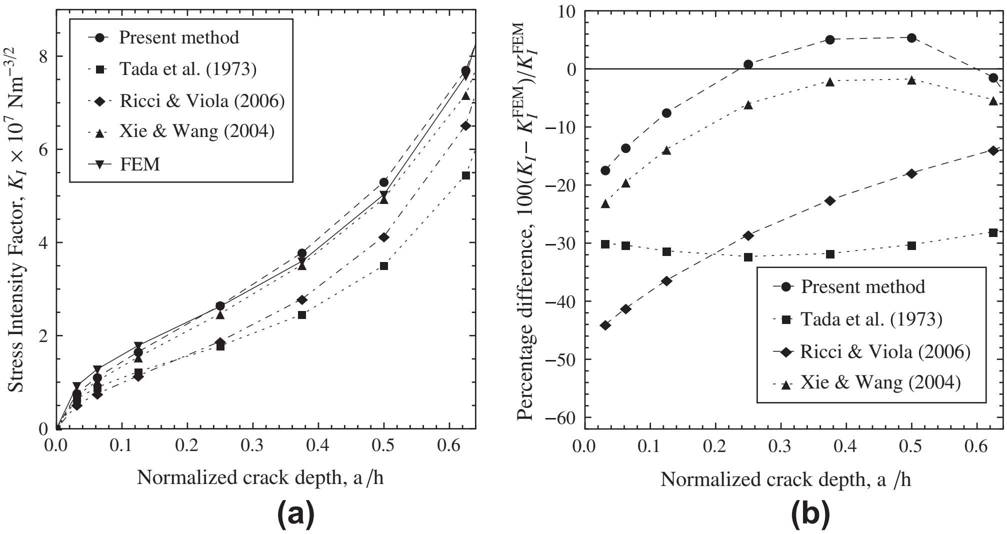

Fig. 5. (a) Stress intensity factor for a cracked thin-walled T beam under bending (M_y = 6 kN m, no warping). (b) Percentage difference with respect to FEM results.

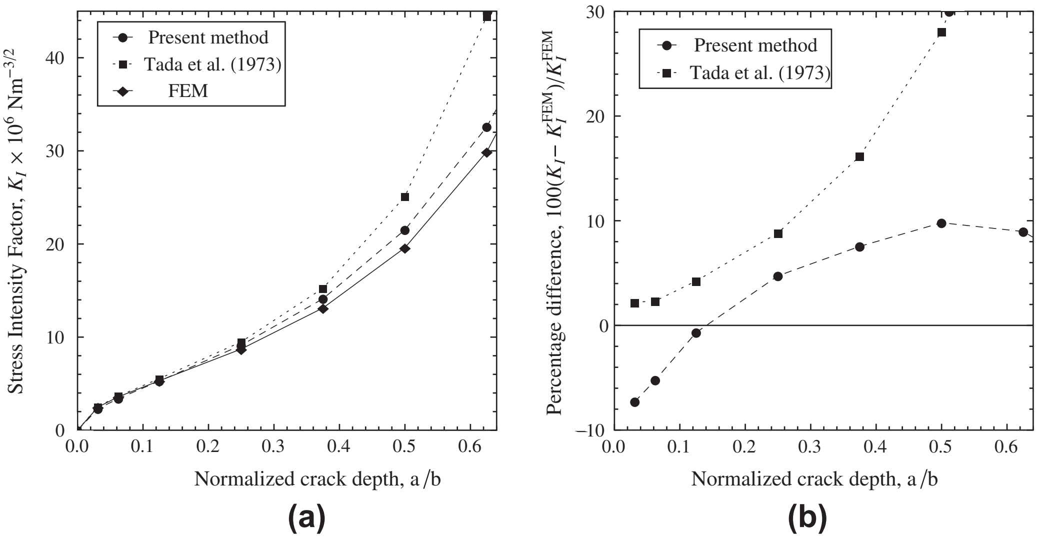

Fig. 6. (a) Stress intensity factor for a cracked thin-walled U beam under bending (M_y = 6 kN m, B = 341 N m^2). (b) Percentage difference with respect to FEM results.

4. Mode I stress intensity factor

Eq. (2) is obtained by solving the G^* integral, while Eq. (15) derives from classic mechanics of materials and thin-walled beam theory. But although both equations derive from different definitions, they both represent the crack mouth widening energy release rate. By equating these expressions of G^*, the mode I stress intensity factor can be obtained as

$$K_I(a) = \sqrt{\frac{\pi E}{t(1-\nu^2)} \boldsymbol{Q_c}|_{x=\xi} \left(\boldsymbol{I}_{10} \int_0^1 \gamma d\lambda - \boldsymbol{G}_{\boldsymbol{c}}^{(0)} \right)} \tag{17}$$

Eq. (17) shows that K_I depends on crack depth a, generalized beam forces in cracked cross-sectional area $\boldsymbol{Q_c}|_{x=\xi}$, material properties E and ν, beam thickness t and properties of cracked and uncracked cross-section, given in γ and $\boldsymbol{G}_{\boldsymbol{c}}^{(0)}$, respectively. Expression (17) is general and can be applied to edge-cracked thin-walled beams of any cross-section.

5. Results and discussion

For cross-sectional shapes in Fig. 3, we present comparisons of Eq. (17) with results from finite element method (FEM) and other authors in open literature. In FEM analysis, we employed ABAQUS 6.7 package [12,13], meshing with 8-node shell elements (S8R5). The elements used in the neighborhood of crack tip were six-node triangular quarter-point elements (STRI65). Around 3000 elements were used for meshing the beams. For both, T and U beams, the dimensions considered were h = 0.2 m, b = 0.1 m, t = 0.01 m and L = 2 m. Crack location was set to $\xi/L = 0.5$.

Also, in all the comparisons, a rearrangement of classical K_I formulas was included [14], which considers the cracked flange as an independent plate (see Appendix B for more details).

Ricci and Viola's formula [3] was considered in the example of the axially loaded T beam with cracked web. Referring to FEM results, the present method shows a good performance for a wide range of crack depths, as can be seen in Fig. 4. Despite of its simplicity, classical formula yields better results for very small cracks (Referenced in Figure as Tada et al.). The approach from Ref. [3] fails to 40% difference with respect to FEM results for very small cracks.

For the T beam with bending load (no warping), the present method and Xie and Wang's formula [1] show the best results, as can be seen in Fig. 5. Both approaches employ G^* integral concept, so they were expected to give similar results.

There are no approaches in the literature for the case of the U beam with a crack only at one flange. Therefore only FEM and adapted classical formula were considered in the comparison of Fig. 6. For this example, in which a flexural–torsional load is considered, the difference among the present method and FEM is less than 10%, regardless of the crack depth. Classical approach shows a good agreement for small cracks, but blows up for moderate to large cracks.

6. Conclusions

A new method to determine the mode I stress intensity factor for cracked thin-walled beams is presented. This approach may be considered as an extension of the Xie's method [4] in order to take into account the influence of the warping effect, a very common feature in thin-walled beams. This is performed by considering the energetic contribution of the bimomental force. The method proves to be more versatile and, in a wide majority of scenarios, more accurate than other methods in the open literature. The proposed technique may represent an important contribution in the failure analysis and health monitoring of slender structures from civil, mechanical and aerospace industry.

In a future article, this approach will be extended in order to consider fiber reinforced plastics as constructive material.

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Appendix A. Components of vector $\boldsymbol{\gamma}$

The vector of shape functions $\boldsymbol{\gamma}$ defined in Eq. (12), is given by

$$\boldsymbol{\gamma} = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7, \gamma_8, \gamma_9, \gamma_{10}\}, \tag{A.1}$$

being its components expressed in terms of the cracked cross-sectional constants as

$$\gamma_1 = \frac{1}{\Psi}\left\{\left(I_{y\omega}^{(c)}\right)^2 I_z^{(c)} + C_w^{(c)}\left[\left(I_{yz}^{(c)}\right)^2 - I_y^{(c)} I_z^{(c)}\right] - 2 I_{yz}^{(c)} I_{y\omega}^{(c)} I_{z\omega}^{(c)} + I_y^{(c)}\left(I_{z\omega}^{(c)}\right)^2\right\}, \tag{A.2}$$

$$\gamma_2 = \frac{2}{\Psi}\left[C_w^{(c)} I_z^{(c)} S_y^{(c)} - \left(I_{z\omega}^{(c)}\right)^2 S_y^{(c)} - C_w^{(c)} I_{yz}^{(c)} S_z^{(c)} + I_{y\omega}^{(c)} I_{z\omega}^{(c)} S_z^{(c)} - I_{y\omega}^{(c)} I_z^{(c)} S_\omega^{(c)} + I_{yz}^{(c)} I_{z\omega}^{(c)} S_\omega^{(c)}\right], \tag{A.3}$$

$$\gamma_3 = \frac{1}{\Psi}\left\{A^{(c)}\left[-C_w^{(c)} I_z^{(c)} + \left(I_{z\omega}^{(c)}\right)^2\right] + C_w^{(c)}\left(S_z^{(c)}\right)^2 + S_\omega^{(c)}\left(-2 I_{z\omega}^{(c)} S_z^{(c)} + I_z^{(c)} S_\omega^{(c)}\right)\right\}, \tag{A.4}$$

$$\gamma_4 = \frac{2}{\Psi}\left[-C_w^{(c)} I_{yz}^{(c)} S_y^{(c)} + I_{y\omega}^{(c)} I_{z\omega}^{(c)} S_y^{(c)} + C_w^{(c)} I_y^{(c)} S_z^{(c)} - \left(I_{y\omega}^{(c)}\right)^2 S_z^{(c)} + I_{yz}^{(c)} I_{y\omega}^{(c)} S_\omega^{(c)} - I_{z\omega}^{(c)} I_y^{(c)} S_\omega^{(c)}\right], \tag{A.5}$$

$$\gamma_5 = \frac{2}{\Psi}\left[A^{(c)}\left(C_w^{(c)} I_{yz}^{(c)} - I_{y\omega}^{(c)} I_{z\omega}^{(c)}\right) - C_w^{(c)} S_y^{(c)} S_z^{(c)} + S_\omega^{(c)}\left(I_{z\omega}^{(c)} S_y^{(c)} + I_{y\omega}^{(c)} S_z^{(c)} - I_{yz}^{(c)} S_\omega^{(c)}\right)\right], \tag{A.6}$$

$$\gamma_6 = \frac{1}{\Psi}\left\{A^{(c)}\left[-C_w^{(c)} I_y^{(c)} + \left(I_{y\omega}^{(c)}\right)^2\right] + C_w^{(c)}\left(S_y^{(c)}\right)^2 + S_\omega^{(c)}\left(-2 I_{y\omega}^{(c)} S_y^{(c)} + I_y^{(c)} S_\omega^{(c)}\right)\right\}, \tag{A.7}$$

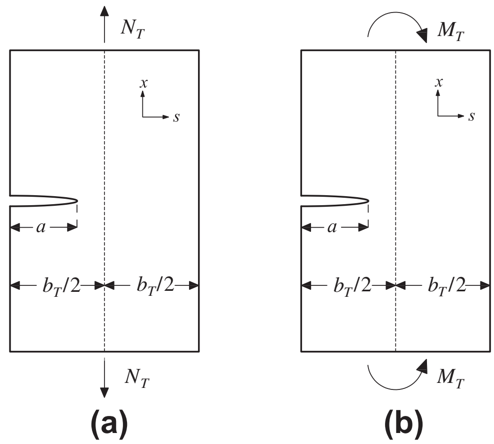

Fig. B1. (a) Cracked plate loaded with an axial force. (b) Cracked plate loaded with a bending moment.

$$\gamma_7 = \frac{2}{\Psi}\left[-I_{y\omega}^{(c)}I_z^{(c)}S_y^{(c)} + I_{z\omega}^{(c)}I_{yz}^{(c)}S_y^{(c)} + I_{y\omega}^{(c)}I_{yz}^{(c)}S_z^{(c)} - I_{z\omega}^{(c)}I_y^{(c)}S_z^{(c)} - \left(I_{z\omega}^{(c)}\right)^2 S_\omega^{(c)} + I_y^{(c)}I_z^{(c)}S_\omega^{(c)}\right], \tag{A.8}$$

$$\gamma_8 = \frac{2}{\Psi}\left[A^{(c)}I_{y\omega}^{(c)}I_z^{(c)} - A^{(c)}I_{z\omega}^{(c)}I_{yz}^{(c)} + I_{z\omega}^{(c)}S_y^{(c)}S_z^{(c)} - I_{y\omega}^{(c)}\left(S_z^{(c)}\right)^2 - I_z^{(c)}S_y^{(c)}S_\omega^{(c)} + I_{yz}^{(c)}S_z^{(c)}S_\omega^{(c)}\right], \tag{A.9}$$

$$\gamma_9 = \frac{2}{\Psi}\left[A^{(c)}I_{z\omega}^{(c)}I_y^{(c)} - A^{(c)}I_{y\omega}^{(c)}I_{yz}^{(c)} + I_{z\omega}^{(c)}\left(S_z^{(c)}\right)^2 + I_{y\omega}^{(c)}S_y^{(c)}S_z^{(c)} + I_{yz}^{(c)}S_y^{(c)}S_\omega^{(c)} - I_y^{(c)}S_z^{(c)}S_\omega^{(c)}\right], \tag{A.10}$$

$$\gamma_{10} = \frac{1}{\Psi}\left\{A^{(c)}\left[\left(I_{yz}^{(c)}\right)^2 - I_y^{(c)}I_z^{(c)}\right] + I_z^{(c)}\left(S_y^{(c)}\right)^2 + S_z^{(c)}\left(-2I_{yz}^{(c)}S_y^{(c)} + I_y^{(c)}S_z^{(c)}\right)\right\}, \tag{A.11}$$

where

$$\begin{aligned}\Psi = E\Big\{&A^{(c)}\left\{\left(I_{y\omega}^{(c)}\right)^2 I_z^{(c)} + C_w^{(c)}\left[\left(I_{yz}^{(c)}\right)^2 - I_y^{(c)}I_z^{(c)}\right] - 2I_{yz}^{(c)}I_{y\omega}^{(c)}I_{z\omega}^{(c)} + I_y^{(c)}\left(I_{z\omega}^{(c)}\right)^2\right\} - \left(I_{z\omega}^{(c)}S_y^{(c)} - I_{y\omega}^{(c)}S_z^{(c)}\right)^2 \\ &+C_w^{(c)}\left[I_z^{(c)}\left(S_y^{(c)}\right)^2 - 2I_{yz}^{(c)}S_y^{(c)}S_z^{(c)} + I_y^{(c)}\left(S_z^{(c)}\right)^2\right] + 2\left(-I_{y\omega}^{(c)}I_z^{(c)}S_y^{(c)} + I_{yz}^{(c)}I_{z\omega}^{(c)}S_y^{(c)} + I_{yz}^{(c)}I_{y\omega}^{(c)}S_z^{(c)} - I_y^{(c)}I_{z\omega}^{(c)}S_z^{(c)}\right)S_\omega^{(c)} \\ &+\left[-(I_{yz}^{(c)})^2 + I_y^{(c)}I_z^{(c)}\right](S_\omega^{(c)})^2\Big\}.\end{aligned} \tag{A.12}$$

Appendix B. Stress intensity factor for thin-walled beams from classical expressions

Classical formula of stress intensity factor proposed by Tada, Paris and Irwin [14] for thin plates is rearranged in order to be used for thin-walled beams. The flange (or web) with a crack is regarded as independent of the rest of the beam. Formula from Ref. [14] can be expressed as

$$K_I^{(T)} = K_I^{(N)} + K_I^{(M)}, \tag{B.1}$$

where

$$K_I^{(N)} = (\pi ba/b_T)^{1/2}\left[0.265(1-a/b_T)^4 + (0.857 + 0.265a/b_T)/(1-a/b_T)^{3/2}\right]N_T/(tb_T), \tag{B.2}$$

$$K_I^{(M)} = [b_T\tan(0.5\pi a/b_T)]^{1/2}\left\{\frac{0.923 + 0.199[1-\sin(0.5\pi a/b_T)]^4}{\cos(0.5\pi a/b_T)}\right\}6M_T/(tb_T^2). \tag{B.3}$$

N_T and M_T correspond respectively to reduced axial force and bending moment, applied in a plate of wide b_T as defined in Fig. B1. In order to consider the beam forces than can be present in a thin-walled beam, those reduced forces are obtained as

$$N_T = t\left(\frac{N}{A}\int_{S^{(R)}} ds + \frac{M_y}{I_y}\int_{S^{(R)}} Zds + \frac{M_z}{I_z}\int_{S^{(R)}} Yds + \frac{B}{C_w}\int_{S^{(R)}} \omega_p ds\right), \tag{B.4}$$

$$M_T = t\left(\frac{N}{A}\int_{S^{(R)}} sds + \frac{M_y}{I_y}\int_{S^{(R)}} sZds + \frac{M_z}{I_z}\int_{S^{(R)}} sYds + \frac{B}{C_w}\int_{S^{(R)}} s\omega_p ds\right), \quad (B.5)$$

where the integration domain $S_{(R)}$ refers to the wide of the plate.

References

[1] Xie YJ, Wang XH. Application of G∗ integral on cracked structural beams. J Constr Steel Res 2004;60:1271–90.
[2] Ghafoori E, Motavalli M. Analytical calculation of stress intensity factor of cracked steel I-beams with experimental analysis and 3D digital image correlation measurements. Engng Fract Mech 2011;78:3226–42.
[3] Ricci P, Viola E. Stress intensity factors for cracked T-sections and dynamic behavior of T-beams. Engng Fract Mech 2006;73:91–111.
[4] Xie YJ, Xu H, Li PN. Crack mouth energy-release rate and its application. Theor Appl Fract Mech 1998;29(3):195–203.
[5] Xie YJ, Wang XH, Lin YC. Stress intensity factors for cracked rectangular cross-section thin-walled tubes. Engng Fract Mech 2004;71:1501–13.
[6] Vlasov VZ. Thin walled elastic beams. Jerusalem: Israel Program for Scientific Translation; 1961.
[7] Gjelsvik A. Theory of thin walled beams. New York: John Wiley & Sons; 1981.
[8] Cortínez VH, Piovan MT. Vibration and buckling of composite thin-walled beams with shear deformability. J Sound Vibrat 2002;258(4):701–23.
[9] Westergaard HM. Bearing pressures and cracks. J Appl Mech 1939;6:A49–53.
[10] Irwin GR. Analysis of stresses and strains near the end of a crack traversing a plate. J Appl Mech 1957;24:361–4.
[11] Dotti FE. Identificación de daño en vigas de pared delgada isótropas y compuestas mediante el análisis de vibraciones, Doctoral Thesis, Universidad Nacional del Sur, Argentina; 2012.
[12] ABAQUS Analysis Manual v6.7. Dassault Systèmes; 2007.
[13] ABAQUS Theory Manual v6.7. Dassault Systèmes; 2007.
[14] Tada H, Paris PC, Irwin GR. Stress analysis of cracks handbook. St. Louis: Del Research Corporation; 1973.