Annals of Operations Research
https://doi.org/10.1007/s10479-022-04688-2

ORIGINAL RESEARCH

Operational transportation planning in the forest industry integrating bucking decisions

Maximiliano R. Bordón[1] · **Jorge M. Montagna**[2] · **Gabriela Corsano**[2,3]

Accepted: 21 March 2022

Abstract
The forest supply chain encompasses different closely related operations. Harvesting and transportation decisions are interdependent, where a modification in the former has a considerable impact on the latter. In the literature, these decisions are usually approached in a decoupled way, leading to suboptimal solutions. In this work, a mixed integer linear programming model that integrates both problems for a weekly planning horizon is presented. In addition to decisions about bucking patterns selection in each harvest area and the trucks routing, the composition of the load and the scheduling of the harvesting crews are considered. In this way, the different involved tradeoffs are simultaneously addressed and solved. Through the obtained results, the capabilities of the proposed model are analyzed.

Keywords Vehicle routing · Harvesting planning · MILP · Forest industry

List of symbols

B	Set of bucking patterns, $b = b_1, b_2, \ldots, b_{max}$
$BF_{b,f}$	Set of bucking patterns b that can be used in harvest area f
C	Set of trucks, $c = c_1, c_2, \ldots, c_{max}$
C_p	Set of trucks belonging to regional base p
D	Set of log diameters, $d = d_1, d_2, \ldots, d_{max}$
F	Set of harvest areas, $f = f_1, f_2, \ldots, f_{max}$

✉ Gabriela Corsano
gcorsano@santafe-conicet.gov.ar

Maximiliano R. Bordón
mbordon@santafe-conicet.gov.ar

Jorge M. Montagna
mmontagna@santafe-conicet.gov.ar

[1] Centro de Investigación y Transferencia (CIT, CONICET-UNRaf), Bv. Roca 989, (S2300), Rafaela, Argentina

[2] Instituto de Desarrollo y Diseño (INGAR, CONICET-UTN), Avellaneda 3657, (S3000), Santa Fe, Argentina

[3] Facultad de Ingeniería Química (Universidad Nacional del Litoral), Santiago del Estero 2829 (S3000), Santa Fe, Argentina

Published online: 13 April 2022

I	Set of plants, $i = i_1, i_2,..., i_{max}$
L	Set of log lengths, $l = l_1, l_2,..., l_{max}$
P	Set of regional bases, $p = p_1, p_2,..., p_{max}$
T	Set of time periods, $t = t_1, t_2,..., t_{max}$
V	Set of possible truck trips, $v = v_1, v_2,..., v_{max}$
$cd_{p,f}$	Cost per travelled kilometer between p and f ($/km)
$cl_{f,i}$	Cost per travelled kilometer between f and i ($/km)
$cu_{i,f}$	Cost per travelled kilometer between i and f ($/km)
$cr_{i,p}$	Cost per travelled kilometer between i and p ($/km)
$cap_{i,t}$	Plant processing capacity during period t (logs)
$capmax_c$	Maximum truck capacity (ton)
$capmin_c$	Minimum truck capacity (ton)
$cfix_{c,p}$	Fixed cost per use of truck ($/truck)
$closs_b$	Cost for loss of wood when applying pattern b ($/ton)
$cqrl_{l,d,f}$	Inventory cost of log of length l and diameter d in f ($/log)
$cstock_{l,d,i}$	Inventory cost of log of length l and diameter d in i ($/log)
$dpf_{p,f}$	Distance between p and f (km)
$dfi_{f,i}$	Distance between f and i (km)
$dif_{i,f}$	Distance between i and f (km)
$dip_{i,p}$	Distance between i and p (km)
$dmin_{l,d,i,t}$	Minimum committed demand (logs)
$dtot_{l,d,i}$	Total (weekly) demand (logs)
$fconv_{l,d,b}$	Log conversion (stem-to-log) by using bucking pattern b (log)
$loss_b$	Loss of raw material after applying bucking pattern b (%)
$maxstock_{l,d,i}$	Storage capacity, by type of log, in plant (logs)
$qcsmax_f$	Maximum amount of stems to cut in harvest area f (stems)
$qcsmin_f$	Minimum amount of stems to cut in harvest area f (stems)
$qend_f$	Desired amount of standing stems at the end of the planning horizon (stems)
$qini_f$	Initial stock of standing stems (stems)
$qminb_b$	Minimum number of times to apply bucking pattern b if used (stems)
$qttup_{l,d,c,v,f,i,t}$	Maximum amount of logs that a truck can load on a trip (logs)
$stockini_{l,d,i}$	Initial stock of logs, by type, in plant i (logs)
$maxt_{c,t}$	Maximum route duration time (h)
$vd_{p,f}$	Average travel speed between p and f (km/h)
$vl_{f,i}$	Average travel speed between f and i (km/h)
$vu_{i,f}$	Average travel speed between i and f (km/h)
$vr_{i,p}$	Average travel speed between i and p (km/h)
$weight_{l,d}$	Weight of log of length l and diameter d (ton)
$CUT_{f,t}$	Binary variable that indicates if harvest area f is cut in period t or not
$Q^{BP}_{b,f,t}$	Continuous variables that indicates the number of times bucking pattern b is used in f in period t
$Q^{CL}_{l,d,f,t}$	Continuous variable that indicates the amount of logs of each type that are generated in harvest area f in period t
$Q^{CS}_{f,t}$	Continuous variable that indicates the amount of stems that are cut in harvest area f in period t
$Q^{PL}_{l,d,i,t}$	Continuous variable that indicates the amount of logs of each type that are processed above the minimum demand in plant i in period t

$Q^{RL}_{l,d,f,t}$	Continuous variable that indicates the amount of logs of each type that remain on the roadside in harvest area f at the end of period t
$Q^{SL}_{l,d,i,t}$	Continuous variable that indicates the amount of logs of each type that are kept in inventory at plant i at the end of period t
$Q^{SS}_{f,t}$	Continuous variable that indicates the amount of stems that are left standing in harvest area f at the end of period t
$Q^{TL}_{l,d,f,t}$	Continuous variable that indicates the amount of logs of each type that are transported from harvest area f during period t
$Q^{TT}_{l,d,c,v,f,i,t}$	Continuous variable that indicates the amount of logs of each type that are transported from harvest area f to plant i on trip v of truck c in period t
$X^{D}_{c,p,f,f}$	Binary variable that indicates whether truck c during trip v travels from p to f in period t or not
$X^{L}_{c,v,f,i,t}$	Binary variable that indicates whether truck c during trip v travels from f to i in period t or not
$X^{U}_{c,v,i,f,t}$	Binary variable that indicates whether truck c during trip v travels from i to f in period t or not
$X^{R}_{c,v,i,p,t}$	Binary variable that indicates whether truck c during trip v travels from i to p in period t or not
$Y^{BP}_{b,f,t}$	Binary variable that indicates whether bucking pattern b is used in harvest area f in period t or not
$Y^{T}_{c,p,t}$	Binary variable that indicates whether truck c belonging to regional base p is used in period t or not
$Y^{TT}_{l,c,v,f,i,t}$	Binary variable that indicates whether logs of length l are loaded into truck c during trip v that travels from f to i in period t or not
Z	Objective function ($)

1 Introduction

Generally speaking, supply chain encompasses all activities related to the flow and transformation of goods, from the raw material extraction stage to the delivery of the final products, as well as the related information flow. Supply chain management is the integration of these activities by enhancing the relationships between the members involved to achieve a sustainable competitive advantage. In the forestry sector, the main activities are related to harvesting, transportation, inventory management and production. However, these activities are usually treated in a decoupled way, and even in different planning horizons (Borges et al., 2014; D'Amours et al., 2008).

When closely related activities are addressed in a decoupled way, each decision made can have a negative impact on the other; therefore, it is essential to tackle the performance of all operations simultaneously. In the forest industry, transportation and harvesting are the most expensive logistics activities (Simon et al., 2020), therefore, the integration of decisions such as the type and quantity of raw material to be harvested, the volume of raw material to be delivered and the level of supply actually required to meet the daily customer demand, can improve the efficiency of the production system and save significant costs.

Harvesting activities at the tactical/operational level are associated with the extraction of trees or stems from forest plantations and their subsequent bucking to convert them into smaller logs, the latter having different destinations (sawmills, paper and pulp mills, among others). The harvesting process involves tree felling, tree delimbing and tree debarking, and its subsequent transport to the sectors destined for stockpiling. The bucking activity consists of the transverse cutting of a stem to produce logs of a certain length and diameter, and it can be performed at the destination of the wood (plant or temporary storage sector) or directly in the forest when the stem is harvested (Rönnqvist et al., 2015).

For each particular stem there are many bucking options, and consequently it must be decided which products (logs) will be generated. The different ways of cutting a stem depend on the characteristics of the tree and the required logs. The most relevant characteristics of a stem are its length, diameter, quality and species. Bucking decisions are irreversible once the stem is cut: a bad bucking decision can mean loss of wood and eventually generate products without demand. The logs generated in the bucking process are collected, loaded onto trucks and transported to their final destination. This transport activity presents different variants depending on the type of the inherent routing problem: characteristics of the fleet, characteristics of the route, pursued objectives, among others.

In the existing literature, from a tactical point of view, decisions related to harvesting are first made and later, the log transportation is planned assuming certain supply capacity. At operational level, bucking and routing problems are also separately addressed. In other words, in a first instance the stems are cut to obtain the logs according to the customer's demand and, in a later instance, the delivery of the logs is decided: as bucking is performed without taking into account the capacity of the vehicles and the information of the routes, it is possible that a greater number of trips can be generated with partial load, and consequently greater distances will be traveled increasing the total cost. For example, in El Hachemi et al. (2014) the authors propose a two-stage solution methodology. In the first stage they determine the destination of the raw material, while in the second stage they plan the routes to be performed by the truck fleet in such a way that the requirements of the plants are satisfied (for a weekly planning horizon). In this structure, no bucking decisions are considered, i.e. logs are available at each raw material site, and hierarchical decisions deals with raw material allocation and truck routing. In Bordón et al. (2020), the authors propose a mathematical model to simultaneously determine the destination of a single type of raw material, the vehicle routing and scheduling, for a daily planning horizon. However, it is also assumed that bucking decisions are fixed (that is, the number of available logs is known). In a later work, Bordón et al. (2021a) propose a column generation approach to solve the raw material allocation and routing problem for fixed raw material availability. Again, no bucking decisions are considered.

When a decoupled approach is employed, many times transportation is considered as a cost or even a capacity constraint, without detailing how trips and routes should be performed to minimize the total costs. For example, Lintafi et al. (2016) present a mathematical programming model for tactical harvest planning, where raw material transportation is considered as a cost without taking into account vehicle routing decisions. In this work, in addition, the authors consider that at most one bucking rule can be applied in each harvest area in each period, strongly limiting the alternatives to satisfy customer requirements.

In Vanzetti et al. (2019), a multi-period mathematical formulation for the production planning of sawmills considering aspects related to bucking activities is proposed. The authors model the log production planning according to a predefined set of bucking patterns. For log transportation, they assume that demands can be satisfied even if the number of logs to be shipped from a given supplier does not complete a minimum truck load. Fuentealba et al. (2019) propose a mathematical formulation to solve a similar problem to the previous one considering the cost of transported volume, without specifying, for example, the percentage of load of each truck or the number of trips. Again, routing decisions are not taken into account in this formulation.

In Dems et al. (2017), the wood procurement problem is addressed, where decisions about the scheduling of harvesting crews and the use of bucking patterns are considered through a proposed priority list. The simple cost per transported volume is assumed, without taking into account the route performed by each truck and the number of required trucks.

This hierarchical way of dealing with these problems has unfavorable consequences both from an optimization point of view (suboptimal solutions) and from an operational point of view (it is not possible to correct a poor bucking operation). For example, when bucking costs are minimized, logs not required by consumers can be obtained, generating stock of logs that will eventually lose commercial value (fungi may appear and deteriorate its quality, or they are used to elaborate products for which they were not be harvested).

As was mentioned before, the effective availability of raw material is determined through the bucking activity. The associated transport activities will then depend on the raw materials being transported, stems or smaller logs. The first case is the simplest to address, where it is enough to determine the destination of the harvested stem—usually considering full-truckloads—(Bordón et al., 2018), while the second case is more complex since transport decisions must necessarily consider aspects related to the load of each truck (logs of the same length must be transported on the same trip, although not necessarily the same diameter). The second approach is addressed in this work.

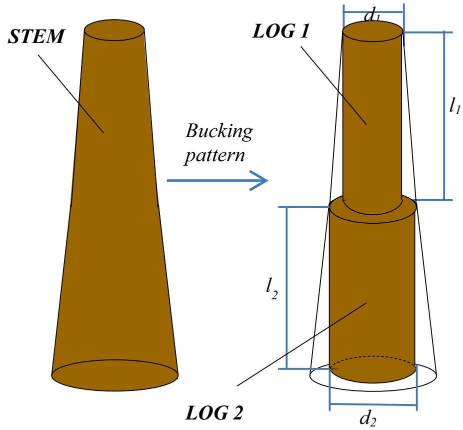
According to Rönnqvist et al. (2015), one of the open problems in the literature is the integration of harvesting and transportation problems at the operational level, i.e. the bucking and routing operations. In line with this, a mixed integer linear programming (MILP) model that integrates both bucking and routing decisions is proposed in this work. The developed MILP model allows to determine, for a weekly planning horizon: the periods in which the harvesting crews will work, the bucking patterns to be used in each harvest area and the number of logs of each type (length-diameter) to be obtained, decisions related to log storage in both harvest areas and plants, the size of the required truck fleet to distribute the generated logs, the routes to be performed by each truck, as well as the composition of the load of each trip. The simultaneous optimization of all these decisions permits an efficient management and coordination of all these tasks. The different tradeoffs among the involved decisions are together evaluated. In this way, this formulation helps to guide the decision making processes for the harvesting and transportation operational planning in the forest industry. To the best of our knowledge, there is no published work where all these decisions are jointly addressed. A preliminary version of this work was presented in the International Conference of Production Research—Americas 2020 (Bordón et al., 2021b).

In the following section the addressed problem is detailed, while in Sect. 3 proposed MILP model is described. In Sect. 4, the model performance is analyzed: through a motivating example, the potentiality of the developed mathematical model is highlighted and, in addition, a large-scale problem is solved by emphasizing the trade-offs between harvesting and transportation activities. Finally, in Sect. 5 the final remarks are highlighted.

2 Problem statement

The considered forest supply chain involves a set of industrial sites $I = \{i_1, i_2,..., i_{max}\}$, whose raw material requirements must be covered. Each industrial site (or plant) demands logs of a certain length $L = \{l_1, l_2,..., l_{max}\}$ and diameter $D = \{d_1, d_2,..., d_{max}\}$. It is assumed that at a higher planning level (that is, annual harvesting planning) the set of harvest areas

Fig. 1 Useful wood after applying bucking pattern (d: useful diameter, l: length)



$F = \{f_1, f_2, \ldots, f_{max}\}$ to be exploited during the considered planning period $T = \{t_1, t_2, \ldots, t_{max}\}$ has already been defined.

At the beginning of the planning horizon, there are a known number of available stems in each harvest area f ($qini_f$). The stems that belong to the same harvest area have the same characteristics since it is considered a planted forest. It is required that no more than a specified number of stems remain standing at the end of the planning horizon ($qend_f$), which depends on the objectives of the company.

The number of stems that can be cut in a harvest area in a given period $Q^{CS}_{f,t}$ is limited by the minimum and maximum stem cut capacity of the assigned harvesting teams in each area ($qcsmin_f$ and $qcsmax_f$, respectively). The costs associated with the harvesting crews are not taken into account since it is assumed that they were considered at a higher planning level (tactical harvest planning).

There is a set of predefined bucking patterns $B = \{b_1, b_2, \ldots, b_{max}\}$. Taking into account the stem size, some harvest areas may not be suitable for the use of some bucking patterns, and then, the set $BF_{b,f}$ that defines this relationship is introduced. By applying a bucking pattern to a stem, logs of certain length and diameter are obtained: the conversion factor of each bucking pattern ($nu_{l,d,b}$) establishes the number of logs of length l and diameter d obtained through the bucking pattern b (see Fig. 1).

Associated with each bucking pattern there is residual material, $loss_b$ measured in tons, and the corresponding cost, $closs_b$, per ton of unused wood. This residue includes both stem fine tip (i.e., wood not suitable for sawing) and losses due to the taper of the log.

If a bucking pattern is applied in a given harvest area, it should be used at least $qminb_b$ times to avoid additional setup times and costs. The logs generated in harvest area f that are not transported at the end of the planning horizon $Q^{RL}_{l,d,f,t}$, are considered as lost raw material with an associated cost, $cqrl_{l,d,f}$.

With regard to the requirements of the plant i, the weekly demand of logs (discriminated by length and diameter) must be satisfied, $dtot_{l,d,i}$. This weekly demand can be covered in any period. However, each plant has a daily minimum requirement for logs ($dmin_{l,d,i,t}$) and a limited processing capacity per period ($cap_{i,t}$). Moreover, each plant has a limited storage capacity given by $maxstock_{l,d,i}$. All these parameters are measured in number of logs.

Logs not used by each plant at the end of the planning horizon have a cost $cstock_{l,d,i}$, because it is assumed that these raw materials may not be required in the future and lose quality, or they may be used for manufacturing products for which they were not intended, affecting the productivity levels of the plant. At the beginning of the planning horizon there is an initial inventory of logs available from previous planning periods, $stockini_{l,d,i}$.

Regarding the transportation activity, there is a heterogeneous fleet of trucks $C = \{c_1, c_2,..., c_{max}\}$ to transport the logs from harvest area to plants. The number of logs to be transported on each trip of each truck depends on its capacity. Each truck has a minimum ($capmin_c$) and maximum ($capmax_c$) load capacity, in tons, while each log has its corresponding weight ($weight_{l,d}$), in tons. In addition, on each trip, trucks can only transport logs that are the same length, although not necessarily the same diameter. Trucks are housed in a set of regional bases $P = \{p_1, p_2,..., p_{max}\}$, to which they must return after completing all the assigned trips. Each truck is assigned to one and only one regional base, where this relationship is given by the set $PC_{p,c}$.

Regarding the composition of truck routes, the definition of Bordón et al. (2018) is considered. In this approach an arc-based formulation is used, where the route performed by each truck consists of a departure movement from its regional base, a succession of movements between harvest areas and plants, and finally a return movement to the regional base from where the route began. Multiple pickups/deliveries are not allowed before making a delivery/pickup, that is, the truck loads logs in a single harvest area and delivers them in a single plant, on each trip. Each truck has a limited working day time ($maxt_{c,t}$) and a maximum number of movements or trips $V = \{v_1, v_2,..., v_{max}\}$ to be performed in each period t. With regard to travel times, these depend on whether the truck is traveling loaded or not. The loading and unloading times of each truck in each node of the network are assumed to be known ($load_{c,f}$ and $unload_{c,i}$, respectively).

Finally, regarding the costs associated with the transport activity, variable costs (per kilometers traveled with and without load) and fixed costs (per use of trucks) are considered.

In summary, the addressed problem determines (see Fig. 2):

- The number of cut stems in each period and remaining standing stems at the end of each period in each harvest area: $Q^{CS}_{f,t}$ and $Q^{SS}_{f,t}$, respectively.
- The periods in which harvesting activities are carried out in each harvest area, $CUT_{f,t}$.
- The selection of bucking patterns and the number of times they are applied in each harvest area in each period, $Y^{BP}_{b,f,t}$ and $Q^{BP}_{b,f,t}$, respectively.
- The number of generated logs of each type (length-diameter combination), per period, in each harvest area, $Q^{CL}_{l,d,f,t}$.
- The number of transported logs of each type, per period, from each harvest area to each plant, $Q^{TL}_{l,d,f,t}$.
- The number of remaining logs of each type, per period, at the roadside in harvest areas, $Q^{RL}_{l,d,f,t}$.
- The number of processed logs of each type (above the minimum demand) in each plant in each period, $Q^{PL}_{l,d,i,t}$.
- The inventory levels of logs of each type, per period, in the storage yard of each plant, $Q^{SL}_{l,d,i,t}$.
- The truck selection in each period, $Y^{T}_{c,p,t}$.
- The type of logs loaded on each trip by each truck and the number of logs (of the same length but not necessarily the same diameter) transported on each trip by each truck from each harvest area to each plant in each period, $Y^{TT}_{l,c,v,f,i,t}$ and $Q^{TT}_{l,d,c,v,f,i,t}$, respectively.
- The route performed by each truck in each period, that is, its initial, loaded, unloaded and return movements, ($X^{D}_{c,p,f,t}$, $X^{L}_{c,v,f,i,t}$, $X^{U}_{c,v,i,f,t}$ and $X^{R}_{c,v,i,p,t}$, respectively). See Bordón et al. (2018) for further details.

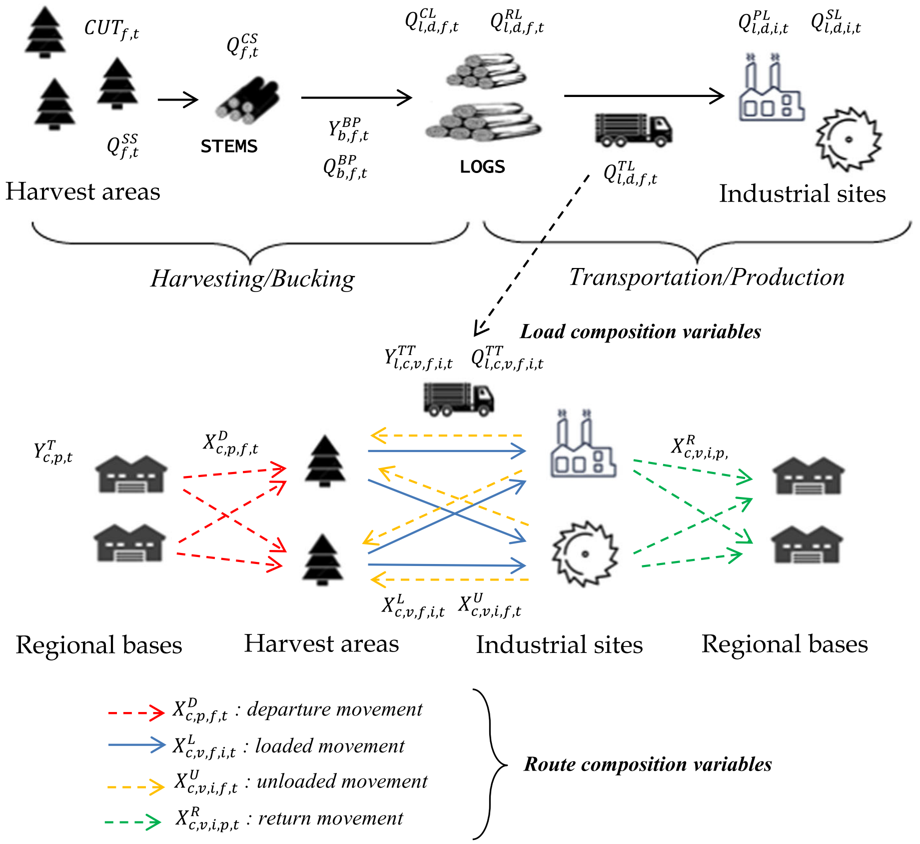


Fig. 2 Harvesting and transportation problem description

3 Mathematical model

3.1 Objective function

The objective function is given by the cost minimization expressed through (1), where: CV^{Transp} establishes the variable cost per traveled kilometer, CF^{Transp} represents the fixed costs per use of trucks, CV^{Loss} defines the cost for raw material loss when applying a bucking pattern, CV^{Stock} sets the cost of keeping logs in plant storage yards at the end of the planning horizon, and CV^{Logs} establishes the cost of not transported logs which remain on the roadside in the harvest areas at the end of the planning horizon. The detailed list of model parameters and variables is presented in the Nomenclature section.

$$\min Z = CV^{Transp} + CF^{Transp} + CV^{Loss} + CV^{Stock} + CV^{Logs} \tag{1}$$

where

$$CV^{Transp} = \sum_{c} \sum_{p \in PC_{p,c}} \sum_{f} \sum_{t} dpf_{p,f} cd_{p,f} X^{D}_{c,p,f,t} + \sum_{c} \sum_{v} \sum_{f} \sum_{i} \sum_{t} dfi_{f,i} cl_{f,i} X^{L}_{c,v,f,i,t}$$

$$+\sum_{c}\sum_{v}\sum_{i}\sum_{f}\sum_{t} dif_{i,f}cu_{i,f}X^{U}_{c,v,i,f,t} + \sum_{c}\sum_{p\in PC_{p,c}}\sum_{v}\sum_{i}\sum_{t} dip_{i,p}cr_{i,p}X^{R}_{c,v,i,p,t} \tag{1a}$$

$$CF^{Transp} = \sum_{c}\sum_{p\in PC_{p,c}}\sum_{t} ctruck_{c,p}Y^{T}_{c,p,t} \tag{1b}$$

$$CV^{Loss} = \sum_{b}\sum_{f\in BF_{b,f}}\sum_{t} closs_{b}loss_{b}Q^{BP}_{b,f,t} \tag{1c}$$

$$CV^{Stock} = \sum_{l}\sum_{d}\sum_{i} cstock_{l,d,i}Q^{SL}_{l,d,i,t=|T|} \tag{1d}$$

$$CV^{Logs} = \sum_{l}\sum_{d}\sum_{f} cqrl_{l,d,f}Q^{RL}_{l,d,f,t=|T|} \tag{1e}$$

The restrictions associated with the problem are presented below.

3.2 Constraints related to harvesting and bucking activities

Constraints (2) and (3) define the balance of stems in each harvest area in each period. Constraint (2) determines that, for the first period, the number of cut and standing stems after the harvesting must be equal to the available stems in the harvest area, while for later periods, constraint (3) establishes that the number of cut and standing stems in the period must be equal to the number of standing stems in the previous period.

$$Q^{CS}_{f,t} + Q^{SS}_{f,t} = qini_{f} \quad \forall f \in F, t = 1 \tag{2}$$

$$Q^{SS}_{f,t-1} = Q^{CS}_{f,t} + Q^{SS}_{f,t} \quad \forall f \in F, t > 1 \tag{3}$$

The number of standing stems at the end of the planning horizon must not exceed a determined target.

$$Q^{SS}_{f,t} \leq qend_{t} \quad \forall f \in F, t = |T| \tag{4}$$

Constraints (5) and (6) establish the minimum and maximum number of stems to be cut, by period, where $CUT_{f,t}$ is the binary variable that indicates whether the harvesting activities are carried out the harvest area or not.

$$Q^{CS}_{f,t} \geq qcsmin_{f}CUT_{f,t} \quad \forall f \in F, \forall t \in T \tag{5}$$

$$Q^{CS}_{f,t} \leq qcsmax_{f}CUT_{f,t} \quad \forall f \in F, \forall t \in T \tag{6}$$

The number of cut stems must match the number of times the bucking patterns are used.

$$Q^{CS}_{f,t} = \sum_{b\in BF_{b,f}} Q^{BP}_{b,f,t} \quad \forall f \in F, \forall t \in T \tag{7}$$

Constraint (8) defines the number of generated logs of each type by using the bucking patterns.

$$\sum_{b\in BF_{b,f}} nu_{l,d,b}Q^{BP}_{b,f,t} = Q^{CL}_{l,d,f,t} \quad \forall l \in L, \forall d \in D, \forall f \in F, \forall t \in T \tag{8}$$

Constraints (9) and (10) determine the minimum and maximum number of times that the bucking patterns can be applied, respectively. It should be noted that the maximum number of times that a bucking pattern can be applied in a harvest area is limited by the maximum number of stems to be cut during a period, since only one bucking pattern can be applied to each stem.

$$Q^{BP}_{b,f,t} \geq qminb_b Y^{BP}_{b,f,t} \quad \forall (b,f) \in BF_{b,f}, \forall t \in T \tag{9}$$

$$Q^{BP}_{b,f,t} \leq qcsmax_f Y^{BP}_{b,f,t} \quad \forall (b,f) \in BF_{b,f}, \forall t \in T \tag{10}$$

Constraints (11) and (12) state that a bucking pattern must be used in a harvest area during a period if and only if a harvesting crew cuts stems in the harvest area during that period.

$$Y^{BP}_{b,f,t} \leq CUT_{f,t} \quad \forall (b,f) \in BF_{b,f}, \forall t \in T \tag{11}$$

$$CUT_{f,t} \leq \sum_{b \in BF_{b,f}} Y^{BP}_{b,f,t} \quad \forall f \in F, \forall t \in T \tag{12}$$

Constraint (13) ensures that harvesting crew cuts in successive periods. These constraints are incorporated into the model to avoid having idle days without operations.

$$CUT_{f,t} \leq CUT_{f,t+1} - CUT_{f,k} + 1 \quad \forall f \in F, \forall t \in T, k \geq t+2 \tag{13}$$

Constraints (14) and (15) state the inventory balance of each type of log in each harvest area, per period. Constraint (14) determines that, for the first period, the amount of cut logs can either be sent to customers or remain in stock in the harvest area, while constraint (15) states a similar condition for later periods.

$$Q^{CL}_{l,d,f,t} = Q^{TL}_{l,d,f,t} + Q^{RL}_{l,d,f,t} \quad \forall l \in L, \forall d \in D, \forall f \in F, t = 1 \tag{14}$$

$$Q^{CL}_{l,d,f,t} + Q^{RL}_{l,d,f,t-1} = Q^{TL}_{l,d,f,t} + Q^{RL}_{l,d,f,t} \quad \forall l \in L, \forall d \in D, \forall f \in F, t > 1 \tag{15}$$

3.3 Constraints related to distribution activities

The number of logs of length l and diameter d shipped from a harvest area must correspond to the total number of logs (of the same type) transported by all trucks from that harvest area to the different customers.

$$Q^{TL}_{l,d,f,t} = \sum_{c} \sum_{v} \sum_{i} Q^{TT}_{l,d,c,v,f,i,t} \quad \forall l \in L, \forall d \in D, \forall f \in F, \forall t \in T \tag{16}$$

Only logs of the same length are allowed to be transported on each trip; therefore, the binary variable $Y^{TT}_{l,c,v,f,i,t}$ indicates if logs of length l are delivery from harvest area f to plant i in trip v performed by truck c in period t. Equations (17) and (18) state this condition, where the parameter $qttup_{l,d,c,v,f,i,t}$ represents the maximum number of logs (of each type) that a truck can transport during a trip (this parameter is calculated considering the logs dimension and the available truck capacity, in m^3).

$$Q^{TT}_{l,d,c,v,f,i,t} \leq qttup_{l,d,c,v,f,i,t} Y^{TT}_{l,c,v,f,i,t} \quad \forall l \in L, \forall d \in D, \forall c \in C, \forall v \in V, \forall f \in F, \forall i \in I, \forall t \in T \tag{17}$$

$$\sum_{l} Y^{TT}_{l,c,v,f,i,t} = X^{L}_{c,v,f,i,t} \quad \forall c \in C, \forall v \in V, \forall f \in F, \forall i \in I, \forall t \in T \tag{18}$$

Constraints (19) and (20) state the minimum and maximum load capacity of trucks, respectively.

$$\sum_{l}\sum_{d} weight_{l,d} Q^{TT}_{l,d,c,v,f,i,t} \geq capmin_c X^{L}_{c,v,f,i,t} \quad \forall c \in C, \forall v \in V, \forall f \in F, \forall i \in I, \forall t \in T \tag{19}$$

$$\sum_{l}\sum_{d} weight_{l,d} Q^{TT}_{l,d,c,v,f,i,t} \leq capmax_c X^{L}_{c,v,f,i,t} \quad \forall c \in C, \forall v \in V, \forall f \in F, \forall i \in I, \forall t \in T \tag{20}$$

Through the restrictions (21) to (30) the routes that each truck must perform are built. These constraints are adapted from Bordón et al. (2018). Constraints (21) and (22) establish that if a truck is used, it must at most leave and return to the corresponding regional base. Constraint (23) assures that a truck has to visit one harvest area when it departs from its regional base. Equations (24) and (25) establishes a truck can perform loaded and unloaded movements, respectively, only if the truck is used. Equation (26) states that if a truck is used, it must perform at least a loaded trip. The appropriate sequence of visits to each node in the network is described through Eqs. (27)–(29). Constraint (27) states that for each trip, an unloaded movement must be performed, if loaded movement is first travelled. Equation (28) denotes that if an unloaded truck from a plant arrives at a harvest site, this truck must visit a plant in the next trip. Constraint (29) states that if a truck completes a loaded movement from harvest area f to plant i, then the truck can either perform an unloaded movement toward some f to begins a new trip or goes toward its base regional p to finish the route. Equation (30) states the travel time limit for each truck.

$$\sum_{f} X^{D}_{c,p,f,t} = Y^{T}_{c,p,t} \quad \forall (c,p) \in PC_{p,c}, \forall t \in T \tag{21}$$

$$\sum_{v}\sum_{i} X^{R}_{c,v,i,p,t} = Y^{T}_{c,p,t} \quad \forall (c,p) \in PC_{p,c}, \forall t \in T \tag{22}$$

$$X^{D}_{c,p,f,t} = \sum_{i} X^{L}_{c,v,f,i,t} \quad \forall (c,p) \in PC_{p,c}, \forall f \in F, \forall t \in T, v = 1 \tag{23}$$

$$X^{L}_{c,v,f,i,t} \leq Y^{T}_{c,p,t} \quad \forall (c,p) \in PC_{p,c}, \forall v \in V, \forall f \in F, \forall i \in I, \forall t \in T \tag{24}$$

$$X^{U}_{c,v,i,f,t} \leq Y^{T}_{c,p,t} \quad \forall (c,p) \in PC_{p,c}, \forall v \in V, \forall f \in F, \forall i \in I, \forall t \in T \tag{25}$$

$$Y^{T}_{c,p,t} \leq \sum_{v}\sum_{f}\sum_{i} X^{L}_{c,v,f,i,t} \quad \forall (c,p) \in PC_{p,c}, \forall t \in T \tag{26}$$

$$\sum_{f} X^{U}_{c,v,i,f,t} \leq \sum_{f} X^{L}_{c,v,f,i,t} \quad \forall c \in C, \forall v \in V, \forall i \in I, \forall t \in T \tag{27}$$

$$\sum_{i} X^{U}_{c,v-1,i,f,t} \leq \sum_{i} X^{L}_{c,v,f,i,t} \quad \forall c \in C, \forall f \in F, \forall t \in T, v > 1 \tag{28}$$

$$\sum_{f} X^{L}_{c,v,f,i,t} = \sum_{f} X^{U}_{c,v,i,f,t} + X^{R}_{c,v,i,p,t} \quad \forall (c,p) \in PC_{p,c}, \forall v \in V, \forall i \in I, \forall t \in T \tag{29}$$

$$\sum_{f}\left(\frac{dpf_{p,f}}{vd_{p,f}}\right) X^{D}_{c,p,f,t} + \sum_{v}\sum_{f}\sum_{i}\left(\frac{dfi_{f,i}}{vl_{f,i}}\right) X^{L}_{c,v,f,i,t}$$
$$+ \sum_{v}\sum_{i}\sum_{f}\left(\frac{dif_{i,f}}{vu_{i,f}}\right) X^{U}_{c,v,i,f,t} + \sum_{v}\sum_{i}\left(\frac{dip_{i,p}}{vr_{i,p}}\right) X^{R}_{c,v,i,p,t}$$

$$+\sum_v \sum_f \sum_i \left(load_{c,f} + unload_{c,i}\right) X^L_{c,v,f,i,t} \le maxt_{c,t} \quad \forall (c,p) \in PC_{p,c}, \forall t \in T \tag{30}$$

3.4 Constraints related to production activities

Equations (31) and (32) set the inventory balances of logs of each type in each plant. Constraint (33) determines the maximum inventory capacity of logs of each type in each sector of the storage yard, while restriction (34) establishes the processing capacity limit of each plant in each period. Equation (35) forces the total demand for each plant to be covered.

$$\sum_c \sum_v \sum_f Q^{TT}_{l,d,c,v,f,i,t} + stockini_{l,d,i} = dmin_{l,d,i,t} + Q^{SL}_{l,d,i,t} + Q^{PL}_{l,d,i,t} \quad \forall l \in L, \forall d \in D, \forall i \in I, t = 1 \tag{31}$$

$$\sum_c \sum_v \sum_f Q^{TT}_{l,d,c,v,f,i,t} + Q^{SL}_{l,d,i,t-1} = dmin_{l,d,i,t} + Q^{SL}_{l,d,i,t} + Q^{PL}_{l,d,i,t} \quad \forall l \in L, \forall d \in D, \forall i \in I, t > 1 \tag{32}$$

$$Q^{SL}_{l,d,i,t} \le maxstock_{l,d,i} \quad \forall l \in L, \forall d \in D, \forall i \in I, \forall t \in T \tag{33}$$

$$\sum_l \sum_d \left(dmin_{l,d,i,t} + Q^{PL}_{l,d,i,t}\right) \le cap_{i,t} \quad \forall i \in I, \forall t \in T \tag{34}$$

$$\sum_t \left(dmin_{l,d,i,t} + Q^{PL}_{l,d,i,t}\right) = dtot_{l,d,i} \quad \forall l \in L, \forall d \in D, \forall i \in I \tag{35}$$

Finally, (36) and (37) establish the nature of the involved variables.

$$CUT_{f,t}, X^D_{c,p,f,t}, X^L_{c,v,f,i,t}, X^R_{c,v,i,p,t}, X^U_{c,v,i,f,t}, Y^{BP}_{b,f,t}, Y^T_{c,p,t}, Y^{TT}_{l,c,v,f,i,t} \in \{0,1\} \tag{36}$$

$$Q^{BP}_{b,f,t}, Q^{CL}_{l,d,f,t}, Q^{CS}_{f,t}, Q^{PL}_{l,d,i,t}, Q^{RL}_{l,d,f,t}, Q^{SL}_{l,d,i,t}, Q^{SS}_{f,t}, Q^{TL}_{l,d,f,t}, Q^{TT}_{l,d,c,v,f,i,t}, Z \ge 0 \tag{37}$$

4 Computational results

4.1 Motivating example

In this section a very simple example is presented in order to highlight the economic and operational impact obtained by jointly addressing both problems. This example is developed for illustrative purposes, and consists of a single plant that demands logs from a unique harvest area. The log demands for each period for each type of log, the total demands and the bucking patterns conversions are displayed in Table 1.

The example is tested using both integrated and hierarchical approaches. Table 2 shows the results corresponding to integrated model solution and describes the number of logs (of each type, i.e., length and diameter combination) that remain in stock in the harvest area, the number of transported logs and logs in stock at the plant, in each case by period. The difference with respect to the solution obtained by the hierarchical approach is detailed in parentheses. For example, the first row of Table 2 in its first three columns describes that,

Table 1 Minimum demand per period, total weekly demand and bucking patterns

Log type	Minimum demand (logs)			Total demand (logs)	Bucking patterns		
	t_1	t_2	t_3		b_1	b_2	b_3
l_1 d_1	0	50	0	100	1	2	2
l_1 d_2	0	80	0	100	2	1	0
l_1 d_3	25	75	0	100	0	1	1
l_1 d_4	0	20	80	125	1	2	0
l_2 d_1	100	20	50	200	0	1	1
l_2 d_2	0	0	90	100	2	0	1
l_2 d_3	50	0	50	150	1	2	0
l_2 d_4	0	50	0	75	2	0	3

after the harvest, there is a total of 620 logs of length l_1 and diameter d_1 t_1 and t_2, and 80 in t_3, for the optimal solution of the simultaneous approach. On the other hand, when the hierarchical approach is performed, the number of remaining logs of this type in the harvest area is 700 in t_1 and t_2, and no logs in t_3. In a similar way, the number of transported logs from the harvest area to the plant and the number of stored logs in the plant for each period and for both approaches are displayed in the second and third group of columns, respectively.

As it was mentioned in the Introduction section, a traditional hierarchical approach solves bucking and routing problems in decoupled way. First, the bucking is solved, and the global quantities of logs transported from harvest areas to plant are determined without specifying trucks load, number of trips, and routes. Then, knowing the quantity of logs obtained from the bucking, the routing is performed.

The model for the first stage considers the minimization of the cost given by raw material losses (due to the use of a bucking pattern, CV^{Loss}), raw material harvested and not transported at the end of the planning horizon (CV^{Stock}) and raw material transported but not processed in the plant at the end of the planning horizon (CV^{Logs}). The involved constraints are Eqs. (2)–(16) and Eqs. (31)–(35). Constraints (16), (31) and (32) must be rewritten since the positive variables $Q^{TT}_{l,d,c,v,f,i,t}$ are defined for each trip v performed by truck c (at this stage, routing decisions are ignored). The modified constraints are as follows:

$$Q^{TL}_{l,d,f,t} = \sum_i Q^{T*}_{l,d,f,i,t} \quad \forall l \in L, \forall d \in D, \forall f \in F, \forall t \in T \tag{16b}$$

$$\sum_f Q^{T*}_{l,d,f,i,t} + stockini_{l,d,i} = dmin_{l,d,i,t} + Q^{SL}_{l,d,i,t} + Q^{PL}_{l,d,i,t} \quad \forall l \in L, \forall d \in D, \forall i \in I, t = 1 \tag{31b}$$

$$\sum_f Q^{T*}_{l,d,f,i,t} + Q^{SL}_{l,d,i,t-1} = dmin_{l,d,i,t} + Q^{SL}_{l,d,i,t} + Q^{PL}_{l,d,i,t} \quad \forall l \in L, \forall d \in D, \forall i \in I, t > 1 \tag{32b}$$

Note that $Q^{TT}_{l,d,c,v,f,i,t}$ are replaced by $Q^{T*}_{l,d,f,i,t}$. $Q^{T*}_{l,d,f,i,t}$ represents the number of logs transported from each harvest area to each plant in each period.

Once the harvesting problem is solved, the routing problem is addressed fixing the solution obtained in the first stage and minimizing the cost given by raw material transportation (variable cost per traveled kilometer, CV^{Transp}) and the use of trucks (fixed cost, CF^{Transp}).

Table 2 Obtained results for motivating example

Log type	# Logs remaining in harvest area			# Logs transported from harvest area			# Logs stored in plant		
	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3
$l_1\ d_1$	620 (80)	620 (80)	80 (– 80)	180 (– 80)	0 (0)	540 (160)	180 (– 130)	80 (– 80)	620 (80)
$l_1\ d_2$	20 (152)	20 (52)	0 (0)	80 (– 80)	0 (100)	20 (52)	80 (– 80)	0 (0)	0 (72)
$l_1\ d_3$	300 (75)	300 (0)	0 (0)	100 (– 75)	0 (75)	300 (0)	75 (– 75)	0 (0)	300 (0)
$l_1\ d_4$	155 (164)	155 (144)	0 (0)	45 (– 20)	0 (20)	155 (144)	45 (– 45)	0 (0)	75 (144)
$l_2\ d_1$	220 (– 220)	180 (– 180)	0 (0)	180 (220)	40 (– 40)	180 (– 180)	50 (250)	70 (180)	200 (0)
$l_2\ d_2$	120 (108)	120 (108)	0 (0)	180 (– 180)	0 (0)	120 (108)	170 (– 170)	170 (– 170)	200 (– 72)
$l_2 d_3$	150 (144)	0 (294)	0 (0)	50 (0)	150 (– 150)	0 (294)	0 (0)	100 (– 100)	50 (144)
$l_2\ d_4$	875 (– 216)	688 (– 79)	614 (– 612)	25 (0)	188 (– 138)	73 (534)	0 (0)	138 (– 138)	211 (396)

The variables that link both problems are $Q^{T*}_{l,d,f,i,t}$. This relationship is given by constraint (38):

$$Q^{T*}_{l,d,f,i,t} = \sum_{c}\sum_{v} Q^{TT}_{l,d,c,v,f,i,t} \quad \forall l \in L, \forall d \in D, \forall f \in F, \forall i \in I, \forall t \in T \tag{38}$$

Therefore, the routing stage model includes constraints (17), (18), (20) to (30) and (38). Constraint (19) is relaxed in order to avoid infeasibilities given by the inability of fitting the number of logs to the truck capacity lower bound (for example, according to Table 3, c9 during trip v_1 in period t_2 transports 50 logs of length l_2 and diameter d_4, which represents approximately 10 tons).

In Table 2 the results obtained from hierarchical approach are displayed in parentheses. They describe the number of logs that differs from the simultaneous approach solution (minus sign means that less quantity is obtained through the hierarchical approach). The total transported logs in the simultaneous solution are 764 less than those determined by the hierarchical approach and, therefore, shorter inventory is generated in the plant in the last period. In both approaches 400 stems are harvested, but different bucking patters are applied. In the hierarchical solution methodology, the bucking patterns b_2 and b_3 are applied 172 and 228 times, respectively, while in the integrated solution, b_2 is applied 100 times and b_3 300 times.

Table 3 Used trucks and transported logs on each trip

Truck-trip	t_1				t_2				t_3			
	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4
Hierarchical solution												
c_1 v_1 l_2	0	0	0	0	0	0	0	0	0	0	12	130
c_1 v_2 l_2	0	0	0	0	0	0	0	0	0	47	0	119
c_2 v_1 l_2	0	0	0	0	0	0	0	0	0	0	0	136
c_3 v_1 l_1	0	0	0	0	0	0	0	0	180	0	120	56
c_3 v_1 l_2	40	0	0	0	0	0	0	0	0	0	0	0
c_3 v_2 l_1	100	0	25	25	0	0	0	0	180	72	180	16
c_5 v_1 l_2	0	0	0	0	0	0	0	0	0	180	102	0
c_5 v_2 l_2	0	0	0	0	0	0	0	0	0	0	180	0
c_6 v_1 l_1	0	0	0	0	0	0	0	0	180	0	0	139
c_6 v_2 l_1	0	0	0	0	0	0	0	0	160	0	0	88
c_9 v_1 l_2	180	0	0	0	0	0	0	50	0	0	0	86
c_9 v_2 l_1	0	0	0	0	0	100	75	20	0	0	0	0
c_9 v_2 l_2	180	0	50	25	0	0	0	0	0	1	0	136
Integrated solution												
c_3 v_1 l_1	0	0	0	0	0	0	0	0	180	0	180	0
c_3 v_2 l_2	0	0	0	0	0	0	0	0	180	120	0	73
c_9 v_1 l_1	180	80	100	45	0	0	0	0	180	20	0	80
c_9 v_1 l_2	0	0	0	0	0	0	0	137	0	0	0	0
c_9 v_2 l_1	0	0	0	0	0	0	0	0	180	0	120	75
c_9 v_2 l_2	180	180	50	25	40	0	150	51	0	0	0	0

Another notable result is the use of trucks and the loading of logs per trip. Table 3 shows the detail of the transported logs per trip in each solution approach.

Table 3 shows that, for the hierarchical solution, 2 trucks are used in the first period (c_3 and c_9, performing 2 trips each), one truck in period 2 (c_9, performing 2 trips) and 6 trucks in the last period (all perform 2 trips except for one truck that performs only one trip). On the other hand, for the integrated solution, one truck is used in the first two periods (c_9, performing 2 trips in each period) and 2 trucks are used in the last period (c_3 and c_9, performing 2 trips each). In summary, through the hierarchical solution methodology, 19 trips are performed throughout the planning horizon, while only 8 trips are performed in the integrated solution. This implies a better use of the truck's load capacity, with the corresponding decrease in the variable costs per travelled kilometer and fixed costs for truck use.

From the economical point of view, the total cost obtained from the simultaneous approach is equal to \$19,563.88 while from the hierarchical approach is \$22,057.48 (12.75% greater). All the considered economical terms are reduced in the simultaneous model solution, except for inventory cost at harvest sites. In this case, the transportation cost is not so significant since both locations are close each other, reaching \$2447.5 for the simultaneous approach and \$5811.5 for the traditional one, which represents an increase of 137.45%.

As can be seen, when dealing with bucking and routing problems in a decoupled way, highly inefficient situations are generated. This motivating example shows the potential savings that can be obtained by applying the mathematical model presented in this work.

4.2 Real-size example

The model performance and capabilities are assessed through the following example. In this case, two plants that demand logs of 4 different lengths (l_1: 3 m, l_2: 3.5 m, l_3: 4.25 m and l_4: 5 m) and 4 different diameters (d_1: 0.1 m, d_2: 0.18 m, d_3: 0.22 m and d_4: 0.3 m) are considered. Table 4 shows the minimum and total demands, for each plant and each type of log during the planning horizon (5 periods). In addition, Table 4 also shows the total weight of the required raw material (in tons). Assuming a conversion factor of 0.8 tons/m^3 of pine, in this example a weekly supply of 1697.5 tons of raw material (equal to 2121 m^3) is considered. According to the Ministry of Agriculture, Livestock and Fisheries (2015), the volume of transported raw material in this example is similar to that handled in large companies. The weight of a given log is calculated assuming a perfect cylinder of length l and diameter d, according to $weight_{l,d}[ton] = 0.8 * \pi * l * (d/2)^2$.

Regarding the storage capacity, it is assumed that each plant has 150 m^3 assigned for each type of log, so the maximum number of logs to be kept in inventory is obtained according to $maxstock_{l,d,i} = 150 * \left(\pi * l * (d/2)^2\right)^{-1}$.

There are 3 harvest areas with different availability of standing stems to obtain the required logs ($qini_f$): 8000 stems in f_1, 7000 stems in f_2 and 6500 stems in f_3.

In each available harvest area, there are two harvesting crews. These harvesting crews can use a set of predetermined bucking patterns, which are presented in Table 5. This table shows the number of logs of each type that can be obtained with each bucking pattern (conversion factor, $fconv_{l,d,b}$) and the percentage of raw material lost when each pattern is used. In addition, the harvest areas where these patterns can be applied are also detailed. For example, the element placed on row "l_1 d_1" and column "b_1 (f_1, f_3)" equal to 1 means that applying the bucking pattern b_1 to stems of harvest areas f_1 and f_3, one log of length l_1 and diameter d_1 is obtained. In the last row of this column, the value 7 represents the percentage of the loss obtained when bucking pattern b_1 is applied to stems in harvest area f_1 and f_3.

Table 4 Minimum demand (per period) and total demand of logs

Log type	Minimum demand (logs, per period)										Total demand (logs)		Total weight (ton)	
	i_1					i_2								
	t_1	t_2	t_3	t_4	t_5	t_1	t_2	t_3	t_4	t_5	i_1	i_2	i_1	i_2
$l_1\ d_1$	0	100	0	0	100	200	200	200	0	0	400	1500	8	30
$l_1\ d_2$	0	0	0	0	0	0	100	100	200	0	250	850	15	51
$l_1\ d_3$	100	100	100	100	0	0	0	0	0	0	700	100	63	9
$l_1\ d_4$	0	100	100	100	100	0	0	0	0	0	550	100	93.5	17
$l_2\ d_1$	100	100	100	0	0	100	100	100	100	100	500	750	10	15
$l_2\ d_2$	0	0	200	0	0	200	200	200	200	200	500	1200	35	84
$l_2\ d_3$	50	100	50	50	50	0	0	0	0	0	650	200	71.5	22
$l_2\ d_4$	0	0	0	0	0	0	0	0	0	0	800	100	160	20
$l_3\ d_1$	100	100	100	100	100	100	200	200	200	200	650	1500	19.5	45
$l_3\ d_2$	0	0	0	0	100	100	0	500	0	0	350	950	31.5	85.5
$l_3\ d_3$	100	300	0	0	0	0	0	0	200	200	500	500	65	65
$l_3\ d_4$	0	0	0	150	0	50	0	0	0	0	350	250	84	60
$l_4\ d_1$	200	0	0	0	100	0	0	0	100	100	500	1500	15	45
$l_4\ d_2$	50	50	50	50	50	150	350	350	0	0	350	1000	35	100
$l_4\ d_3$	0	0	0	0	0	0	0	0	0	200	200	500	30	75
$l_4\ d_4$	0	100	50	200	0	0	0	0	0	0	600	250	168	70
Total weekly demand (ton)													940	793.5

Table 5 Bucking patterns available in each harvest area and stem-to-log conversion factor

Log type	Bucking patterns									
	$b_1\ (f_1, f_3)$	$b_2\ (f_1, f_3)$	$b_3\ (f_1, f_3)$	$b_4\ (f_1, f_3)$	$b_5\ (f_1, f_3)$	$b_6\ (f_2, f_3)$	$b_7\ (f_2, f_3)$	$b_8\ (f_2, f_3)$	$b_9\ (f_2, f_3)$	$b_{10}\ (f_3)$
$l_1\ d_1$	1	2	2	2	1	0	0	0	0	1
$l_1\ d_2$	2	0	0	0	0	0	1	1	1	0
$l_1\ d_3$	0	0	0	1	0	0	1	0	1	0
$l_1\ d_4$	0	0	0	0	0	0	2	1	0	1
$l_2 d_1$	0	1	1	0	0	0	0	1	0	0
$l_2\ d_2$	0	0	1	1	0	1	1	0	0	0
$l_2\ d_3$	1	2	0	0	0	1	0	1	0	0
$l_2\ d_4$	0	0	0	0	0	1	0	2	1	0
$l_3\ d_1$	0	0	1	1	0	0	1	0	0	0
$l_3\ d_2$	0	1	0	0	1	0	0	1	1	0
$l_3\ d_3$	3	0	0	0	1	0	0	0	1	0
$l_3\ d_4$	0	0	1	0	1	0	0	0	1	1
$l_4\ d_1$	0	0	0	1	0	1	0	0	1	0
$l_4\ d_2$	0	0	0	0	0	2	0	0	0	1
$l_4\ d_3$	0	1	0	1	0	0	1	0	0	1
$l_4\ d_4$	0	0	1	0	2	0	0	0	0	1
Loss (%)	7	4	2	2	7	7	9	9	0	9

It is assumed that each harvesting crew can process a maximum of 50 stems per hour, so the maximum harvest capacity is 100 stems per hour. In addition, if the harvesting crew works in a certain period, it is desirable that it operates at least 50% of its capacity.

There is a fleet of 20 trucks with a minimum and maximum load capacity of 18 and 27 tons, respectively. This fleet is distributed in 10 different regional bases. Each truck can perform a maximum of 3 trips with load on its route, as long as the working time limit allows it (8 h per day).

Truck loading times in harvest areas and truck unloading times in plants are considered 30 min (on average). The distances between each of the nodes of the supply network are presented in Table 6. This table also shows the availability of trucks in each regional base and its associated fixed cost. The remaining parameters are detailed in Table 7.

The model is implemented and solved in GAMS (Rosenthal, 2020) 24.7.3 version, with the CPLEX 12.6.3 solver, in an Intel(R) Core(TM) i7–8700, 3.20 GHz.

Table 6 Distances between nodes (in kilometers), trucks hosted in each regional base and associated fixed cost (in $)

	Distances (km)					Fixed cost ($)
	f_1	f_2	f_3	i_1	i_2	
p_1 (c_1)	100	80	95	35	70	132
p_2 (c_2)	80	30	150	55	65	134
p_3 (c_3)	45	105	65	40	80	136
p_4 (c_4)	65	70	43	80	51	138
p_5 (c_5)	49	57	80	75	37	140
p_6 (c_6)	63	21	80	60	50	142
p_7 (c_7)	75	106	59	95	45	144
p_8 (c_8)	50	70	60	104	80	146
p_9 (c_9)	49	83	72	25	65	148
p_{10} $(c_{10}$ *a* $c_{20})$	84	72	76	76	63	150
i_1	50	110	85			
i_2	75	65	80			

Table 7 Model parameters considered for real-size example

$\mathrm{qttup}_{l,d,c,v,f,i,t}$	180 (logs)	qminb_b	0 (logs)
$\mathrm{stockini}_{l,d,i}$	0 (logs)	$\mathrm{cd}_{p,f}$	2.5 ($/km)
$\mathrm{cstock}_{l,d,i_1}$	17*$\mathrm{weight}_{l,d}$ + 5 ($/log)	$\mathrm{cl}_{f,i}$	1.5 ($/km)
$\mathrm{cstock}_{l,d,i_2}$	17*$\mathrm{weight}_{l,d}$ + 10 ($/log)	$\mathrm{cu}_{i,f}$	2.5 ($/km)
cqrl_{l,d,f_1}	15, 33*$\mathrm{weight}_{l,d}$ + 7 ($/log)	$\mathrm{cr}_{i,p}$	2.5 ($/km)
cqrl_{l,d,f_2}	15, 33*$\mathrm{weight}_{l,d}$ + 14 ($/log)	$\mathrm{vd}_{p,f}$	55 (km/h)
cqrl_{l,d,f_3}	15, 33*$\mathrm{weight}_{l,d}$ + 21 ($/log)	$\mathrm{vl}_{f,i}$	40 (km/h)
closs_b	9.55 ($/ton)	$\mathrm{vu}_{i,f}$	50 (km/h)
$\mathrm{cap}_{i,t}$	5000 (logs)	$\mathrm{vr}_{i,p}$	55 (km/h)

The proposed example contains 11,910 binary variables, 29,966 continuous variables, and 42,959 constraints. The obtained results, after 15 min of execution, are presented below, where the found solution presents an optimality gap of 3.42%. The value of the objective function is $118,017.32.

Regarding the harvest stage, the amount of cut stems in harvest area f_1 is 780, 657 and 541 for periods t_1, t_2 and t_3, respectively, while in harvest area f_2 404 and 400 stems are cut in periods t_1 and t_2, respectively. In the case of harvest area f_3, 542, 474 and 400 stems are cut in periods t_1, t_2 and t_3, respectively. No cutting activities are carried out in periods t_4 and t_5 in any of the harvest areas.

Due to in this example no initial stock of logs is considered in any of the nodes of the supply chain, it is necessary to assign at least one harvesting crew to the first period to cover the minimum demand of the plants in that period. And, because harvesting crews are required to work in successive periods, forest production (harvesting) will be concentrated in the early planning periods.

Since the harvesting crews perform the cutting activities during the first 3 periods, in the last periods the demands of the plants are completely covered with stocks (both in harvest areas and in plants). It is worth to mention that only stocks at the end of the planning horizon (logs remaining at forest sites and plants in t_5) have cost.

The used bucking patterns and the number of times they are applied in each period and in each harvest area are detailed in Table 8.

From Table 8, it can be seen, for example, that the bucking pattern b_4 is used 360 times in the harvest area f_1 and 73 times in the harvest area f_3, during the period t_1; in turn, it is used 187 times in f_1 and 134 times in f_3 during period t_2; finally, it is used 199 times in f_3 during the third period.

The detailed bucking and routing weekly plan can be obtained from the model solution. Next, some results are described and the corresponding analysis is presented in order to have a concise view of the approach capabilities. Due to the large amount of information, only for some types of logs the results are exposed.

Table 8 Number of times bucking pattern b is applied in harvest area f, per period

Period /Harvest area	Bucking pattern									
	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}
t_1										
f_1	73	274	6	360	67	0	0	0	0	0
f_2	0	0	0	0	0	165	130	0	109	0
f_3	0	0	185	73	0	0	107	0	177	0
t_2										
f_1	0	245	0	187	225	0	0	0	0	0
f_2	0	0	0	0	0	399	0	0	1	0
f_3	0	0	0	134	0	111	229	0	0	0
t_3										
f_1	0	0	540	0	1	0	0	0	0	0
f_2	0	0	0	0	0	0	0	0	0	0
f_3	0	0	0	199	0	0	0	0	201	0

Table 9 Fulfillment of demand, for logs of length l_3

l_3	i_1					i_2				
	Q^{TL}	mindem	Q^{PL}	Q^{SL}	covdem	Q^{TL}	mindem	Q^{PL}	Q^{SL}	covdem
d_1										
t_1	180	100	0	80	100	360	100	260	0	360
t_2	197	100	0	177	200	334	200	134	0	694
t_3	93	100	0	170	300	360	200	0	160	894
t_4	0	100	0	70	400	130	200	90	0	1184
t_5	180	100	150	0	650	316	200	116	0	1500
d_2										
t_1	0	0	0	0	0	172	100	0	72	100
t_2	206	0	0	206	0	136	0	0	208	100
t_3	34	0	0	240	0	352	500	0	60	600
t_4	0	0	240	0	240	110	0	0	170	600
t_5	110	100	10	0	350	180	0	350	0	950
d_3										
t_1	171	100	0	71	100	134	0	0	134	0
t_2	231	300	2	0	402	19	0	0	153	0
t_3	0	0	0	0	402	57	0	0	210	0
t_4	0	0	0	0	402	0	200	0	10	200
t_5	98	0	98	0	500	290	200	100	0	500
d_4										
t_1	112	0	0	112	0	50	50	0	0	50
t_2	114	0	0	226	0	90	0	90	0	140
t_3	201	0	0	427	0	0	0	0	0	140
t_4	0	150	0	277	150	57	0	57	0	197
t_5	0	0	200	77	350	53	0	53	0	250

Table 9 shows the fulfillment of the demand for each diameter corresponding to length l_3, for each plant. It details information about the number of logs transported to each plant ("Q^{TL}"), the minimum demand to cover ("*mindem*"), the level of demand covered above the minimum committed demand ("Q^{PL}"), the inventory level of logs in plant at the end of the period ("Q^{SL}") and the weekly cumulative demand level ("*covdem*").

It is interesting to analyze the operations on a log type. For example, for the case of the log of length l_3 and diameter d_2 for plant i_1, no logs of this type are transported in the first period. In the second period, 206 logs are transported, which remain in stock (they are not used). In the third period 34 logs are transported, increasing the total logs in stock to 240. In the fourth period, no logs of this type are transported and the 240 logs in stock are used to cover part of the weekly demand (240 out of 350). In the last period, 110 logs are transported, of which 100 correspond to the minimum committed demand for that period and the 10 extra logs are used to cover the entire weekly demand (350 in this case).

Regarding the log inventory levels, Figs. 3 and 4 show the obtained results for logs of length l_3 in harvest areas and in plants, respectively.

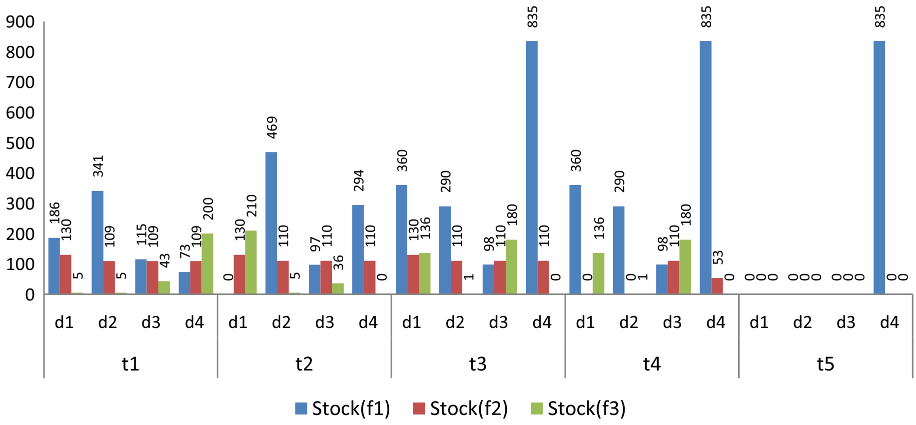


Fig. 3 Stored logs, by period, in each harvest area

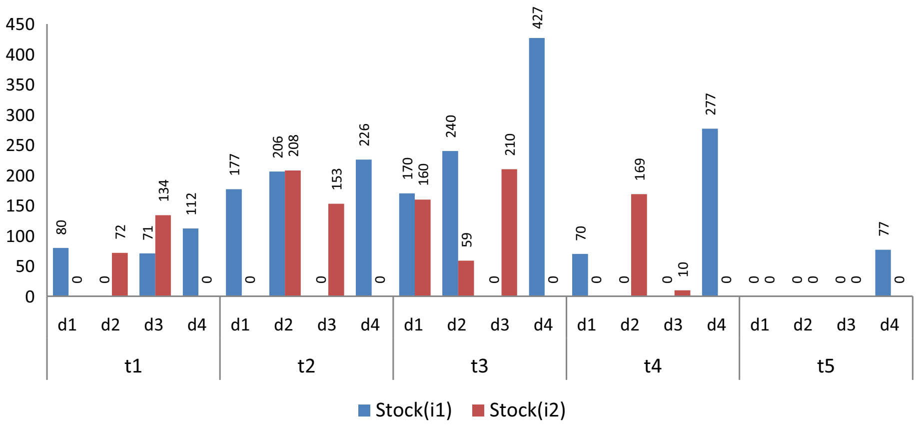


Fig. 4 Stored logs, by period, in each plant

In Fig. 4, it can be seen that, since the stored logs in the plant at the end of the planning horizon have a penalty cost, the stocks are minimized in period t_5. Furthermore, as the cost for keeping a type of log in stock at plant i_2 is higher than in plant i_1, the model tends to use the stock capacity in the last period in plant i_1 (note that in plant i_2 the stock level in the last period is zero). A similar situation occurs for harvest areas, since harvest area f_1 has the lowest maintenance cost, and therefore, its inventory level is the highest in the last period (see Fig. 3).

As can be seen, in this solution many logs of length l_3 and diameter d_4 are generated that are not transported during the planning horizon. This is because the model chooses to use those bucking patterns that have the best performance (for example, b_3 and b_9, see Table 8).

In the third period, for example, 540 times b_3 (in f_1) and 201 times b_9 (in f_3) are used, which generates a total of 741 logs of type l_3-d_4 (this type of log has a relatively low demand, 600 logs, as can be seen in Table 4), i.e., some bucking patterns are used because a certain log is needed and in these patterns logs with diameter d_4 are also generated (as a by-product).

As was previously mentioned, bucking pattern b_3 (see Table 8) is used 540 times in the third period in harvest area f_1. All those logs generated by b_3 are eventually transported to the plants, with the exception of logs with diameter d_4 (with higher associated inventory costs, due to their weight). As is to be expected, in the last period, the logs required to satisfy the weekly demand are mostly transported, leaving the logs generated by excess on the roadside in the harvest areas. This phenomenon can be seen in Table 9 for period t_5, where the total number of logs transported to the plants is practically consumed in this last period.

Table 10 shows the number of logs of each type sent from each harvest area and the number of logs transported to each plant in each period.

Finally, regarding the use of vehicles, 15 trucks are used in period t_1, 15 in period t_2, 13 in period t_3, 11 in period t_4 and 13 in period t_5. Table 11 shows the load (in number of logs and total weight) corresponding to each trip of the used trucks in the last period (t_5).

The route composition for each truck can be gathered from this table. For example, in the last period, truck c_9 performs two trips. On the first trip it transports logs of length l_1 from f_1 to i_2, with a minimum load (18.03 tons). The load is composed of 178 logs of diameter d_1 and 161 logs of diameter d_3. On the second trip, the truck transports logs of length l_2 from f_1 to i_1 with full load (26.99 tons) carrying 23 logs of diameter d_1, 103 of d_2 and 180 of d_3.

For this particular example, when traditional approach is applied decomposing in two steps, with harvesting optimization first, and solving the transportation problem in a second stage, the total costs worse 32.3%, with a very significant difference in the transportation cost, which doubles it value (125% increase).

5 Final remarks

In the forest industry, decisions associated with harvesting are closely related to production planning decisions in plants. The plants, based on the commitments assumed with the customers, plan detailed raw material supply programs to guarantee production. The harvesting crews receive these supply schedules and harvest the raw material accordingly. Transportation activities, in general, are not taken into account in this structure and have a great influence, not only on the efficiency of the system but also on the cost. Therefore, the problem should be addressed through a supply chain approach.

Previous works found in the literature usually address these issues separately, which leads to suboptimal and highly inefficient solutions. These inefficiencies are related to unnecessary log inventory levels both in harvest areas and in plants, and with the under-utilization of truck loading capacity due to inefficient coordination, generating a large number of trips and the corresponding increase in the number of used trucks. The foregoing has been made explicit through the motivating example developed for this purpose.

In order to face this drawback, in this work a MILP model was proposed that allows making decisions related to: the operational harvest planning and the inventory levels of logs at each node of the supply chain, the delivery of logs of different types from each harvest area to each plant, the load composition for each truck and the routes to perform by each truck to distribute the corresponding logs. The mathematical model considers heterogeneous fleet of trucks, multiple depots and multiple periods.

Simultaneous optimization of all these decisions leads to an approach where bucking, allocation of raw material (logs) and transportation activities are coordinated and efficiently addressed. The different tradeoffs among these decisions are jointly assessed and effectively evaluated. This approach represents a useful tool for operational planning of these tasks

Table 10 Number of logs transported from harvest areas and to plants, by type and by period

	Logs delivered from f_1				Logs delivered from f_2				Logs delivered from f_3				Logs receive in i_1				Logs receive in i_2			
	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4
t_1																				
l_1	180	0	162	0	0	0	64	123	516	169	220	214	180	0	406	158	516	169	40	179
l_2	280	229	212	0	0	168	93	26	0	0	0	0	180	59	177	0	100	338	128	26
l_3	180	0	171	0	0	0	0	0	360	172	134	162	180	0	171	112	360	172	134	50
l_4	360	0	19	140	274	312	0	0	205	0	162	81	429	162	162	75	410	150	19	146
t_2																				
l_1	0	0	0	0	0	179	176	0	268	344	360	183	0	179	536	124	268	344	0	59
l_2	245	268	193	0	0	179	0	208	0	0	0	0	65	272	178	208	180	175	15	0
l_3	373	342	243	4	0	0	0	0	158	0	7	200	197	206	231	114	334	136	19	90
l_4	7	0	46	70	360	360	20	0	175	180	57	76	7	180	103	70	535	360	20	76
t_3																				
l_1	0	0	0	0	0	0	0	0	0	10	180	58	0	10	180	58	0	0	0	0
l_2	220	283	188	0	0	168	131	142	185	349	0	115	55	445	304	216	350	355	15	41
l_3	180	180	0	0	0	0	0	0	273	206	57	201	93	34	0	201	360	352	57	0
l_4	0	0	0	285	0	360	5	0	178	34	118	0	0	0	0	285	178	394	123	0
t_4																				
l_1	360	146	224	0	0	61	0	137	180	20	180	135	180	61	341	193	360	166	63	79
l_2	120	153	42	0	0	179	170	117	0	168	0	212	0	168	170	257	120	332	42	72
l_3	0	0	0	0	130	110	0	57	0	0	0	0	0	0	0	0	130	110	0	57
l_4	0	0	0	95	0	0	0	0	180	8	260	28	0	8	172	95	180	0	88	28

Table 10 (continued)

	Logs delivered from f_1				Logs delivered from f_2				Logs delivered from f_3				Logs receive in i_1				Logs receive in i_2			
	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4	d_1	d_2	d_3	d_4
t_5																				
l_1	178	0	161	0	0	0	0	0	218	171	180	82	40	0	180	57	356	171	161	25
l_2	200	160	359	0	0	0	170	181	0	521	111	162	200	681	640	343	0	0	0	0
l_3	360	290	98	0	0	0	110	53	136	0	180	0	180	110	98	0	316	180	290	53
l_4	180	0	0	75	40	96	105	0	157	0	145	0	180	0	0	75	197	96	250	0

Table 11 Load composition of each trip (for the last period, t5)

Truck/trip	Origin/destiny	Length	Logs				Total load (ton)
			d_1	d_2	d_3	d_4	
c_1-v_1	f_2-i_1	l_2	0	0	0	136	26.90
c_2-v_1	f_2-i_1	l_2	0	0	170	45	26.99
c_3-v_1	f_1-i_1	l_4	180	0	0	75	26.85
c_3-v_2	f_1-i_1	l_3	180	110	98	0	26.98
c_4-v_1	f_3-i_2	l_3	136	0	180	0	26.84
c_5-v_1	f_1-i_1	l_2	177	57	179	0	27.00
c_5-v_2	f_1-i_2	l_3	180	180	0	0	20.38
c_6-v_1	f_2-i_2	l_4	40	96	105	0	26.98
c_6-v_2	f_2-i_2	l_3	0	0	110	53	26.94
c_7-v_1	f_3-i_2	l_4	157	0	145	0	26.97
c_9-v_1	f_1-i_2	l_1	178	0	161	0	18.03
c_9-v_2	f_1-i_1	l_2	23	103	180	0	26.99
c_{10}-v_1	f_3-i_1	l_2	0	177	111	13	26.98
c_{11}-v_1	f_3-i_1	l_1	40	0	180	57	26.84
c_{14}-v_1	f_3-i_2	l_1	178	171	0	25	18.02
c_{16}-v_1	f_3-i_1	l_2	0	176	0	73	26.97
c_{20}-v_1	f_3-i_1	l_2	0	168	0	76	26.99

in the forest industry, and the good computational performance allows appraising diverse harvesting-routing scenarios for guiding the decision maker.

Forest supply chain is confronted to several sources of uncertainty, such as stem dimensions in harvest areas, log demands, produced logs,, stems accessibility due to climate conditions (harvest sites are located in a tropical zone and rains complicate transportation), etc. Although the most works presented in the literature for the problem considered in this article assume deterministic model parameters, a more realistic representation must take into account uncertain parameters. This assumption will be considered in future works as well as methodologies to efficiently solve stochastic optimization.

Another future work from the planning point of view is considering long time periods, i.e. the harvesting and transportation planning over several weeks. This assumption increases the model size and therefore, complicates the model solution. In order to overcome this drawback, the so-called rolling-horizon approach can be adopted. This method aims to solve the problem periodically, including information of the certain periods. In this way, the model presented in this work can be repeatedly solved for each next period fixing the variables of previous periods to their optimal values prior obtained.

Finally, improving the computational efficiency to solve the proposed formulation represents a challenging task. Although the performance of the solved real size problem has a small optimality gap, it is desirable to arrive at optimal solutions and be able to tackle larger problems. The development and application of methodologies for obtaining better bounds on decision variables and consequently, improving the model performance, will be addressed in future works.

Acknowledgements This work was supported by the CONICET, FONCyT and UTN through their projects PIP 1401, PICT 2017-4004 and PID SIUTIFE0005246TC, respectively.

References

Bordón, M. R., Montagna, J. M., & Corsano, G. (2018). An exact mathematical formulation for the optimal log transportation. *Forest Policy and Economics, 95*, 115–122. https://doi.org/10.1016/j.forpol.2018.07.017

Bordón, M. R., Montagna, J. M., & Corsano, G. (2020). Mixed integer linear programming approaches for solving the raw material allocation, routing and scheduling problems in the forest industry. *International Journal of Industrial Engineering Computations, 11*(4), 525–548. https://doi.org/10.5267/j.ijiec.2020.5.001

Bordón, M. R., Montagna, J. M., & Corsano, G. (2021a). Solution approaches for solving the log transportation problem. *Applied Mathematical Modelling, 98*, 611–627. https://doi.org/10.1016/j.apm.2021.06.003

Bordón, M. R., Montagna, J. M., Corsano, G. (2021b). An integrated approach for solving the bucking and routing problems in the forest industry. In: D. A. Rossit, F. Tohmé, G. Mejía Delgadillo (eds) *Production Research. ICPR-Americas 2020. Communications in Computer and Information Science*, 1408. Springer. https://doi.org/10.1007/978-3-030-76310-7_16

Borges, J. G., Diaz-Balteiro, L., McDill, M. E., & Rodriguez, L. C. E. (2014). *The management of industrial forest plantations, managing forest ecosystems*. Springer.

D'Amours, S., Rönnqvist, M., & Weintraub, A. (2008). Using operational research for supply chain planning in the forest products industry. *Information Systems and Operations Research, 46*(4), 47–64. https://doi.org/10.3138/infor.46.4.265

Dems, A., Rousseau, L. M., & Frayret, J. M. (2017). Annual timber procurement planning with bucking decisions. *European Journal of Operational Research, 259*(2), 713–720. https://doi.org/10.1016/j.ejor.2016.10.039

El Hachemi, N., El Hallaoui, I., Gendreau, M., & Rousseau, L.-M. (2014). Flow-based integer linear programs to solve the weekly log-truck scheduling problem. *Annals of Operations Research, 232*, 87–97. https://doi.org/10.1007/s10479-014-1527-4

Fuentealba, S., Pradenas, L., Linfati, R., & Ferland, J. A. (2019). Forest harvest and sawmills: An integrated tactical planning model. *Computers and Electronics in Agriculture, 156*, 275–281. https://doi.org/10.1016/j.compag.2018.11.011

Linfati, R., Pradenas, L., & Ferland, J. A. (2016). Planificación agregada en la cosecha forestal: Un modelo de programacion matemática y solucion. *Maderas Ciencia y Tecnología, 18*, 555–566. https://doi.org/10.4067/S0718-221X2016005000048

Ministry of Agriculture, Livestock and Fisheries (2015). *National census of sawmills*. Retrieved March, 2021, from https://www.magyp.gob.ar/sitio/areas/ss_desarrollo_foresto_industrial/censos_inventario/_archivos/censo//000000_Informe%20Nacional%20de%20Aserraderos%202015.pdf

Rosenthal R. E. (2020). *A GAMS tutorial*. Retrieved March, 2021, from https://www.gams.com/latest/docs/UG_Tutorial.html

Rönnqvist, M., D'Amours, S., Weintraub, A., Jofre, A., Gunn, E., Haight, R. G., Martell, D., Murray, A. T., & Romero, C. (2015). Operations research challenges in forestry: 33 open problems. *Annals of Operations Research, 232*, 11–40. https://doi.org/10.1007/s10479-015-1907-4

Simon, F., Girard, A., Krotki, M., & Ordoñez, J. (2020). Modelling and simulation of the wood biomass supply from the sustainable management of natural forests. *Journal of Cleaner Production, 282*, 124487. https://doi.org/10.1016/j.jclepro.2020.124487

Vanzetti, N., Broz, D., Montagna, J. M., & Corsano, G. (2019). Integrated approach for the bucking and production planning problems in forest industry. *Computers and Chemical Engineering, 125*, 155–163. https://doi.org/10.1016/j.compchemeng.2019.03.008