

Evolutionary timescales from the AGB to the CSPNe phase

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Abstract. The transition from the asymptotic giant branch (AGB) to the final white dwarf (WD) stage is arguably the least understood phase in the evolution of single low- and intermediate-mass stars ($0.8 \lesssim M_{\text{ZAMS}}/M_{\odot} \lesssim 8\text{--}10$). Here we briefly review the progress in the last 50 years of the modeling of stars during the post-AGB phase. We show that although the main features, like the extreme mass dependency of post-AGB timescales were already present in the earliest post-AGB models, the quantitative values of the computed post-AGB timescales changed every time new physics was included in the modeling of post-AGB stars and their progenitors. Then we discuss the predictions and uncertainties of the latest available models regarding the evolutionary timescales of post-AGB stars.

Keywords. stars: AGB and post-AGB, planetary nebulae: general, stars: evolution, stars: mass loss

1. Modeling the evolution after the AGB. A historical introduction

The transition from the asymptotic giant branch (AGB) to the final white dwarf (WD) stage is arguably the least understood phase in the evolution of single low- and intermediate-mass stars ($0.8 \lesssim M_{\text{ZAMS}}/M_{\odot} \lesssim 8\text{--}10$). This transition phase includes the so-called proto-planetary nebulae (PPNe) central stars and some OH/IR objects (also collectively known as *post-AGB stars*, [van Winckel 2003†](#)) as well as the hotter central stars of planetary nebula (CSPNe) and other UV-bright stars. In the most simple picture low- and intermediate-mass stars undergo strong and slow stellar winds ($10^{-8}M_{\odot}/\text{yr} \lesssim \dot{M} \lesssim 10^{-4}M_{\odot}/\text{yr}$ and $3 \text{ km/s} \lesssim v_{\text{wind}} \lesssim 30 \text{ km/s}$, see [Höfner & Olofsson 2018](#)) at the end of the AGB which lead to the almost complete removal of the H-rich envelope of the AGB star—see [Shklovsky \(1957\)](#), [Abell & Goldreich \(1966\)](#) and [Paczynski \(1970\)](#). After this point, the stars contract at constant luminosity (L_{\star}) increasing their effective temperatures (T_{eff}) by almost two orders of magnitude, and if there is enough material surrounding the star, a PN is formed in the process.

Since the very first stellar evolution models of CSPNe were computed by [Paczynski \(1970\)](#) it became clear that the evolution from the AGB to the WD phase is extremely mass dependent. In fact, the early models of [Paczynski \(1970\)](#) suggested that the time to “cross” the HR diagram decreased by 4 orders of magnitude just by increasing the mass of the CSPN model by a factor of two (see Fig. 1). The early models by [Paczynski \(1970\)](#),

† Note that throughout this paper we refer to the whole evolutionary stage between the end of the AGB and the beginning of the WD phase as the *post-AGB phase*. This should not be confused with the so called *post-AGB stars* which are defined as stars that have already departed from the AGB but are not yet hot enough to ionize the circumstellar material, i.e. $T_{\text{eff}} \lesssim 30\,000\text{K}$.

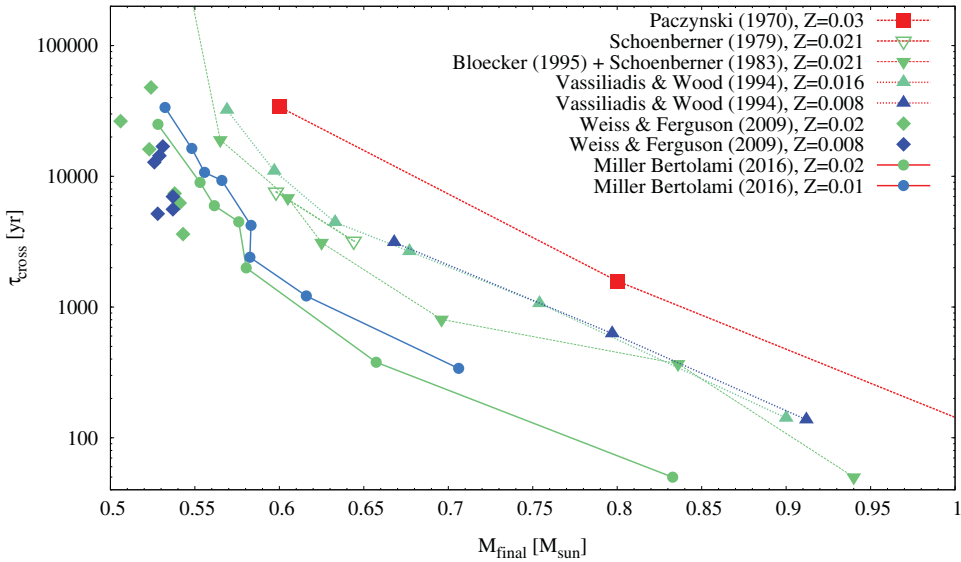


Figure 1. Evolutionary post-AGB time between the point at $\log T_{\text{eff}} = 4$ to the point of maximum effective temperature in the HR diagram; τ_{cross} .

1971) were constructed by artificially fitting H-envelopes to core structures all obtained from a flash suppressed AGB sequence of $M_i = 3M_{\odot}$ ($Z = 0.03$) and evolved to obtain CO-cores at desired final remnant mass ($M_f = 0.6, 0.8, 1.2M_{\odot}$). Schönberner (1979, 1981) computed for the first time the transition from the AGB to the CSPNe phase by assuming a steady wind according to the mass loss prescription by Reimers (1975) and including a detailed computation of the thermally pulsating (TP) AGB phase. This was done for two full sequences with initial masses $M_i = 1$ and $1.45 M_{\odot}$ (final masses $M_f = 0.598$ and $0.644M_{\odot}$ respectively) and $Z = 0.021$. These two sequences already showed post-AGB timescales to be about 4.5 times faster than predicted by the early Paczyński (1970) models (see Fig. 1). Later, Schönberner (1983) computed two more sequences of initial masses $M_i = 0.8$ and $1M_{\odot}$ (initial metallicity $Z = 0.021$, final masses $M_f = 0.546$ and $0.565M_{\odot}$ respectively) by including for the first time a “superwind” phase at the end of the AGB with mass-loss rates of $\dot{M} \gtrsim 10^{-4}M_{\odot}/\text{yr}$. Although it only covered a small mass range, Schönberner’s post-AGB models were the first to include a detailed treatment of the TP-AGB, showing the importance of AGB modeling for the computation of accurate post-AGB stellar models Schönberner (1987). The next grid of models, which covered a wider range of remnant masses ($0.6 \leq M_f/M_{\odot} \leq 0.89$), was computed by Wood & Faulkner (1986). These models were constructed by artificially stripping most of the H-envelope from red giant models computed through many thermal pulses on the AGB but from a single progenitor sequence of $M_i = 2M_{\odot}$ ($Z = 0.02$). Wood & Faulkner (1986) computed the end of the TP-AGB by assuming two different extreme mass-loss rates in their computations. However, the lack of a realistic initial-final mass relation (IFMR Weidemann 1987) had consequences in the predicted post-AGB evolution and was criticized by Blöcker & Schönberner (1990).

The following generation of post-AGB models came in the 90’s when both Vassiliadis & Wood (1994) and Blöcker (1995) published grids of post-AGB models, covering the whole mass range of CSPNe, which included a detailed treatment of the winds during the TP-AGB phase—see Vassiliadis & Wood (1993) and Blöcker (1995). In particular these grids adopted different initial masses to produce different CSPNe, as expected from early determinations of the Initial/Final Mass Relation (see Weidemann 1987 and

references therein). These grids of post-AGB stellar evolution models represented a great improvement over the previous Paczyński (1970, 1971) and Wood & Faulkner (1986) models and confirmed the previous result by Schönberner (1979, 1981) that post-AGB timescales were several times shorter than predicted by Paczyński (1970), as can be seen in Fig. 1. It is worth noting, however, that neither Vassiliadis & Wood (1994) nor Blöcker (1995) incorporated the impact of core-overshooting in the upper main sequence, which was already known at the time to be significant, e.g. Schaller *et al.* (1992). Neither did these models make use of the updated radiative opacities computed by the OPAL (Rogers & Iglesias 1992) and OP (Seaton *et al.* 1994) projects that revolutionized stellar astrophysics during the early nineties.

About a decade later another significant improvement in AGB and post-AGB stellar evolution models was made by Kitsikis (2008) (later published in Weiss & Ferguson 2009, from now on KWF). These authors incorporated several features for the first time, both in the computation of the AGB and post-AGB stellar evolution models. First, following Marigo (2002), these authors included for the first time C-rich molecular opacities in the computation of full AGB stellar evolution models. In addition, they also incorporated a separated treatment of C-rich and O-rich dust-driven winds. Most importantly, these authors included both the impact of convective boundary mixing on the main sequence, helium core-burning stage and TP-AGB evolutionary stages, as well as the inclusion radiative opacities from the OPAL project. Probably because of the latter, many convergence problems prevented KWF from computing a large grid of post-AGB stellar evolutionary models. In spite of the lack of a large grid of post-AGB sequences, the models computed by KWF already showed a clear trend, the post-AGB evolution of these models was much faster than those computed by Vassiliadis & Wood (1994) or Blöcker (1995), see Fig. 1. Again, as it happened with Schönberner's post-AGB models more than two decades before, an improvement in the modeling of previous evolutionary stages lead to much shorter post-AGB timescales. Finally, following the approach of KWF, Miller Bertolami (2015, 2016) computed a larger grid of post-AGB stellar evolution models. The main difference between this work and that of KWF is that mixing at convective boundaries from the ZAMS to the TP-AGB were calibrated trying to reproduce several observables on the main sequence and on the TP-AGB and post-AGB evolutionary stages. In particular, the models computed by Miller Bertolami are able to reproduce the width of the main sequence, the C/O ratios of the AGB and post-AGB stars and the He, C and O abundances observed in PG1159 stars. Most importantly, the IFMR of the theoretical models computed by Miller Bertolami (2016) are closer to the semi empirically derived ones than those of KWF (see Fig. 2). In agreement with the findings of KWF the post-AGB models computed by Miller Bertolami (2016) are significantly faster and slightly brighter than earlier models of similar final mass (see Fig. 1).

2. Post-AGB timescales and definitions

The terminology used to define the various stages after the departure from the AGB is sometimes confusing. For example, stellar evolution studies usually refer to the whole stage between the end of the AGB and the beginning of the white dwarf phase as the post-AGB stage (e.g. Vassiliadis & Wood 1994, Blöcker 1995 and Miller Bertolami 2016), while observationally is common to refer as *post-AGB stars* to those stars that have already departed from the AGB but are not yet hot enough to ionize the circumstellar material, i.e. $T_{\text{eff}} \lesssim 30\,000$ K (van Winckel 2003; Szczerba *et al.* 2007). Also, from the observational point of view it is usual to split the evolution after the AGB and before the WD phase into the Proto-Planetary Nebulae (PPNe) and the Central Star of Planetary Nebulae (CSPNe) phases. Within this classification the PPNe phase corresponds to the early evolution from the end of the AGB to the beginning of the ionization of the surrounding

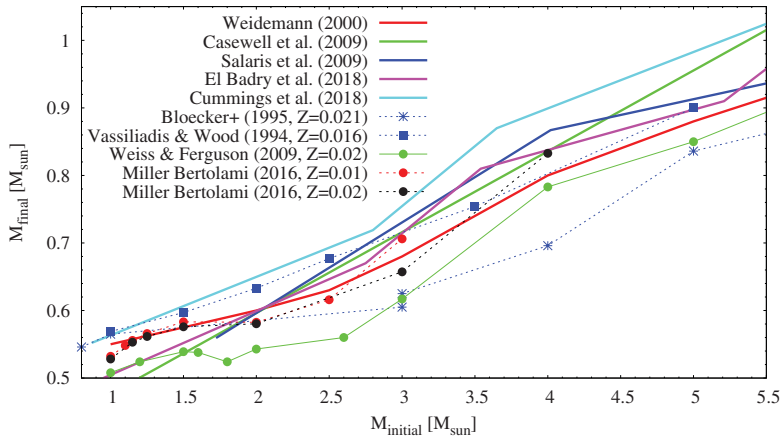


Figure 2. Initial-Final Mass Relation (IFMR) of different post-AGB stellar evolution models as compared with the latest semi empirical determinations Casewell *et al.* (2009); Salaris *et al.* (2009); El-Badry *et al.* (2018); Cummings *et al.* (2018), and the classic semi empirical relation of Weidemann (2000).

nebulae at about $T_{\text{eff}} \sim 30\,000$ K. During the PPNe phase it is expected that many central stars would still be enshrouded in dust and not visible in the optical, see Szczerba *et al.* (2007). The CSPNe phase would then correspond to the phase from the moment the central star attains $T_{\text{eff}} \sim 30\,000$ K to the beginning of the white dwarf cooling track. Yet, from the point of view of the evolution of the central star, this classification is of little use, as it relies on the properties of the surrounding material. Even more, very low mass stars might not evolve fast enough or eject significant amounts of material during the late AGB phase to produce a visible PNe.

In order to be able to quantify the properties of the models during the post-AGB phase precise definitions are required. In particular it is worth noting that the very idea of the end of the AGB is not easy to define from the point of view of stellar evolution, as stars continuously evolve away from the AGB during the late AGB evolution but without any sudden change in the stellar properties. From the point of view of the structure of the central star the main change that takes place during the departure from the AGB is the transition from an expanded giant-like configuration into a dwarf-like one. This is caused by the reduction of the H-rich envelope below the critical value required to sustain a giant-like structure (see Fig. 3 and Faulkner 2005 for an extended discussion of this problem). This leads to a continuous increase in the heating rate of the stellar surface from $\dot{T}_{\text{eff}} \lesssim 0.1$ K/yr on the AGB to $1\text{K/yr} \lesssim \dot{T}_{\text{eff}} \lesssim 10\,000$ K/yr once the star attained $T_{\text{eff}} \gtrsim 10\,000$ K, see Fig. 4.

In this context, and in order to discuss the properties of the computed stellar models, different authors choose to divide the transition from the AGB to the WD phase according to different arbitrary definitions. As the relative mass of the envelope is a key feature determining the end of the AGB, Miller Bertolami (2016) choose to define the end of the AGB phase as the moment in which $M_{\text{env}}/M_{\star} = 0.01$ (see Fig. 3). At this moment, models have already moved significantly to the blue ($T_{\text{eff}} \sim 3700\text{--}5000$ K), which is true at all masses and metallicities. Although this choice defines the end of the AGB in a homogeneous way for all masses and metallicities, and is based on the underlying physical reason behind the departure from the AGB, the choice remains arbitrary.

In order to disentangle the impact of different uncertainties and definitions we define two different timescales: the transition time scale τ_{trans} corresponding to the early (and

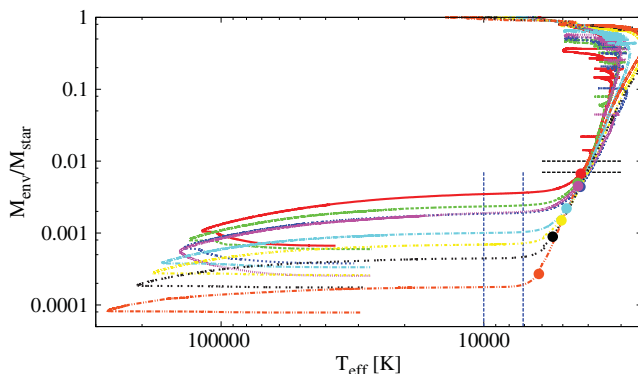


Figure 3. Mass of the H-rich envelope of the H-burning models computed by Miller Bertolami (2016). The vertical dashed lines indicate the zero points at $\log T_{\text{eff}} = 3.85$ ($\log T_{\text{eff}} = 4$) adopted by Miller Bertolami (2016) Vassiliadis & Wood (1994) for the computation of the post-AGB crossing times (τ_{cross}). Horizontal dashed lines indicate two alternative envelope masses adopted by Miller Bertolami (2016) as a definition of the end of the AGB. Color circles indicate the end of the AGB as estimated from the suggestion by Soker (2008).

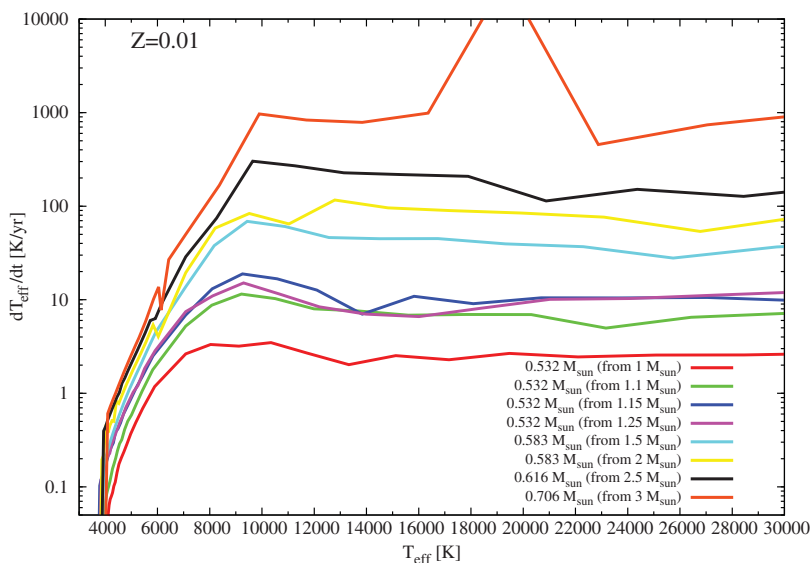


Figure 4. Heating speed of the stellar evolution models during the early post-AGB phase. Note the exponential increase in the heating rate below $T_{\text{eff}} \lesssim 10\,000$ K as the almost constant heating rate at $10\,000\text{K} \lesssim T_{\text{eff}} \lesssim 30\,000\text{K}$.

slow) evolution from the end of the AGB ($M_{\text{env}}/M_{\star} = 0.01$) to the point at $\log T_{\text{eff}} = 3.85$, and the crossing timescale τ_{cross} corresponding to the late (fast) evolution from $\log T_{\text{eff}} = 3.85$ to the point of maximum effective temperature. In what follows we discuss the properties and uncertainties of these two post-AGB timescales.

2.1. The crossing time: τ_{cross}

The uncertainties in τ_{cross} (Fig. 1) are mostly related to uncertainties in the previous evolution, and not to uncertainties of the physics during the post-AGB phase itself. In particular, with the exception of very luminous CSPNe one should not expect

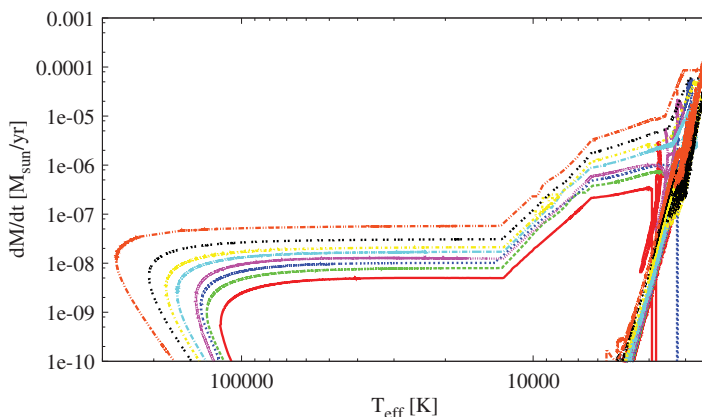


Figure 5. Mass-loss rates of the sequences computed by Miller Bertolami (2016) for $Z = 0.01$.

hot radiative-driven winds to be of any importance for the value of τ_{cross} in H-burning post-AGB stars. Fig. 5 shows the evolution of the mass-loss rate $\dot{M}_{\text{winds}}^{\text{env}}$ adopted in the computation of the sequences computed by Miller Bertolami (2016) for $Z = 0.01$. The rate of reduction of the H-rich envelope by winds have to be compared with the rate of H-consumption by nuclear burning which in the case of CNO-burning is of $\dot{M}_{\text{H}}/(M_{\odot}\text{yr}^{-1}) \sim 10^{-11} L_{\text{H}}/L_{\odot}$. Consequently, for typical post-AGB stars, in the range $\log L/L_{\odot} = 3...4$, the H-rich envelope is consumed by nuclear burning at $\dot{M}_{\text{env}}^{\text{burn}} \sim 1.4 \times 10^{-8} \dots 2 \times 10^{-7}$. Consequently, and with the exception of the more massive and luminous model, winds affect the rate of consumption of the H-rich envelope by less than a 20%, see Table 3 of Miller Bertolami (2016), and do not play a significant role in the determination of τ_{cross} . The value of τ_{cross} is consequently determined by the mass of the H-rich envelope at which the model departs from the AGB and the intensity of the H-burning shell. While these two properties are to some extent affected by the phase of the thermal pulse cycle at which the star departs from the TP-AGB, they are much more affected by the degeneracy level of the CO-core and intershell (see Blöcker 1995) as well as by the chemical composition of the H-rich envelope (see Tuchman *et al.* 1983 and Marigo *et al.* 1999). In turn these properties are mostly affected by the micro physics adopted in the models and the macro physics (winds and convective boundary mixing prescriptions) which affect the efficiency of third dredge up as well as the length of the TP-AGB phase and the initial-final mass relation. It is worth emphasizing that convective boundary mixing the main sequence also affects the final post-AGB timescales, as it has an important impact in the initial final mass relation, see Salaris *et al.* (2009) and also Wagstaff *et al.* in preparation.

Additionally, due to the fast evolution from $T_{\text{eff}} = 7\,000$ K to $10\,000$ K (see Fig. 6) different choices of the zero point (like those of Vassiliadis & Wood 1994, Blöcker 1995 and Miller Bertolami 2016) have a negligible impact in the actual value of τ_{cross} . This, together with the previously discussed points, link all important uncertainties and differences in the computations of τ_{cross} (see Fig. 1) directly to the modeling of previous evolutionary stages.

2.2. The transition time: τ_{trans}

At variance with what happens with τ_{cross} the value of the transition time τ_{trans} is directly affected by the intensity of stellar winds during this phase. As can be seen

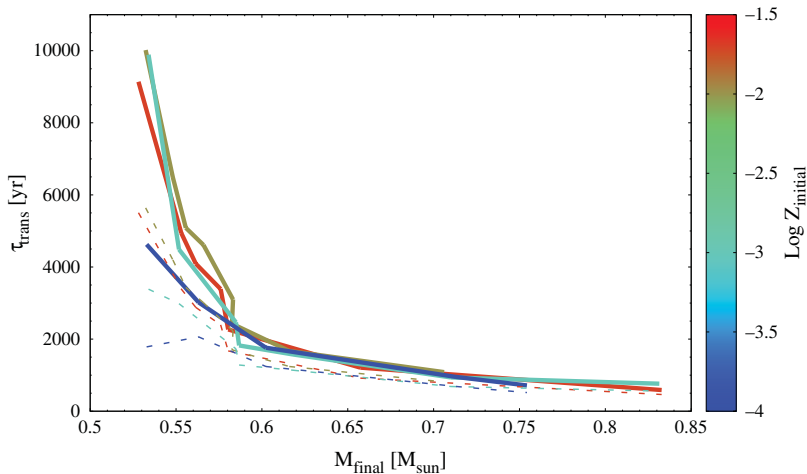


Figure 6. Solid lines: Transition times τ_{trans} from the end of the AGB defined at $M_{\text{env}} = 0.01M_{\star}$ to the point at $\log T_{\text{eff}} = 3.85$ during the post-AGB evolution for sequences of different final masses and metallicities Miller Bertolami (2016). Dashed lines: Same as the solid lines but with the end of the AGB defined as the point at $M_{\text{env}} = 0.007 M_{\star}$.

in Fig. 5 mass-loss rates are well above the threshold of $10^{-8} \dots 10^{-7} M_{\odot}/\text{yr}$ and the evolution is then dominated by the intensity of stellar winds. To make things worst, winds during this transition phase are completely uncertain. Also, attempts to measure the evolutionary speed of these objects by means of the study of the period drift in PPNe are not still possible (see Hrnivak *et al.*, these proceedings). For example, Miller Bertolami (2016) adopted during this stage mostly the wind prescription for cold giant winds by Schröder & Cuntz (2005) which have only been validated for much cooler stars $T_{\text{eff}} \lesssim 4500$ K (see Schröder & Cuntz 2007), so inaccuracies of a factor of a few are not unthinkable. Note that, as τ_{trans} is basically determined by speed of the reduction of the remaining H-rich envelope by winds, any error in the wind intensity in this transition regime ($4000 \text{ K} \lesssim T_{\text{eff}} \lesssim 7000 \text{ K}$) will directly translate into errors in the computed value of τ_{trans} .

In addition to our current lack of knowledge of winds during this early post-AGB phase the arbitrariness in the definition of the end of the AGB (and thus of the beginning of this early stage) directly affects the value of τ_{trans} . Fig. 6 shows that the value of τ_{trans} would change by a factor of ~ 2 if the end of the AGB would have been defined as $M_{\text{env}} = 0.007M_{\star}$ (dashed lines) as compared with the $M_{\text{env}} = 0.01M_{\star}$ adopted by Miller Bertolami (2016). In this connection it is worth recalling the suggestion by Soker (2008) of quantitative definition for the end of the TP-AGB based on the ratio Q of the dynamical and envelope timescales of the star. Fig. 3 shows a simple estimation of the point at which Q reaches its maximum value (from Eq. 6 in Soker 2008 and under the assumption of $\beta = 1$). Fig. 3 suggests that the criterion proposed by Soker (2008) might indeed be able to capture key aspect of the transition from the AGB to the post-AGB, as it defines the end of the AGB close to the point where the fast post-AGB phase starts. It remains to be seen to which extent it agrees with the observational definitions of the post-AGB phase, but it certainly deserves further examination.

In view of the previous discussion, the values of τ_{trans} are only qualitatively useful. In particular, an interesting result from Fig. 6 is that τ_{trans} changes by more than an order of magnitude when going from $M_f \sim 0.5M_{\odot}$ to $M_f \gtrsim 0.7M_{\odot}$. Note, in particular, that for $M_f \gtrsim 0.6M_{\odot}$ this stage lasts for $\tau_{\text{trans}} \lesssim 2000$ yr for all metallicities.

3. Final comments

During the last 50 years our modeling of post-AGB stars has been slowly but continuously improved as better physics (both micro- and macro-physics) have been included in the modeling both of the post-AGB and previous evolutionary phases. In particular, it seems that with each current improvement post-AGB timescales became shorter. Current models that have been calibrated to reproduce several observables in the evolution of low- and intermediate-mass stars [Miller Bertolami \(2016\)](#) indicate that the time required to cross the HR-diagram from $T_{\text{eff}} \sim 7000$ K to $T_{\text{eff}} \gtrsim 7000$ K is of only $\tau_{\text{cross}} \sim 10000$ yr for remnant stars of $M_f \sim 0.55M_{\odot}$ of $\tau_{\text{cross}} \lesssim 2000$ yr $M_f \gtrsim 0.58M_{\odot}$ and less than a few hundred years for objects with $M_f \gtrsim 0.70M_{\odot}$. The fast post-AGB evolution of this models helps to explain the observed existence of single CSPNe of low mass (e.g. [Althaus et al. 2008](#), [Henry et al. 2018](#) and [Miller et al. 2018](#)), as well as to understand the properties of CSPNe in the Galactic Bulge ([Gesicki et al. 2014](#)). In addition the faster evolution of current post-AGB models might be key to understand the mystery of the invariance of the planetary nebulae luminosity cut-off mystery ([Gesicki et al. 2018](#)) and, may be, the dearth of post-AGB stars in M32 ([Brown et al. 2008](#)). However, although the current models are able to reproduce several observables of AGB and post-AGB populations ([Miller Bertolami 2016](#)), some significant discrepancies are still present. The most important ones are the inability of the current models to reproduce the total lifetime of intermediate luminosity M-stars and C-stars at about the LMC luminosity (see [Miller Bertolami 2016](#)) and the systematically lower final masses of current models in the range $M_i \sim 2...3M_{\odot}$ when compared with the latest determinations of the initial-final mass relation, see Fig. 2 ([Casewell et al. 2009](#), [Salaris et al. 2009](#), [El-Badry et al. 2018](#), [Cummings et al. 2018](#)). A lower intensity of third dredge up processes and a lower intensity of the mass loss during the C-rich phase might help to solve both problems ([Wagstaff et al.](#) in preparation).

Still, the largest uncertainty in our current understanding of the post-AGB evolution in single stars concerns the intensity of winds during the departure from the AGB $4000\text{K} \lesssim T_{\text{eff}} \lesssim 7000$ K which strongly affects the evolutionary speed of the models during the transition stage (τ_{trans}).

Finally it should be mentioned that all post-AGB stellar evolutionary sequences are based on the assumption that the final ejection of the envelope occurs through steady winds. This leads to a well defined relationship between the mass of the remnant and the mass of the remaining H-rich envelope. The strong dependency of the critical mass of the envelope at which the stars depart from the AGB (Fig. 3) is key in the determination of the mass dependency of the crossing timescale (τ_{cross} , Fig. 1). If some objects eject their envelopes by means of a dynamical phase due to binary interaction or, for example, the ingestion of a substellar companion then the remnant might depart from the AGB with smaller envelope masses and out of thermal equilibrium (e.g. [Hall et al. 2013](#)), and evolve through the post-AGB phase at a much faster pace. In particular, this implies that any comparison of post-AGB stellar models like those computed by [Vassiliadis & Wood \(1994\)](#); [Blöcker \(1995\)](#); [Miller Bertolami \(2016\)](#) with CSPNe in close binary systems should be addressed with strong skepticism.

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Discussion

D'ANTONA: From your very complete presentation, do I understand correctly that, in spite of the difficulties with the determination of the transition time, massive planetary nebulae nuclei cannot be found in the high luminosity crossing phase, like in the old Paczynski models?

MILLER BERTOLAMI: Yes, indeed. According to my models, remnants with masses between 0.7 and $0.8 M_{\odot}$ (the largest ones I computed) cross the HR diagram in only ~ 100 to ~ 10 yr and then start to decrease their luminosity towards the white dwarf phase. So they should be very rare. In addition, one might wonder whether they would not be still highly obscured by circumstellar material.

VENTURA: Which are the typical timesteps and mass-loss rates which you use during the transition time?

MILLER BERTOLAMI: During the transition phase I forced the code not to use the extremely small timesteps our algorithm would naturally suggest (which might be as small as 10^{-4} yr). So I usually forced timesteps to be between 0.1 and 1 yr during the transition phase.



