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Cosmological parameters and non–vacuum initial states

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Abstract. A class of spatially flat models with a cosmological constant and a primordial broken scale invariant (BSI) spectrum of adiabatic perturbations is confronted with the most up-to-date observational data of CMB and matter power spectrum. The theoretical model includes a parameter $n_{\rm b}$ for the number of quanta in the non–vacuum initial state, and a privileged scale leading to the existence of a feature in the primordial power spectrum. This feature is located at comoving wavenumber $k_{\rm b}$ and its profile is characterized by a step in k with steepness α, the full set $\{n_{\rm b}, k_{\rm b}, \alpha\}$ being taken as free parameters in our numerical study.
We present here preliminary results of a detailed Markov Chain Monte Carlo analysis with CAMB and CosmoMC of the latest CMB and P(k) measurements, including the 3-year WMAP and the final 2dFGRS catalogue, where we derive joint constraints on eight out of the many relevant primary parameters –both cosmological and feature– of our BSI adiabatic model.

Resumen. Realizamos una comparación detallada de modelos inflacionarios con estados iniciales de no–vacío para las perturbaciones cosmológicas con las más recientes observaciones de la radiación cósmica del fondo de microondas y surveys de la estructura a gran escala del universo.

1. Introduction

The recent development in cosmological observations, mainly from the measurements of the Cosmic Microwave Background (CMB) radiation and the large scale structure of the Universe, have revolutionized our knowledge of the values of the basic cosmological parameters, helping to establish the ΛCDM as the standard cosmological model. According to this model, the Universe is roughly spatially flat with approximately one-third of its energy content in the form of mass (most of which is of a type not yet found in the laboratory) and the remaining two-thirds in the form of some sort of dark energy. The primordial cosmological fluctuations that seeded the astrophysical structures we see today were adiabatic, gaussian–distributed and had a nearly scale invariant power spectrum, in consistency with standard inflationary models.

A built–in characteristic scale is one of the simplest ways to generalize the standard vacuum initial state for the cosmological perturbations of quantum mechanical origin which possesses no priviledged scale. It also opens up the window to consider general BSI models of the early universe and to see whether those constitute a class of models that can fit present–day observations better than what standard inflation does.

It is our aim here to concentrate ourselves in a particular BSI model, namely, a model where a feature in the matter power spectrum is located at an arbitrary comoving wavenumber $k_{\rm b}$ with a profile characterized by a step in k with steepness α. We improve over previous analyses by adding to the feature parameters many more cosmological parameters in a Markov Chain Monte Carlo (MCMC) analysis, and we study how much the primordial model that best agrees with current data in constrained by this.

2. Power spectrum from non–vacuum initial states

The models we are studying here belong to a class with a BSI power spectrum for the matter density and were analyzed in detail in (Gangui et al. 2002). In this section we give a brief overview of the most essential features of this model. In the theory of cosmological perturbations of quantum-mechanical origin, the perturbed inflaton field and the Bardeen potential are both linked through the Einstein equations. As such, it is clear that they should be placed in the same quantum state. Let us consider a quantum scalar field living in a spatially flat Friedmann–Lemaître–Robertson–Walker background. The expression of the corresponding quantum operator for the field is

$$\varphi(\eta,\mathbf{x}) = \frac{1}{a(\eta)}\frac{1}{(2\pi)^{3/2}}\int\frac{\mathrm{d}^3\mathbf{k}}{\sqrt{2k}}\left[\mu_k(\eta)c_{\mathbf{k}}(\eta_{\rm i})e^{i\mathbf{k}\cdot\mathbf{x}}+\mu_k^*(\eta)c_{\mathbf{k}}^\dagger(\eta_{\rm i})e^{-i\mathbf{k}\cdot\mathbf{x}}\right]. \quad (1)$$

In this expression, $c_{\mathbf{k}}(\eta_{\rm i})$ and $c_{\mathbf{k}}^\dagger(\eta_{\rm i})$ are the annihilation and creation operators, respectively, which satisfy the commutation relation $[c_{\mathbf{k}}, c_{\mathbf{p}}^\dagger] = \delta(\mathbf{k}-\mathbf{p})$, and $a(\eta)$ is the scale factor depending on conformal time η. The equation of motion for the mode function $\mu_k(\eta)$ can be written as $\mu_k'' + (k^2 - \frac{a''}{a})\mu_k = 0$, where the "primes" stand for derivatives with respect to conformal time. The above is the characteristic equation of a parametric oscillator whose time-dependent frequency depends both on the scale factor and its derivative.

In order to define a general non–vacuum initial state, let $\mathcal{D}(\sigma)$ be a domain in momentum space, such that if $\mathbf{k}$ is between 0 and σ, the domain $\mathcal{D}(\sigma)$ is filled by n quanta, while otherwise $\mathcal{D}$ contains no quanta (we will change notation, n to $n_{\rm b}$, in the last part of this section). The state characterized by n quanta, $|\Psi_1(\sigma,n)\rangle$, is defined by

$$|\Psi_1(\sigma,n)\rangle \equiv \prod_{\mathbf{k}\in\mathcal{D}(\sigma)}\frac{(c_{\mathbf{k}}^\dagger)^n}{\sqrt{n!}}|0_{\mathbf{k}}\rangle\bigotimes_{\mathbf{p}\notin\mathcal{D}(\sigma)}|0_{\mathbf{p}}\rangle = \bigotimes_{\mathbf{k}\in\mathcal{D}(\sigma)}|n_{\mathbf{k}}\rangle\bigotimes_{\mathbf{p}\notin\mathcal{D}(\sigma)}|0_{\mathbf{p}}\rangle. \quad (2)$$

The state $|n_{\mathbf{k}}\rangle$ is an n-particle state satisfying, at conformal time $\eta = \eta_{\rm i}$: $c_{\mathbf{k}}|n_{\mathbf{k}}\rangle = \sqrt{n}|(n-1)_{\mathbf{k}}\rangle$ and $c_{\mathbf{k}}^\dagger|n_{\mathbf{k}}\rangle = \sqrt{n+1}|(n+1)_{\mathbf{k}}\rangle$. From this we obtain the normalization $\langle\Psi_1(\sigma,n)|\Psi_1(\sigma',n')\rangle=\delta(\sigma-\sigma')\delta_{nn'}$, as well as the following 2-point functions for all the combinations of annihilation and creation operators

$$\begin{aligned}\langle\Psi_1(\sigma,n)|c_{\mathbf{p}}c_{\mathbf{q}}|\Psi_1(\sigma,n)\rangle &= \langle\Psi_1(\sigma,n)|c^\dagger_{\mathbf{p}}c^\dagger_{\mathbf{q}}|\Psi_1(\sigma,n)\rangle=0,\\ \langle\Psi_1(\sigma,n)|c_{\mathbf{p}}c^\dagger_{\mathbf{q}}|\Psi_1(\sigma,n)\rangle &= n\delta(\mathbf{q}\in\mathcal{D})\delta(\mathbf{p}-\mathbf{q})+\delta(\mathbf{p}-\mathbf{q}),\\ \langle\Psi_1(\sigma,n)|c^\dagger_{\mathbf{p}}c_{\mathbf{q}}|\Psi_1(\sigma,n)\rangle &= n\delta(\mathbf{q}\in\mathcal{D})\delta(\mathbf{p}-\mathbf{q}).\end{aligned}\tag{3}$$

In these formulas, $\delta(\mathbf{q}\in\mathcal{D})$ is a function that is equal to 1 if $\mathbf{q}\in\mathcal{D}$ and 0 otherwise. From these last expressions we already notice that, when computing mean values for relevant quantities, extra terms proportional to the number of quanta n appear all the time, as we will see below is the case for the power spectrum of the Bardeen potential.

However, as it was emphasized in (Gangui et al. 2002), we see from the definition of $|\Psi_1\rangle$ that the transition between the empty and the filled modes is sharp. In order to smooth out the state $|\Psi_1\rangle$, one is led to consider a new state $|\Psi_2\rangle$, defined as a quantum superposition of $|\Psi_1\rangle$. In doing so, we introduce an a priori arbitrary function $g(\sigma;k_{\rm b})$ of σ. The definition of the state $|\Psi_2(n,k_{\rm b})\rangle$ is

$$|\Psi_2(n,k_{\rm b})\rangle\equiv\int_0^{+\infty}\mathrm{d}\sigma g(\sigma;k_{\rm b})|\Psi_1(\sigma,n)\rangle,\tag{4}$$

where the new function $g(\sigma;k_{\rm b})$ defines the privileged scale $k_{\rm b}$. We then assume that the state is normalized and we obtain therefore $\int_0^{+\infty}g^2(\sigma;k_{\rm b})\mathrm{d}\sigma=1$. This privileged scale is the one that will characterize our BSI model to be analyzed in detail in the next section.

From this expression for $|\Psi_2(n,k_{\rm b})\rangle$ one can compute all (non–vacuum) expectation values for all relevant (observable) quantities, which are given in terms of products of annihilation and creation operators $c_{\mathbf{k}}(\eta_{\rm i})$ and $c^\dagger_{\mathbf{k}}(\eta_{\rm i})$. In particular, one can compute the expectation value of the squared of the quantum inflaton field perturbation of Eq. (1) and, from it, obtain the expectation for the squared of the quantum Bardeen potential operator $\Phi(\eta,\mathbf{x})\sim\int\mathrm{d}\mathbf{k}[c_{\mathbf{k}}(\eta_{\rm i})f_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}+c^\dagger_{\mathbf{k}}(\eta_{\rm i})f^*_k(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}]$. In this last equation, the mode function $f_k(\eta)$ is related to the mode function $\mu_k(\eta)$ of the perturbed inflaton field through the perturbed Einstein equations. In the case of power-law inflation and in the long wavelength limit, the function $f_k(\eta)$ is given in terms of the amplitude A_s and the spectral index $n_{\rm s}$ of the induced density perturbations by $k^3|f_k|^2=A_sk^{n_{\rm s}-1}$.

We only have now to compute the non-vacuum expectation value for the squared of $\Phi(\eta,\mathbf{x})$, using $|\Psi_2(n,k_{\rm b})\rangle$ and Eqs. (3) and (4), to get the corrected expression for the matter power spectrum we will employ to test the new model. The details are somewhat lenghly and not worth to be recalled for this present work, but we refer the reader to (Sánchez et al. 2007; Gangui et al. 2002) where the full treatment is provided. At the end of the day, and writing $n_{\rm b}$ in the place of n for the number of quanta in the non–vacuum initial state for the cosmological perturbations, in these models the total power spectrum of the Bardeen potential can be written as $k^3|\Phi_k|^2=A_s\left(\frac{k}{k_0}\right)^{n_s-1}\left[1+2n_{\rm b}\bar{h}(k)\right]$, where now A_s is the amplitude of the scalar fluctuations at a scale k_0, n_s is the scalar spectral index and, using the same notation as in previous works, $\bar{h}(k)=\frac{1}{2}\left[1-\tanh\left(\alpha\ \ln\frac{k}{k_{\rm b}}\right)\right]$.

In this equation, $k_{\rm b}$ represents the privileged (comoving) wavenumber already introduced in Eq. (4) and α is a free parameter which controls the sharpness of the "step" function $\bar{h}(k)$. Restrictions imposed by the theoretical model imply that the steplike feature $\bar{h}(k)$ cannot be arbitrary, but ought to obey the requirements of being always positive, monotonically decreasing with k and vanishing at infinity. In the following section, we compute the theoretical CMB and matter power spectra using a version of CAMB adapted to include the modification to the primordial power spectrum given above.

Table 1. Marginalized 68 % (for α) and 95 % (for $\log(k_{\rm b})$) interval constraints on the feature parameters for different M_n models analyzed.

parameter	$n_{\rm b}=1$	$n_{\rm b}=2$	$n_{\rm b}=3$	$n_{\rm b}=4$	$n_{\rm b}=5$	$n_{\rm b}=6$
α	$4.2^{+2,8}_{-1,9}$	$4.0^{+2,9}_{-2,1}$	$4.4^{+2,6}_{-1,8}$	$4.4^{+2,6}_{-1,8}$	$4.5^{+2,5}_{-1,8}$	$4.5^{+2,5}_{-1,7}$
$\log(k_{\rm b})$	$< -3{,}26$	$< -3{,}41$	$< -3{,}50$	$< -3{,}55$	$< -3{,}58$	$< -3{,}62$

3. Preliminary analysis and outlook

We classify the cosmological models studied in this work by the number of quanta in the non-vacuum initial states, $n_{\rm b}$. It has been shown that this number cannot be large. Hence, in the present analysis we will consider values of $n_{\rm b}$ up to 6. We refer to these models as M_n. For each of these models, we restrict our parameter space to the standard six-dimensional set to which we add α and $\log(k_{\rm b})$. Therefore, we parameterize our cosmological model by $\mathbf{P} = (\omega_b, \omega_{cdm}, \Theta, \tau, n_s, A_s, \alpha, \log(k_{\rm b}))$, where $\omega_b = \Omega_b h^2$ is the baryon density in units of the critical density (h is the Hubble constant in units of $100\,{\rm km\,s^{-1}Mpc^{-1}}$), $\omega_{cdm} = \Omega_{cdm} h^2$ is the density of cold dark matter, Θ is the ratio of the comoving sound horizon at decoupling to the angular diameter distance to the surface of last scattering and τ is the reionization optical depth. We explore the parameter space [ω_b: 0.018 − 0.032; ω_{cdm}: 0.04 − 0.16; Θ: 0.98 − 1.1; τ: 0 − 0.5; n_s: 0.8 − 1.2; $\ln(10^{10} A_s)$: 2.7 − 4; α: 0.1 − 7; $\log(k_{\rm b})$: −4 − −0,7] by the construction of a MCMC generated with CosmoMC modified to include the additional parameters characterizing the feature in the spectrum. In order to constrain the parameters in our cosmological model, we use the 3-year observations from WMAP combined with the power spectrum of galaxy clustering measured from the final 2dFGRS catalogue (Cole et al. 2005), as in Sánchez et al. 2006.

Outlook: Our preliminary results (Table 1.) show that the parameter $k_{\rm b}$ that controls the position of the step in the primordial power spectrum is tightly constrained and is restricted to very large scales, where it has no effect on the CMB observations (that is, scales larger than the observable Universe). As the number of quanta in the initial state $n_{\rm b}$ increases the allowed region for $k_{\rm b}$ gets even smaller. As a consequence, the constraints on α, which controls the sharpness of the step, are very poor, since it has no impact on the observations.

References

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