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Crime and punishment: Does it pay to punish?

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ABSTRACT

Crime is the result of a rational distinctive balance between the benefits and costs of an illegal act. This idea was proposed by Becker more than forty years ago (Becker (1968) [1]). In this paper, we simulate a simple artificial society, in which agents earn fixed wages and can augment (or lose) wealth as a result of a successful (or not) act of crime. The probability of apprehension depends on the gravity of the crime, and the punishment takes the form of imprisonment and fines. We study the costs of the law enforcement system required for keeping crime within acceptable limits, and compare it with the harm produced by crime. A sharp phase transition is observed as a function of the probability of punishment, and this transition exhibits a clear hysteresis effect, suggesting that the cost of reversing a deteriorated situation might be much higher than that of maintaining a relatively low level of delinquency. Besides, we analyze economic consequences that arise from crimes under different scenarios of criminal activity and probabilities of apprehension.

1. Introduction

Most of the books and movies referring to criminality discuss the offenders point of view: Is crime rewarding? If one sets aside the ethic problem, as discussed by Dostoyevsky in "Crime and Punishment" one can say that crime is rewarding if you are not caught. However, the problem here is not if crime is rewarding but instead if punishment is rewarding or, in other words, if the expenses with police, lawyers, judges, prisons, prison guards, etc., compensate the wealth rubbed off by crime. This is the point discussed by Becker [1] in 1968. But let us first define what kind of crimes are we talking about and what is the retribution for those crimes.

One of the widely accepted definitions states that committing a crime is breaking one or more rules or laws, defined by a social group through some kind of social contract. All along history, human societies try to cope with crime either by prevention or punishment. However, these approaches are not independent. Prevention relies on some common moral or religious standards, and in part on the fear of a penalty. Even religion, which provides a moral guide for individuals and societies, makes use of a reward (paradise) and a punishment (hell). In some societies rewards for not committing an offense exist. In Brazil, for example, drivers that are not caught in contravention with transit regulations during the current year have a discount in the cost of the car annual permit. Thus, crime prevention has as its main arguments the risk of punishment and its real application if the offense is committed.

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We are not going to discuss the causes of crime, nor the effectiveness of different policies of punishment. There is an extensive literature on crime in different fields: Law, Economics, Politics, Religion, Mathematics and Physics. Gordon et al. [2] discuss thoroughly the common characteristics of criminal activity and various attempts for explaining, preventing and deterring criminality.

We are interested here in a class of crimes that are of economic nature. No physical harm or death of the victim will be considered. We restrain ourselves to an economic analysis of crime, starting from the idea that the decision to commit a crime results from a trade off between the expected benefit and the risk of punishment. In a now classical article Becker [1] presented an economic analysis of costs and benefits of crime, with the aim of developing optimal policies to combat illegal behaviors. Considering the social losses from crimes (which depend on their number and on the produced harm), the cost of apprehension and conviction, and the probability of punishment per crime, the model tries to determine how many crimes should be permitted and how many offenders should go unpunished, through minimization of the social loss function. Using a similar point of view, Ehrlich [3,4] applies economic theory for explaining the participation in illegitimate activities. The author assumes that an individual's decision to participate in an illegal activity is motivated by the relation between costs and gain, or risks and benefits, which result from such activity. This model seems to provide strong empirical evidence of the deterrent effectiveness of sanctions.

In the present paper we simulate an artificial society where the agent's decision to commit a crime depends on the expected profit compared with the underlying risk, as proposed by Becker [1], and in addition, on an idiosyncratic factor that we call the "honesty" index (the higher the honesty the lower the probability of attempting to perpetrate a crime). The victim is considered as a virtual entity that may be a person, a company, a bank, or money obtained by and/or from corrupted officers. We also assume that crime is a rational activity, and that "*most criminal acts are not undertaken by deviant psychopathic individuals, but are more likely to be carried out by ordinary people reacting to a particular situation with a unique economic, social, environmental, cultural, spatial and temporal context. It is these reactionary responses to the opportunities for crime which attract more and more people to become involved in criminal activities rather than entrenched delinquency*" [5]. We evaluate the minimum percentage of crimes that should be punished in order to guarantee a relatively low level of criminal activity; the cost of returning back to a low criminality state once the situation has deteriorated and the society suffers a wave of high criminality; and the economic cost of crime and punishment, measured both through the cumulated wealth of the society and by the cost of the law enforcement apparatus.

Partially anticipating the results, we will show that even if the society can attain a state of very low criminality with no need of punishing all and every offense, a sharp transition as a function of the probability of retribution is observed, i.e. there is a threshold in the punishment probability: below this threshold criminality is high, above it, it is low. In Section 2 we describe the model. In Section 3 we present the results of the simulations. In Section 4 we discuss the results and present conclusions and suggestions.

2. Model

We consider a population with a fixed number N of heterogeneous agents. Agents earn different incomes (wages), have different predispositions for committing a crime (honesties), and modify their behavior according to the risk of apprehension. If caught, the offender is punished. In this artificial society we do not consider profits from capital, neither manufacturing, nor banks, nor unemployment. We describe the parameters and the dynamics of the model in the next sections.

2.1. Agents' characteristics

Each agent receives a periodic (monthly) wage W_i, which may be seen as the remuneration of some productive activity or a rent. Wages are drawn at random from a finite support distribution $p_W(W_i)$ ($W_i \in [W_{\min}, W_{\max}]$). The distribution of income in many developed countries follows a power law (Pareto law) for the high income fraction of the population and a log-normal law for the low-income sector [6]. In order to keep the stylized aspects of the distribution, we assume a triangular distribution p_W with $W_{\min} = 1$ and $W_{\max} = 100$ (in some arbitrary monetary unit), and with its maximum at the lowest wage $W_{\min}$. The mean value of this triangular distribution can be easily calculated and is $\langle W \rangle = W_{\min} + (W_{\max} - W_{\min})/3$.

We assume that in this Utopian society the minimum wage is enough to provide for the minimum needs of each agent. The resources not spent are saved by the agents without further financial manipulations. Wages and booties from successfully committed crimes constitute the sole income source of the agents.

In addition, each agent is characterized by an honesty index H_i, which quantifies his inclination to abide by the law. This honesty may be the result of psychological and ethical characteristics or qualities, or a reflection of his educational level and/or socio-economical environment, in a way similar to Bourguignon et al. [7]. However, we assume that this index is not an intrinsic and static characteristic of the agent: it evolves in time according to the risk of apprehension upon committing a crime [8]. The initial values of H_i are drawn at random from a compact support distribution $p_H(H_i)$ ($H_i \in [H_{\min}, H_{\max}]$), *without any correlation with the wages*. This is justified by the lack of empirical evidence that poorer agents are more or less law abiding than the rich. In the simulations, we use a triangular distribution $p_H(H_i)$, with $H_{\min} = 0$ and the maximum at $H_{\max} = 100$, assuming that a majority of people in the society are honest. Besides, an initial finite fraction of "hard-core" corrupt agents with honesty index $H_{\min}$ is assumed.

2.2. Crime dynamics

In this section, we present the main elements of the model's dynamics. In order to define an appropriate time scale the basic time unit we adopt is the "month". We assume that each month every agent receives his salary and also can try (or not) to commit a crime. By the end of the month we compute the number of successful and unsuccessful crimes and other indicators of the consequences of criminal activities. So, in the following, we quantify the number of criminal attempts, and also the crime characteristics: the booty, the probability of punishment, the utility expected from a criminal act and, finally, the evolution of the honesty.

Criminal attempts. In order to determine the number of criminal attempts, we adopt a simple hypothesis: there are no intrinsically honest agents, i.e. every agent has the possibility of attempting to commit a crime. Thus, the number of criminal attempts per month is just the number of non imprisoned (free) agents. Any agent who is not in prison may commit a crime, and the decision depends on the utility expected from crime and from the honesty of the agent. In the simulation, initially all agents are free, so that in the first period the number of attempts is equal to the total number of agents. In the successive months a number of agents, N_j, will be in prison, and the number of free agents will be $N_{fr} = N - N_j$. One can argue that in many countries agents in prison can order and even commit crimes. However, it is reasonable to assume that in macroscopic level most of the inmates are unable to participate in crimes. Formally, we define the number of criminal attempts per month m as $A(m)$:

$$A(m) = N_{fr}. \tag{1}$$

Each attempt implies that an agent will evaluate the possibility of committing a crime. As we are interested in the criminal action only, we neither determine who is the victim, nor consider illicit criminal associations (gangs). Crime is the action of an individual, even if in a future generalization of the model the agents should act in groups or mobs.

Becker [1] argued that the motive for committing a crime is the expected gain. Confirming this idea, Shikida [9] interviewed convicted criminals and showed that the motive to commit a crime is monetary, but also is the opportunity to obtain "easy money", even if the offender does not really need it. Thus, one may conclude that an attractive utility and a low risk of punishment will further stimulate the offender to continue his criminal "career".

In order to determine the expected utility we should take into account both the gains obtained if the crime is successful and the losses if the offender is caught. So, before calculating the expected utility, we need to estimate the booty, the impact of the punishment and the probability of apprehension.

Booty. The value of the booty, S, is taken at random in a range going from the minimum wage, for small larcenies, to 10 times the average wage for important crimes. The upper limit is arbitrary, although we verified that results are robust against any change in this parameter. Then, the booty of a particular crime, S, is drawn at random from a uniform probability distribution between 0 and $10 * \langle W \rangle$, where $\langle W \rangle$ is the average value of the wage distribution. Also, we define the average booty, $\langle S \rangle = 5 \langle W \rangle$, which, together with the average wage, $\langle W \rangle$, will serve as reference values in the simulations.

Imprisonment and fines. We calculate the expected losses in case the offender is caught and punished, using the assumptions of Gordon et al. [2]: a punished offender is imprisoned for a number of months $\tau = 1 + [S/\langle W \rangle]$, i.e. proportional to the booty, and is forced to return the entire booty plus a fine. In real life examples this might be unfeasible, yet we assume that the offender gets caught "hands in the cookie jar", and does not have time to spend the spoils.

Furthermore, a monetary punishment is inflicted to the offender by means of a fine, or duty, proportional to the booty. The proportionality factor, f_D is assumed to be positive and the fine is $D = f_D S$, which is deduced from the wealth of the caught offender. In order to avoid a negative value for the remainder wealth, if $(1 + f_D)S > K_k$ (where K_k is the wealth of the offender *after* the crime) the fine is reduced to $D = K_k$, i.e. confiscating the total wealth of the offender. In the simulations, the fine represents 25% of the stolen amount, i.e. $f_D = 0.25$.

To this extent, if a criminal is caught, he is put into jail for a period of time that depends on the magnitude of the booty, and his wealth is cut down of an amount larger than the booty.

We remark that in our setup the caught criminals always get punished. We do not consider the possibility of an early release, nor the failures of the law enforcement system. Maintaining each criminal in prison bears a fixed monthly cost to the society, that we evaluate. Further simplifying assumptions are adopted concerning the inmates: agents in prison cannot commit crimes, they do not earn wages[1], and their needs are covered by the society by means of the fines collected through the punishments. Therefore, in a very crude approach, the economic burden of criminality is evaluated on the basis of the total number of months that inmates remain in prison. The larger the number of inmates, and the longer the sentences, the higher is the cost for the society. Strictly speaking, we neglect the loss of handwork due to imprisonment, the victim's prejudice and other social losses like the cost of maintaining police and judicial systems.

The probability of apprehension. In contrast with most models in the literature, we assume that criminals are caught and punished with a probability that depends on the magnitude of the booty. The probability of imprisoning and punishing an offender depends on the importance of the felony. Small crimes are easily tolerated and many times the victim does not even produce a report. This is in agreement with Becker's theory that small crimes should be tolerated [1]. On the other hand,

[1] Even so, in some countries inmates, if civil state servants, keep their salaries during the period that they remain condemned.

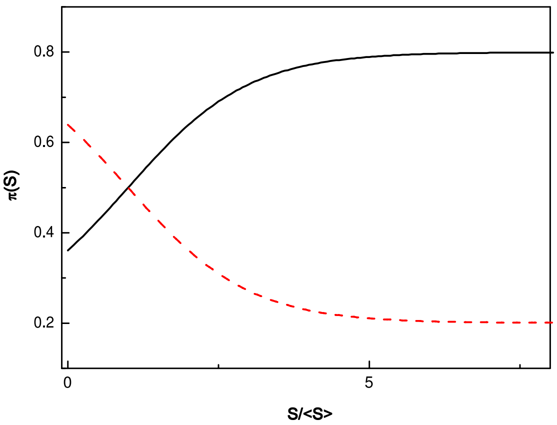

Fig. 1. The probability of punishment as a function of the magnitude of the booty S. Solid line: $p_0 = 0.2$ and $p_1 = 0.8$. Dashed line: $p_0 = 0.8$ and $p_1 = 0.2$.

important crimes are generally reported and the police and prosecutors dedicate more time and efforts to their solution. Also, when caught, the authors of the crimes are punished (or should be) in proportion with the magnitude of the felonies. This assertion is in agreement with the classical theory about punishment [10], which states that the penalty should be proportional to the importance of the offense. Thus we define a probability p_0 of punishing small crimes and p_1 of punishing important crimes, and assume a continuous variation of the probability of apprehension as a function of the booty, defined as:

$$\pi(S) = p_1 + \frac{(p_0 - p_1)}{1 + \mathrm{e}^{\frac{S - \langle S\rangle}{\langle S\rangle}}}, \tag{2}$$

which is a logistic function that starts at its minimal value $(ep_0 + p_1)/(1 + e)$ (for $S = 0$), increases with S, goes through $\pi(S) = 1/2(p_0 + p_1)$ for $S = \langle S\rangle$ and gets close to the asymptotic value p_1 for larger booties, see Fig. 1. The two curves in Fig. 1 correspond to the cases with (a) $p_0 < p_1$ (the "reasonable" scenario where important crimes are punished with higher probability than small ones), and (b) $p_0 > p_1$ (the sometimes *undesirable* scenario where important crimes are punished with a probability lower than small ones).

The expected utility. The decision to commit a crime depends on its expected utility. We define the utility as the difference between the gains obtained if the crime is committed, and the gains obtained otherwise. If the crime is committed, an offender expects to obtain:

$$U_{\text{crime}} = [(1 - \pi)(S + \tau W_i) - \pi(1 + f_D)S] - H_i. \tag{3}$$

If agent i is not punished, he gets $S + \tau W_i$ with probability $(1 - \pi)$, i.e. the booty plus the wage that he would have lost if imprisoned. If agent i is punished, then, with probability π, he returns the booty S and pays the fine $f_D S$. Finally, we discount from the utility of crime the value of the honesty (which acts as a threshold).

If crime is not committed, then the agent gets his wage as utility: $U_{\text{no crime}} = \tau W_i$.

Thus, we define the expected utility as $U_{\text{crime}} - U_{\text{no crime}}$:

$$U = [(1 - \pi)(S + \tau W_i) - \pi(1 + f_D)S] - H_i - \tau W_i. \tag{4}$$

The crime will be committed if the utility is positive.

Summary. Summing up, the dynamics of the model is:

- An offender i is selected among free agents at random. This agent earns wage W_i and has honesty H_i.
- The booty S is estimated as $r * 10 * \langle W\rangle$, where $r \in [0, 1]$ is a random number.
- The probability of apprehension π is given by Eq. (2).
- Agent i commits the crime if the expected utility is positive.
- If i is not punished, he adds S to his wealth. Otherwise, i spends $\tau = 1 + [S/\langle W\rangle]$ months in jail (without salary), and his wealth is reduced by $(1 + f_D)S$.

2.3. Honesty dynamics

As we discussed in the previous section honesties and the fear of punishment have a dissuasive effect on the population. Following the empirical results of Eide [8], we assume that honesty indexes H_i are modified proportionally to the number of punished (or unpunished) crimes, and do not depend on the severity of the punishment. At the end of the month, if most of the committed crimes are punished, the honesty indexes of all free agents increase; otherwise, they decrease. We assume a simple linear relation between the variations of honesty and the number of punished crimes:

$$H_i = H_i + \frac{(2N_p - N_k)}{N_k}\delta H, \tag{5}$$

where N_k is the number of crimes committed in the month, N_p is the number of crimes punished, and δH is the maximum change of honesty, set to $\delta H = 10$ in simulations. If only half of the committed crimes are punished the honesties do not change, and their values decrease or increase if the number of punished crimes is less (greater) than half of the committed crimes. Negative values for honesties are not allowed, the minimum value is 0. However, the lower bound of the distribution is not absorbing. Variants that are worth of testing are: (a) modifying the honesty indexes of criminals proportionally to the importance of the booty, and (b) considering the negative effect of prisons (''contagion'' effect).

3. Simulations

We simulate a system of $N = 1000$ agents for a period of 240 months. This time interval seems to be reasonable for studying the evolution of the society over 20 years, and long enough for a system to arrive to a stable state. Starting with the initial conditions for honesties and wages, defined in Section 2, there are $A(m)$ criminal attempts each month m, as given in Section 2.2.

The evolution of the system is very sensitive to the probability of apprehension. As a result of criminal activity, the final wealth distributions, as well as the honesty index distributions of the population, are shifted and distorted. In the next sections we discuss the effects produced by changing the probability of apprehension. In Section 3.1 we study the scenario in which the probabilities of apprehension are fixed during the monthly evolution of the system, while in Section 3.2 the probabilities change in time within the total interval of time. The results are important for when one tries to cope with a deteriorated situation with high delinquency, and looks for the necessary changes to be introduced in order to diminish criminal activity.

3.1. Static dependence on the punishment probability

In the present section we discuss the states of the society as a function of the probability of punishment. In order to make comparable experiments we study societies with the same initial conditions under different levels of punishment. The results presented in this section are *averages* over the last 120 months of the total 240 months of evolution. We consider the last 120 months in order to avoid the effect of initial transients in the calculation of the averages.

We begin with small and constant values of p_0 (probability of punishment of small crimes), and study the variation of several social indicators as a function of p_1 (probability of punishment of important crimes). We observe that the system reaches the states with either a high crime-low honesty population (for p_1 less than a critical value), or with a low crime-high honesty population (above the same critical value). The transition, apparent in all the studied quantities, as may be seen in Fig. 2, corresponds to a swing between the regime with high criminality to one with moderate criminality.

In Fig. 2(a) the number of crimes and convictions per capita is plotted for three representative values of p_0: 0.2, 0.3 and 0.4. There is a sharp transition in the criminality when p_1 increases, at a critical value which depends on the value of p_0. When for $p_0 = 0.4$ the transition is smoother and happens at $p_1 \simeq 0.5$, for $p_0 = 0.2$ the critical value reaches $p_1 = 0.7$[2]. This suggests that the permissiveness in coping with small crimes has a deleterious effect, because the probability of punishment needed to deter important crimes increases. This might be seen as *a posteriori* justification for zero tolerance policies. However, it is possible to see that Becker's ideas may be correct, as for a relatively tolerant situation, i.e. $p_0 = 0.3$, and $p_1 \geq 0.7$, the criminality is under control.

In Fig. 2(b) the average honesty of the population is plotted against p_1. We observe that in the region with high criminality the average honesty goes to zero, which means that the society perceives the situation as a kind of a ''permission'' to wrongdoing, i.e. when crimes remain unpunished the feedback effects reduce the honesties. On the other hand, in the low criminality region, the average honesty is high, and gets higher for larger values of p_0. We observe that the average honesty exhibits a peak for very high values of p_1. This is due to the fact that the honesties increase when the number of crimes punished is greater than half of the total number of crimes committed (see Eq. (5)). But when there are no (or just a few) crimes, there is no modification of the honesty indexes, then the average honesty is slightly lower than in the case with moderate criminality.

In Fig. 2(c) we plot the sum of the total number of months spent by inmates in prison during 120 months. This quantity is a good indicator to measure the total amount of money spent by the society in crime prevention. The curve exhibits a clear symmetry with the criminality measures: when the number of crimes is high, the more resources are needed in order to maintain the law enforcement system. It is clear that a low probability of punishment is more expensive for the society than

[2] A simple argument considering the variation of the honesty as a function of the number of punished crimes allows us to understand the abrupt transition found in the simulations [2,11].

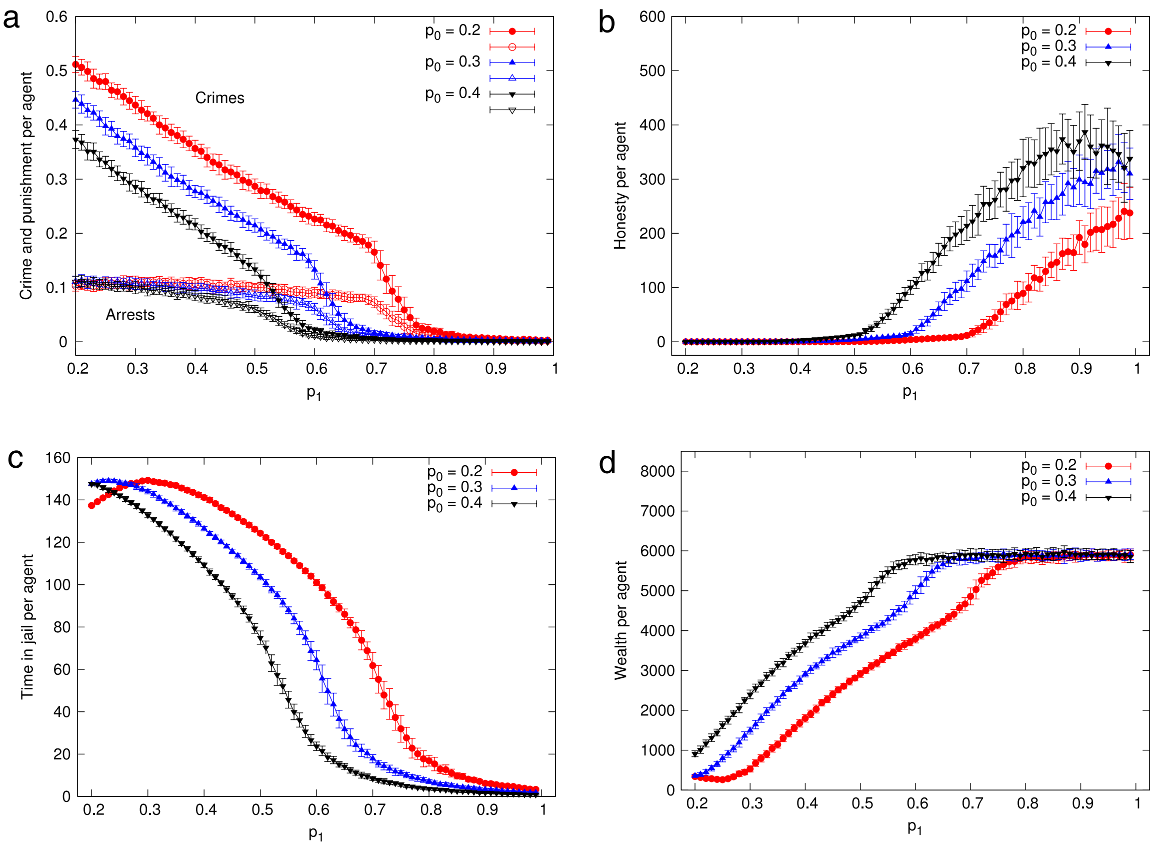

Fig. 2. Panel (a): average number of crimes and convicted criminals per capita vs. p_1. Panel (b): average honesty per capita as a function of p_1. The results in (a) and (b) are averages over the last 120 months and 40 samples. Panel (c): total number of months spent in prison by all inmates, per capita, vs. p_1. The results are summed over the last 120 months, and averaged over 40 samples. Panel (d): total wealth per capita vs. p_1. Results are averaged over 40 samples.

a relatively strict control of criminality, and that the permissiveness to cope with small crimes is also a factor of increasing expenses.

Finally, in Fig. 2(d) we plot the average wealth of the society. The effect of wrongdoing is evident. From a strictly economic point of view the worse situation arises in the high criminality scenario. Even if the majority of crimes remain unpunished, the extra resources introduced by criminality are not enough for the increase of the total wealth of the population, which is decreased because inmates in prison pay fines and do not earn wages. On the other hand, when criminality is under control, the total wealth of the society reaches a maximum level. We also calculated the Gini coefficient but we do not plot it as it looks very similar to previous results [2]: inequality is high when criminality is high and low when criminality is low; there is no Robin Hood effect.

The results obtained follow trends similar to the results published in Ref. [2], however, the transition happens at larger values of p_1, suggesting that both (a) the consideration of the utility of committing a crime instead of the importance of the booty only, and (b) the consideration of the honesty index as a parameter in the utility instead of considering it as a separate probability factor only, induce a shift in the transition probability. We can renormalize the honesty scale in order to change the value of the transition probability, however, a sharp transition always occurs, and the value at which it happens depends on the scale of the parameters. Nevertheless there are important simplifications and less parameters in the present model compared to the one in Ref. [2].

Summarizing the results: if p_0 is small, when p_1 increases the system suffers an abrupt transition between a state with *high crime – low honesty* to a state with *low crime – high honesty*. In the high criminality state, cumulated earnings are small and the total cost of maintaining inmates in jail is high.

For completeness, we study how criminality changes when the probability of retribution of small crimes p_0 changes, in the case where this probability can be greater than p_1 (when a "big" criminal risks less retribution than a "chicken thief"). In Fig. 3(a) we plot the average number of crimes after 240 months as a function of p_0, for two values of p_1. There is no sharp transition here, the crime rates decrease when p_0 increases, but due to the fact that big crimes are less punished ($p_1 < p_0$) the number of crimes is higher, for example, for ($p_0 = 0.8, p_1 = 0.4$)in Fig. 3(a) and for $p_0 = 0.4, p_1 = 0.8$ in Fig. 2(a). This suggests that a zero tolerance policy is not enough to control criminality, because even when $p_0 \to 1$ the low value of

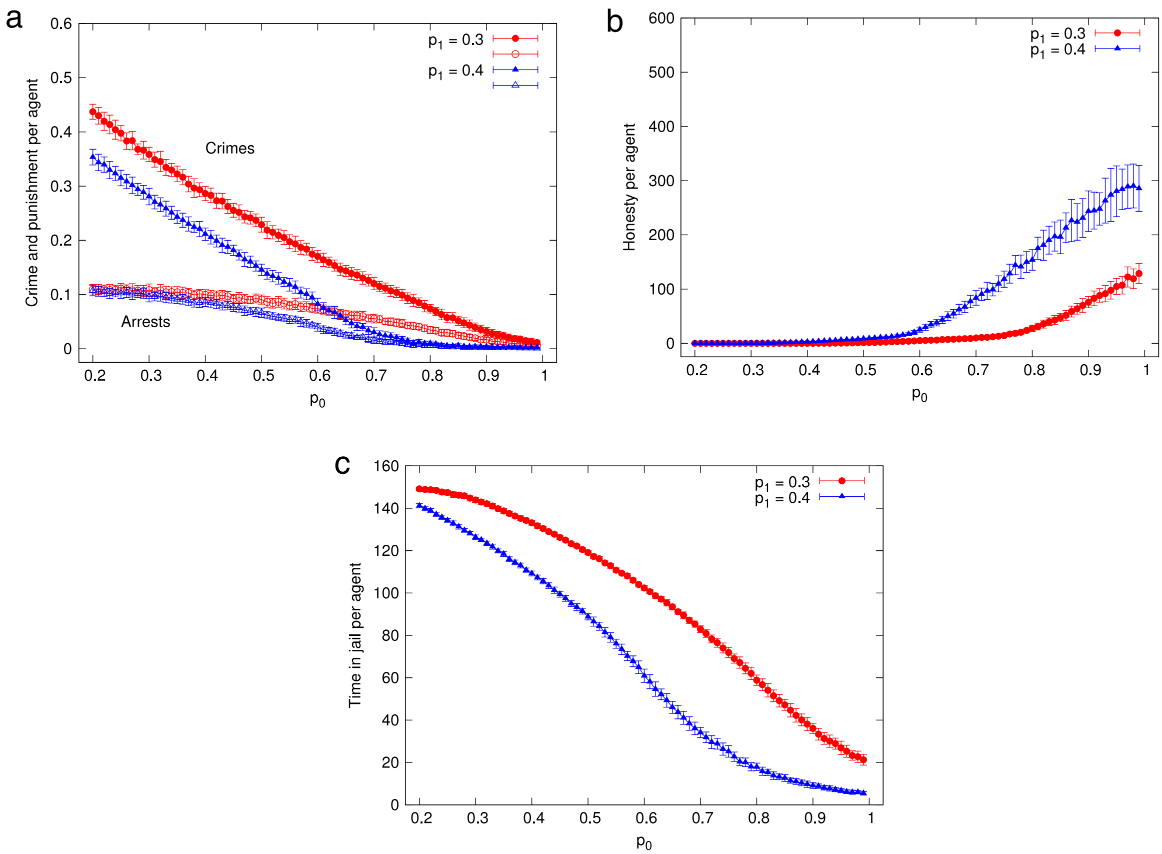

Fig. 3. Panel (a): average number of crimes and convicted criminals per capita vs. p_0. Panel (b): average per capita honesty of the population as a function of p_0. The results are averaged over the last 120 months and over 40 samples. Panel (b): total number of months spent in prison by all inmates, per agent, vs. p_0. The results are summed over the last 120 months, and averaged over 40 samples.

p_1 guarantees a certain degree of impunity to white collar offenders. If important crimes remain unpunished or, as it is the case for white collar crimes, small larcenies are penalized stronger than the big crimes, a situation of high criminality may develop and settle down. As a consequence honesty also compares badly. While for high values of p_1 the average values of honesty can go up to almost 400 (see Fig. 2(b)), for high values of p_0 and $p_1 = 0.4$ honesty increases up to only 250 (see Fig. 3(b)).

One would expect that the time in prison will exhibit a significant change depending on whether $p_0 < p_1$ or $p_0 > p_1$, because it is proportional to the booty S (see Section 2.2). If only small crimes are punished, then the average booty $\langle S \rangle$ will be small, and thus, this would cost less to the society. However, as a result of the underlying dynamics of the model, agents spend less time in prison, but there are more crimes and convictions than when $p_1 > p_0$. Then the total number of months in prison is comparable to the preceding case for low values of p_0 but does not go through a maximum and decreases continually when increasing p_0 (see Fig. 3(c)).

In the next section we discuss how we can mend a situation with high criminality, i.e. how we can reduce the criminality starting from a deteriorated state. We show that the role of honesties is a determinant factor for reducing criminality.

3.2. Hysteresis: Changing p_1 in time

The results of the previous section correspond to the states of the system reached after 240 months, and the probability of apprehension was constant during this evolution. We expect that modifying either p_0 or p_1, or both, as a function of time (as it may happen in actual societies in order to correct an abnormal increase of criminality), might produce different results due to the different initial conditions at the moment of modifying the probabilities.

We simulate a situation in which the society is in the state with high criminality, and it is necessary to increase the probability of punishment p_1 in order to cope with the high criminal activity. We begin with a deteriorated stable situation, like the one presented in Fig. 2 for low values of p_1, and start to increase the probability p_1 by a small step every 50 months. Although the total number of steps will be larger than in the previous section, the order of magnitude is comparable. In any case we need a reasonable number of intermediate steps in order for the system to reach a stable state.

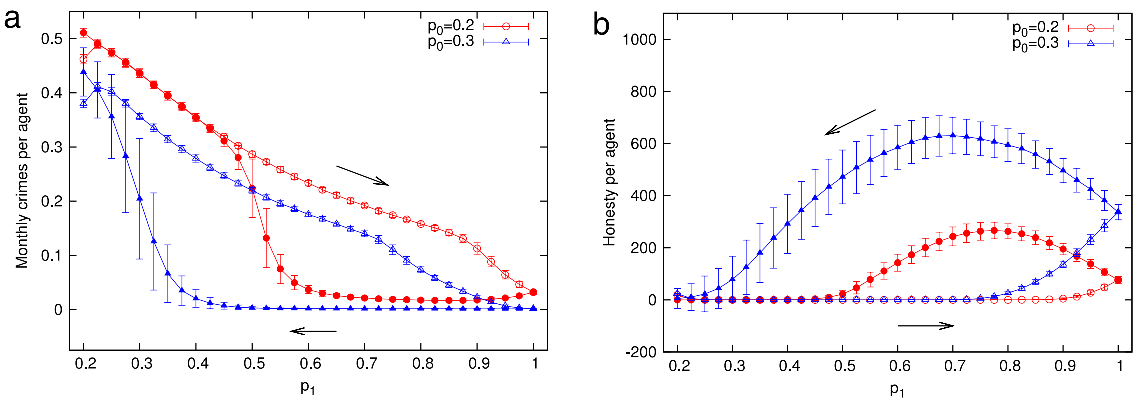

Fig. 4. Panel (a): variation of criminality when the probability of punishment p_1 changes in time. The step is 0.025. Circles: $p_0 = 0.2$. Triangles: $p_0 = 0.3$, the arrows point when p_1 is increasing or decreasing. Panel (b): variation of the average honesty. The results are averages over 100 samples.

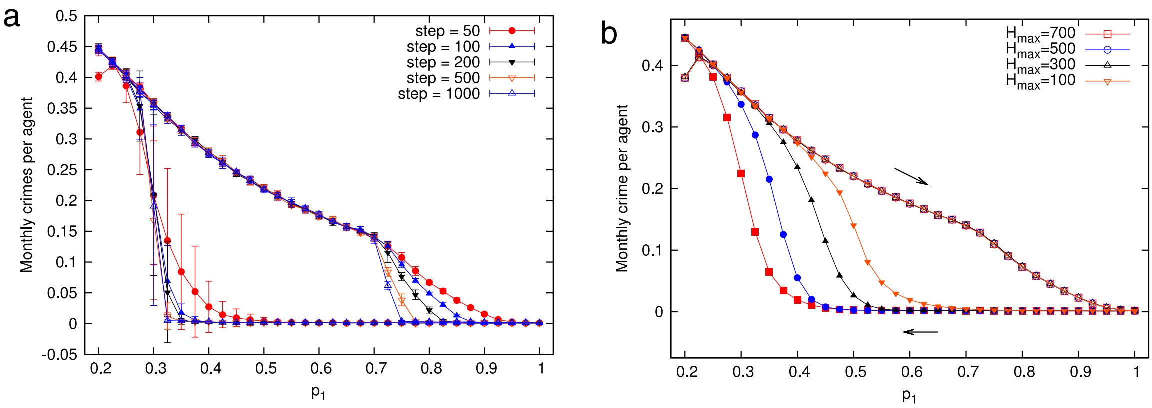

Fig. 5. The hysteresis cycles caused by modifying: (a) the time separation between two changes of p_1, (b) the upper limit of the honesty coefficient.

The results are plotted in Fig. 4. We observe that the criminality remains at high levels even for relatively high values of p_1, and only for $p_1 \sim 0.9$ and $p_0 = 0.2$, and $p_1 \sim 0.7$ for $p_0 = 0.3$ the crime rates start to decrease. The latter suggests that it is more difficult to cope with a situation of crime incidence than to keep the crime at low levels. Indeed, when decreasing p_1 from an initial high value, one observes that it is possible to keep low criminality indexes down to $p_1 \sim 0.6$ ($p_1 \sim 0.4$). The effect produced by changing p_0 is impressive. Increasing p_0 from 0.2 to 0.3 induces a shift in the hysteresis cycle to the left, as it is shown in Fig. 4(a). The average honesty coefficient also exhibits a hysteresis effect, as shown in Fig. 4(b), but in a rather continuous way, demonstrating that one does not need a discontinuous change in honesty to induce a jump in criminality, a smooth reasonable variation of honesty may induce a drastic change in criminality.

One may wonder whether this hysteresis is a proof that the sharp transition in criminality as a function of p_1 is a first order transition. In order to verify the stability of the hysteresis cycle we performed the following test: we changed the number of intermediate steps. If the time between the changes in p_1 goes to infinity there should be no hysteresis. This is not confirmed in Fig. 5(a), where one can see that the hysteresis remains unchanged even for a very long transient between changes in p_1. Moreover, the cycle is more accentuated as the jumps are more abrupt when increasing the number of steps between two values of p_1. Another factor should be taken into account: we considered a lower bound for the honesty index but not an upper bound. This means that long periods of time with low criminality can drive the average honesty to very high values, which are the initial conditions when we begin to decrease p_1 again. In Fig. 5(b) we show the hysteresis cycle with an upper bound for the honesty: the hysteresis cycle is narrower, as expected, when the honesty upper bound is lower (we remark that the modifications arises in the curves with p_1 decreasing, as expected). Nevertheless, the hysteresis effect survives even for the lower value of the upper bound, $H_{max} = 100$, indicating that for finite times there is a significant hysteresis.

One should compare these results with the ones of Nadal et al. [11], because using a mean field approach they obtain no hysteresis. However the settings of the model are different, first they consider that honesty has a finite number of discrete values, and the honesty changes a finite amount with each crime. Finally the mean field approach is equivalent to consider infinite time in the evolution. So we do not expect to recover their results for finite times, and with the honesty changing *proportionally* to the relative number punished crimes.

Finally we would like to comment the variations in the error bars in Fig. 4(a). The fluctuations are larger when going from or to a region with high criminality, and this can be seen as a precursor of the transition, in the way discussed by Donangelo et al. [12]. In the state with low criminality, high fluctuations in the number of punished crimes may be an indicator of the rise of the crime activity.

In any case, if the society is in the state with high criminality, a very high probability of punishment (much higher than the critical values presented here) would be needed in order to reduce the criminality back to acceptable levels. Preventive policies should be much less expensive and easier to apply than recovering from a very deteriorated security situation. This seems to contradict Becker's idea that a low level of crime activity should be tolerated as it would be less expensive for the society than carrying out the zero tolerance policy.

4. Discussion and conclusion

We summarize the main hypothesis of the model:

- Crimes have an underlying economic motive: the expected utility depends on the benefits and losses of crime.
- Agents are characterized by an honesty index which quantifies the willingness to abide by the law.
- The probability of punishment depends on the importance of the booty.
- If caught, offenders are punished with fines and imprisoned, both proportional to the booty.
- The average honesty of the population changes proportionally to the success of the crime preventive actions: if many crimes remain unpunished the average level decreases, and increases otherwise.

We emphasize a particularity of the last hypothesis: the honesty level of the population is correlated with the mere existence (or absence) of punishment, and not with its importance (which is proportional to the size of the booty). Thus, punishing small crimes is as effective as punishing large crimes, in order to increase the population's honesty globally. Since small crimes have a lower probability of being punished, the public effort on crime deterrence depends on the importance of the crime through the probability of punishment.

The model shows an interesting abrupt transition of the criminality level beyond a critical value of p_1, that depends on p_0. When small crimes have a high probability of being punished, the value of p_1 needed to reduce crime is lower. Therefore, assuming a proportionality between the size of the booties and the expenses of the law enforcement system, we can conclude that being too permissive with small crimes may lead to high expenditures for controlling important crimes. However, even if the existence of a sharp transition might be an indicator that a deteriorated situation with high criminality may be recovered with relatively small changes in the probability of punishment, hysteresis results show that the probability of retribution should be increased significantly above the transition value. On the other hand, after a situation of low criminality is established, the level of retribution can be relaxed. Finally, we would like to stress that those results are general and independent of the detailed parameters of simulations.

So, we presented a simple model for crime and punishment that stands on the assumption that punishment has a deterrent effect on criminality. Our main result is that a big tolerance with respect to small felonies has global negative consequences because it requires bigger efforts to cope with important crimes in order to keep a given level of honesty. We also observe an avalanche effect since a small change in the probability of punishment may reduce or increase the average criminality significantly. A more detailed model should include the effect of a particular treatment (or not) of recidivism. This is a point of discussion in countries like France, where some legislators ask for a minimum sentence for relapse. Also, one should be careful when treating more "sophisticated" criminal activities, like organized crime, or criminals that choose their victims according to the expected booty. It would be interesting to include the effect of imprisonment: either to recover or to increase the inmates criminal tendencies. We are presently working on these points, which are of great importance.

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