

# SET-MEMBERSHIP ESTIMATION THEORY FOR COUPLED MIMO WIENER-LIKE MODELS

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**Abstract**— In this paper, an approach for identifying coupled multiple-input, multiple-output (MIMO) Wiener-like models is presented. Each multiple-input, single-output (MISO) model structure contained in the MIMO model is parameterized using finite sets of discrete Laguerre transfer functions followed by High Level Canonical Piecewise Linear (HLCPL) that represents the static memoryless nonlinear block. For each MISO model, the parameters of the HLCPL functions are found via Set-membership (SM) estimation theory, under mild error constraints. In this way, each MISO Wiener-like model is described as a set of parameters for the nonlinear static subsystem, whose values are obtained by solving a linear programming problem. The MIMO Wiener-like model structure is then represented as a set of coupled input-output MISO models, converting the identification of a coupled MIMO system into the identification of MISO systems. In order to validate the proposed identification algorithm, an illustrative example is provided.

**Keywords**— Wiener like-models, nonlinear systems identification, HLCPL functions, set-membership estimation theory, UBB error.

## I. INTRODUCTION

Controller design, fault detection, optimization, prediction and simulation are based on models that describe the dynamic behavior of a real system, making the identification of nonlinear dynamical systems from multiple-input, multiple-output data a very important subject.

Block-oriented models have proved useful as simple nonlinear models that describe the behavior of a great number of nonlinear dynamical systems over a large operating range. They consist of the interconnection of a linear, time-invariant dynamical block ( $\mathcal{L}$ ) and a static memoryless nonlinear block ( $\mathcal{N}$ ). One advantage of these models is that they combine a simple

parametrization as well as good approximation properties. Furthermore, they require low computational effort and are suitable for controller design, making them attractive as models for control. For a complete state of the art and recent advances in this area, the reader is referred to Giri and Bai (2010).

In this work we assume that a parametric coupled MIMO system is identified using a Wiener-like structure for each MISO nominal model contained in the MIMO model. The linear dynamic part of each MISO model is represented by a finite number of Laguerre filters (Álvarez *et al.*, 2011), while the static nonlinearity is represented by HLCPL functions (Lin *et al.*, 1994; Julián *et al.*, 1999).

This model has the advantage of reducing the identification of each MISO nonlinear dynamical system to two simpler steps: the identification of a linear dynamic system and the approximation of a nonlinear static function using some adequate structure.

In the literature, several approaches can be found to perform the nominal identification of block-oriented nonlinear models. Within this class, two of the most widely used model structures are the Wiener model, where  $\mathcal{L}$  precedes  $\mathcal{N}$  (Wigren, 1993), and the Hammerstein model, where  $\mathcal{N}$  is followed by  $\mathcal{L}$  (Haber and Unbehauen, 1990). These structures are present in a wide variety of fields with applications in the area of communications (Hadjiloucas *et al.*, 2004), medicine (Celka and Colditz, 2002), biology (Hunter and Korenberg, 1986) and chemical engineering (Zhu, 1999); Visala *et al.*, 1999). A detailed review of applications can be found in Janczak (2005).

Results for the Wiener models identification using different linear dynamic representation, as well as different nonlinear approximation approaches, have been presented (Billings, 1980; Castro *et al.*, 2003; Agamenonni, 2004). The main difficulty in the identification of nonlinear block-oriented systems is that the internal signal is not available for measurement. Most of the contributions assume invertibility of the nonlinearity (Gomez and Baeyens, 2004). Identification of SISO Wiener models with non-invertible nonlinearity, when

the measurement errors are assumed to be unknown but bounded, have been provided in (Cerone *et al.*, 2003; Cerone and Regruto, 2005).

This paper extends the results of Álvarez *et al.* (2011) to MIMO dynamical systems, where the identification structure proposed for a SISO system is a Wiener-like model. In this model, the dynamic linear part is realized by using Laguerre filters and the method used for evaluating the minimum number of them is based on the Lipschitz coefficients (He and Asada, 1993). The static nonlinearity is represented by HLCPL functions whose parameters are identified by using Set membership estimation theory, assuming that the parameters of Laguerre filters have been already identified. Furthermore, in Álvarez *et al.* (2011) it is proved that this identification structure allows any discrete SISO nonlinear system with fading memory to be uniformly approximated. The model is nonlinear, easily identifiable from input/output (I/O) data and can predict the response of the system on arbitrarily large time horizons. These features make the actual approximation structure particularly suitable for nonlinear predictive control.

In order to find the optimal approximating model for a fixed number of Laguerre functions, we have to address the problem of choosing the optimal pole. In this case, we use the I/O data of the system to identify a first order ARX model, where the parameters are adjusted by using a least-squares method. The desired pole can be obtained with these parameters and this is associated to the pole of these basis functions. The reader is referred to Oliveira e Silva (2000) for some procedures to solve this problem.

The approximation of a static nonlinear function using a suitable structure requires the choice of an adequate representation that should satisfy several restrictions. First, it should allow any continuous function to be represented uniformly, and secondly it must be efficient from the point of view of the number of parameters needed to approximate a given function. Another relevant factor is the availability of a suitable algorithm for automatically adjusting the parameters. The identification of the static nonlinear functions with HLCPL functions uses the least number of parameters and the algorithm for computing the approximation is very efficient.

In order to estimate the parameters of the HLCPL functions, the problem is formulated within the Set-membership (SM) framework (see, *e.g.* Milanese (1989); Milanese and Vicino (1991) for surveys on the topic). In SM estimation theory, the uncertainty is described using an unknown but bounded additive noise, called UBB error. In this context, the estimation of the model that minimizes the worst-case identification error, is usually referred to as SM identification. In this work, the problem of estimating the parameters of the HLCPL functions will be described in the SM identification formalism, assuming UBB error.

The paper is organized as follows. In Section II the model structure is presented. The concepts and results about HLCPL functions and Wiener modeling are briefly summarized and the approximation structures are described. In Section III the approach for identifying the HLCPL mapping within the framework of Set-membership estimation theory is presented and the model structure is addressed. This is the main contribution of this paper: a mechanism for MIMO Wiener-like model characterization that converts the problem of identifying the coupled MIMO dynamical system into the identification of MISO ones by applying SM theory for estimating the parameters. In Section IV an example that shows the identification capabilities of the presented structure is reported. Finally, in Section V some concluding remarks are drawn and future work is outlined.

## II. MODEL STRUCTURE

In the description of block-oriented models, several approaches can be found in the literature for performing the identification process. In this article, we focus on a particular and widely-used type of block-oriented nonlinear models, the Wiener-like model, and assume it has a parametric representation.

The approach herein allows for the identification of coupled MIMO nonlinear systems to be described as the identification of a set of MISO ones.

Suppose that a Lipschitz continuous, nonlinear coupled dynamic system is excited by a sequence of multiple inputs  $\mathbf{u}(t) = (u_1(t), u_2(t), \dots, u_p(t)) \in R^p$ .  $\mathbf{y}(t) = (y_1(t), y_2(t), \dots, y_q(t)) \in R^q$  are the respective outputs of the real system at each instant  $t$ , being  $0 \leq t \leq N - 1$ .

Let  $\mathbf{u}_i = (u_i(0), u_i(1), \dots, u_i(N - 1))$ ,  $\mathbf{y}_j = (y_j(0), y_j(1), \dots, y_j(N - 1))$ , the pairs of input/output vectors corresponding to each SISO nonlinear dynamic system contained in the given coupled MIMO one.

Figure 1 depicts the structure used for the SISO Wiener-like model. It consists of a finite set of linear filters for each input vector  $\mathbf{u}_i$ ,  $1 \leq i \leq p$ , followed by a nonlinear static mapping  $f_{pwl}$ , where  $f_{pwl}$  is a HLCPL function. That is, the linear model  $\mathcal{L}$  maps the input vector  $\mathbf{u}_i \in R^N$  into the intermediate signal  $Z = \mathbf{z}[\mathbf{u}_i]$ , where  $Z$  is a  $n_i \times N$  matrix. The number of rows depend of the number  $n_i$  of Laguerre filters used in each input vector  $\mathbf{u}_i$ , and the overall model output is the output  $\mathbf{y}_j$  of the nonlinear block. It is assumed that each measured output vector  $\mathbf{y}_j$  contains a UBB additive noise component  $\mathbf{e}_j$ . Then the model structure is given by

$$\tilde{\mathbf{y}}_j(t + 1) = f_{pwl}(\mathbf{z}[u_i(t)]), \quad (1)$$

for  $1 \leq i \leq p$  and for each  $1 \leq j \leq q$ . The HLCPL functions are defined on a compact domain  $\mathbf{S}$  of the form

$$\mathbf{S} \doteq \{\mathbf{x} \in \mathbb{R}^n : a_i \leq x_i \leq a_i + \delta \text{ndiv}, 1 \leq i \leq n\}, \quad (2)$$

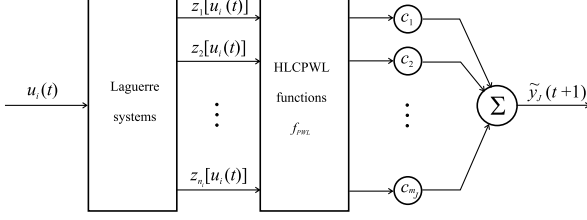


Figure 1: SISO Wiener-like model structure.

where  $a_i \in \mathbb{R}$ ,  $\delta$  is the grid size and  $\text{ndiv} \in \mathbb{Z}_+$  is the number of divisions associated with the  $x_i$  axis. The domain  $\mathbf{S}$  is partitioned by means of a simplicial boundary configuration  $H$  (Julian *et al.*, 1999).

The space  $PWL_H[\mathbf{S}]$  of all continuous PWL mappings defined over the domain  $\mathbf{S}$  partitioned with a simplicial boundary configuration  $H$  (Chien and Kuh, 1977), is a linear vector space. According to Julian (1999), any  $f_{pwl} \in PWL_H[\mathbf{S}]$  can be written as

$$f_{pwl}(\mathbf{x}) = \mathbf{c}^T \Lambda(\mathbf{x}), \quad (3)$$

where  $\mathbf{c}$  is a vector of parameters and  $\Lambda$  is the matrix of the HLCPWL basis functions defined on  $\mathbf{S}$ .

Starting from Eq. (1) we propose a model structure for the given coupled MIMO nonlinear system as a set of coupled input-output MISO ones. These are composed by the union of  $p$  finite sets of discrete-time Laguerre filters applied to each entry  $\mathbf{u}_i$ ,  $1 \leq i \leq p$ , followed by a HLCPWL function. The output model is given by

$$\tilde{y}_j(t+1) = f_{pwl}(\mathbf{z}[u_1(t)], \mathbf{z}[u_2(t)], \dots, \mathbf{z}[u_p(t)]), \quad (4)$$

for each  $1 \leq j \leq q$ , where  $(\mathbf{z}[\mathbf{u}_1], \dots, \mathbf{z}[\mathbf{u}_p])^T$  is a  $n \times N$  matrix with  $n = \sum_{i=1}^p n_i$ .

Then, from Eqs. (3) and (4) we obtain

$$\tilde{y}_j(t+1) = \mathbf{c}_j^T \Lambda(\mathbf{z}[u_1(t)], \dots, \mathbf{z}[u_p(t)]), \quad (5)$$

where  $\mathbf{z}[u_i(t)] = (z_1[u_i(t)], z_2[u_i(t)], \dots, z_{n_i}[u_i(t)])$ , for  $i = 1, 2, \dots, p$  are the outputs of the Laguerre filters for each input  $u_i(t)$  and  $\mathbf{c}_j \in \mathbb{R}^{m_j}$  is the vector of unknown parameters of the nonlinear block which must be estimated.

In order to obtain the MISO model structure we determine the parameter  $\mathbf{c}_j$  by applying SM theory.

### III. NONLINEAR MAPPING IDENTIFICATION

Assuming that

$$\mathbf{u}_i \in \mathbf{K} \doteq \{\mathbf{u}_i : \|\mathbf{u}_i\|_\infty \leq U, U > 0\} \subset l^\infty, \quad (6)$$

it can be easily verified that the outputs of the Laguerre filters satisfy the bound

$$\|\mathbf{z}_r[\mathbf{u}_i(\cdot)]\|_\infty \leq C \cdot U, \forall r = 1, \dots, n_i, 1 \leq i \leq p, \quad (7)$$

where  $C \doteq \|\mathbf{z}_1\|_1$ . Therefore, the domain of the HLCPWL functions can be defined as the set

$$\mathbf{S} \doteq \{\mathbf{z} \in \mathbb{R}^n : \|\mathbf{z}\|_\infty \leq C \cdot U\}, \quad (8)$$

that is partitioned as described above.

Also, the input to the nonlinear mapping, *i.e.* the output of the linear filters, should be spread over all the simplices. Depending on the input, it is possible to have a large concentration of input/output data in a few simplices and a small number of data in most of them. To overcome this problem, we can use a simple algorithm (Castro, 2001) that removes data points from the simplices with high quantity of data points making its distribution more uniform. Besides, it guarantees the identification of all the simplices and keeps the computations easy on the number of samples.

Based on SM estimation theory (Milanese and Tempo, 1985; Kaciewicz *et al.*, 1997; Kaciewicz, 1999; Garulli *et al.*, 2000; Casini, 2004), in this section we describe the procedure for identifying the nonlinear mapping of the model.

We assume that the noise vector is UBB in  $l^\infty$  norm, *i.e.*

$$|e_j(t)| \leq \epsilon_j, \quad \text{for given } \epsilon_j > 0, \forall t. \quad (9)$$

We have to determine a HLCPWL function to approximate the mapping  $g$  between the Laguerre filters output  $\mathbf{z}[u_1(t)], \dots, \mathbf{z}[u_p(t)]$  and the system output  $y_j(t+1)$  related by

$$y_j(t+1) = g(\mathbf{z}[u_1(t)], \dots, \mathbf{z}[u_p(t)]) + e_j(t), \quad (10)$$

for all  $0 \leq t \leq N-1$ . In order to apply the SM estimation theory for finding the parameters of the HLCPWL function, we need to define different spaces.

Let  $(Y, \|\cdot\|_\infty)$  the normed space of dimension  $N$  that contains the noisy measurements  $y_j(t)$ . The finite-dimensional normed space  $X$  is the space of unknown parameters  $\mathbf{c} = [c_1, c_2, \dots, c_{m_j}]^T$ ,  $m_j = (\text{ndiv} + 1)^n$ ,  $n = \sum n_i$  is the number of Laguerre filters and  $\text{ndiv}$  is the number of cells associated with the  $x_i$  axis in Eq. (2).

According to Eq. (5), the *information operator* can be chosen as  $F(\mathbf{c}_j) = \mathbf{c}_j^T \Lambda(\mathbf{z}[u_1(t)], \dots, \mathbf{z}[u_p(t)])$ . Therefore, in this case  $F(\mathbf{c}_j)$  is linear.

In what follows, we calculate a *set estimator* and the *central algorithm* for the estimation of the parameters  $\mathbf{c}_j$ .

The set of *Feasible Parameter Set* (Casini, 2004) is defined, according to Eq. (9), as

$$FPS_{y_j} = \{\mathbf{c}_j \in X : \|\mathbf{y}_j - F(\mathbf{c}_j)\|_\infty \leq \epsilon_j\} \subseteq \mathbb{R}^{m_j}, \quad (11)$$

and we can assure that the set  $FPS_{y_j}$  is a convex polytope, given that  $F(\mathbf{c}_j)$  is linear and the norm in the space  $Y$  is  $l^\infty$ .

Since the set  $FPS_{y_j}$  might have a very complex shape, great attention has been devoted to algorithms

that approximate it. Most common approaches use orthotopes or ellipsoids as approximating regions. In this paper we use minimum volume outer box (MOB) and maximum volume inner box (MIB) with faces parallel to the coordinate axis.

For each  $k$ , both boxes can be computed by finding the maximum ranges of possible variations of the feasible values  $c_j^k$ , given by the values uncertainty intervals ( $VUI_j^k$ ) defined as

$$VUI_j^k = [\inf_k c_j^k, \sup_k c_j^k], k = 1, 2, \dots, m_j. \quad (12)$$

The sets  $VUI_j^k$  give the length of each side, along the corresponding  $k$ -th coordinate axis, of the axis aligned box of minimal volume containing  $FPS_{y_j}$ . This requires the solution of  $2m_j$  linear programming problems with  $m_j$  variables and  $2N$  inequality constraints, for each  $j = 1, \dots, q$  as stated in the next theorem.

**Theorem III.1** Let  $f_k(\mathbf{c}_j) = c_j^k$ ,  $k = 1, 2, \dots, m_j$ . For each  $k$ , the intervals  $VUI_j^k$  can be found by solving the following linear programming problems

$$\begin{cases} \min_{\mathbf{c}_j \in FPS_{y_j}} f_k(\mathbf{c}_j), \\ \max_{\mathbf{c}_j \in FPS_{y_j}} f_k(\mathbf{c}_j), \end{cases} \quad k = 1, 2, \dots, m_j \quad (13)$$

subject to

$$\|\mathbf{y}_j - F(\mathbf{c}_j)\|_\infty \leq \epsilon_j. \quad (14)$$

**Proof :** The proof is immediate taking into account Eqs. (11) and (12).

To obtain the central algorithm in  $l^\infty$  norm, it suffices to compute the *Chebyshev center* of the previously determined box, as stated in (Milanese and Tempo, 1985; Casini, 2004). The worst-case identification error is then given by the *Chebyshev radius* (Casini, 2004).

#### IV. NUMERICAL EXAMPLE

##### Steam Generation Unit

The process studied in this example consists of a 200 MW drum boiler. The model for this unit developed by Ray and Dutta Majumder (1983) is given by:

$$\begin{aligned} \frac{dP}{dt} = & -0.00193SP^{1/8} + 0.014524F \\ & - 0.000736w_c + 0.00121L + 0.000176T_e \end{aligned} \quad (15)$$

$$\frac{dS}{dt} = 10c_v P^{1/2} - 0.785716S \quad (16)$$

$$\begin{aligned} \frac{dL}{dt} = & 0.00863w_c + 0.002F + 0.463c_v - 610^{-6}P^2 \\ & - 0.00914L - 8.210^{-5}L^2 - 0.007328S. \end{aligned} \quad (17)$$

The states of the boiler's model are the drum pressure  $P$ , the steam flow to the high pressure turbine  $S$  and the drum level  $L$ . The states  $P$  and  $L$  are the

controlled variables. It has two optimization variables, that are also manipulated variables: the fuel input  $F$  and the feed water input  $w_c$ , and two disturbances: the feed water temperature  $T_e$  and the control valve setting  $c_v$ . The stationary values of the state variables are presented in Table 1.

Henceforth we will call  $\mathbf{y}_1 = P$ ,  $\mathbf{y}_2 = L$  and  $\mathbf{u}_1 = F$ ,  $\mathbf{u}_2 = w_c$ .

We generated a data set of 30000 samples with sampling time of 20 seconds and a random input signal was added to their nominal values. These random disturbances can take values between  $\pm 20\%$  of the nominal value. Each value is maintained for a number of samples, in order to identify the steady-state gain. To illustrate the situation, unfavorable acquisition noise measurements are introduced, namely output data are corrupted with noise bounded by  $\epsilon_1 = 0.5 \text{randn}(\text{length}(\mathbf{y}_1))$  and  $\epsilon_2 = 0.5 \text{randn}(\text{length}(\mathbf{y}_2))$ .

The 20000 first samples are devoted to identification and other 10000 to validation. The identification

VARIABLE	VALUE
$F(kg/s)$	38,5736
$w_c(kg/s)$	190,9620
$T_e(K)$	310
$c_v$	0,8

Table 1: Variables of UGVD

process was carried following the description given in Álvarez *et al.* (2011). In this case, a Laguerre expansion of order 1 was selected with poles 0.95 and 0.8.

The nominal model  $f_{pwl}$  was identified using estimators based on the feasible system set Eq. (11), the inputs being the outputs of the Laguerre filters. The number of identified parameters for each nonlinear structure was 9, which is equal to the number of HLCPL functions in the representation. The estimated noise bounds were  $\epsilon_1 = 3.54$  and  $\epsilon_2 = 2.71$ , for the available measurements  $\mathbf{y}_1$ ,  $\mathbf{y}_2$ , respectively.

For the output system  $\mathbf{y}_1$ , the root mean square error and the *FIT* index, defined by  $FIT = 1 - \frac{\|\tilde{\mathbf{y}} - \mathbf{y}\|_2}{\|\mathbf{y} - \tilde{\mathbf{y}}\|_2} 100$ , attained by the model are  $RMSEs = 0.8866$ ,  $FIT = 94.08$  and  $RMSEs = 1.0174$ ,  $FIT = 93.5216$  on the identification and validation data sets, respectively. The worst-case optimal error for the identification is  $E_y(\Phi_{cc}, \epsilon) = 2.3974$ .

For the output system  $\mathbf{y}_2$ , the  $RMSEs = 0.5947$ ,  $FIT = 90.3804$  on the identification data set and  $RMSEs = 0.6464$ ,  $FIT = 89.7152$  on the validation data sets. The worst-case optimal error for the identification is  $E_y(\Phi_{cc}, \epsilon) = 2.4240$ .

Figures 2 and 3 presents the dynamic behavior of the models  $\tilde{\mathbf{y}}_1$  and  $\tilde{\mathbf{y}}_2$  compared to the system outputs  $\mathbf{y}_1$  and  $\mathbf{y}_2$ .

A validation study was performed on the validation set. In Figures 4 and 5 it can be seen that the model process mismatch is very small. It should also be noted

that a first-order Laguerre model and only nine parameters for each MISO systems were used.

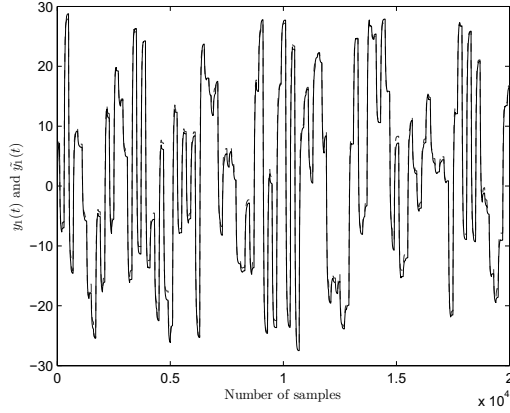


Figure 2: Laguerre-HLCPWL model. System ( $y_1$ : solid line), model ( $\tilde{y}_1$ : dotted line).

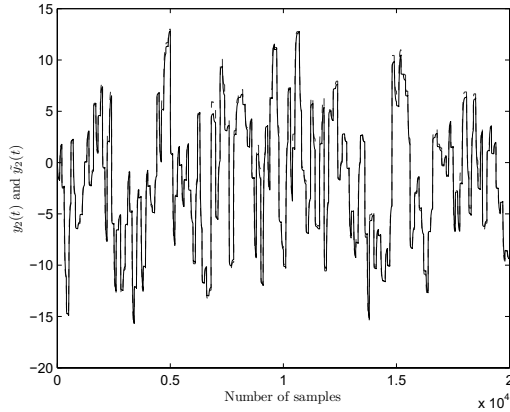


Figure 3: Laguerre-HLCPWL model. System ( $y_2$ : solid line), model ( $\tilde{y}_2$ : dotted line).

## V. CONCLUSIONS

A model evaluation from input-output data of a coupled MIMO nonlinear dynamic system using Set-membership estimation theory has been presented. The identified model is a Wiener-like structure, a special type of block oriented models, which is represented as a set of input-output MISO models. The linear part of each MISO model can be represented in a parametric way using rational orthonormal bases and the static nonlinearity is represented parametrically. In this work, discrete-time Laguerre filters applied to each entry  $u_i$  of the dynamic system describe the main dynamic characteristics and HLCPWL functions describes the static nonlinearity.

The parameters of the HLCPWL representation were identified based on a SM approach. The advan-

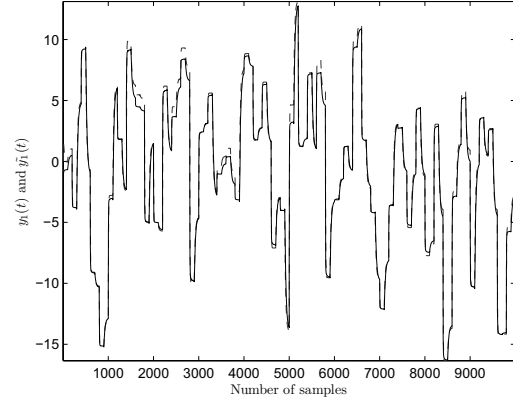


Figure 4: Laguerre-HLCPWL model on the validation set. System ( $y_1$ : solid line), model ( $\tilde{y}_1$ : dotted line).

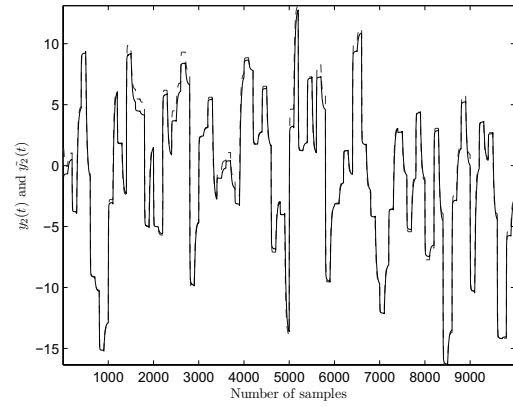


Figure 5: Laguerre-HLCPWL model on the validation set. System ( $y_2$ : solid line), model ( $\tilde{y}_2$ : dotted line).

tage of this approach is that the only assumption on the noise is that it has to be bounded, in contrast with standard approaches, that rely on statistical assumptions of the noise such as stationarity, uncorrelated, etc. Estimation was stated as an optimization problem which is a linear programming one. Results show the potential of the proposed modeling technique and that a good approximation can be attained with just a few parameters. It is important to note that the HLCPWL functions have a simple electronic implementation, and thereby it is possible to develop a real-time implementation of this model structure for a large class of systems. The study of an optimal input design for the identification process within the Set-membership framework is a point of further research.

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