THE CLOSING FUNCTION IN THE WATERHAMMER MODELING

P. G. PROVENZANO, F. BARONI and R. J. AGUERRE †

† *Departamento de Tecnología, Universidad Nac. de Luján (CONICET), 6700 Luján, Argentina, rjaguerre@gmail.com*

***Abstract*— The closing valve law is a mathematical function that describes the speed variation of the fluid as it is closing. This reduction of the speed determines the shape of the pressure wave during the development of the water-hammer. A wide variety of closing modes exists, depending on the valve type and their operation, each one is mathematically given by a function. A generic function was formulated that allows to model an extensive variety of closing laws by means of a polygonal segmented structure. An algorithm has been generated that includes this closing law as boundary condition for describing the transitory. The pressure wave shape and amplitude depends on closing function in a unique relationship.**

***Keywords*— Water-hammer, valve, closing law, pressure wave, transitory.**

I. INTRODUCTION

The water-hammer has its origins in accidental or programmed perturbations on hydraulic systems. The operation of a valve generates a transitory in the hydrodynamic system that produces changes on the flow conditions. These changes are observed in the ondulatory behavior that shows the pressure and the fluid speed, an alternate succession of crests and troughs that attenuates in time. This transitory is known as water-hammer.

The earlier studies on the water-hammer phenomena have been found in the works of Young (1808), Wertheim (1848) and Michaud (1878) among others.

Joukowsky (1900, 1904) published the results of the experimental studies carried out on the water system distribution of Moscow. In that work it was extended the transitory description to the total time of duration. Established a rational expression for the complex variations of pressure that experiences a pipe net taking into account the reflection and transmission in pipe boundaries, and introduced the pipe period concept. These works were based on the observations and developments of Wertheim (1848), von Helmholtz (1847), Korteweg (1878) and Lamb (1898).

During that period (1903-1913) Lorenzo Allievi was an active investigator in this theme, arriving to the same results as Joukowsky, assuming pipe flow without friction, uniform section, homogeneous wall and uniform distribution of speed of the fluid. Allievi has developed the wave equations, rejecting the convective terms and solving it by the general method proposed by Riemann and D'Alembert. Allievi (1925) has extended the Joukowsky results to the cases of non instantaneous valve closing, that is to say, closing times higher than the pipe period being able to predict the variations of pressure along the pipe and not only on the valve (Murga and Molina; 1997).

Wood (1938) introduced the Heaviside operational calculus and presented a solution for a simple pipe with instantaneous valve closing. Rich (1945) proposed the use of the Laplace-Mellin transform for the same system.

With the advancement in computing technology appeared the firsts numeric methods for the Water-hammer modeling (Harding, 1966). The Method of the Characteristics is a particularly appropriate technique for the solution of hyperbolic partial differential equations (Abbott, 1966). Gray (1953, 1954), Ezekial and Paynter (1957) and Streeter *et al.* (1962, 1967, 1972, 1983) have found useful the use of this technique.

In Europe, Fox (1968), Evangelisti (1969) and Swaffield (1970) were the precursors in the use of this method, which has settled down as a standard technique for the transitory analysis (Brunone *et al.,* 1991).

Some authors have indicated the influence of the closing perturbation on the pressure wave transient, however the closing functions that have been included in successive works in models of the Water-hammer are restricted to the instantaneous, the lineal and the cosenoidal closing (Hager, 2001).

The objective of the present work is the development of a mathematical model that takes into account the closing law, and the formulation of an algorithm that allows to describe a wide range of closing functions.

II. THE MODEL

The most common cause of Water-hammer is an accidental (a pump getting out of service) or programmed (valve closing) flow perturbation. The location where the perturbation takes place is considered a frontier of the system. Before and after this location, the flow changes are different. A π angle shift appears between the upstream and downstream pressure waves that travel across the pipe. These characteristics allow to evaluate the valve closing (or any device that generates a perturbation) as a boundary condition.

A water-hammer mathematical model was developed using a simple system, constituted by a reservoir, a single horizontal constant diameter conduction with a valve at the pipe end (Fig. 1).

The set of equations that define the analytic pattern of water-hammer is:

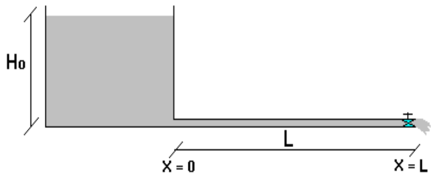

Figure 1: model system

$$\frac{\partial^2 p}{\partial x^2} = -\delta \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial t}\right) \quad (1.a)$$

$$\frac{1}{a^2}\frac{\partial p^2}{\partial^2 t} = -\delta \frac{\partial}{\partial t}\left(\frac{\partial V}{\partial x}\right) \quad (1.b)$$

Applying the Laplace Transform to the equation system (1) and solving:

$$\overline{P} = A \cdot cosh\left(\frac{sx}{a}\right) + B \cdot sinh\left(\frac{sx}{a}\right) + \frac{P_0}{s} \quad (2)$$

The generic equation was obtained, where $\overline{P}$ is the pressure transient in Laplace's Transform field.

The particular solution is obtained determining the expressions of the integration constants A and B of the Eq. (2), applying the boundary conditions. These boundary conditions are:

for $t > \tau$ $\quad t = 0,\quad V = V_0$

for $t \geq 0$ $\quad x = 0,\quad P = P_{static} =$ constant

for $0 < t \leq \tau$ $\quad x = L,\quad V = V(t)$ (closing law)

There are many ways in which the fluid speed can be modified by action of a valve (closing law). These closing functions, like the operations of flow reduction, can be classified into:

- Convex closing laws: A low decrease of the speed flow during the early closing time (τ) that increases with time.
- Concave closing laws: An initial high decrease of the speed flow at the early closing time (τ) that follows with a slow reduction that involves most of that time.
- Linear closing law: The speed flow reduction is uniform during the whole closing time.
- Instantaneous closing law: The flow speed changes instantly from the régime value to the final value or zero.

A stepped procedure was developed, by means of which all closing functions can be approached by polygonal closing functions constructed by an ordered succession of segments. The closing time (τ) was stepped in k equal parts (β).

The following closing function was proposed (Fig. 2):

$$V(t) = \left(V_0 - V_\tau\right)\left[1 - \left(\frac{t}{\tau}\right)^m\right] + V_\tau \cdot \quad (3.a)$$

It can be steeped as follows:

$$V_i = \left(V_0 - V_\tau\right)\left[1 - \left(\frac{i\beta}{\tau}\right)^m\right] + V_\tau \quad (3.b)$$

where $0 \leq i \leq N$, $\tau = N\beta$, $0 \leq m < \infty$ and V_τ is the fluid speed at the end of the closing operation.

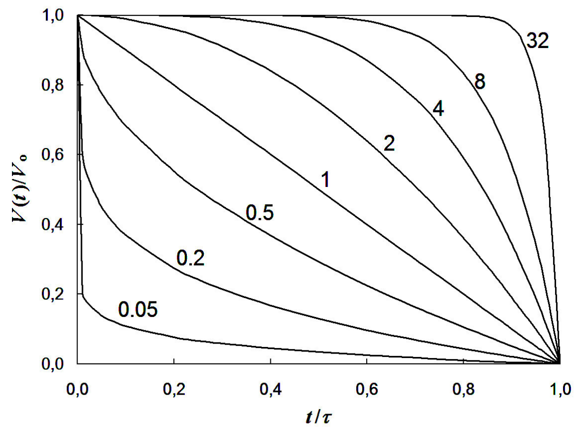

Figure 2: Closing laws corresponding to different m values. (Eq. 3.a)

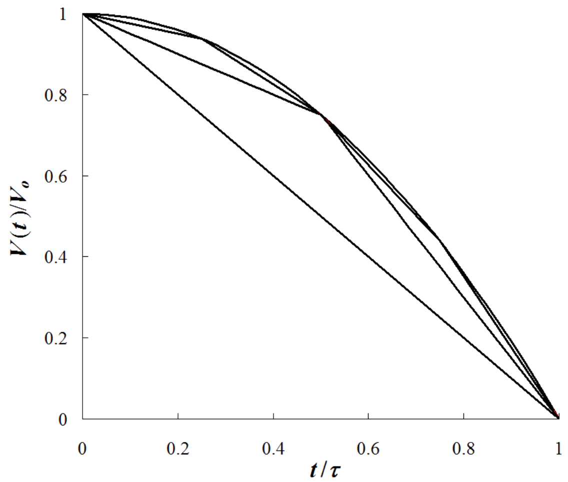

Figure 3: Convex closing, $m = 2$, and their polygonal approach by means of 1, 2 and 4 segments.

Equation (3.b) allows to find the speed values at each of the stepped times $i\beta$.

The m exponent in Eq. (3.a) and Eq. (3.b) determines the closing curve law and the polygonal approaching at true closing law:

$m = 0$ instantaneous closing,

$0 \leq m < 1$ concave closing,

$m = 1$ lineal closing,

$1 \leq m < \infty$ convex closing.

Figure 3 shows a convex closing function and their approach by means a polygonal with one, two and four segments.

The closing law model gets better as more segments are included in the polygonal approach; therefore a generic solution for k segments was developed.

Applying the polygonal approach procedure to the closing function and the Laplace Transform to the Eq. (2) with boundary conditions (closing law) and solving it, a solution that models the Water-hammer was obtained.

The solution for times before the closing end ($0 < t \leq \tau$) is:

$$p(x,t) = p_o + \frac{\rho}{g} a \sum_{i=1}^{j<k-1} \frac{(V_i - V_{i-1})}{\beta} \frac{8L}{\pi^2 a} \left\{ \sum_{n=1}^{\infty} (-1)^n \frac{\cos\left((2n-1)\pi \frac{a}{2L}\left(t-(i-1)\beta\right)\right)\sin\left((2n-1)\pi\frac{x}{2L}\right)}{(2n-1)^2} - \sum_{n=1}^{\infty} (-1)^n \frac{\cos\left((2n-1)\pi . \frac{a}{2L}\left(t-i\beta\right)\right)\sin\left((2n-1)\pi\frac{x}{2L}\right)}{(2n-1)^2} \right\}$$
$$- \frac{\rho}{g} a \frac{\left(V_{j+2} - V_{j+1}\right)}{\beta} \left\{ \frac{x}{a} - \frac{8.L}{\pi^2 a} \sum_{n=1}^{\infty} (-1)^n \frac{\cos\left((2n-1)\pi \frac{a}{2L}\left(t-(j+1)\beta\right)\right)\sin\left((2n-1)\pi\frac{x}{2L}\right)}{(2n-1)^2} \right\} \quad (4)$$

and for times after the closing end ($t > \tau$) is:

$$p(x,t) = p_o + \frac{\rho}{g\tau} a \sum_{i=1}^{k} \frac{V_i - V_{i-1}}{\beta} \frac{8L}{\pi^2 a} \left\{ \sum_{n=1}^{\infty} (-1)^n \frac{\left(\cos(2n-1)\pi \frac{a}{2L}\left(t-(i-1)\beta\right)\right)\sin\left((2n-1)\pi\frac{x}{2L}\right)}{(2n-1)^2} - \sum_{n=1}^{\infty} (-1)^n \frac{\left(\cos(2n-1)\pi \frac{a}{2L}\left(t-i\beta\right)\right)\sin\left((2n-1)\pi\frac{x}{2L}\right)}{(2n-1)^2} \right\} \quad (5)$$

III. RESULTS AND DISCUSSION

The Water-hammer analysis was carried out for different closing functions, by means of the Eq. (4) and Eq. (5). Different transient pressure wave shapes and over-pressure peaks were obtained, some of them are shown in Fig. 4.

The Fig. 4a) to Fig. 4h) give information about the wave shape as a function of the closing law.

The shape of pressure wave evolves from square for small values of m ($m < 0.05$) toward trapezoidal shapes ($0.05 < m < 0.3$) with right bias.

When the value of m increases ($0.3 < m < 8$) the wave shape is practically triangular evolving toward trapezoidal shapes with left bias for $8 < m < 48$, and acquiring square form for higher m values. The wave phase in each case has stayed unalterable, without observable shift.

The wave presented a strictly triangular form for $m = 1$ and the over-pressure peak has been 36% of the maximum, constituting the lower over-pressure peak registration of the whole series.

It has been possible to model, by means of this development, the water-hammer from the beginning of the closing operation.

IV. CONCLUSIONS

The inclusion of the closing function in water-hammer analysis gives a mathematical solution that provides information about the shape, over-pressure peak and phase of the pressure wave during the transient development.

The application of the shift operator and an adequate combination of segments (polygonal), can appropriately model the behavior of any mechanism at the end of a pipe. The polygonal modeling procedure of the closing law has allowed the analysis of a wide variety of closing shapes that appears in the operation of hydraulic systems.

For the linear law ($m = 1$), triangular type waves are predicted giving a lower over-pressure peak than other closing laws.

The solution obtained has also allowed to explore the behavior of the pressure during the closing period, extending the analysis at the beginning of the valve operation.

Quasi linear functions present a defined wave oscillation inside the closing time τ, and it reaches the peak value in the first oscillation, that appears at $t = \tau / 3$.

The convex functions show very moderate oscillations during closing and the pressure reaches the first peak value at $t = \tau$. The concave closing functions produce the maximum pressure peak inside the closing interval at $t = \tau/3$. This result reinforces the importance of the transitory evaluation from the beginning of the closing operation. However in most of the models and experimental studies only the transitory description for $t > \tau$ is considered

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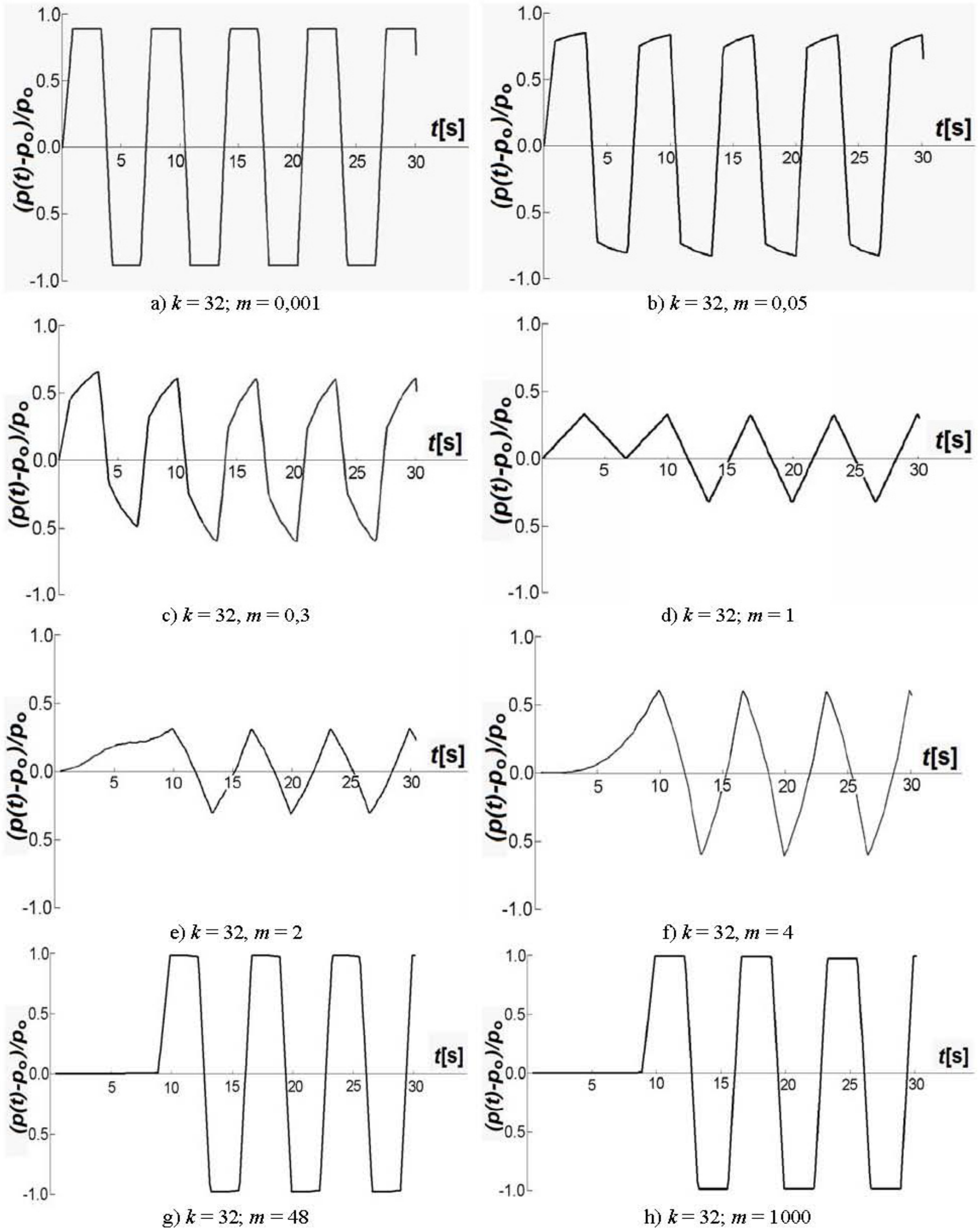

Figure 4: Transient pressure waves for different closing laws, at x = L (Fluid: water; Fluid speed: 3.66 m/s; Conduction length (L): 1520 m; Wave velocity: 915 m/s; Closing time (τ): 10 s.)

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