Noise induced decoherence in a solid-state physics

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The interaction between solid-state qubits and their enviromental degrees of freedom produces non-unitary effects like decoherence, dissipation, and fluctuations. Uncontrolled decoherence is one of the main obstacles that must be overcommed in quantum information processing. It depends on the interaction between the qubit and its environment and the spectral composition of the noise. If the spectral density is well known, it is possible to successfully fight against decoherence, for example, applying sequences of inversion pulses to the qubit system. We study the dynamically decay of coherences in solid-state qubits coupled to arbitrary environments by means of the use of a Markovian master equation. We analyze the effect introduced by thermal Ohmic envirromments including the case of a zero-temperature bath, in which noise fluctuations also induce decoherence. We also analyze low-frequency 1/f noise as coming from spin-fluctuator baths. We focus on the effect of longitudinal and transversal noise on the superconducting qubit's dynamics. Our results can be used to design experimental future setups when manipulating superconducting qubits.

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I. INTRODUCTION

The scaling-down of microelectronics into the nanometer range will inevitably make quantum effects such as tunneling and wave propagation important. The use of these quantum devices in gate operations enhances the need of controlling decoherence. Noise from the environment may cause fluctuations in both qubit amplitude and phase, leading to relaxation and decoherence. External perturbations can influence a two-level system in typically two ways: either shifting the individual energy levels (which changes the transition energy and therefore, the qubit's phase) or inducing energy levels transitions (which changes the level populations). Decoherence is a major hurdle in realizing scalable quantum technologies in the solid state.

Decoherence in qubit systems falls into two general categories. One is an intrinsic decoherence caused by constituents in the qubit system, and the other is an extrinsic decoherence caused by the interaction with uncontrolled degrees of freedom, such as an environment. Understanding the mechanisms of decoherence and achieving long decoherence times is crucial for many fields of science and applications including quantum computation and quantum information [1]. Most theoretical investigations of how the system is affected by the presence of an environment have been done using a thermal reservoir, usually assuming Markovian statistical properties and defining bath correlations [2, 3]. However, there has been some growing interest in modeling more realistic environments, sometimes called composite environments, or environments out of thermal equilibrium [4–6].

Lately, there have been many studies focusing in decoherence in solid state-qubit. The same physical structures that make these superconducting qubits easy to manipulate, measure, and scale are also responsible for coupling the qubit to other electromagnetic degrees of freedom that can be a source of decoherence via noise and dissipation. Thus, a detailed mechanism of decoherence and noise due to the coupling of Josephson devices to external noise sources is still required. In [7] authors reviewed the effect of 1/f noise in nano-devices with emphasis on implications for solid-state quantum information. It has been shown that low frequency noise is an important source of decoherence for superconducting qubits. Generally, this noise is described by fluctuations in the effective magnetic field which are directed either in the z axis -longitudinal noise- or in a transverse direction -transversal noise. Both types of noise have been phenomenologically modeled by making different assumptions on these fluctuations, such as being due to a stationary, Gaussian and Markovian process [8]. In Ref. [9], the influence of $1/f$ noise by random telegraph processes was modeled, also showing that depending on the parameters of the environment, the model can describe both Gaussian and non-Gaussian effects of noise. Ref. [10] presented a phenomenological model for superconducting qubits subject to noise produced by two-state fluctuactors whose coupling to the qubit are all roughly the same. In [11] has been studied the influence of an environment and an adiabatically changing external field, where temperature effects are also considered. However, they resorted to a secular approximation to solve the master equation, which seems to have controversial results, namely that decoherence is null at zero temperature. Decoherence at zero temperature does occur contrary to what is most commonly believed. There are simple examples in Literature which demonstrate that decoherence is induced even by a reservoir at zero temperature [12–15]. In general, a small system coupled to an environment fluctuates even in the zero-T limit. These fluctuations can take place without generating an energy trace in the bath. The fluctuations in energy of the small system are a peculiar fact of the entanglement with the quantum environment. However, the suppression of the interferences is not as fast as it is at high temperature limit. In the latter case, it is expected to happen, for a quantum Brownian particle of mass M, at times of $O(1/2M\gamma_0 k_B T L)$ while it occurs at times smaller than $O(1/\gamma_0)$ when the environment is at zero temperature [12] (where γ_0 is the dissipation constant, T is the environmental temperature, and L the distance between classical trajectories of the particle).

In this manuscript, we consider a two-level system coupled to a general enviroment. We develop an unified frame for studying the dynamically decay of coherences in a solid-state qubit coupled to an arbitrary environment by means of the use of a Markovian master equation. We analyze the effect induced in the system by thermal Ohmic environment, through a non-purely dephasing process, including the case of a zero-temperature bath. We also analyze low-frequency $1/f$ noise as coming from a fluctuator environment, by defining the corresponding spectral density. The comprehension of the decoherence and dissipative processes should allow their further suppression in future qubits designs or experimental setups. In section II we describe the model used for the analysis of a superconducting qubit coupled to an environment. In Sec. III we present the master equation approach by considering different spectral densities to describe the environment. By means of a general master equation for the reduced density matrix of the qubit, we follow the nonunitary evolution characterized by fluctuations, dissipation and decoherence. In Sec.IV, we study the decoherence process due to an Ohmic environment both at high and low temperature. In Sec.V we study the decoherence induced by an $1/f$ noise. In both cases, we particularly study the difference between the longitudinal and transversal couplings and provide analytical estimations of decoherence time when possible. Finally in Sec.VI, we summarize our final remarks.

II. MODEL FOR A SOLID-STATE QUBIT

Experimental observation of Rabi oscillations in driven quantum circuits have shown several periods of coherent oscillations, confirming the validity of the two-level approximation and possibility of coherently superimpose the computational two states of the system. Nevertheless, the unavoidable coupling to a dissipative environment surrounding the circuit represents a source of relaxation and decoherence that limit the performances of the qubit for quantum computation tasks. Therefore, for the implementation of superconducting circuits as quantum bits, it is necessary to understand the way the system interacts with the environmental degrees of freedom, and to reduce their effect, if possible.

When the two lowest energy levels of a current biased Josephson junction are used as a qubit, the qubit state can be fully manipulated with low and microwave frequency control currents. Circuits presently being explored combine in variable ratios the Josephson effect and single Cooper-pair charging effects. In all cases the Hamiltonian of the system can be written as,

$$H = \frac{\hbar}{2}\omega_a \sigma_z + \hbar\Omega_{\rm R}\cos(\omega t + \varphi_{\rm R})\sigma_x, \tag{1}$$

where $\hbar\Omega_{\rm R}$ is the dipole interaction amplitude between the qubit and the microwave field of frequency ω and phase $\varphi_{\rm R}$. $\Omega_{\rm R}/2\pi$ is the Rabi frequency. This Hamiltonian can be transformed to a rotating frame at the frequency ω by means of an unitary transformation and, after the rotating wave approximation, resulting in a new effective Hamiltonian of the form

$$H_{\rm eff} = \frac{\hbar}{2}\left(\Delta\sigma_z + \Omega_x\sigma_x + \Omega_y\sigma_y\right), \tag{2}$$

where $\Omega_x = \Omega_{\rm R}\cos\varphi_{\rm R}$ and $\Omega_y = \Omega_{\rm R}\sin\varphi_{\rm R}$. Then, we shall consider the dynamics of a generic two-level system steered by a system's Hamiltonian of the type (where we have set $\hbar = 1$ all along the paper)

$$H_{\rm Total} = H_q + H_{\rm int} + H_{\mathcal{E}}, \quad \text{with} \tag{3}$$

$$H_q = \frac{1}{2}\left(\Omega\sigma_x + \Delta\sigma_z\right) \tag{4}$$

where we have defined a qubit Hamiltonian H_q similar to that of a solid-state qubit Eq.(2) - setting $\varphi_{\rm R} = 0$ for simplicity-, and $H_{\mathcal{E}}$ is the Hamiltonian of the bath. The interaction Hamiltonian is thought as some longitudinal and transverse noise coupled to the main system:

$$H_{\rm int} = \frac{1}{2}\left(\hat{\delta\omega_1}\sigma_x + \hat{\delta\omega_0}\sigma_z\right). \tag{5}$$

By considering this interaction Hamiltonian we are implying that the superconducting qubit is coupled to the environment by a coupling constant in the $\hat{z}$ direction, called longitudinal direction, and in $\hat{x}$, the transverse direction. This type of bidirectional coupling has been recently used in [16] to compute the geometric phase of a supercomputing qubit. It is important to note that the derivation of a master equation has not been done before for a Hamiltonian of the form of Eq.(3).

III. MASTER EQUATION APPROACH

We shall derive the master equation in the Born-Markov approximation, for general noise terms $\hat{\delta\omega_1}$ and $\hat{\delta\omega_0}$. We will consider a weak coupling between system and environment and the bath sufficiently large to stay in a stationary state. In other words, the total state $\rho_{\mathcal{SE}}$ (system and environment) can be split as $\rho_{\mathcal{SE}} \approx \rho(t)\times\rho_{\mathcal{E}}$, for all times. It is important to stress that due to the Markov regime, we will restrict to cases for which the self-correlation functions generated at the environment (due to the coupling interaction) would decay faster than typical variation scales in the system [17]. In the interaction picture, the evolution of the total state is ruled by the Liouville equation

$$\dot{\rho}_{\mathcal{SE}} = -i\left[H_{\rm int}, \rho_{\mathcal{SE}}\right], \tag{6}$$

where we have denoted the state $\rho_{\mathcal{SE}}$ in the interaction picture in the same way than before, just in order to simplify notation. A formal solution of the Liouville equation can be obtained perturbatively using the Dyson expansion. From this expansion, one can obtain a perturbative master equation, up to second order in the coupling constant between system and environment for the reduced density matrix $\rho = {\rm Tr}_{\mathcal{E}}\rho_{\mathcal{SE}}$. In the interaction picture the formal solution reads as

$$\begin{aligned}\rho(t) \approx \rho(0) - i\int_0^t ds {\rm Tr}_{\mathcal{E}}\left([H_{\rm int}(s), \rho_{\mathcal{SE}}(0)]\right) \quad (7)\\ - \int_0^t ds_1 \int_0^{s_1} ds_2 {\rm Tr}_{\mathcal{E}}\left([H_{\rm int}(s), [H_{\rm int}(t), \rho_{\mathcal{SE}}(0)]]\right).\end{aligned}$$

In order to obtain the full master equation for a superconducting qubit, it is necessary to perfom the temporal derivative of the previous equation and assume that the system and the environment are not initially correlated. In addition, we consider that the $\hat{\delta\omega}_i$ of the $H_{\rm int}$ (Eq.(5)) are operators acting only on the Hilbert space of the environment (and the Pauli matrices applied on the system Hilbert space). Finally, the master equation explicitely reads,

$$\begin{aligned}\dot{\rho} = &-i\left[H_q, \rho\right] - d_{xx}(t)\left[\sigma_x, [\sigma_x, \rho]\right] - f_{xy}(t)\left[\sigma_x, [\sigma_y, \rho]\right]\\ &- f_{xz}(t)\left[\sigma_x, [\sigma_z, \rho]\right] - f_{zx}(t)\left[\sigma_z, [\sigma_x, \rho]\right]\\ &- f_{zy}(t)\left[\sigma_z, [\sigma_y, \rho]\right] - d_{zz}(t)\left[\sigma_z, [\sigma_z, \rho]\right]\\ &+ i\gamma_{xy}\left[\sigma_x, \{\sigma_y, \rho\}\right] + i\gamma_{xz}\left[\sigma_x, \{\sigma_z, \rho\}\right]\\ &+ i\gamma_{zx}\left[\sigma_z, \{\sigma_x, \rho\}\right] + i\gamma_{zy}\left[\sigma_z, \{\sigma_y, \rho\}\right],\end{aligned} \tag{8}$$

where the noise effects are included as the normal (d_{xx} and d_{zz}) and anomalous diffusion coefficients (f_{ab} with $a, b = x, y, z$). The dissipative effects are included in the corresponding coefficients (γ_{ab} with $a, b = x, y, z$). Eq.(8) is the first master equation derived that considers both diffusion and dissipation effects for a superconducting qubit.

$$\begin{aligned}&d_{xx}(t) = \int_0^t ds\, \nu_1(s)\, X_1(-s), \quad d_{zz}(t) = \int_0^t ds\, \nu_0(s)\, Z_0(-s), \quad f_{xy}(t) = \int_0^t ds\, \nu_1(s)\, Y_1(-s),\\ &f_{xz}(t) = \int_0^t ds\, \nu_1(s)\, Z_1(-s), \quad f_{zx}(t) = \int_0^t ds\, \nu_0(s)\, X_0(-s), \quad f_{zy}(t) = \int_0^t ds\, \nu_0(s)\, Y_0(-s)\\ &\gamma_{xy}(t) = \int_0^t ds\, \eta_1(s)\, Y_1(-s), \quad \gamma_{xz}(t) = \int_0^t ds\, \eta_1(s)\, Z_1(-s)\\ &\gamma_{zx}(t) = \int_0^t ds\, \eta_0(s)\, X_0(-s), \quad \text{and} \quad \gamma_{zy}(t) = \int_0^t ds\, \eta_0(s)\, Y_0(-s).\end{aligned} \tag{9}$$

These coefficients are defined in terms of the noise and dissipation kernels, $\nu(t)$ and $\eta(t)$, respectively. These kernels are generally defined, for unspecified operators $\hat{\delta\omega}_0(t)$ and $\hat{\delta\omega}_1(t)$, as

$$\nu_{0,1}(t) = \frac{1}{2}\langle\left\{\hat{\delta\omega}_{0,1}(t), \hat{\delta\omega}_{0,1}(0)\right\}\rangle, \tag{10}$$

$$\eta_{0,1}(t) = \frac{1}{2}\langle\left[\hat{\delta\omega}_{0,1}(t), \hat{\delta\omega}_{0,1}(0)\right]\rangle. \tag{11}$$

The functions $X_{0,1}, Y_{0,1}$, and $Z_{0,1}$ appearing in Eq.(9) are derived by obtaining the temporal dependence of the Pauli operators σ_i in the Heisenberg representing through the differential equations. Their explicit forms can be found in Ref.[16]. It is easy to check that if the Rabi frequency is zero and $\delta\hat{\omega}_1 = 0$, we recover the dynamics of a spin-1/2 precessing a biased field vector $\mathbf{R}$.

The intention is to study environmental-induced decoherence by means of the use of the master equation. In this equation there are dissipation, diffusion, and possible driven (unitary) effects, all consistently included within the Markov approximation to the dynamics of the environment. We will mainly concentrate on two types of decoherence sources. On one side, we shall consider that the environment is characterized by an Ohmic spectral density, as the one commonly used in models of Quantum Brownian Motion (QBM) or in the well known spin-boson model [18, 19]. In these examples, the environment is represented by an infinite set of harmonic oscillators at thermal equilibrium. On the other side, we shall analyze decoherence induced effects coming from spin-environments, for example spin-fluctuator models, that give us the possibility to study $1/f$ noise-effects via the master equation approach, without resorting to classical statistical evolutions or phenomenological models. Once the coefficients in Eqs.(9) are defined, we can numerically solve the master equation and obtain the evolution in time of the reduced density matrix.

IV. OHMIC ENVIRONMENT

A relevant contribution to decoherence in solid-state qubits, is introduced by the electromagnetic noise of the control circuit, typically Ohmic noise at low frequencies. In this Section, we model this kind of environments by means of an infinite set of harmonic oscillators with an Ohmic spectral density. An environment composed by harmonic oscillators at thermal equilibrium at temperature T is commonly introduced in order to take into account dissipative effects, additionally to noise or fluctuations effects.

It is easy to see that in the case that the environment is modeled by a set of harmonic oscillators, the noise (Eq. (10)) and dissipation (Eq.(11)) kernels become

$$\nu_{0,1}(t) = \frac{1}{2}\sum_n \lambda_{0,1,n}^2 \langle\{q_n(t), q_n(0)\}\rangle, \tag{12}$$

$$\eta_{0,1}(t) = \frac{1}{2}\sum_n \lambda_{0,1,n}^2 \langle[q_n(t), q_n(0)]\rangle, \tag{13}$$

where q_n are the position operators for the environmental degrees of freedom.

The noise correlations can be defined by their spectral density $J_a(\omega) = 1/(2\pi)\int dt e^{i\omega t}\langle\delta\hat{\omega}_a(0)\delta\hat{\omega}_a(-s)\rangle_{\mathcal{E}}$ with $a = 0, 1$. If we assume the environment is composed by an infinite set of harmonic oscillators, it is useful to use the relation

$$\sum_n^N \frac{\lambda_n^2}{2m_n\omega_n} f(\omega_n) = \int_0^\infty J(\omega) f(\omega) d\omega, \tag{14}$$

in order to express kernels in terms of integrals in frequency. For example, using Eqs. (12) and (13), the noise and dissipation kernels can be written as

$$\nu_a(t) = \int_0^\infty J_a(\omega)\cos(\omega t)\coth(\frac{\beta\omega}{2})d\omega, \tag{15}$$

$$\eta_a(t) = \int_0^\infty J_a(\omega)\sin(\omega t)d\omega, \tag{16}$$

where $\beta = 1/k_B T$ is the equilibrium temperature of the environment.

In this model, we use $J_a(\omega) = \gamma_a\omega\exp[-\omega/\Lambda]$ as the spectral density of the environment. This definition allows to calculate the noise and dissipation kernels from Eqs. (15) and (16) [19]. In the definition of $J(\omega)$, Λ is a physical ultraviolet cutoff, which represents the biggest frequency present in the environment. Starting with an arbitrary initial superposition state, parametrized by $|\psi\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)|1\rangle$ (where with the angle θ we localize the state in the Bloch sphere), we numerically solve the master equation at different environmental temperatures. In the high temperature limit, one can expand the $\coth(\beta\omega/2)$ for small β, and obtain the noise kernel (the one that depends on temperature) as $\nu_a(t) = \gamma_a k_B T\delta(t)$ (where, with the sub-index $a = 0, 1$ we denote the longitudinal and transversal noise, respectively). In this limit, it is trivial to evaluate the diffusion terms in Eq.(9), to obtain that $d_{xx} = 2\gamma_1 k_B T$, $d_{zz} = 2\gamma_0 k_B T$, and all $f_{ij} = 0$ (there is no anomalous diffusion terms). In the opposite case, when $T \to 0$, the diffusion kernel yields $\nu_a(t) = \gamma_a(t\Lambda\sin(\Lambda t) + \cos(\Lambda t) - 1)/t^2$. With this kernel all the diffusion coefficients in Eq.(9) can be obtained. It is important to remark that they are all different from zero and contribute to the master equation We do not present here the explicit expression of them since their form is not relevant. The dissipative coefficients for the Ohmic environment, can be all calculated from the dissipation kernel Eq.(16). Thus, these kernels are given by $\eta_a(t) = \gamma_a\delta'(t)$, which are independent on temperature. Therefore, dissipation coefficients (in Eq. (9)) are $\gamma_{xx} = 0$, $\gamma_{xy} = -2\Delta\gamma_1$, $\gamma_{zx} = 0$, and $\gamma_{zy} = 2\Omega\gamma_0$ for all temperature.

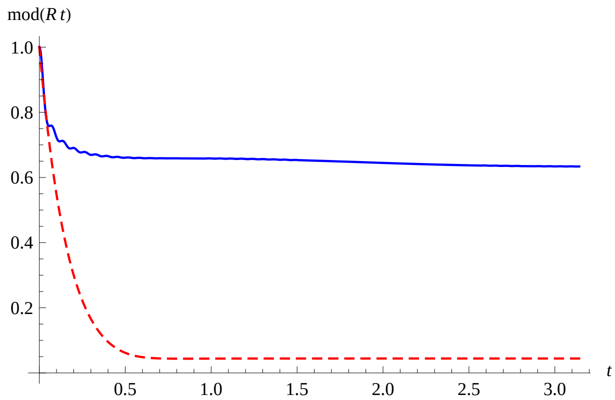

FIG. 1. (Color online) Temporal evolution of the Bloch vector for different environments coupled to the superconducting qubit. The red dashed line represents an ohmic environment in the high temperature limit while the blue solid line represents an ohmic environment a zero temperature. It is easy to note that the state vector of the system is more affected by the influence of the high temperature environment. However, we must note that the initially pure state looses purity even at zero temperature for the same values of γ. Parameters used: $\Omega = 0.5\Delta,\ \gamma_1 = 0.03 = \gamma_0,\ \Lambda = 100\Delta,\ T =$.

In Fig.1 we present the module of the Bloch vector of the state system as a function of time for more than one period $\tau = 2\pi/\tilde{\Omega}$, with $\tilde{\Omega} = \Delta/(\Delta^2 + \Omega^2)$. Qualitatively, decoherence can be thought of as the deviation of probabilities measurements from the ideal intended outcome. Therefore, decoherence can be understood as fluctuations in the Bloch vector $\mathbf{R}$ induced by noise. Since decoherence rate depends on the state of the qubit, we will represent decoherence by the change of $|\mathbf{R}|$ in time, starting from $|\mathbf{R}| = 1$ for the initial pure state, and decreasing as long as the quantum state losses purity. The red dashed line is the evolution of the Bloch vector of a qubit evolving under a high temperature ohmic environment. This kind of environment is very destructive and the state vector is soon removed from the surface of the sphere (where purity states lie). The blue solid line represents the behavior of the Bloch vector when the qubit is evolving under the influence of a zero temperature Ohmic environment. It is easy to note that the state losses purity even at zero temperature, though the influence of the environment is not as drastic as when the temperature is high.

The reduced density matrix can be represent as

$$\rho_{\rm r}(t) = \begin{pmatrix} a_{11}(t) & a_{12}(t)\mathcal{F}(t) \\ a_{21}(t)\mathcal{F}(t)^* & a_{22}(t) \end{pmatrix},$$

where $\mathcal{F}(t)$ can be called decoherence factor. Independently of its expression, we know it must be a decaying function by which after some time bigger than the decoherence time $t > t_D$, the coherences of the density matrix can be neglected. Therefore, we shall look the behavoir of the coherences $\rho_{r01}(t) = a_{12}(t)\mathcal{F}(t)$ to study the influence of the environment on the system's evolution.

In Fig.2 we present the evolution in time of the coherence of the qubit-system ($\rho_{r01}(t)$) as a function of dimensionless time (Δt), in the case of high and zero temperature and for different dissipation constants in the weak coupling limit. The red dotted line is the solution of the master equation in the limit of high temperature (for dimensionless parameter $T = 50$). As expected, off-diagonal terms in the reduced density matrix decay quickly to their minimum value, reaching a steady state of minimum coherence. A relevant result is the one obtained in the limit of $T = 0$ environmental temperature represented by the black dashed line $\gamma_i = 0.003$ and the blue solid one $\gamma_i = 0.03$. In this case, we can see that, after a few large amplitude oscillations, the coherences in the system decay (more slowly than in the case of high temperature) with time, reaching an asymptotic value of minimum coherence, different from the value corresponding to the high-T limit. This is mainly due to the presence, in the master equation for T = 0, of diffusion and dissipation coefficients, which are absent in the case of high-T. Nevertheless, we show that fluctuations at zero-T also induce decoherence in the solid-state qubit, with a lower efficiency than in the thermal case, but strong enough to destroy the unitary evolution. If we set the decoherence timescale as the time at which coherence reach its asymptotic value, it is easy to see that decoherence time in the case of high-T environment is shorter than the timescale associated to the decay of the coherence in the presence of a zero-T environment.

In order to have a rough analytical estimation of decoherence times, we consider that the qubit is solely coupled in the longitudinal direction, and that there is no anomalous and dissipation terms in the master equation. This means that for the moment we neglect the effect of the tunneling term (proportional to σ_x) in the main system Hamiltonian. Thus, we may follow the result given in Refs.[12, 19] for the purely dephasing model. There, decoherence time in the high temperature approximation can be estimated as $t_D \sim 1/(k_B T \gamma_0)$, which does not depend on the frequency cutoff Λ. Using parameters in

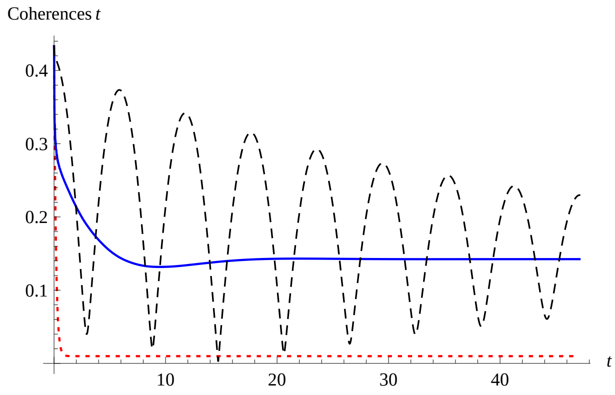

FIG. 2. (Color online) Temporal evolution of the coherence ($a_{12}(t)$) of the qubit system. The red dotted line is the evolution of the coherences in the case the qubit is coupled to a high-T environment ($\gamma_0 = \gamma_1 = 0.03$ and $T = 50$ in dimensionless units). In this case, the system losses coherence quickly and with no possibility of re-coherence. The blue solid line, shows the evolution of the coherence in the limit of zero environmental temperature (with cutoff in frequencies $\Lambda = 100\Delta$ and $\gamma_0 = \gamma_1 = 0.03$). Finally, the blach dashed oscilatory line representes the evolution of the coherence for a smaller value of γ_i at zero temperature ($\gamma_0 = \gamma_1 = 0.003$). Even there are oscillations in the evolution, the system also losses coherence when the environment is at zero temperature. The final value of the off-diagonal term differs from the one in the high-T case, due to the presence of difussion and other dissipative coefficients in the master equation. We have considered $\Omega = 0.5\Delta$.

Fig. 2, one can estimate decoherece time be $\Delta t_D \sim 0.7$ (in units of Δ), in good agreement with the corresponding plot in Fig. 2. For the Ohmic case at zero temperature, the decoherence time scales as $t_D \sim e^{1/\gamma_0}/\Lambda$ for times $\Lambda t \geq 1$. In this case, decoherence is delayed as γ_0 decreases.

As we are considering a bidirectional coupling in our model, it is interesting to see if there is a direction in which decoherence becomes more important. As the value of γ_0 and γ_1 imply the coupling with the environment, we can turn off one of the couplings to study the effect of noise in the longitudinal and transverse directions. In Fig.3 we show three curves corresponding to each of the cases considered: only longitudinal coupling, only transversal coupling and both couplings. The red dashed line is the evolution of the coherences in the case the qubit is coupled to a high-T environment only in the longitudinal direction, i.e. through $\delta\omega_0$. This means $\gamma_1 = 0$ and for example, $\gamma_0 = 0.03$. The red solid line represents the evolution of the coherence when the qubit is coupled only in the transverse direction, namely through $\delta\omega_1$. In this case, we use $\gamma_1 = 0.03$ and $\gamma_0 = 0$. Finally, the black dotted line is the evolution of the coherence when the qubit is equally coupled in both directions, i.e. $\gamma_0 = 0.03$ and $\gamma_1 = 0.03$. It is easy to see that decoherence is mainly ruled by the longitudinal direction which means that noise in the $\hat{z}$-direction affects more the unitary dynamics of the system than noise in $\hat{x}$ direction.

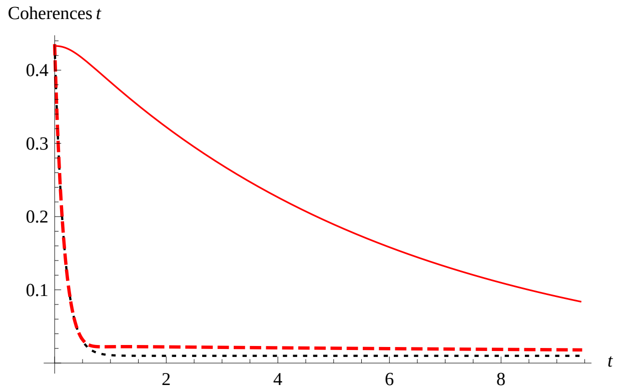

FIG. 3. (Color online) Temporal evolution of the coherence ($a_{12}(t)$) of the qubit system coupled to a high temperature ohmic environment. The red dashed line is the evolution of the coherences in the case the qubit is coupled to a high-T environment only in the longitudinal direction, i.e. through $\delta\omega_0$. This means $\gamma_1 = 0$ and for example, $\gamma_0 = 0.03$. The red solid line represent the evolution of the coherence when the qubit is coupled only in the transverse direction, namely through $\delta\omega_1$. In this case, we use $\gamma_1 = 0.03$ and $\gamma_0 = 0$. Finally, the black dotted line is the evolution of the coherence when the qubit is equally coupled in both directions, i.e. $\gamma_0 = 0.03$ and $\gamma_1 = 0.03$. It is important to mention that this behavior can not be seen in the zero-T environment. We have considered $\Omega = 0.5\Delta$.

V. 1/F NOISE

Much effort has been spent recently to understand how noise at low frequencies affects the dynamics of superconducting qubit, both from a theoretical and an experimental point of view. In solid-state systems decoherence is potentially strong due to numerous microscopic modes. Noise is dominated by material-dependent sources, such as background-charge fluctuations or variations of magnetic fields and critical currents, with given power spectrum, often known as $1/f$. This noise is difficult to suppress and, since the dephasing is generally dominated by the low-frequency noise, it is particularly destructive.

The $1/f$ noise is frequently modeled by an ensemble of two-level systems or fluctuators and describes both Gaussian or non-Gaussian effects [21, 22]. Then, the noise is described as coming from N uncorrelated fluctuators, that we call here $\delta\hat{\omega}_N = \sum_i^N \chi_i(t)$, where $\chi_i(t)$ is a random telegraph process. The variable $\chi_i(t)$ takes the values $-\xi_i$ or ξ_i. Thus, $\chi_i(t)^2 = \xi_i^2 = \text{const}$.

By assuming a random process, there is no dissipation contribution. In order to obtain the diffusion coefficients of the master equation, we need to evaluate the noise correlation functions from Eq.(10), for each of the interaction terms -the longitudinal and the transversal-, characterized by the subindex 0 and 1, respectively. We refer to these as

$$\langle \delta\hat{\omega}_{N,0}(t)\delta\hat{\omega}_{N,0}(s)\rangle = \sum_{i=1}^{N} \xi_{i,0}^2 e^{-2\zeta_{i,0}|t-s|},$$
$$\langle \delta\hat{\omega}_{N,1}(t)\delta\hat{\omega}_{N,1}(s)\rangle = \sum_{i=1}^{N} \xi_{i,1}^2 e^{-2\zeta_{i,1}|t-s|}.$$

Following Ref.[**?**], we define the effective random telegraph process for $N \gg 1$, as $\delta\hat{\omega}_a(t) = lim_{N\to\infty}\delta\hat{\omega}_{N,a}(t)$, considering a continuous distribution of amplitudes (ξ) and switching rates (ζ). Assuming that for an individual fluctuator, the correlation relations are given by

$$\langle \chi_i(t)\rangle = 0,$$
$$\langle \chi_{i,a}(t)\chi_{j,a}(s)\rangle = \frac{\sigma_a^2}{N}\delta_{ij}e^{-2\zeta_a|t-s|}, \tag{17}$$

where $\sigma_a^2 = \lim_{N\to\infty} N\xi_a^2$. For $N \to \infty$, the effective random process becomes a Gaussian Markovian process with an exponential correlation function

Finally, we consider that the noise correlation is defined by

$$\langle \delta\hat{\omega}_a(t)\delta\hat{\omega}_a(s)\rangle = {\sigma_a}^2 e^{-2\zeta_a|t-s|}, \tag{18}$$

where index $a = 0, 1$, indicates longitudinal and transversal couplings between the fluctuator and the qubit. By using the noise correlation functions of Eq.(18), we compute the diffusion coefficients Eq. (9) of the master equation and solve it numerically to obtain the qubit dynamics.

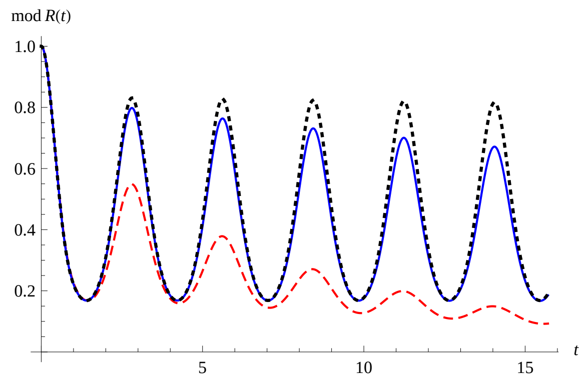

FIG. 4. (Color online) Temporal evolution for the Bloch vector in time for the case of 1/f noise. We consider an amplitude $\sigma_0^2 = \sigma_1^2 = 1\Delta$, and switching rates $\zeta_i = 0.001\Delta$ for the dotted black line, $\zeta_i = 0.01\Delta$ for the blue solid line and $\zeta_i = 0.1\Delta$ for the red dashed curve ($i = 0, 1$).

In Fig.4 we present the temporal evolution of the Bloch vector while the qubit is evolving under the presence of $1/f$ noise. We consider $\zeta_i = 0.001\Delta$ for the dotted black line, $\zeta_i = 0.01\Delta$ for the blue solid line and $\zeta_i = 0.1\Delta$ for the red dashed curve ($i = 0, 1$). We can see that as the value of ζ_i becomes bigger sooner is purity lost.

In Fig.5 we show the behaviour of the coherence $\rho_{r_{01}}(t)$ of the reduced density matrix as a function of time (in

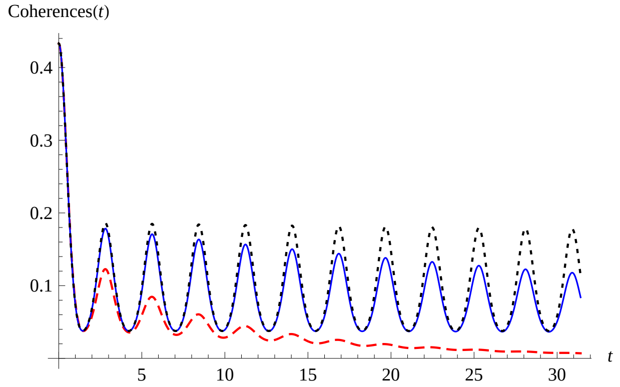

FIG. 5. (Color online) Coherence decay in time for the case of 1/f noise. We consider amplitudes $\sigma_0^2 = \sigma_1^2 = 1\Delta$, and switching rates $\zeta_i = 0.001$ for the dotted black line, $\zeta_i = 0.01\Delta$ for the blue solid line and $\zeta_i = 0.1\Delta$ for the red dashed curve $(i = 0, 1)$.

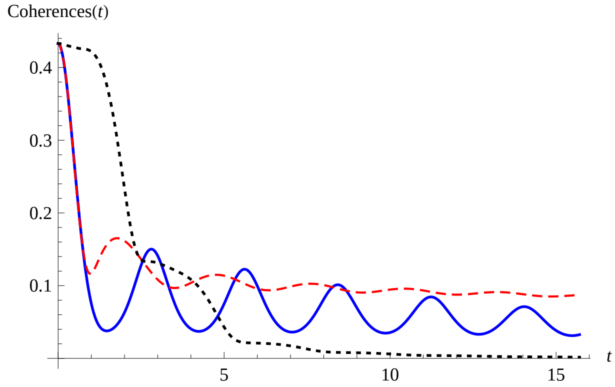

FIG. 6. (Color online) Coherence decay in time for the case of 1/f noise. We consider different couplings. The blue solid line is a bidirectional coupling of the same value ($\sigma_1 = \sigma_0 = 1\Delta$ and $\zeta_0 = 0.5\Delta = \zeta_1$). The red dashed line represents a qubit coupled to the environment in a longitudinal direction only $\zeta_0 = 1\Delta$ while the black dotted line is the qubit coupled to the environment in the transverse direction only $\zeta_1 = 0.1\Delta$. In both cases $\zeta_0 = 0.5\Delta = \zeta_1$.

units of Δ) in the large amplitude limit (i.e. when $\sigma_a^2 = 1\Delta$), for switching rates given by $\zeta_0 = \zeta_1 = 0.001\Delta$ in the dotted black line, $\zeta_0 = \zeta_1 = 0.01\Delta$ in the blue solid line, and $\zeta_i = 0.1\Delta$ for the red dashed curve $(i = 0, 1)$. Similar to the case of an Ohmic environment at high-T, the system losses coherence well before any oscillation when the value of ζ_i is bigger. The final state keeps a small amplitude oscillatory behaviour but not comparable at all to the unitary evolution. The 1/f noise is very efficient in inducing decoherence on the system.

Considering just longitudinal coupling and no anomalous difussion terms, makes it possible to obtain a decoherence time through a rough estimation. Then, assuming that $\zeta_a t \leq 1$, to obtain that $t_D \sim \sqrt{2}/\sigma_a$, independent of the switching rate. With the parameters used in the figures, it is possible to check that decoherence time scales as $\Delta t_D \sim 1.4$ (in units of Δ), which is in agreement with the numerical results. It is worthy to note that this is the temporal scale in which coherences abruptly decay from the pure-case value. This estimation sets a bound on the decoherence time. The asymptotic value is reached in a longer time. On the contrary, when $\zeta t \geq 1$, decoherence time scales as $t_D \sim 1/\zeta_a$, independently of the value of σ_a.

Finally, we consider the effect of noise in both directions. It is important to note that in the case of $1/f$ noise, the coupling constant is included in parameter σ_i of the model. Here, we will study the behavior of the coherence to infer how is decoherence induced in each case. In Fig.6 we present the coherence decay for different coupling situations. The red dashed line represents a qubit coupled to the environment in a longitudinal direction only $\sigma_0 = 0.5\Delta$ while the black dotted line is the qubit coupled to the environment in the transverse direction only $\sigma_1 = 0.1\Delta$. The blue solid line is a bidirectional coupling of the same value. As expected, it is easy to see that coherences decay faster as when the coupling to the system is bigger, i.e a bidirectional coupling of the qubit induces more decoherence on the system. The red dashed line shows that having a coupling in the $\hat{z}$ direction induces more decoherence in the system that having a coupling only in the $\hat{x}$ direction. This is due to the fact that in the latter case, the transverse noise induces transition between the qubit states.

VI. FINAL REMARKS

The interaction of solid-state qubits with environmental degrees of freedom strongly affects the qubit dynamics, and leads to decoherence. In quantum information processing with solid-state qubits, decoherence significantly limits the performances of such devices. These degrees of freedom appear as noise induced in the parameters entering the qubit Hamiltonian and also as noise in the control currents. These noise sources produce decoherence in the qubit, with noise, mainly, at microwave frequencies affecting the relative population between the ground and excited state, and noise or low-frequency fluctuations affecting the phase of the qubit. It is important to study the physical origins of decoherence by means of noise spectral densities and noise statistics. Therefore, it is necessary to fully understand the mechanisms that lead to decoherence.

Superconducting devices show quantum behavior at low temperatures, and the qubit is encoded in the two lowest energy levels of a superconducting circuit. We have derived a master equation for a superconducting qubit coupled to an external source of noise, including the combined effect of noise in the longitudinal and transversal directions. We considered different types of noise by defining their correlation function in time.

For an Ohmic environment, we have considered thermal effects. We have solved the master equation and presented the dynamics of the superconducting qubit in the presence of a high temperature environment and a zero temperature one. In both cases, we have presented the corresponding diffusion and dissipation coefficients and derived some analytical rough estimation of the decoherence time when possible. Decoherence can be understood as fluctuations in the Bloch vector $\mathbf{R}$ induced by noise. Since decoherence rate depends on the state of the qubit, we have represented decoherence by the change of $|\mathbf{R}|$ in time, starting from $|\mathbf{R}| = 1$ for the initial pure state, and decreasing as long as the quantum state losses purity. As expected, an environment at high temperature is an effective coherence destructor and pure state vector is soon removed from the surface of the Bloch sphere. In addition, we have shown that decoherence is still induced in the qubit when the environment is at zero temperature. This process is less drastic and takes longer times compered to the high temperature limit. However, it is important to remark that the decoherence process exists induced by the vacuum fluctuactions of the environment and can be seen for example, in the loss of purity of the state vector. This result is in contrast to some recent publications [11], where the effect of the environment at zero temperature is neglected. We have also focused on the effect of longitudinal and transversal noise. As expected, when the qubit is coupled to both directions, namely longitudinal and transverse, the influence of the environment is bigger as observed in the destruction of the coherences. However, it is important to note that having a transverse coupling only, does not imply a decoherence process as important as the one induced by the system when the coupling is longitudinal. This result is novel and should help in future qubits designs or experimental setups.

For a noise $1/f$, modeled herein by an ensemble of two-level fluctuators, we have also presented a master equation approach. From the definition of the noise correlation function of the environment, we have computed the diffusion coefficients and solve numerically the dynamics of the qubit. We have studied how this type of noise affects the coherences of the reduced density matrix and how the state vector is removed from the surface of the Bloch sphere. We have seen that this noise can be very destructive, depending on the value of the free parameter ζ. We have provided some rough analytical estimations of the decoherence timescale that agree with the numerical solutions presented here. As for the effect of longitudinal and transversal noise, when the coupling is bidirectional the effect of noise is bigger on the coherences of the qubit. However, the behavior of the couplings separately is observed to be opposite to the one observed in the Ohmic environments. This may be adduced to the fact that the real coupling constant between the qubit and environment does not have a crucial role in the model.

The analysis of the decoherence timescales may provide additional information about the statistical properties of the noise. The comprehension of the decoherence and dissipative processes should allow their further suppression in future qubits designs or experimental setups.

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