

An energy-conserving, particle dominated, time-dependent model of 3C58 and its observability at high-energies

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ABSTRACT

We present a time-dependent spectral model of the nebula 3C 58 and compare it with available data. The model is for a leptonic nebula, in which particles are subject to synchrotron, inverse Compton, self-synchrotron Compton, adiabatic, and bremsstrahlung processes. We find that 3C 58 is compatible with being a particle dominated nebula, with a magnetic field of $35\mu\text{G}$. A broken power law injection fits well the multi-frequency data, with a break energy at about 40 GeV. We find that 3C 58 is not expected to appear in VERITAS or MAGIC II, unless the local IR background is a factor of ~ 20 off Galactic models averages. For cases in which the CMB dominates the inverse Compton contribution, we find that 3C 58 will not be visible either for the Cherenkov Telescope Array.

Subject headings: ISM: individual (3C 58)

1. Introduction

3C 58 was suggested to be plausibly associated with the 831 years-old supernova SN 1181 (see e.g., Stephenson 1971, and Stephenson & Green 2002). However, recent investigations of dynamical models for the pulsar wind nebula (PWN, Chevalier 2005) and the velocities of both, the expansion rate of the radio nebula (Bietenholz 2006) and the optical knots (Fesen et al. 2008) imply an age of several thousand years. This is closer to the characteristic age of the pulsar in the nebula, PSR J0205+6449 (Murray et al. 2002). A recent rekindling of the low age has been put forward by Kothes (2010), based on a new estimation of the nebula distance.

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3C 58 has a flat-radio spectrum with a spectral break between the radio and IR bands (Green & Scheuer 1992). X-ray observations reveal a non-thermal spectrum that varies with radius, becoming steeper towards the outer regions (Slane et al. 2004). PSR J0205+6449 is one of the most energetic pulsars known in the Galaxy. The pulsar powers a faint jet and is surrounded by a toroidal structure apparently associated with flows downstream of the pulsar wind termination shock (Slane et al. 2004). The shell of the thermal X-ray emission that was seen in 3C 58, e.g., by Gotthelf et al. (2007), is smaller than the maximum extent of the PWN. Therefore, this emission is likely associated with supernova ejecta swept up by the expanding PWN rather than the original forward shock from the supernova. The pulsar has been recently detected at high-energy gamma-rays by *Fermi*, but only upper limits were imposed for the nebula emission (Abdo et al. 2009). Similarly, *Whipple* (Hall et al. 2001), and both MAGIC (Anderhub et al. 2010) and VERITAS (Konopelko et al. 2007) observed the nebula, but only upper limits were imposed at TeV energies.

3C 58 and the Crab Nebula differ significantly both in luminosity and size. 3C 58 is larger, but less luminous, e.g., its TeV luminosity is at least ~ 100 (Anderhub et al. 2010), its X-ray luminosity is ~ 2000 (Torii et al. 2000), and its radio luminosity is ~ 10 times smaller than Crab. The similarity is in fact coming from morphology (e.g., Slane et al. 2004).

PWN models for 3C 58 have been presented before by a few authors, e.g., Bednarek & Bartosik (2003, 2005), Bucciantini et al. (2011), with some disparity in the results, particularly at the high-energy end of the spectrum. These studies use different assumptions for the primary particles assumed to populate the wind, and differ also in the treatment of the radiative physics. With 3C 58 being a candidate for observations in the current or forthcoming generation of Cherenkov telescopes, it is interesting to study under what conditions 3C 58 is observable at high energies. Here we present an analysis of the nebula evolution using a detailed radiative, time-dependent, leptonic code.

2. The PWN model and results

The PWN model (see Martín, Torres & Rea 2012) uses the diffusion-loss equation $\partial N(\gamma, t)/\partial t = -\partial/\partial\gamma [\dot{\gamma}(\gamma, t)N(\gamma, t)] - N(\gamma, t)/\tau(\gamma, t) + Q(\gamma, t)$. $\dot{\gamma}(\gamma, t)$ contains the energy losses due to synchrotron, (Klein-Nishina) inverse Compton, bremsstrahlung, and adiabatic expansion. $Q(\gamma, t)$ represents the injection of particles per unit energy and unit volume in a certain time, and $\tau(\gamma, t)$ is the escape time (assuming Bohm diffusion). The model computes the luminosity produced by synchrotron, inverse Compton (IC, with the cosmic-microwave background as well as with IR/optical photon fields), self-synchrotron Compton (SSC), and bremsstrahlung processes, all devoid of any radiative approximations.

We assume that the spin-down of the pulsar powers the nebula: $L(t) = 4\pi^2 I \dot{P} / P^3 = L_0 (1 + t/\tau_0)^{-(n+1)/(n-1)}$. P and \dot{P} are the period and its first derivative and I is the pulsar moment of inertia. The spin-down power can also be written (see second equality above) in terms of the the initial luminosity L_0 , the initial spin-down timescale τ_0 , and the braking index n . τ_0 is given by (e.g., Gaensler & Slane 2006), $\tau_0 = P_0 / [(n-1)\dot{P}_0] = 2\tau_c / [n-1] - t_{age}$, where P_0 and \dot{P}_0 are the initial period and its first derivative and τ_c is the characteristic age of the pulsar, $\tau_c = P / 2\dot{P}$. The braking index is unknown for the great majority of pulsars, and assumed to be ~ 3 (corresponding to a dipole spin-down rotator).

We adopt a broken power-law for the injection of particles,

$$Q(\gamma, t) = Q_0(t) \begin{cases} \left(\frac{\gamma}{\gamma_b}\right)^{-\alpha_1} & \text{for } \gamma \leq \gamma_b, \\ \left(\frac{\gamma}{\gamma_b}\right)^{-\alpha_2} & \text{for } \gamma > \gamma_b, \end{cases} \quad (1)$$

where γ_b is the break energy, the parameters α_1 and α_2 are the spectral indices. The maximum Lorentz factor of the particles is limited by confinement, with the Larmor radius being smaller than the termination shock, $\gamma_{max}(t) = (\varepsilon e \kappa / m_e c^2) \sqrt{\eta L(t)} / c$, where e is the electron charge and ε is the fractional size of the radius of the shock. The Larmor Radius is $R_L = (\gamma_{max} m_e c^2) / (e B_s)$, where B_s is the post-shock field strength, defined as $B_s \sim (\kappa (\eta L(t) / c)^{0.5}) / R_s$, with R_s the termination radius. We have fixed κ , the magnetic compression ratio, to 3 (Venter & de Jager 2006, Holler et al. 2012). The normalization of the injection function is obtained from $(1 - \eta)L(t) = \int_0^\infty \gamma m c^2 Q(\gamma, t) d\gamma$, where $\eta = L_B(t) / L(t)$ is the magnetic energy fraction, assumed constant along the evolution, with $L_B(t)$ being the magnetic power, and B is the average field in the nebula.

We have adopted a $B(t)$ resulting from

$$\int_0^t \eta L(t') R_{PWN}(t') dt' = (4\pi/3) R_{PWN}^4(t) B^2(t) / (8\pi). \quad (2)$$

This equation is equivalent to $(dW_B/dt) = \eta L - W_B(dR_{PWN}/dt) / R_{PWN}$ where $W_B = (4\pi/3) R_{PWN}^3(t) B^2(t) / (8\pi)$, which includes the adiabatic losses due to nebular expansion (e.g., Pacini & Salvati 1973).

We have adopted the free expanding expansion phase in the model by van der Swaluw et al. (2001, 2003), where the radius of the PWN is $R_{PWN}(t) \sim (L_0 t / E_0)^{1/5} V_{ej} t$, with $V_{ej} = \sqrt{10 E_0 / 3 M_{ej}}$ and where E_0 and M_{ej} are the energy of the supernova explosion and the ejected mass, respectively. V_{ej} is determined requiring that the kinetic energy of the ejecta equals E_0 . These assumptions encompass self-similarity of the ejecta flow, a linear velocity profile as well as a uniform density ejecta, and it is the same to that taken by Bucciantini et al. (2011) with their density distribution parameter $\alpha = 0$. Adding more

Table 1: Physical magnitudes

From observations, computed, or assumed	
Magnitude	Value
Period (ms), $P(t_{age})$	65.7
Period derivative (s s^{-1}), $\dot{P}(t_{age})$	1.93×10^{-13}
Characteristic Age (yr), τ_c	5397
Moment of inertia (g cm^2), I	10^{45}
Spin-down luminosity now (erg/s), $L(t_{age})$	2.7×10^{37}
Braking index, n	3
SN explosion energy (erg), E_0	10^{51}
Ejected mass (M_\odot), M_{ej}	8
Hydrogen density (cm^{-3}), n_H	0.1
Adopted age and distance, derived quantities	
Age (yr), t_{age}	2500
Distance (kpc), d	3.2
Initial spin-down age (yr), τ_0	2897
Initial spin-down luminosity (erg/s), L_0	9.32×10^{37}

freedom to the dynamics by changing α does not have a significant impact in the results. We adopt a mass of the ejecta comparable to that of Crab, motivated by estimates of the total mass of the precursor (Rudie & Fesen 2007).

Not all estimates for the age and distance of 3C 58 (see Fesen et al. 2008 for a summary) can be consistently encompassed within the expansion model of the nebula. Consider first an age of ~ 5000 years or more (as in Murray et al. 2002, Bietenholz 2006, Slane et al. 2002) and a distance of 3.2 kpc (as in Roberts et al. 1993). Following Fesen et al. (2008) for the angular size of the nebula, its physical size at that distance is 6×9.5 pc. We can extrapolate these magnitudes to the spherical case by matching the projected area of the nebula to that of a circle, and so we obtain a radius of 3.7 pc. Using P and \dot{P} , the parameters of the first panel of Table 1, and the formulae above, we would however obtain a physical size of about 18 pc; a result that worsens for larger ages. On the contrary, if we assume the observed size and compute the ejected mass needed to have a radius of ~ 3.7 pc, we would find an inconsistently large value. Similarly problematic results the scenario when we change the initial spin-down power, since it would be impossible to reach the current $L(t)$, being the initial one smaller than the current power. Such innuendos are not solved by assuming a different value of braking index, and are also stable (producing sizes in excess of 10 pc) for up to one order of magnitude variations in I . If 3C 58 is closer to Earth, the mismatch would

be larger, given that the physical size of the nebula would be smaller than at 3.2 kpc.

Consider next a 830 years-old nebula (as in Stephenson 1971, and Stephenson & Green 2002) and a distance of 2 kpc (as in Kothes 2010). Geometry implies that the physical size of the nebula should be around 2.3 pc, but using the observed, derived, and assumed parameters of Table 1 we obtain a size of 0.8 pc, a factor of 3 smaller. This result, similarly to the larger age case above, is stable against changes in n , I , or other parameters. The only way to recover a larger nebula would be to assume a mass of the ejecta of the order of $1 M_{\odot}$, but this would be inconsistent with estimates based on the observed filamentary knots (which account already for a large fraction of $1 M_{\odot}$) or with evolutionary models (e.g., Rudie & Fesen 2007, Fesen et al. 2008, Bocchino et al. 2001, Slane et al. 2004). A larger distance to 3C 58 would imply a larger physical size of the nebula, making the mismatch more severe.

We are a priori favorable to the case of an age of 2500 years and a distance of 3.2 kpc. For this set of parameters, the size of the nebula can be easily accommodated within the model described in the previous section. Variations of the parameters in a reasonable manner maintain this conclusion stable. In addition, the shock velocity agrees with estimates coming from the thermal X-ray emission (e.g., Bocchino et al. 2001) and at the same time, the swept-up mass resulting from these model parameters $M_{sw} = M_{ej}(R_{PWN}/V_{ej}t)^3 \sim 0.26M_{\odot}$ is in line with the measurements of the mass contained in filaments (Bocchino et al. 2001, Slane et al. 2004); i.e., we assume that the filamentary structure roughly corresponds to the swept-up shell of the ejecta. Similar conclusions have been reached by Chevalier (2004, 2005) or Bucciantini et al. (2011) with others arguments.

Multi-frequency results of our model for a 3C 58 age of 2500 years located at 3.2 kpc are shown in Fig. 1.¹ We also show for comparison the results corresponding to the Crab Nebula at different ages, i.e., 940 (where the data points are fit with a corresponding model, see Martín et al. 2012), 2000, and 5000 years. The differences in the data of Crab and that of 3C 58 are evident. Fig. 1 shows results for three different assumptions regarding the dominance of the IC contribution. In the first panel, only the CMB is assumed as background for IC. The resulting parameters are given in Table 2, and they show that a broken power-law fits the current radio to X-ray data. The magnetic field of the nebula results in $35 \mu\text{G}$. The contribution of SSC to IC is sub-dominant to bremsstrahlung under the assumption of a low medium density of 0.1 cm^{-3} . As there is no clearly detected SNR shell, we cannot

¹The data for 3C 58 in that Figure comes from Green (1986), Morsi and Reich (1987), Salter et al. (1989) [in radio], Green (1994), Slane et al. (2008) [in IR], Torii et al. (2000) [in X-rays], Abdo et al. (2009) [in GeV], and Hall et al. (2001), Konopelko (2007) and Anderhub et al. (2010) [in TeV]. Data for Crab have been collected by Martín et al. (2012).

reliably estimate the ISM density. It is impressive how low the prediction in the GeV and TeV regimes is, far beyond the reach of *Fermi*-LAT and Cherenkov telescopes.

Taking into account that the IR/FIR background is uncertain, we consider two limiting situations by exploring how large the energy density at 20 K should be for its corresponding IC contribution to reach the sensitivity of the Cherenkov Telescope Array (Actis et al. 2011) and of the currently operating MAGIC II (Aleksic et al. 2012). The additional parameters shown in the last two panels of Table 2 are used in order to achieve the fits shown in the middle and bottom plots of Fig. 1. They represent energy densities ranging from 3 to 20 times that of the CMB. These can be compared with the typical background at that frequency from e.g., GALPROP models (see Fig. 1 of Porter et al. 2006), to see that the case where 3C 58 appears as an in-principle-observable nebula in MAGIC II or VERITAS would be highly unexpected. In both of these cases, still, the GeV emission would be far below the *Fermi*-LAT upper limit.

3. Discussion

Fig. 2 focuses on the CMB-only model. The first panel shows the timescales for the losses at the current adopted age, showing a clear domination of synchrotron losses at high, and adiabatic losses at low energies. This is a generic feature of all models. Losses results comparable to those in the Crab Nebula today. The second panel shows the evolution of the electron spectra in time, whereas the third panel does it with the SED. The electron distribution is compared with that of Crab today, and it can be seen both a change in the peak as well as in normalization (differences amounting between 1 and 2 orders of magnitude across all the energies) between the two nebulae.

We have explored models having different braking indices (keeping age and distance fixed at 2500 years and 3.2 kpc, respectively, as the testbed). For these models, the overall quality of the fit is unchanged; some of the fit parameters are slightly modified, however. Lower values of n implies changes in the initial spin-down power (from 9.3, to 7.3, to 5.9×10^{37} erg s⁻¹ for $n = 3, 2.5,$ and $2,$ respectively), initial spin-down age (from 2897, to 4696 to 8294 years) and the PWN radius today (from 3.7 to 3.5 to 3.4 pc). The magnetic fraction changes from 0.23, for $n = 2$ to 0.22 for $n = 2.5,$ to 0.21 for $n = 3.$ The magnetic field today has a value between 35 and 37 μ G in all these cases. Letting the ϵ parameter vary allows to explore the ability of the fit to adapt to the X-ray measured spectra better. But again we find a very small parameter dependence. For instance, for $\epsilon = 0.5, 0.3,$ and $0.1,$ we find the maximum Lorentz factor changing from 10.2, to 7.3, to $2.5 \times 10^9,$ with the high energy index, magnetic field, and magnetic fraction being practically unchanged to attain

fits of similar quality of those in Fig. 1. None of these models increment significantly the high-energy yield of the nebula.

The total energetics is conserved in our model in exact terms, since particles have a fraction $(1-\eta)$, and the magnetic field a fraction η , of the total power. 3C 58 features a 21% magnetic fraction in our model, significantly higher than the one we obtain for Crab ($\sim 3\%$). It is still a particle-dominated nebula. These results differ from those in Bucciantini et al.’s (2011) work, where the total energy was not conserved by $\sim 30\%$, leading to a nebula in equipartition, and where it was said that models with energy conservation would always under predict the radio flux.

Note the two contiguous IR measurements around 10^{14} GHz. Because of their location, it is nearly impossible to fit them both at once. Indeed, there appears to be two subsequent steepening of the spectrum, one just beyond the radio band and one additional in the IR band (see Slane et al. 2008). We have explored an injection containing another break, but results do not improve the fit significantly. We also tried to improve the fit using an injection model based on the PIC simulations done by Spitkovsky (2008) keeping the ratios of the additional parameters as in Holler et al. (2012). These are not devoid of significant extrapolations and include a number of additional parameters for which we have no constraints. We used a value for the energy break of $\gamma_b = 2 \times 10^4$ to have an acceptable fit of the radio points, but it is not possible to fit the IR and X-ray points correctly, even changing the ratios of the parameters. In all these cases for the injection, the fits are of similar (2 breaks power-law) or lower (PIC motivated) quality than the ones presented above; and in none, the high-energy yield is significantly affected.

Observations at the smallest angular resolution make these studies preliminary to a yet-lacking 3D MHD/radiative/time-evolving code which could at once deal with morphology and spectral evolution. Another caveat may reside in that 3C 58 nebula is assumed to be in the free expansion phase: Green (1987) reported an increase in 3C 58 flux density at radio frequencies between 1967 and 1986, which might be the result of the reverse shock encountering the PWN shock around 3C58, with the PWN being compressed and therefore the flux density raising. This report has not, however, been subsequently confirmed (Bietenholz 2006).

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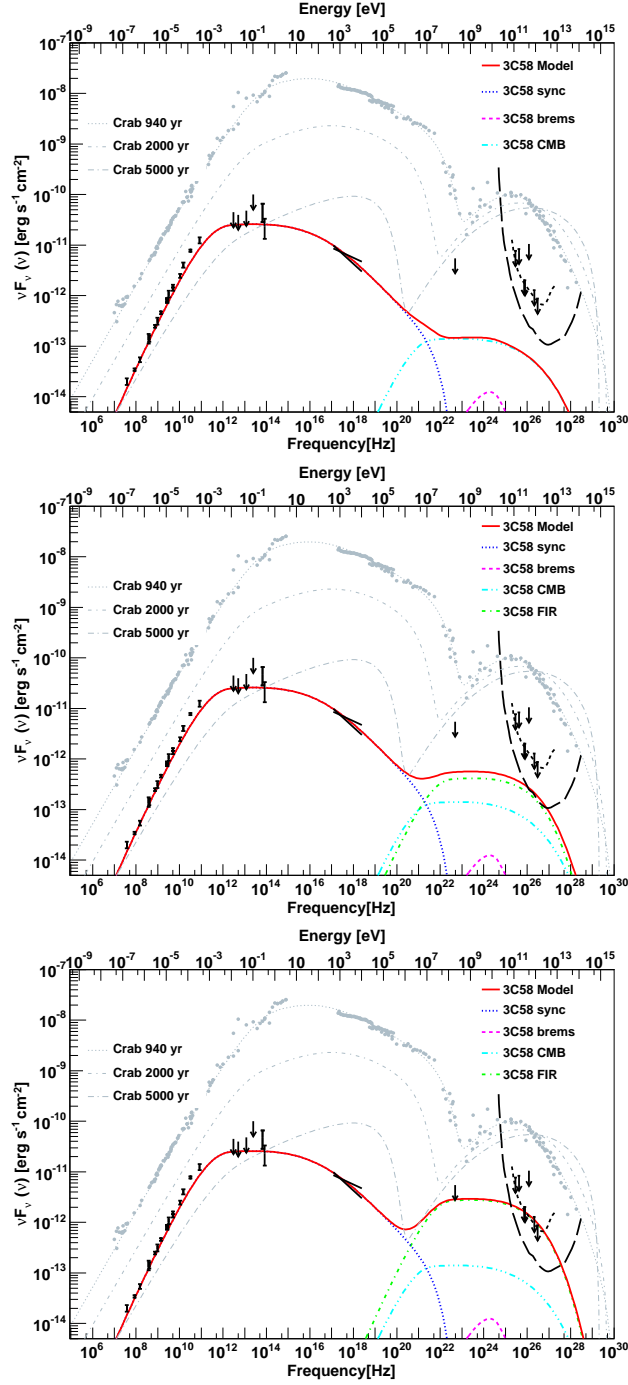


Fig. 1.— Multi-frequency models of the PWN 3C 58 under different assumptions for the background photon fields. Top: CMB-only. Middle: IR energy density up to the level where the emission of 3C 58 reaches the CTA sensitivity. Bottom: *ibid*, for the MAGIC II sensitivity. Fitting parameters are giving in Table 2. The SSC contribution is not visible in this scale.

Table 2: Fit values

Without IR contribution to IC	
Magnitude	Value
Max. Lorentz factor today, $\gamma_{max}(t_{age})$	7.3×10^9
Break Lorentz factor, γ_b	0.78×10^5
Low energy index, α_1	1.05
High energy index, α_2	2.91
Shock radius fraction, ϵ	0.3
Magnetic field today (μG), $B(t_{age})$	35
Magnetic fraction, η	0.21
PWN radius today (pc), $R_{PWN}(t_{age})$	3.7
IR contribution up to CTA sensitivity	
IR temperature (K), T_{FIR}	20
IR energy density (eV/cm^3), w_{FIR}	0.75
IR contribution up to MAGIC-II sensitivity	
IR temperature (K), T_{FIR}	20
IR energy density (eV/cm^3), w_{FIR}	5.0

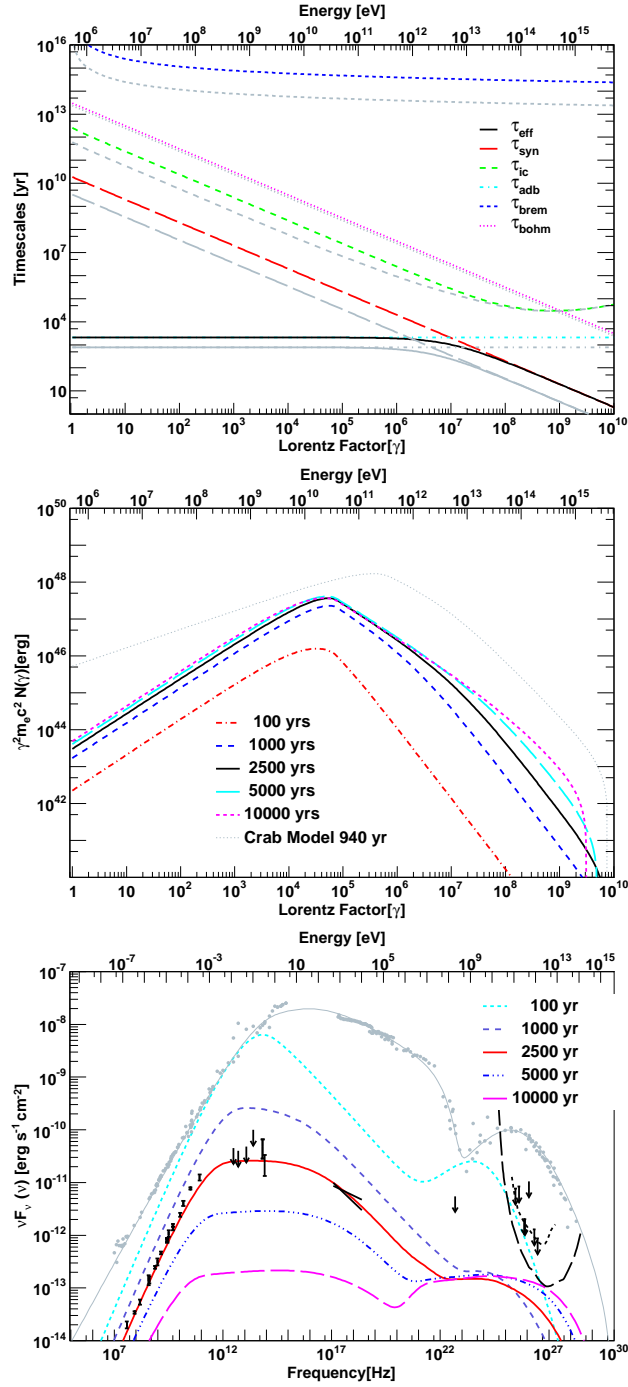


Fig. 2.— CMB dominated model for the 3C 58 multi-frequency emission. The first panel shows the timescales for the losses under the different processes at the current age. The second panel shows the evolution of the electron spectra in time, whereas the third panel does it with the SED. Corresponding results for the Crab Nebula at its current age are shown in grey.