On the Iterated Biclique Operator

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Abstract: A biclique of a graph G is a maximal induced complete bipartite subgraph of G. The biclique graph of G, denoted by $KB(G)$, is the intersection graph of the bicliques of G. We say that a graph G diverges (or converges or is periodic) under an operator F whenever $\lim_{k\to\infty} |V(F^k(G))| = \infty$ ($\lim_{k\to\infty} F^k(G) = F^m(G)$ for some m, or $F^k(G) = F^{k+s}(G)$ for some k and $s \geq 2$, respectively). Given a graph G, the iterated biclique graph of G, denoted by $KB^k(G)$, is the graph obtained by applying the biclique operator k successive times to G. In this article, we study the iterated biclique graph of G. In particular, we classify the different behaviors of $KB^k(G)$ when the number of iterations k grows to infinity. That is, we prove that a graph either diverges or converges under the biclique operator. We give a forbidden structure characterization of convergent graphs, which yield a polynomial time algorithm to decide if a given graph diverges or converges. This is in sharp contrast with the situsation for the better known clique operator, where it is not even known if the corresponding problem is decidable.
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1. INTRODUCTION

Intersection graphs of certain special subgraphs of a general graph have been studied extensively. Let us mention, for example, the case of line graphs (which are the intersection graphs of the edges of a graph), interval graphs (defined as the intersection graphs of intervals of the real line), and, in particular, clique graphs (defined below) [3, 4, 8, 11, 12, 22, 23].

The *clique graph* of G, denoted by $K(G)$, is the intersection graph of the family of all maximal cliques of G.

Clique graphs were introduced by Hamelink in [15] and characterized by Roberts and Spencer in [26]. It was proved in [1] that the clique graph recognition problem is NP-complete.

As the clique graph construct can be thought of as an operator between graphs, the *iterated clique graph* $K^k(G)$ is the graph obtained by applying the clique operator k successive times. It was introduced by Hedetniemi and Slater in [16]. Much work has been done on the scope of the clique operator, looking at the different possible behaviors. The associated problem is deciding whether an input graph converges, diverges, or is periodic under the clique operator, when k grows to infinity. In general, it is not clear that the problem is decidable. However, partial characterizations have been given for convergent, divergent, and periodic graphs, restricted to some classes of graphs. Some of these lead to polynomial time recognition algorithms. For the clique-Helly graph class, graphs which converge to the trivial graph have been characterized in [2]. Cographs, P_4-tidy graphs, and circular-arc graphs are examples of classes where the different behaviors are characterized [5, 17]. Divergent graphs were also considered. For example, in [24], families of divergent graphs are shown. Periodic graphs were studied in [8, 21]. In particular, it is proved that for every integer i, there exist graphs with period i and convergent graphs which converge in i steps. More results about iterated clique graph can be found in [9, 10, 18, 19, 20, 25].

The *biclique graph* of a graph G, denoted by $KB(G)$, is the intersection graph of the family of all maximal bicliques of G. It was defined and characterized in [13]. However, no polynomial time algorithm is known for recognizing biclique graphs. As for clique graphs, the biclique graph construct can be viewed as an operator KB between graphs.

In this article, we introduce the *iterated biclique graph* $KB^k(G)$, i.e., the graph obtained by applying iteratively the biclique operator KB k times to G. We are interested in the possible behavior of a graph under the biclique operator. Indeed, we prove that a graph G is either divergent or convergent, but it is never periodic (with period bigger than 1). In addition, we give general characterizations for convergent and divergent graphs. These results are based on the fact that if a graph G contains a clique of size at least 5, then $KB(G)$ contains a clique of larger size. Therefore, G diverges. Similarly, if G contains the so-called *gem* or *rocket* as an induced subgraph, then $KB(G)$ contains a clique of size 5, and again, G diverges. Otherwise, it is shown that, after removing false twin vertices of $KB(G)$, the resulting graph is a clique on at most four vertices, in which case, G converges. Moreover, we prove that if a graph G converges, it converges to the graphs K_1

or K_3, and it does so in at most three steps. These results are very different from the ones known for the clique operator. Our characterizations lead to an $O(n^4)$ time algorithm for deciding if a given graph converges or diverges under the biclique operator.

This work is organized as follows. In Section 2, we give the required definitions. In Section 3, we present known and new results about biclique graphs. Also, we give characterizations for convergent and divergent graphs under the biclique operator. Finally, we give a polynomial time algorithm for deciding the behavior of a general graph under the biclique operator. The study of the convergence and divergence of the trees is presented in Section 4. Section 5 contains some concluding remarks.

2. PRELIMINARIES

We will assume that all graphs discussed are connected (clearly, this is no lose of generality for the problems at hand). Let G be a graph with vertex set $V(G)$ and edge set $E(G)$, and let $n = |V(G)|$ and $m = |E(G)|$. A *clique* of G is a maximal complete induced subgraph, while a *biclique* is a maximal complete bipartite induced subgraph of G with no empty bipartition. The *open neighborhood* of a vertex $v \in V(G)$, denoted $N(v)$, is the set of vertices adjacent to v. A vertex $v \in V(G)$ is *universal* if it is adjacent to all of the other vertices in $V(G)$. A *path* with k vertices is denoted by P_k and a cycle with k vertices is denoted by C_k.

A *diamond* is a complete graph with four vertices minus an edge. A *gem* is an induced path with four vertices plus an universal vertex. A *rocket* is a complete graph with four vertices and a vertex adjacent to two of them. The graph O_3 is the complement of the union of three copies of K_2.

Given a family of sets $\mathcal{A}$, the *intersection graph* of $\mathcal{A}$ has as vertices the set of $\mathcal{A}$ and the edges correspond to the pairs of sets from $\mathcal{A}$ with a nonempty intersection. We remark that any graph is an intersection graph [27].

Let F be any graph operator. Given a graph G, the iterated graph under the operator F^k is defined iteratively as follows: $F^0(G) = G$ and for $k \geq 1$, $F^k(G) = F^{k-1}(F(G))$. We say that a graph G diverges under the operator F whenever $\lim_{k\to\infty} |V(F^k(G))| = \infty$. We say that a graph G converges under the operator F whenever $\lim_{k\to\infty} F^k(G) = F^m(G)$ for some m. We say that a graph G is periodic under the operator F whenever $F^k(G) = F^{k+s}(G)$ for some $k, s, s \geq 2$. The associated problem is to decide whether G converges, diverges, or is periodic under F.

In the article, we will use the terms convergent or divergent meaning convergent or divergent under the biclique operator KB.

By convention, we arbitrarily say that the trivial graph K_1 is convergent under the biclique operator (observe that this remark is needed, since the graph K_1 does not contain bicliques).

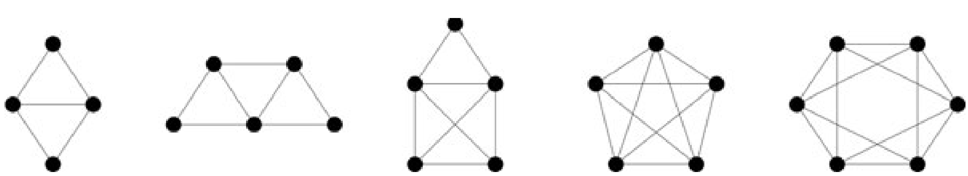

FIGURE 1. The graphs *diamond*, *gem*, *rocket*, K_5, and O_3.

3. CONVERGENCE AND DIVERGENCE OF THE ITERATED BICLIQUE GRAPH

In this section, we classify the behavior of a graph under the operator KB. We remark that if G is connected, then $KB(G)$ is connected too. Our main result is the following theorem.

Theorem 3.1. *If $KB(G)$ contains either K_5 or the gem or the rocket as an induced subgraph, then G is divergent. Otherwise, G converges to K_1 or K_3 in at most three steps.*

For the proof of Theorem 3.1, we shall use the following results.

Observation 3.2. *If G is an induced subgraph of H, then $KB(G)$ is a subgraph (not necessarily induced) of $KB(H)$.*

Proof. Let $b_1, b_2, ..., b_k$ be the bicliques of G. Each biclique b_i, either is a biclique of H, or it is contained in a biclique B_i of H. Remark that if B_i contains b_i, it cannot contain b_j, $j \neq i$. Then, if $b_i \cap b_j \neq \emptyset$ in G, $B_i \cap B_j \neq \emptyset$ in H $\forall i, j = 1, ..., k$. Then $KB(G)$ is a subgraph of $KB(H)$. ■

Next, we examine the bicliques of $KB(K_n)$, for $n \geq 4$. (Note that since the bicliques of K_n are precisely its single edges, $KB(K_n)$ is just the line graph of K_n.)

Lemma 3.3. *Let $n \geq 4$. All bicliques of $KB(K_n)$ are isomorphic to C_4 and each vertex of $KB(G)$ belongs to exactly $(n-2)(n-3)$ different bicliques.*

Proof. It is clear that K_n contains $\frac{n(n-1)}{2}$ bicliques. Consider an edge $e \in E(K_n)$, $e = vw$. The edge e is not adjacent in K_n to any edge that belongs to a complete subgraph of $n-2$ vertices. Let $e' = v'w'$ be an edge of G not adjacent to e. Consider edges $e_1 = vv'$, $e_2 = vw'$, $e_3 = v'w$, and $e_4 = ww'$ as vertices in $KB(G)$. Then, $B_1 = \{e, e'\} \cup \{e_1, e_4\}$ and $B_2 = \{e, e'\} \cup \{e_2, e_3\}$ induce complete bipartite subgraphs in $KB(K_n)$ isomorphic to C_4. Since three not adjacent edges cannot be adjacent to a common edge, B_1 and B_2 are indeed bicliques in $KB(K_n)$. Next, we show that each vertex of $KB(K_n)$ belongs to $(n-2)(n-3)$ bicliques. As before, each edge e of K_n is not adjacent to $\frac{(n-2)(n-3)}{2}$ other edges of K_n, therefore each one of these along with e induce two different bicliques in $KB(K_n)$. Then, each vertex of $KB(K_n)$ belongs to $(n-2)(n-3)$ different bicliques of $KB(K_n)$. ■

As a consequence, since $KB(G)$ has $\frac{n(n-1)}{2}$ vertices and each vertex belongs to $(n-2)(n-3)$ bicliques isomorphic to C_4, we conclude that $KB(G)$ has $\frac{n(n-1)(n-2)(n-3)}{8}$ bicliques.

Proposition 3.4. *Let G be a graph that contains K_n as a subgraph, for some $n \geq 4$. Then, $K_{2n-4} \subseteq KB(G)$ or $K_{(n-2)(n-3)} \subseteq KB^2(G)$.*

Proof. If $G = K_n$, then by Lemma 3.3, each vertex of $KB(G)$ belongs to $(n-2)(n-3)$ different bicliques and then $K_{(n-2)(n-3)} \subseteq KB^2(G)$. Otherwise, $G \neq K_n$ and let v be a vertex in $G - K_n$ adjacent to some k_v vertices of K_n. Consider first the case $1 < k_v < n-1$. Let $A = \{v_1, v_2, ..., v_{k_v}\}$ be the vertices of K_n adjacent to v and let $B = \{v_{k_v+1}, v_{k_v+2}, ..., v_n\}$ be the vertices of K_n not adjacent to v. Therefore, there are $k_v(n-k_v)$ edges with endpoints in A and B. Let $e_{ij} = v_iv_j$, $1 \leq i \leq k_v$, $k_v + 1 \leq j \leq n$, be those edges. Since v is adjacent to each vertex of A, then for each i and j, v and e_{ij} are contained in a biclique B_{ij} of G. Clearly, $B_{ij} \neq B_{i'j'}$ if $i \neq i'$ or $j \neq j'$. Finally, there are $k_v(n-k_v)$ different bicliques in G that contain v. It follows that $K_{k_v(n-k_v)} \subseteq KB(G)$. Now, let $F : [2, n-2] \subseteq \mathbb{N} \to \mathbb{N}$ be a function defined by $F(k_v) = k_v(n-k_v)$. It is easy to see that F reaches its minima in the extremes of the interval, i.e., for $k_v = 2$ and $k_v = n-2$. Then, $K_{2(n-2)} = K_{2n-4} \subseteq K_{k_v(n-k_v)} \subseteq KB(G)$.

Before considering the remaining cases, we make the following observation:

Observation 3.5. *Let G be a graph that contains K_n as a subgraph for some $n \geq 4$, and let e and e' be two edges of K_n that are not adjacent. Let B and B' be two different bicliques of G such that e belongs to B and e' belongs to B'. If $B \cap B' \neq \emptyset$, then there exists a vertex $v \in B \cap B'$ such that v is adjacent to exactly one endpoint of e and one endpoint of e' see Figure 2.*

Now, suppose for every $v \in G - K_n$, either $k_v = 1$ or $k_v = n-1$ or $k_v = n$. We prove that in any case, $K_{(n-2)(n-3)} \subseteq KB^2(G)$. Consider $B_1, B_2, ..., B_s$ the bicliques of G such that each B_i contains an edge of the K_n and no two of them contain the same edge of K_n. Clearly, $s = \frac{n(n-1)}{2}$. Let $H \subseteq KB(G)$ be the subgraph induced by the vertices of $KB(G)$ corresponding to the bicliques $B_1, B_2, ..., B_s$. We show that H is isomorphic to $KB(K_n)$ and then by Observation 3.2 and Lemma 3.3, $K_{(n-2)(n-3)} \subseteq KB(H) \subseteq KB^2(G)$. Suppose that it is not true, then there exist two adjacent vertices of H that are not adjacent in $KB(K_n)$. By Observation , if B_i and B_j are different bicliques in G that contain the

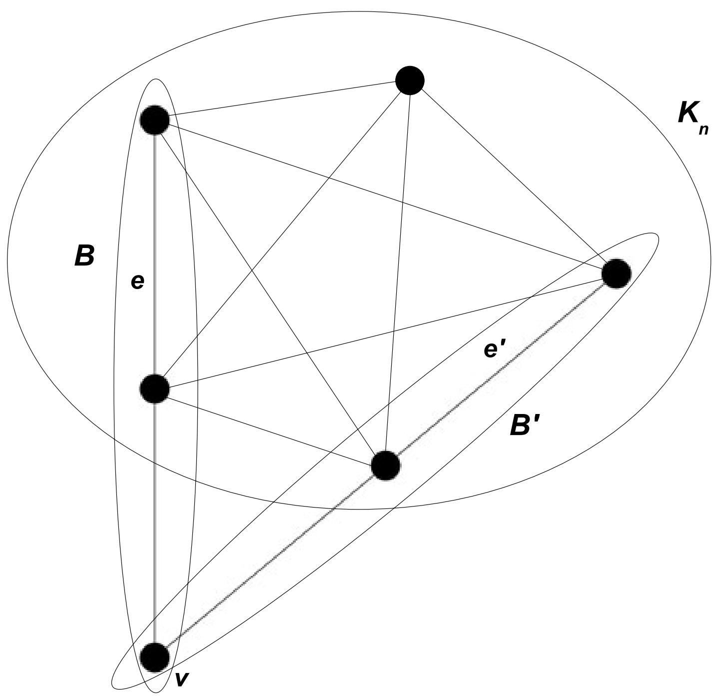

FIGURE 2. The P_3 in bold edges is contained in an induced *diamond* and in an induced *gem*, respectively.

nonadjacent edges e_i and e_j of K_n, respectively, and $B_i \cap B_j \neq \emptyset$, then there exists a vertex v in $B_i \cap B_j$, $v \notin K_n$, v adjacent to exactly one endpoint of each edge and therefore not adjacent to the others. This contradicts cases $k_v = 1$, $k_v = n-1$, and $k_v = n$. Then H is isomorphic to $KB(K_n)$ and therefore $K_{(n-2)(n-3)} \subseteq KB(H) \subseteq KB^2(G)$.

Two vertices u, v are *false twins* if $N(u) = N(v)$. Consider all maximal sets of false twin vertices $Z_1, ...Z_k$ and let $\{z_1, z_2, ..., z_k\}$ be the set of *respresentative vertices* such that $z_i \in Z_i$. We call *Tw(G)* to the graph obtained by the deletion of all vertices of $Z_i \setminus \{z_i\}$, for $i = 1...k$. Observe that $Tw(G)$ has no false twin vertices.

Proposition 3.6. *For any graph G, we have $KB(G) = KB(Tw(G))$.*

Proof. Let v, w be false twin vertices of G. It will suffice to prove that $KB(G) = KB(G - \{v\})$. It is clear that every biclique of G either contains both v and w, or does not contain either of them. Let $B_1, B_2, ..., B_k$ be bicliques of G that do not contain v and w. Clearly, they are bicliques of $G - \{v\}$. Consider B a biclique of G containing vertices v and w. Consider $\widetilde{B} = B - \{v\}$. If $\widetilde{B}$ is not a biclique, there is a vertex adjacent to w and not to v or there is a vertex adjacent to v and not to w, contradicting the hypothesis. Then $\widetilde{B}$ is a biclique of $G - \{v\}$, and any two bicliques of G containing v are bicliques in $G - \{v\}$ containing w and they are adjacent vertices of $KB(G - \{v\})$. Then, $KB(G) = KB(G - \{v\})$ and clearly $KB(G) = KB(Tw(G))$ since we can repeat the same argument for all false twin vertices in G. ■

It is shown in [13, 14] that a graph G such that $G = KB(H)$ for some H has the following property. Every induced P_3 of G is contained in an induced *diamond* or in an induced *gem* of G (Fig. 3). We now show that this property holds when we consider $Tw(G)$.

Lemma 3.7. *Let $G = Tw(KB(H))$. Then every induced P_3 of G is contained in an induced diamond* or an induced *gem of G*.

Proof. Let $Z_1, Z_2, ..., Z_k$ be the maximal sets of false twin vertices in $KB(H)$. Let z_i, z_j, z_l be the representative vertices of an induced P_3 in G. Then, there exists three vertices v, w, x in $KB(H)$ such that $v \in Z_i$, $w \in Z_j$, and $x \in Z_l$. The vertices v, w, x induce a P_3 in $KB(H)$, and therefore by [13, 14] it is contained in an induced *diamond* or *gem* of $KB(H)$. Suppose that this P_3 is contained in a *diamond*. Then there exists a vertex u in $KB(H)$ adjacent to v, w, and x. Let Z_m be the set of false twin vertices that contains u, where $m \neq i, j, l$. Then, if z_m is the corresponding vertex of the set Z_m in G, z_i, z_j, z_l, z_m induce a *diamond* in G that contains the P_3. The case in which the P_3 is contained in a *gem* is similar, considering two vertices of $KB(H)$ instead of one.

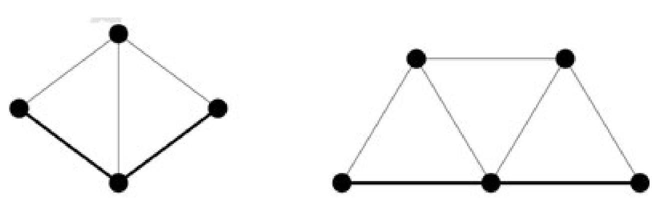

FIGURE 3. The edges e and e' are not adjacent and the bicliques B and B' that contain them intersect in vertex v.

■

Now, we are able to prove Theorem 3.1.

Proof of Theorem 3.1. Suppose $K_n \subseteq KB(G)$, $n \geq 5$. According to Proposition 3.4, we know that $K_{2n-4} \subseteq KB(KB(G))$ or $K_{(n-2)(n-3)} \subseteq KB^2(KB(G))$. Now, define $f(x) = 2x - 4$ and $g(x) = (x-2)(x-3)$. It is clear that $x < f(x)$ and $x < g(x)$ for $x \geq 5$. Therefore, the graph $KB(G)$ grows under the biclique operator, in the worst case, every two iterations. Consequently, $\lim_{s\to\infty} |V(KB^s(G))| = \infty$, i.e., $KB(G)$ is divergent. Now, suppose the *gem* or the *rocket* are induced subgraphs of $KB(G)$. Since $KB(gem) = K_5$ and $K_5 \subseteq KB(rocket)$, that is, $K_5 \subseteq KB(KB(G))$ and $KB(G)$ diverges and consequently, G diverges.

Otherwise, $KB(G)$ does not contain K_5, the *gem* or the *rocket* as induced subgraphs and clearly, $Tw(KB(G))$ does not contain them either. We prove that $Tw(KB(G))$ is isomorphic to K_1, K_2, K_3, or K_4. Suppose that it is false, i.e., $Tw(KB(G)) \neq K_n$, for any $n = 1, ..., 4$. Then, $Tw(KB(G))$ contains an induced P_3 that is contained in an induced *diamond*, by Lemma 3.7. Let u, v, w, x be the vertices that induce such a *diamond* in $Tw(KB(G))$, where u, w are not adjacent vertices (Fig. 4).

Since $Tw(KB(G))$ has no false twin vertices, there exists a vertex $y \in Tw(KB(G))$ adjacent to u, not adjacent to w (Fig. 4A). If y is adjacent to v or x, then u, v, w, x, y induce a *gem* or a *rocket*, a contradiction (Fig. 4B). Otherwise, v, u, y induce a P_3 which, by Lemma 3.7, is included in an induced *diamond*. Then, there is a vertex z adjacent to y, u, v (Fig. 4C). If z is adjacent to x then u, v, w, x, z induce a *rocket*, otherwise, they induce a *gem*, a contradiction (Fig. 4C). We conclude that $Tw(KB(G))$ has no P_3 as induced subgraph, and then, $Tw(KB(G)) = K_n$, for $n = 1, ..., 4$. Recall that $KB(G)$=$KB(Tw(G))$. Since $KB(K_3) = K_3$, $KB(K_4) = O_3$, and $KB(O_3) = K_3$, the result follows. ■

As a direct corollary of Theorem 3.1, we obtain the following result, which will be the bases of our algorithm.

Corollary 3.8. *A graph G is convergent if and only if $Tw(KB(G))$ has at most four vertices.*

Note that if some vertex lies in five bicliques, then $KB(G)$ contains a K_5 and then G diverges. Therefore, the Corollary 3.8 gives a polynomial time algorithm to test convergence of G: if some vertex lies in five bicliques, we answer that G is divergent. Else, the

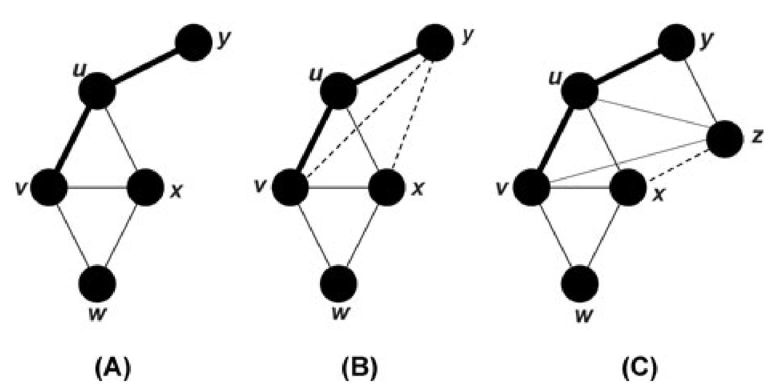

FIGURE 4. Diamond u, v, w, x and the different cases.

computation of $KB(G)$ and $Tw(KB(G))$ is polynomial. If $Tw(KB(G))$ has at most four vertices, answer that G is convergent, otherwise, we answer that G is divergent.

Constructing $KB(G)$ takes $O(n^4)$ time, since for the case that is done, the input graph G has at most $2n$ bicliques and generating each biclique is $O(n^3)$ [6, 7]. To build $Tw(KB(G))$ can be done in $O(n^2)$ time. We conclude that the algorithm runs in $O(n^4)$ time.

4. DIVERGENT AND CONVERGENT TREES

In this section, we analyze convergence and divergence for a special class of graphs, namely, the trees.

Lemma 4.1. *Let T be a tree with $n \geq 3$ vertices and k leaves. Then T has $n - k$ bicliques.*

Proof. Since bicliques in trees are all $K_{1,r}$ (one vertex adjacent to r ones), with $r \geq 1$, each nonleaf vertex represents a unique biclique and the theorem follows. ■

Lemma 4.2. *Let T and T' be nontrivial trees. If T is an induced subgraph of T', then $KB(T)$ is an induced subgraph of $KB(T')$.*

Proof. By Observation 3.2, it is clear that $KB(T)$ is a subgraph of $KB(T')$. We will prove that, in fact, it is an induced subgraph. For a contradiction, suppose that b_1, b_2 are not adjacent in $KB(T)$ but they are in $KB(T')$. Then, if B_1, B_2 are the associated bicliques of b_1, b_2 in T', there exists some vertex $v \in B_1 \cap B_2$ such that $v \in T'$ and $v \notin T$. Now, since bicliques in trees are all $K_{1,r}$, with $r \geq 1$, let t_1, t_2 be the vertices of B_1, B_2, respectively, that are adjacent to v. Observe that $t_1, t_2 \in T$. Now, since T is connected, there exists a path P between t_1, t_2 in T. Clearly, P is also a path in T', however, there is another path $P' = t_1, v, t_2$ between the vertices t_1, t_2 in T' contradicting the fact that T' is a tree. Finally, $KB(T)$ is an induced subgraph of $KB(T')$. ■

Theorem 4.3. *Let T be a tree. Then T diverges if and only if T has at least five bicliques* .

Proof. Observe that if T has at least five bicliques then it has at least five internal vertices and therefore it contains one of the graphs in Figure 5A as induced subgraphs.

In any case, $KB(T)$ contains the *gem*, the *rocket*, or the K_5 as induced subgraphs (Lemma 4.2), and hence, by Theorem 3.1, T diverges. The converse is straightforward, since if T diverges, then, by Theorem 3.1, $KB(T)$ contains a *gem*, a *rocket*, or a K_5 as induced subgraphs, i.e., T has at least five bicliques. ■

We remark that for trees, the running time of the algorithm for deciding if a tree either converges or diverges is $O(n)$. This follows from Lemma 4.1.

5. CONCLUSIONS

In this article, we have studied the behavior of a graph under the biclique operator, in particular, we have studied the limit $\lim_{k\to\infty} |V(KB^k(G))|$. We proved that the limit is 1, 3, or infinity. We presented characterizations of convergent and divergent graphs, and gave

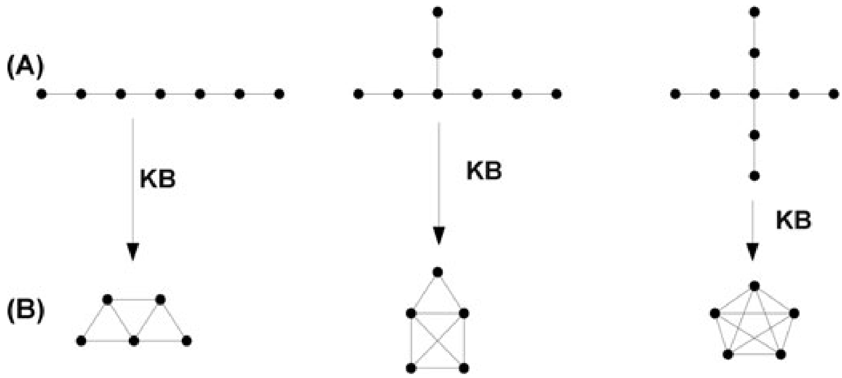

FIGURE 5. (A) Minimal nonisomorphic trees with five bicliques. (B) Their biclique graphs.

a polynomial time algorithm for their recognition. It is worth contrasting these results with those for the clique operator. For the clique operator, there is no characterization for divergent, periodic, or convergent graphs, and the computational complexity of the associated decision problem is open.

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