International Game Theory Review, Vol. 12, No. 1 (2010) 19–35

DOI: 10.1142/S0219198910002490

SHAPLEY VALUE IN A MODEL OF INFORMATION TRANSFERAL

PATRICIA LUCIA GALDEANO, JORGE OVIEDO*
and LUIS GUILLERMO QUINTAS

Departamento de Matemáticas
Facultad de Ciencias Físico Matemáticas y Naturales
Instituto de Matemática Aplicada
Universidad Nacional de San Luis y CONICET
Ejército de los Andes 950. 5700
San Luis, Argentina
**joviedoeunsl.edu.al*

In this paper we analyze the value of the information in a cooperative model. There is an agent (the innovator), having relevant information which can be sold to some potential buyers. The n potential users of the information share a market. The expected utility of each of them can be improved by obtaining the information. The whole situation is modelled as a $(n+1)-$person cooperative game.

We study the properties of the characteristic function of this game. It fulfills a weak version of the superadditivity property, namely 0-monotonicity. The game is proved to be monotonic.

We compute the Shapley value and we prove it is an imputation for the game. We compare the Shapley value with the equilibrium studied in a noncooperative model by Quintas (1995). We also study some limit cases when the potential buyers are completely informed or uninformed. It includes Big Boss Games (Muto *et al.* (1988)) and other limit cases.

We conclude that in a cooperative environment the buyers avoid being exploited by the innovator. Conversely the innovator obtains a higher utility in a noncooperative game.

Keywords: Models of information transferal; cooperative games; 0-monotonicity; Shapley value.

JEL Classification: C71

1. Introduction

Information has a remarkable characteristic as a commodity. While its production usually requires a cost, it can be reproduced easily as many times as possible once it is acquired. The cost for reproduction is generally negligible. Thus, as soon as the information is sold in the market by a monopolistic producer, the competition among resellers should force the price to fall down to zero. The reward to the

*Corresponding author.

original producer will be injured thereby, so that it will not be traded in the market (Arrow(1962)).

Patent licensing is a common legal arrangement to provide a proper incentive for trading information such as technological innovations. A significant portion of the existing literature on information trading has been devoted to analyzing economic effects of patent licensing or protection (Kamien and Tauman (1984, 1986); Gilbert and Shapiro (1990); Katz and Shapiro (1985, 1986); Poddar and Sinha (2004); Stamatopoulos and Tauman (2005) and Muto (1986, 1993)). There exist many papers studying the value of the information when the information holder acts strategically, Kamien (1992) surveys most of these studies.

In this paper we consider the following problem: There are n firms with similar characteristics. There exists an agent having relevant information for the firms. This information can be sold to the firms. The innovator is not going to use the information for himself and the firms acquiring the information will be better than before obtaining it.

A typical example would be n farmers which are willing to obtain information about the upcoming weather. Thus they could know what is the best seed they should use for the upcoming season. They could pay for this information, in order to improve their expectations of choosing the right seed. Another example is the case of an innovator offering a new technology to n firms. They can then reduce their production costs. The information holder is not part of the market, but he can act strategically in order to maximize his utilities by selling the information

The problem can be modelled as a $n+1$ players game. This game can be cooperative or noncooperative. Quintas (1995) considered in a noncooperative framework under what conditions it was optimal and stable to sell the information to all the firms. However the situation was not so appealing for the buyers. The information should be bought for all the buyers, but the utilities they obtained were equal to the case when each of them was the unique uninformed player. Nevertheless they could not ignore the existence of the information, thus it was concluded that they should buy it. This result reflects many real situations where the introduction of a new technology produces serious damage in the local market. On the other hand it might be expected that in some cases the firms could act in a cooperative way in order to prevent the general damage mentioned above. We study this problem in a cooperative characteristic form game of $n + 1$ players.

We make some assumptions on the Information Market: A fixed market, the use of the information by all the agents, a conservative point of view in the computation of informed players utilities, and the same previous information level of all uninformed agents. These assumptions avoid modelling the problem as a game with externalities. The study of games with externalities has been done by Macho-Stadler, Pérez Castillo and Wettstein (2006); de Clippel and Serrano (2005), however in this approach it is still difficult to justify the use of the Shapley value as a solution concept. We compute the Shapley value (Shapley (1953)). When we consider a game in characteristic form function, superadditivity is usually assumed, and thus the Shapley value can be computed and it gives an imputation. However in our approach we use a weak version of superadditivity: 0-monotonicity and we show that it is still possible to compute the Shapley value. It results an imputation for the game. We give conditions for the 0-monotonicity and we study the implications of those conditions on the resulting market. The games is also proved to be monotonic.

We compare the utilities of the agents in the cooperative and noncooperative model and we observe that the Shapley value gives a better utility for the users that what they obtained in the noncooperative model. It means that they avoid being exploited by the innovator. An opposite situation is observed from the innovator point of view because his utility is lower than in the noncooperative model.

2. The Information Market

We consider a market with n firms ($n \geq 2$) and an innovator who posses a patent or an information.

The set of agents will be denoted by: $N = \{1, 2, \ldots, n+1\}$, where : $I = \{1\}$ (the innovator) is the agent having a new information and $U = \{2, \ldots, n+1\}$(users) are the firms who could be willing to obtain the new information.

The n users or firms, interact in the same market, producing or performing the same activity, with the same technology or the same information. Thus all the users have the same incentives for the acquisition of the new information or technology. We will make certain assumptions about the problem we want to study:

S.1: The n users of the information are the same before and after the information holder offers the new technology. It indicates that there are no exits or incoming agents in the market.

S.2: All the players that acquire the new information make use of it. This is a natural assumption in a noncooperative environment, and it is assumed in a cooperative model to avoid the formation of monopolies.

S.3: From the point of view of the players that acquire the information, their utilities will be computed under a conservator point of view, assuming that the uninformed agents take the right decision.

S.4: All the users have the same previous information level. For instance, in the case of the farmers, they all have the same knowledge about the upcoming weather, or in the case of firms producing a good, they all have the same technology. It can be formalized as follows: For each $j \in U$ there exists a probability p_j of having success (taking the right decision). We also assume that this probability is the same for all players: $p_j = c \,\forall\, j \in U$.

S.5: The utility they obtain depends on how many players take a right decision no matter the identity of them. Thus if r players make a right decision (for instance, if they are farmers choosing the right seed for the upcoming weather) each utility function will be $a(r)$. In particular, $a(r)$ is a decreasing function, because when more agents take the right decision, each agent obtains a lower utility level i.e., if $r \leq k$ then $a(r) \geq a(k)$. We also assume that: $a(1) = 1$ and $a(r) \geq 0$. The agents making a wrong decision obtain no utility.

We will model this problem as a cooperative game, where each agent can decide to acquire the information. If an agent is uniformed, the probability of making a right decision (or success) can be described by a binomial probability distribution, being c the probability of sucess. The probability that k among n players take the right decision is: $p(k,c,n) = \binom{n}{k} c^k (1-c)^{n-k}$. The utility obtained by the agents in U is: $p(k,c,n)a(k)$. The success of each agent is independent of the remaining agents. The aggregated utility of k players which succeeded is: $kp(k,c,n)a(k) = k\binom{n}{k} c^k (1-c)^{n-k} a(k)$. Thus, we have:

$C.1$: If the agents in a coalition S do not have the information (the innovator is not a member of S), we have that: $v(S) = w(S) = \sum_{k=0}^{s} k\binom{s}{k} c^k (1-c)^{s-k} a(n-s+k)$ with $s = |S|$. It is so, because if k players have succeeded, each one has an expected utility: $p(k,c,n)a(n-s+k)$, since we assumed that the $(n-s)$ players outside the coalition also success.

$C.2$: If the agents in S have the information (the innovator belongs to S): $v(S) = u(S) = (s-1)a(n)$ with $s = |S|$. It is so because s players have succeeded and their expected utility will be $a(n)$.

Definition 1. An n person game in characteristic function form is given by (N, v), where $N = \{1, 2, \ldots, n+1\}$ is the set of players and $v : 2^N \to \Re$ is the characteristic function.

In our case, by $(C.1)$ and $(C.2)$ the characteristic function ν is:

$$v(S) = \begin{cases} 0 & \text{if } S = \emptyset. \\ u(S) = (s-1)a(n) & \text{if } 1 \in S. \\ w(S) = \displaystyle\sum_{j=0}^{s} j \binom{s}{j} c^j (1-c)^{s-j} a(n-s+j) & \text{if } 1 \notin S, \text{ and } S \neq \emptyset \\ & \qquad \text{for all } S \subseteq N. \end{cases}$$

The set of all characteristic functions games v with the set of players N is denoted by G^N. In this paper, we will restrict our attention to a subset of G^N consisting of all monotonic games. This set is denoted by $MG^N = \{\nu \in G^N | v(S) \leq v(T) \text{ when } S \subseteq T\}$. It is immediate to observe that:

Remark 1. (1) $v(S) \geq 0$ for all $S \subseteq N$.

(2) $v(\{1\}) = 0$, because when the innovator do not sell the information, he obtains no utility by the use of it.

(3) The function v depends only on the fact that the innovator belongs (or not) to it, and the number of agents in the coalition. Thus we will keep the notations $v(S)$, $w(S)$ and $u(S)$, but in all these cases they are functions depending only on the cardinality s of the set S, we will denotes it by $v(s)$ (where $s = |S|$).

2.1. *Properties of the characteristic function*

We will first present a result which analyzes the incentives of uninformed players to join an informed players coalition. This result is used in the study of the 0-monotonic property. We will then study the monotonic property. An usual assumption on the function v is the superadditivity property.

Definition 2. A game (N, v) is superadditives if for all sets $A \subseteq N$ and $B \subseteq N$ with $A \cap B = \emptyset$, we have that: $v(A \cup B) \geq v(A) + v(B)$.

It gives the players proper incentives to form bigger coalitions. We will consider a weaker version of the superadditivity property, the so called 0-monotonic property (Pérez Castillo and Wettstein (2001)).

Definition 3. A game (N, v) is 0-monotonic if for all sets $A \subseteq N$ and for all $i \notin A$, we have that: $v(A \cup \{i\}) \geq v(A) + v(\{i\})$.

In a 0-monotonic game there are no negative externalities when a single player joins a coalition.

2.1.1. 0-*monotonic games*

Theorem 1. *If the innovator is not in the coalition $S(1 \notin S)$ and he belongs to $T(1 \in T)$ such that $S \cap T = \emptyset$, then $v(S \cup T) \geq v(S) + v(T)$ if and only if*

$$v(S) = w(S) \leq u(S \cup \{1\}) = v(S \cup \{1\}) \tag{1}$$

Proof. First, we are going to prove that if $u(S \cup T) \geq w(S) + u(T)$ then $w(S) \leq u(S \cup \{1\})$. If $u(S \cup T) \geq w(S) + u(T)$ by Definition 1, we have: $(s + t - 1)a(n) \geq \sum_{k=0}^{s} k\binom{s}{k} c^k (1-c)^{s-k} a(n - s + k) + (t - 1)a(n)$, simplifying $(t - 1)a(n)$ we have:

$$s\, a(n) \geq \sum_{k=0}^{s} k \binom{s}{k} c^k (1-c)^{s-k} a(n - s + k). \tag{2}$$

By Definition 1;

$$u(S \cup \{1\}) = s\, a(n) \quad \text{and} \quad w(S) = \sum_{k=0}^{s} k \binom{s}{k} c^k (1-c)^{s-k} a(n - s + k) \tag{3}$$

Then using (3) in (2) we have: $w(S) \leq u(S \cup \{1\})$. This proves the first part.

Now we are going to prove that if $w(S) \leq u(S \cup \{1\})$ then $u(S \cup T) \geq w(S) + u(T)$. If $w(S) \leq u(S \cup \{1\})$ then: $u(S \cup T) - w(S) - u(T) \geq u(S \cup T) - u(S \cup \{1\}) - u(T)$, by Definition 1: $u(S \cup T) - u(S \cup \{1\}) - u(T) = (s + t - 1) a(n) - s\, a(n) - (t - 1)a(n) = 0$ then: $u(S \cup T) - w(S) - u(T) \geq 0$, hence $u(S \cup T) \geq w(S) + u(T)$. □

This theorem indicates two outcomes. Firstly, the players in a uninformed coalition $S \subseteq U$ have incentives to join a informed coalition $T \subseteq N$, if the utility they obtain is less than they would obtain buying the information. We do not need a restriction on the set T because by assumption S.3, for the computation of the characteristic function $v(T)$ we assumed that the uninformed agents outside the coalition take the right decision. Thus it is always better for them to join the coalition. Secondly, if $s = |S| = 1$ then $w(S) = 0$ and $u(S \cup \{1\}) = a(n)$. Thus $a(n) \geq 0$. It indicates that a sole uninformed player always have incentives to buy the information.

Now we will analyze what happens if $s > 1$. We analyze the restrictions (1) given in Theorem 1 depending on the number of agents in the market.

For each set S we have an inequation, thus we have $\binom{n}{s}$ inequations, but as all the sets having the same cardinality s give the same inequation, we have only n relevant equations.

If $w(S) \leq u(S \cup \{1\})$, then using Definition 1, and $0 < c < 1$. We have:

$$a(n) \geq \frac{\displaystyle\sum_{j=0}^{s-1} j \binom{s}{j} c^j (1-c)^{s-j} a(n-s+j)}{s(1-c^s)} \tag{4}$$

The first term in the numerator of the right side in (4) is 0. Thus we will consider $j \geq 1$.

We will use in (4): $\frac{j}{s}\binom{s}{j} = \binom{s-1}{j-1}$ and $(1-c^s) = (1-c)\sum_{j=0}^{s-1} c^j$, then we have:

$$a(n) \geq \frac{\displaystyle\sum_{j=1}^{s-1} \binom{s-1}{j-1} c^j (1-c)^{s-j-1} a(n-s+j)}{\displaystyle\sum_{j=0}^{s-1} c^j} \tag{5}$$

For each s with $2 \leq s \leq n$ we have by (5), an inequation. Then we have:

$$\begin{cases} a(n) \geq 0 \\ a(n) \geq \dfrac{c}{1+c} a(n-1) \\ \vdots \\ a(n) \geq \dfrac{\displaystyle\sum_{j=1}^{k-1} \binom{k-1}{j-1} c^j (1-c)^{k-j-1} a(n-k+j)}{\displaystyle\sum_{j=0}^{k-1} c^j} \\ \vdots \\ a(n) \geq \dfrac{\displaystyle\sum_{j=1}^{n-1} \binom{n-1}{j-1} c^j (1-c)^{n-j-1} a(j)}{\displaystyle\sum_{j=0}^{n-1} c^j} \end{cases} \tag{6}$$

We also have the general assumption: $0 \leq a(n) \leq a(n-1) \leq \cdots \leq a(2) \leq 1$. We should solve this system finding the variation of the variables $a(n), a(n-1), \ldots, a(2)$. We will use the Fourier method for inequations. It consists of eliminating in each step a variable obtaining a equivalent system with the remaining variables (Bertsimas and Tsitsiklis (1997)). Thus, we have:

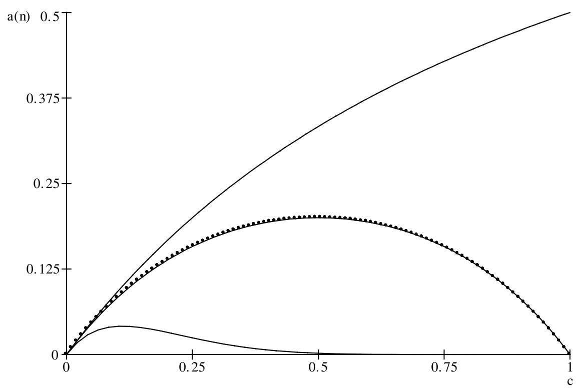
Proposition 1. *The solution to the system* (6) *is*: $\frac{c(1-c)^{n-2}}{1+c(1-c)^{n-2}} \leq a(n) \leq a(n-1) \leq \cdots \leq a(3) \leq a(2) \leq 1$.

The proof is done by induction and it appears in the Appendix.

Remark 2. This proposition shows that the restriction of Theorem 1 can be simplified, giving only a new inequality

$$\frac{c(1-c)^{n-2}}{1+c(1-c)^{n-2}} \leq a(n), \tag{7}$$

We analyze the left side restrictions of inequality (7) for some values of n with c fixed. In the following figure we show the graphics for $n = 2$ (solid line), $n = 3$ (dot dash) and $n = 10$ (dot-dot dash).



Restriction (7) indicates that when the number of players grows it gives a lower value. Thus we have more freedom for choosing $a(n)$. On the other hand, if we analyze restriction (7) for the different values of c and n, we have that for a fixed n: The $\max(\frac{c(1-c)^{n-2}}{1+c(1-c)^{n-2}})$ holds when $c = \frac{1}{n-1}$. Moreover, for a fixed n, we have: $\lim_{c\to 0+} \frac{c(1-c)^{n-2}}{1+c(1-c)^{n-2}} = \lim_{c\to 1-} \frac{c(1-c)^{n-2}}{1+c(1-c)^{n-2}} = 0$, thus in these extreme cases we only have the basic restriction $a(n) \geq 0$.

The following theorem resumes the above studies cases. It shows under what conditions v is 0-monotonic.

Theorem 2. *v is 0-monotonic if and only if* $a(n) \geq \frac{c(1-c)^{n-2}}{1+c(1-c)^{n-2}}$.

Proof. Let $S \subseteq N$ and $i \notin S$. If $S = \emptyset$ and $v(\emptyset) = 0$ $v(\emptyset) + v(\{i\}) = 0 + v(\{i\}) = v(\emptyset \cup \{i\})$. Thus we have that: $v(S \cup \{i\}) \geq v(S) + v(\{i\})$. If $S \neq \emptyset$ and $i \notin S$, we have the following 3 cases:

Case 1. Let S $(1 \in S)$ be an informed players coalition and i an user $i \notin S$ then: $v(S \cup \{i\}) \geq v(S) + v(\{i\})$. By hypothesis we have that $1 \in S$ and $i \neq 1$ then by Definition 1: $u(S) = (s-1)a(n)$ and $w(\{i\}) = ca(n)$, then $u(S) + w(\{i\}) = (s - 1 + c)a(n)$. Then using $0 < c < 1$, we have that: $(s - 1 + c)a(n) \leq sa(n)$. By Definition 1, $u(S \cup \{i\}) = sa(n)$, then $u(S \cup \{i\}) \geq u(S) + w(\{i\})$.

Case 2. If the innovator is not in the coalition S $(1 \notin S)$ and i is an user $i \notin S$ then: $v(S \cup \{i\}S) + v(\{i\})$.

The proof of this case follows by using Definition 1 and properties of combinatoric numbers.

Case 3. If the innovator is not in the coalition S $(1 \notin S)$ and i is the innovator $(i = 1)$ then: $u(S \cup \{1\}) \geq w(S) + u(\{1\}) = w(S)$ if and only if $a(n) \geq \frac{c(1-c)^{n-2}}{1+c(1-c)^{n-2}}$.

This is a particular case of Theorem 1 and Proposition 1, when $T = \{1\}$. □

2.1.2. *Monotonic property*

The following theorem shows that each function v determines a monotonic game $(N, v) \in MG^N$.

Theorem 3. *Given a game* (N, v), *with* v *given by Definition* 1, *and* $0 < c < 1$, *then* $v(S) \leq v(T)$ *for all* $S \subseteq T$.

Proof. Let us first prove $v(S) \leq v(S \cup \{i\})$ for all $i \in N$. If $i = 1$, by Theorem 2, $v(S \cup \{1\}) \geq v(S) + v(\{1\})$ and by Definition 1, $v(\{1\}) = 0$ we have $v(S \cup \{1\}) \geq v(S)$. If $i \neq 1$, by Theorem 2, $v(S \cup \{i\}) \geq v(S) + v(\{i\})$ and by Definition 1, $v(\{i\}) = c\ a(n) \geq 0$ we have $v(S \cup \{i\}) \geq v(S) + ca(n) \geq v(S)$. Now using repeatedly the above result it follows that: $v(S) \leq v(T)$ for all $S \subseteq T$. It completes the proof. □

The following Lemma gives a property of v that will be used in Subsection 4.1.

Lemma 1. *Let* $(N, v) \in MG^N$ *where* v *is given by Definition* 1 *and* S *is an informed players coalition* $(1 \in S)$, *then*: $v(N) - v(S) = \sum_{i \in N \setminus S}(v(N) - v(N \setminus \{i\}))$.

It implies that for every coalition not containing $i = 1$, its contribution to the grand coalition is equal than the sum of the contributions of its players to the grand coalition.

Proof. Let $1 \in S$ and $i \neq 1$. Then by Definition 1, we have: $\sum_{i \in N \setminus S}(v(N) - v(N \setminus \{i\})) = \sum_{i \in N \setminus S}(na(n) - (n-1)a(n)) = \sum_{i \in N \setminus S} a(n) = (n + 1 - s)a(n)$, at the same time $v(N) - v(S) = n\ a(n) - (s-1)a(n) = (n + 1 - s)a(n)$, thus $v(N) - v(S) = \sum_{i \in N \setminus S}(v(N) - v(N \setminus \{i\}))$. □

3. Solution of the Cooperative Game

We compute the Shapley value (Shapley (1953)) for the given characteristic function v. It is defined by:

Definition 4. Given a game (N, v), the Shapley value is defined by the following vector $\varphi(v) = (\varphi_1(v), \ldots, \varphi_{n+1}(v))$ where:

$$\varphi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!\,(n-s)!}{(n+1)!}[v(S \cup \{i\}) - v(S)]$$

with $|S| = s$ and $|N| = n + 1$.

In the following theorem we give an explicit formulation for the Shapley value, when v is given as in Definition 1.

Theorem 4. *Given a game (N, v), where v is given by Definition 1, with $S \subseteq U$ and $0 < c < 1$, then*:

The Shapley value for the users ($i \neq 1$) is given by: $\varphi_i(v) = \frac{1}{2}a(n) + \sum_{s=0}^{n-1} \frac{n-s}{n(n+1)}(w(S \cup \{i\}) - w(S))$.
The Shapley value for the innovator is given by: $\varphi_1(v) = \frac{1}{2}u(N) - \frac{1}{n+1}\sum_{s=0}^{n} w(S)$.

Proof. Let $i \neq 1$, splitting the sums in informed and uninformed coalitions, and by Definition 1 we have:

$$\varphi_i(v) = \sum_{\substack{S \subseteq N \setminus \{i\} \\ 1 \in S}} \frac{s!\,(n-s)!}{(n+1)!}[u(S \cup \{i\}) - u(S)] + \sum_{\substack{S \subseteq N \setminus \{i\} \\ 1 \notin S}} \frac{s!\,(n-s)!}{(n+1)!}[w(S \cup \{i\}) - w(S)]. \tag{8}$$

Now we analyzes how many subsets S are in each sum (8):

1. If $S \subseteq N \setminus \{i\}$, with $1 \in S$, we count how many subsets we have of the type $S \setminus \{1\} \subseteq N \setminus \{1, i\}$. The innovator is a fixed player in all the coalitions S we could form, they are $\binom{n-1}{s-1}$. Then $\binom{n-1}{s-1}\frac{s!(n-s)!}{(n+1)!} = \frac{s}{n(n+1)}$, with $|S| = s = 1, \ldots, n$.
2. If $S \subseteq N \setminus \{i\}$, with $1 \notin S$, then there exists $\binom{n-1}{s}$ subsets with $S \subseteq N \setminus \{i, 1\}$, so that $\binom{n-1}{s}\frac{s!(n-s)!}{(n+1)!} = \frac{n-s}{n(n+1)}$, with $|S| = s = 0, \ldots, n-1$.
3. As the function $v(S)$ depends only on the cardinality s of the set S, (Remark 1), we denotes $u(s)$ and $u(s+1)$ instead of $u(S)$ and $u(S \cup \{i\})$ respectively, and $w(s)$ and $w(s+1)$ instead of $w(S)$ and $w(S \cup \{i\})$.

Using 1, 2 and 3 in (8) we have:

$$\varphi_i(v) = \sum_{\substack{s=1\\1\in S}}^{n} \frac{s}{n(n+1)}[u(s+1) - u(s)] + \sum_{\substack{s=0\\1\notin S}}^{n-1} \frac{n-s}{n(n+1)}[w(s+1) - w(s)]. \qquad (9)$$

As a consequence of Definition 1 $(u(s+1) - u(s)) = a(n)$ and $\sum_{s=1}^{n} \frac{s}{n(n+1)} = 1/2$ in (9). Thus we have:

$$\varphi_i(v) = \frac{1}{2}a(n) + \sum_{s=0}^{n-1} \frac{n-s}{n(n+1)}(w(s+1) - w(s)).$$

Now consider $i = 1$. Working as in the previous, we have that: $\varphi_1(v) = \frac{1}{2}u(N) - \frac{1}{n+1}\sum_{s=0}^{n} w(s)$. □

We will prove that under certain conditions, φ results a payoff distribution for the game (N, v).

Definition 5. An imputation or payoff distribution for the game (N, v) is a vector $x = (x_1, \ldots, x_{n+1})$ satisfying: $\sum_{i\in N} x_i = v(N)$ and $x_i \geq v(\{i\})$ for each $i \in N$.

When the game is superadditive it is well known that φ results an imputation (Shapley (1953)). We will show that it is also true when v is 0-monotonic.

Theorem 5. *Given a game (N, v), where v given by Definition 1, with $0 < c < 1$ then, $\varphi(v) = (\varphi_1(v), \ldots, \varphi_{n+1}(v))$ is an imputation for the game (N, v).*

The proof appears in the Appendix.

3.1. *Comparison of the Shapley value with the equilibrium outcome in the noncooperative game*

The cooperative game studied in this paper was analyzed by Quintas (1995) form a noncooperative point of view and it was observed that the innovator obtained a neat profile by selling the information to the n firms. However the situation was not so appealing for the buyers. The expected utility each one finally obtained after buying the information was that one he would have obtained if he was the only uninformed agent. Nevertheless they couldn't ignore the existence of the information and they should buy it. The main result of the noncooperative study mentioned above states as follows:

Theorem 6. *Given $N = I \cup U$ the set of players, if $p_j = c$ for all $j \in U$ (S.4), then the price P that the innovator can ask to the n users such that all them acquire the information, is determined by an ε — Nash equilibrium of the noncooperative game. This price is: $P = (1-c)a(n) - \varepsilon$, with $\varepsilon \geq 0$ arbitrarily small, and the payoff $(n+1)-$ upla is: $((1-c)na(n) - n\varepsilon, ca(n) + \varepsilon, \ldots, ca(n) + \varepsilon)$.*

As the innovator can choose $\varepsilon \to 0$, then the payoff utilities converge to the $(n+1)-upla$: $((1-c)na(n), ca(n), \ldots, ca(n))$. We compare it with the utilities obtained in the cooperative model.

Theorem 7. *Given a game* (N, v), *with* v *given by Definition 1 and* $0 < c < 1$ *then*:

(1) *The innovator prefers the noncooperative model. It is*: $\varphi_1(v) \leq n\,(1-c)\,a(n)$.
(2) *The users prefer the cooperative model. It is*: $\varphi_i(v) \geq ca(n)$.

Proof. Let us prove 1: As $\varphi(v) = (\varphi_1(v), \ldots, \varphi_{n+1}(v))$ is an imputation for the game (N, v), it satisfies.

(I). $\sum_{i\in N} \varphi_i = v(N) = na(n)$ and (II). $\varphi_i \geq v(\{i\})$ for each $i \in N$.

If $i \in U$ by (II) and Definition 1: $\varphi_i \geq v(\{i\}) = ca(n)$. Then in (I) we have: $na(n) = \sum_{i\in N} \varphi_i = \varphi_1 + \sum_{i\in N\setminus\{1\}} \varphi_i \geq \varphi_1 + nca(n)$. The inequality holds because φ_i is the same for all the users. Then $na(n) \geq \varphi_1 + nca(n)$. We have that $\varphi_1 \leq n\,(1-c)\,a(n)$. Then the Shapley value gives a lower utility for the innovator than the utility obtained in the noncooperative model.

Let us prove 2: We give the Shapley value for the users and using Theorem 2, we have:

$$\varphi_i(v) = \frac{1}{2}\,a(n) + \sum_{s=0}^{n-1} \frac{n-s}{n(n+1)}(w(S\cup\{i\}) - w(S))$$
$$\geq \frac{1}{2}\,a(n) + w(\{i\}) \sum_{s=0}^{n-1} \frac{n-s}{n(n+1)},$$

by Definition 1 $w(\{i\}) = ca(n)$ and $\sum_{s=0}^{n-1} \frac{n-s}{n(n+1)} = 1/2$ we have that: $\varphi_i(v) \geq \frac{1}{2}a(n) + \frac{1}{2}\,ca(n) = \frac{1}{2}(1+c)\,a(n)$. As $0 < c < 1$ then: $\frac{1}{2}(1+c)\,a(n) \geq ca(n)$. Then the Shapley value gives a better utility for the users than the utility obtained in the noncooperative model. □

4. Limit Cases

In this subsection we will study two limits cases: completely uninformed users $(c = 0)$ and completely informed users $(c = 1)$.

4.1. *Users with no previous information* $(c = 0)$. *Big Boss Games*

We will show that in this case the game (N, v) is a Big Boss Games (Muto, Nakayama, Potters and Tijs (1988)).

Definition 6. Let (N, v) be a game in MG^N, is called a Big Boss Games if there is one player, denoted by i^*, satisfying the following two conditions. $(B1)$ $v(S) = 0$ if $i^* \notin S$, and $(B2)$ $v(N) - v(S) \geq \sum_{i\in N\setminus S}(v(N) - v(N\setminus\{i\}))$ if $i^* \in S$.

The Big Boss Games are denoted by $BBG_{i^*}^N$. $(B1)$ implies that one player i^* is very powerful; it is, coalitions not containing i^* cannot get anything. $(B2)$ implies that for every coalition not containing i^*, its contribution to the grand coalition is not less than the sum of the contributions of its players to the grand coalitions. Hence, the weak players may increase their influences by forming coalitions. We here notice that a Big Boss Games v is *superadditive* (Definition 2), because of the monotonicity of v and $B1$. The characteristic function $v : 2^N \to \Re$ results:

$$v(S) = \begin{cases} u(S) = (s-1)a(n) & \text{if } 1 \in S \\ w(S) = 0 & \text{if } 1 \notin S \end{cases} \quad \text{for all } S \subseteq N \text{ and } |S| = s. \tag{10}$$

Here player i^* is the innovator $i^* = 1$.

Theorem 8. *Let* (N, v) *with* v *is given by* (10), *then* $(N, v) \in BBG_1^N$.

Proof. We observe that $v(S) \leq v(T)$ for all $S \subseteq T$, By (10), then $(N, v) \in MG^N$. Condition B1 follows immediately by (10). Let us prove $B2$. If $1 \in S$ and $i \neq 1$, then by Lemma 1, we have: $v(N) - v(S) = \sum_{i \in N \setminus S}(v(N) - v(N \setminus \{i\}))$. Thus it holds with equality. □

4.1.1. *The Shapley value*

Theorem 9. *Given a game* $(N, v) \in MG^N$, *with* v *given by* (10), *then*:

The Shapley value for the users is given by: $\varphi_i(v) = \frac{1}{2}a(n)$.
The Shapley value for the innovator is given by: $\varphi_1(v) = \frac{1}{2}u(N) = \frac{na(n)}{2}$.

It is similar to the proof of Theorem 4, using (10).

4.1.2. *Comparison of the Shapley value with the noncooperative case.*

If the users have no previous information ($c = 0$) then the equilibrium payoff utilities of the noncooperative game is given by: $(\frac{na(n)}{2}, \frac{1}{2}a(n), \ldots, \frac{1}{2}a(n))$, while in the cooperative games we have: $(na(n), 0, \ldots, 0)$. Thus, in the noncooperative game the users are pushed down to 0 while in the cooperative case they obtain $\frac{1}{2}a(n)$. On the other hand the innovator looses $\frac{1}{2}$ of his utilities obtained in the noncooperative model.

4.2. *Completely informed users* ($c = 1$)

In this case the characteristic function $v : 2^N \to \Re$ results:

$$v(S) = \begin{cases} u(S) = (s-1)a(n) & \text{if } 1 \in S \\ w(S) = sa(n) & \text{if } 1 \notin S \end{cases} \quad \text{for all } S \subseteq N \text{ and } |S| = s. \tag{11}$$

As an immediate consequence of (11) we have:

Remark 3. $(N, v) \in MG^N$ and v is superadditive. If $c = 1$ the users have much previous information and then the payoff vector in the cooperative game is: $(0, a(n), \ldots, a(n))$. It is also so in the noncooperative game. The users obtain the same payoff in both cases and the role of the innovator is not relevant.

5. Conclusions

We observe that the Shapley value gives a better utility for the users that what they obtained in the equilibrium of the noncooperative model. It means that they avoid being exploited by the innovator. An opposite situation is observed from the innovator point of view because his utility is lower than in the noncooperative model.

Using a characteristic function that takes into account condition S.2 (saying that the players that acquire the information always make use of it), avoids monopolies formation. It models real situations where there exists antimonopoly laws. The resulting games can be nonsuperadditive. The superadditivity assumptions is usual in cooperative studies, however we show that under a weaker form of superadditivity it is still possible to use the Shapley value. It is used in Theorem 2, and it reinforces the relevance of Theorem 5. The study on the conditions for the 0-monotonicity helps understanding the implications of those conditions on the resulting market.

Acknowledgment

The authors thank the comments and suggestions of an associate editor and two anonymous referees.

References

Arrow, K. [1962] Economic welfare and the allocation of resources for invention, in R. R. Nelson (ed) *The Rate and Direction of Inventive Activity*, Princeton, N.J., 609–625.

Bertsimas, D. and Tsitsiklis, J. N. [1997] *Introduction to Linear Optimization*, Athena Scientific, Betmont, Massachusetts, 70–75.

De Clippel, G. and Serrano, R. [2005] Marginal contributions and externalities in the value, *Economics* working papers we057339, Universidad Carlos III, Departamento de Economía.

Gilbert, G. and Shapiro, C. [1990] Optimal patent length and breadth, *RAND Journal of Economics* **21**, 106–112.

Kamien, M. I. [1992] *Patent Licensing*, Handbook of Game Theory, Vol. 1, 332–335.

Kamien, M. I. and Tauman, Y. [1984] The private value of a patent: A game theoretical analysis, *Journal of Economics, Supp.* **4**, 93–118.

Kamien, M. I. and Tauman, Y. [1986] Fees versus royalties and the private value of a patent, *Quarterly Journal of Economics* **101**, 471–491.

Katz, M. L. and Shapiro, C. [1985] On the license of innovations, *Rand Journal of Economics* **16**, 504–520.

Katz, M. L. and Shapiro, C. [1986] How to license intangible property, *Quarterly Journal of Economics* **101**, 567–589.

Macho-Stadler, I., Pérez Castillo, D. and Wettstein, D. [2006] Efficient bidding with externalities, *Games and Economic Behavior* **57**, 304–320.

Muto, S. [1986] An information good market with symmetric externalities, *Econometrica* **54**, 295–312.

Muto, S. [1993]. On licensing policies in Bertrand competition, *Games Economic Behavior* **5**, 257–267.

Muto, S., Nakayama, N., Potters, J. and Tijs, S. [1988] On Big boss games, *The Economic Studies Quarterly* **39**, 303–321.

Pérez Castillo, D. and Wettstein, D. [2001] Bidding for the surplus: A non-cooperative approach to the Shapley value, *Journal of Economic Theory* **100**, 274–294.

Pérez Castillo, D. and Wettstein, D. [2005] Implementation of the ordinal Shapley value for a three-agent economy, *Economics Bulletin* **3**, 1–8.

Poddar, S., Sinha, U. B. [2002] The role of fixed fee and royalty in patent licensing, Working paper No. 0211, Department of Economics, National University of Singapore.

Poddar, S. and Sinha, U. B. [2004] On patent licensing in spatial competition, *Econ. Rec.* **80**, 208–218.

Quintas, L. G. [1995] How to sell private information, *Modelling, Measurement and Control*, D. **11**, 11–28.

Shapley, L. S. [1953] A value for n-person games, *Annals of Mathematics Study* **28**, 307–317.

Stamatopoulos, G., Tauman, Y. [2005] Licensing of a quality-improving innovation, working paper, Stony Brook University.

Appendix

Proof of Proposition 1. In order to solve the system we use the Fourier method. It consists of eliminating one variable in each step, obtaining a new system equivalent to the previous one but with one variable less.

1st step: We eliminate $a(n)$.

1. We reorder the system (6) as follows:

$$\begin{cases} a(n) \geq 0 \\ a(n) - \dfrac{c}{1+c} a(n-1) \geq 0 \\ a(n) - \dfrac{\sum_{j=1}^{2} \binom{2}{j-1} c^j (1-c)^{2-j} a(n-3+j)}{\sum_{j=0}^{2} c^j} \geq 0 \\ \vdots \\ a(n) - \dfrac{\sum_{j=2}^{n-1} \binom{n-1}{j-1} c^j (1-c)^{n-j-1} a(j)}{\sum_{j=0}^{n-1} c^j} \geq \dfrac{c\,(1-c)^{n-2}}{\sum_{j=0}^{n-1} c^j} \end{cases} \tag{A.1}$$

With $0 \leq a(n) \leq a(n-1) \leq \cdots \leq a(2) \leq 1$,

2. We determine the variation of $a(n)$. We also have $a(n) \leq a(n-1)$, then (A.1) is equivalent to:

$$\max \left\{ \begin{array}{l} 0, \quad \dfrac{c}{1+c} a(n-1), \quad \dfrac{\sum_{j=1}^{2} \binom{2}{j-1} c^j (1-c)^{2-j} a(n-3+j)}{\sum_{j=0}^{2} c^j}, \ldots, \\ \dfrac{\sum_{j=2}^{n-1} \binom{n-1}{j-1} c^j (1-c)^{n-j-1} a(j)}{\sum_{j=0}^{n-1} c^j} + \dfrac{c(1-c)^{n-2}}{\sum_{j=0}^{n-1} c^j} \end{array} \right\}$$

$$\leq a(n) \leq a(n-1).$$

3. As $0 < c < 1$ and $a(n-1) \leq 1$ the variation of $a(n)$, is bounded. Then analyzing the upper and lower bound we obtain a new system in the variable $a(n-1), \ldots, a(2)$:

$$\begin{cases} a(n-1) \geq 0 \\ a(n-1) - \dfrac{c(1-c)}{1+c(1-c)} a(n-2) \geq 0 \\ \vdots \\ a(n-1) - \dfrac{\sum_{j=2}^{n-2} \binom{n-1}{j-1} c^j (1-c)^{n-j-1} a(j)}{\sum_{j=0}^{n-1} c^j - (n-1)\, c^{n-1}} \geq \dfrac{c(1-c)^{n-2}}{\sum_{j=0}^{n-1} c^j - (n-1)\, c^{n-1}} \end{cases}$$

2nd step: Eliminating $a(n-1)$ and working as in the first step we obtain the following system:

$$\begin{cases} a(n-2) \geq 0 \\ a(n-2) \geq \dfrac{c(1-c)^2}{1+c(1-c)^2} a(n-3) \\ \vdots \\ a(n-2) \geq \dfrac{\sum_{j=2}^{n-3} \binom{n-1}{j-1} c^j (1-c)^{n-j-1} a(j) + c\,(1-c)^{n-2}}{\sum_{j=0}^{n-1} c^j - (n-1)\, c^{n-1} - (n-2)\, c^{n-2}} \end{cases}$$

which $0 \leq a(n-2) \leq \cdots \leq a(2) \leq 1$. We observe that in each step we obtain an equivalent system with one variable less.

Step (n-3): Here we have the following system:

$$\begin{cases} a(3) \geq 0 \\ a(3) \geq \dfrac{c(1-c)^{n-3}}{1+c(1-c)^{n-3}} a(2) \\ a(3) \geq \dfrac{(n-1)\, c^2\, (1-c)^{n-3} a(2) + c(1-c)^{n-2}}{\displaystyle\sum_{j=0}^{n-1} c^j - \sum_{j=3}^{n-1} \binom{n-1}{j-1} c^j (1-c)^{n-j-1}} + \end{cases}$$

We want to eliminate $a(3)$. Using properties of combinatorics numbers and operating we have:

$$a(2) \geq \frac{c(1-c)^{n-2}}{1+c(1-c)^{n-2}}$$

As $0 \leq a(2) \leq 1$, the variation of $a(2)$ is bounded by: $\frac{c(1-c)^{n-2}}{1+c(1-c)^{n-2}} \leq a(2) \leq 1$. Working backwards we obtain the variation of the others variables: $\frac{c(1-c)^{n-2}}{1+c(1-c)^{n-2}} \leq a(n) \leq \cdots \leq a(2) \leq 1$ □

Proof of Theorem 5. We should prove that:

1. $\sum_{i \in N} \varphi_i(v) = v(N)$ and 2. $\varphi_i(v) \geq v(\{i\})$ for each $i \in N$.

Let us prove 1. Splitting the sums in the Shapley value for the innovator and using that all the $\varphi_i(v)$ have the same value, we obtain:

$$\sum_{i \in N} \varphi_i(v) = \varphi_1(v) + \ n\varphi_i(v). \tag{A.2}$$

Using Theorem 4 in (A.2),

$$\frac{1}{2} na(n) - \frac{1}{n+1} \sum_{s=0}^{n} w(S) + n \times \left[\frac{1}{2} a(n) + \sum_{s=0}^{n-1} \frac{n-s}{n(n+1)} (w(S \cup \{i\}) - w(S))\right] \quad \text{with } i \in U. \tag{A.3}$$

As the function $w(S \cup \{i\})$ depends only on the cardinality s of the set S, (Remark 1), we denotes $w(s)$ and $w(s+1)$, and replacing in (A.3) we have:

$$na(n) - \frac{1}{n+1} \sum_{s=0}^{n} w(s) + n \left[\sum_{s=0}^{n-1} \frac{n-s}{n(n+1)} (w(s+1) - w(s))\right]. \tag{A.4}$$

Operating in (A.4) we have: $\sum_{i \in N} \varphi_i(v) = na(n) - 0$ and $na(n) = v(N)$, thus we have: $\sum_{i \in N} \varphi_i(v) = v(N)$.

Let us prove 2. $\varphi_i \geq v(\{i\})$ for each $i \in N$. Let us consider the Shapley value for the innovator ($i = 1$):

$$\varphi_1(v) = \sum_{s=0}^{n} \frac{1}{n+1}(u(S \cup \{1\}) - w(S)) \geq \sum_{s=0}^{n} \frac{1}{n+1}(w(S) + w(\{1\}) - w(s)) = 0.$$

The last inequality follows from Theorem 2.

By the Definition $1 : v(\{1\}) = 0$, we obtain: $\varphi_1(v) \geq v(\{1\})$.

Let us consider the Shapley value for the users,($i = 2, \ldots, n+1$). Uusing Theorem 2 and by the Definition 1, we have: $\varphi_i(v) \geq \frac{1}{2}a(n) + \frac{1}{n(n+1)}\sum_{s=0}^{n-1}(n-s)$ $ca(n)$. Using that $\sum_{s=0}^{n-1}(n-s) = \frac{n(n+1)}{2}$ and $0 < c < 1$ in the last sum $\varphi_i(v) \geq$ $ca(n)$. Then by the Definition 1, $w(\{i\}) = ca(n)$ we have that: $\varphi_i(v) \geq v(\{i\})$. □