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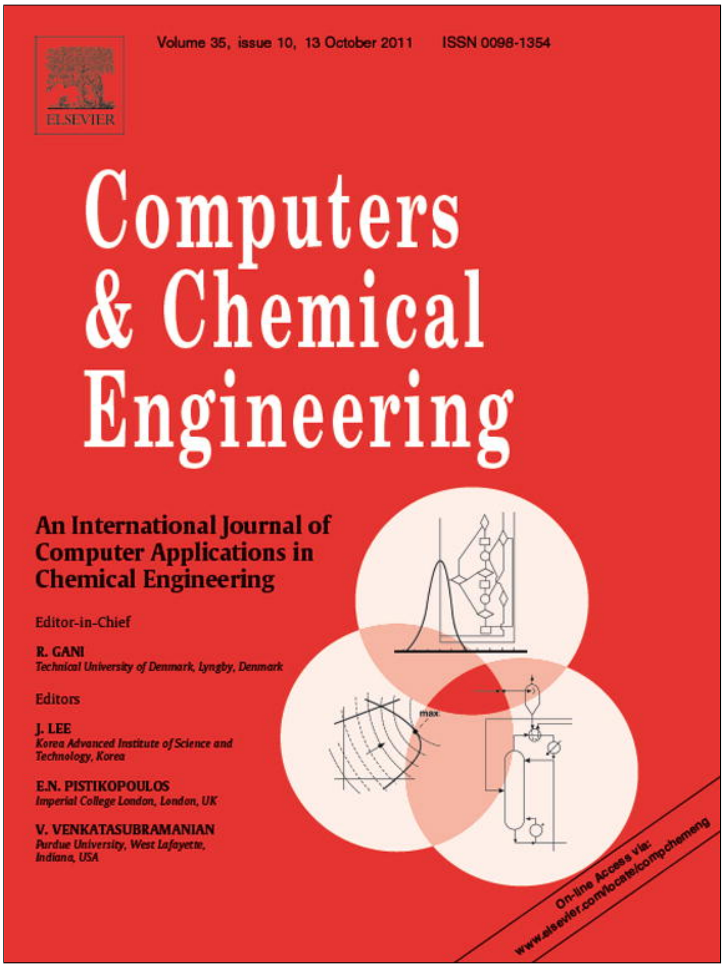

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Plant-wide control strategy applied to the Tennessee Eastman process at two operating points

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ABSTRACT

This work presents a new plant-wide control strategy able to be applied on large scale chemical plants. It is based on an extension of the non square relative gain array (NRG) theoretical concepts, introduced by Chang and Yu (1990), and the generalized relative disturbance gain (GRDG) presented in Chang and Yu (1992). The extension of the NRG is useful for searching the best group of controlled variables (CVs) independently of the problem dimensionality. Meanwhile, the extension of the GRDG allows configure the loops pairing by considering the trade-off between servo and regulator behavior. It can be done thanks to define a proper function, named net load effect, accounting both set point and disturbances effects. Even though these concepts are not new, the main contribution of this paper is the selection of the adequate objective function. It is mathematically expressed in a new way, in terms of Frobenius norm of specific matrices related with the models of the plant and very useful for evaluating the process interaction. Then, it drives the search supported by genetic algorithms (GA), which evaluates all the possible combinations of input–output variables. It allows to solve successfully and with less computational effort the combinatorial optimization problem, even though the high dimension usually involved in large scale chemical plants. The use of the relative gain array (RGA) can also be considered for pairing purpose, but in some cases it could drive to a less effective structure. The use of relative normalized gain array (RNGA) for pairing the selected CVs with the most suitable MVs is able to lead to best control structures only if a dynamic model of the plant is available. Therefore, it must be emphasized that this approach is developed for working in cases where only steady-state plant information is available. However, if a dynamic model is disposable too the algorithm is extended to use it. In addition, a mathematical demonstration is presented so as to understand why is possible to find a well conditioned control structure. The methodology is tested in the Tennessee Eastman (TE) process at the base case proposed by Downs and Vogel (1992), and at an optimized working point presented by Ricker (1995). Both working points show two quite different scenarios. Thus, a set of dynamic simulations for both cases and the hardware requirements compared to the previous suggested are given to proof the capacity of this approach.
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1. Introduction

Generally, from the process synthesis stage some partial control objectives are defined (i.e. product quality, product rate, stabilization of some inherent unstable behavior, active constraints, etc). But, there may be hundreds or thousands of additional variables to decide which could/should be controlled to improve the overall process behavior (either for set point changes or disturbances) is not a trivial problem. The plant-wide control structure selected affects the investment cost (amount of sensors, actuators, controllers) and the final dynamic control performance. Reducing both the heuristic load and the design complexity at this stage (using a systematic methodology) may be helpful to develop suitable control policies quickly and effectively.

Process control researchers have developed many systematic plant-wide control methodologies and applied them to chemical processes. These methodologies can be classified based on heuristic and mathematic tools (Larsson & Skogestad, 2000).

On the heuristic approaches the pioneering work of Buckley (1964) is reported in the first place on plant-wide control. The main issues on this area were introduced there and presented what is still in the industrial approach to plant-wide control nowadays. Other important author on this category is Luyben, Tyreus,

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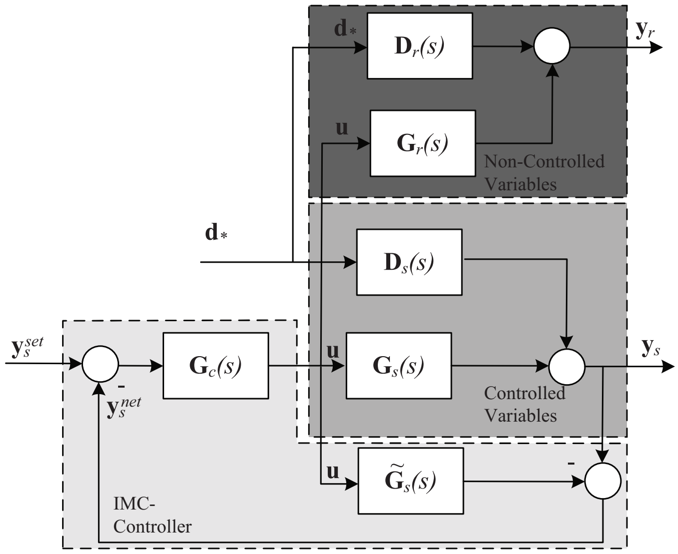

Fig. 1. IMC-controller structure.

and Luyben (1998) with a nine-step approach, that was developed based on the process experience of the group. More recently, Konda, Rangaiah, and Krishnaswamy (2005) proposed to obtain the plant-wide control structure using an integrated framework of simulation and heuristics which uses steady-state and dynamic simulation to support the taken decisions. All the proposed methodologies mentioned above are systematic in nature and addressed many of the major issues involved in the plant-wide control problem, such as the effects of recycles and energy integration. The main result from these methodologies are different ways to achieve a decentralized plant-wide control structure.

The second category relies on a rigorous mathematical framework of dynamic theory, constrained optimization and systems analysis. The mathematical approaches, although rigorous, are often very difficult to formulate for large scale systems and, in many cases, they turn computationally very intensive. However, several authors suggest approaches in this category. Cao and Saha (2005) developed an improved and more efficient algorithm of the “branch and bound (BAB)” that provided a global ranking to all possible input and output combinations. Based on this ranking an efficient control structure with least complexity for stabilizing control is detected which leads to a decentralized proportional controller. Cao and Kariwala (2008) presented a bidirectional BAB algorithm, i.e. the branch pruning is considered in both upward (gradually increasing subset size) and downward directions simultaneously. Thus, the total number of subsets evaluated is reduced dramatically for efficient handling of large-scale processes. Robinson, Chen, McAvoy, and Schnelle (2001) presented an approach based on splitting the optimal controller gain matrix that results from solving an output optimal control problem into feedback and feedforward parts, and from these parts can be obtained the information to designing decentralized plant-wide control system architectures, or if it is preferred a model predictive control (MPC). Jorgensen and Jorgensen (2000) presented the problem of selecting the control structure formulating a special mixed integer linear program (MILP) employing cost coefficients which are computed using Parseval’s theorem combined with relative gain arrays (RGA) and internal model control (IMC) concepts.

As an example of a combination between mathematical and heuristic approaches can be cited Skogestad (2000, 2004). In those works, the focus is done on the selection of the controlled variables that keeping them constant, the process is maintained close to the optimum when disturbances and control errors are present. The control that accomplish with this goal is called self-optimizing. Alstad and Skogestad (2007) showed how to select the CVs as a linear combination of measurements in order to obtain a minimum deviation from the optimal value, i.e. a control structure with better self-optimizing characteristics. More recently, in Suraj Vasudevan, Rangaiah, Konda, and Tay (2009) have found the control structures following three of the approaches cited before for a vinyl acetate monomer plant. The analysis of the results indicated that while all the procedures gave stable control structures, the integrated framework and self-optimizing in Konda et al. (2005) and Skogestad (2004) respectively gave more robust control structures than the resulting from the practical experience and engineering judgment approach proposed by Luyben et al. (1998).

It is remarkable that in the majority of the works cited above reported the use of the pioneering work of Bristol (1966), the relative gain array (RGA) based techniques for control-loop configuration. Even though, it is well-known and has a widespread industry applications, some limitations are recognized. Based on these, several new extensions of the RGA have appeared in the literature. Particularly, in this work, the non square relative gain array (NRG), introduced by Chang and Yu (1990), the generalized relative disturbance gain (GRDG) presented in Chang and Yu (1992) and the recently developed relative normalize gain array (RNGA) together with the RGA have been considered to generate the most critical stages of the new systematic approach presented here. The stage dedicated to determine the best group of controlled variables (CVs) is implemented on the basis of an extension of the NRG theoretical concepts. In the work presented by Chang and Yu (1990) the NRG calculation is limited to the problem dimensionality. In this work, the objective function to be minimized for defining the CVs is the sum of the square deviations in steady-state of the uncontrolled output variables (SSD_{yr}). The optimal solution can be found by the use of genetic algorithm (GA) independently of the problem dimension. It allows to solve successfully the combinatorial optimization problem, even though the high dimension involved in large scale chemical plants. On the other side, the GRDG theoretical concepts presented in Chang and Yu (1992) were limited to consider only disturbance effects, one by time and did not consider the set point changes effect. Thus, the extension given here, allows configure the loops pairing by considering the trade-off between servo and regulator behavior. It can be done thanks to define a proper function, named net load effect, containing both set point and critical disturbance effects. The relative importance between servo and regulator behavior is handled through specific weighting parameters. For selecting the best pairs between CVs and the manipulated variables (MVs) the minimization of the net load effect is done subjected to find a proper plant model, in the context of IMC design.

The complete methodology presented here takes into account two possibilities: the availability or not of the process dynamic model. For the first option it is recommended checking the control structure suggested by the RGA with that proposed by the relative normalize gain array (RNGA), recently introduced by He, Cai, Ni, and Xie (2009). The second one corresponds to the procedure detailed above. Therefore, the paper presented here summarizes, integrates and extends a systematic methodology for plant-wide control which has been tested previously, by the authors, in several academic case studies (Molina, Zumoffen, & Basualdo, 2009; Nieto, Zumoffen, Basualdo, & Outbib, 2010; Zumoffen & Basualdo, 2009; Zumoffen, Basualdo, & Ruiz, 2009; Zumoffen, Molina, & Basualdo, 2010).

The approach presented here is applied in the Tennessee Eastman (TE) process, it is selected because is a very useful case for performing rigorous comparisons because several authors have tested their plant-wide control techniques on it. On the other hand, this plant represents a good step ahead to generalize the

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methodology for large scale systems. The results are analyzed for two working points, called base case and optimal case. It is done because in the work of Ricker (1995) was proposed to improve the TE control problem by considering that the operating point given in Downs and Vogel (1992) must be modified to the optimal one found there. Hence, each of them presents quite different scenarios and some authors show the plant-wide control to the optimal or to the base case. All the control structures (CSs) proposed in the literature on the TE process, have similar dynamic responses and fulfill with the main objectives stated opportunely by Downs and Vogel (1992). In other words, all the CSs proposed for TE for both working points demonstrated to be able for achieve the main objectives and for this reason all of them were published. In this context, the major contribution of the paper proposed here is the selection of the specific objective function, mathematically expressed in terms of the trace of the matrices representing the models of the plant and driving the search by GA, which evaluates all the possible combinations of input–output variables. Both elements are very useful to avoid as much as possible heuristic considerations, so as to find new alternatives of pairing that would be unthinkable for practitioners of plant-wide control. Thus, a new feasible control structure (CS) can be obtained despite the use of tools which are not new for the process control community. Finally, a comparison with other plant-wide control structures, previously published, applied to the emblematic case of the TE process is included so as to demonstrate the potentiality of the method presented here.

2. Background and tools

2.1. Definitions

Let's consider the process transfer function matrix $\mathbf{G}(s)$ with n inputs and m outputs:

$$\mathbf{y}(s) = \mathbf{G}(s)\mathbf{u}(s) + \mathbf{D}(s)\mathbf{d}_*(s) \tag{1}$$

where $\mathbf{G}(s)$ and $\mathbf{G}_d(s)$ are process transfer function matrixes of $m \times n$ with $m > n$ and $m \times p$ respectively; $\mathbf{y}(s)$, $\mathbf{u}(s)$ and $\mathbf{d}_*(s)$ are the output, input and disturbance vector of $m \times 1$, $n \times 1$ and $p \times 1$ respectively. The plant model can be divided into two subsystems: one square, that includes the n output variables, and other, generally non-square one, that includes the remaining variables. Mathematically:

$$\mathbf{y}_s(s) = \mathbf{G}_s(s)\mathbf{u}(s) + \mathbf{D}_s(s)\mathbf{d}_* \tag{2}$$

$$\mathbf{y}_r(s) = \mathbf{G}_r(s)\mathbf{u}(s) + \mathbf{D}_r(s)\mathbf{d}_*, \tag{3}$$

where $\mathbf{G}_s(s)$, $\mathbf{D}_s(s)$, $\mathbf{G}_r(s)$ and $\mathbf{D}_r(s)$ are the process transfer function matrixes of $n \times n$, $n \times p$, $(m-n) \times n$ and $(m-n) \times p$ respectively. It is shown in Fig. 1 which is controlled via internal model control (IMC), where $\tilde{\mathbf{G}}_s(s)$ is the nominal model of $\mathbf{G}_s(s)$ and $\mathbf{G}_c(s)$ is an $n \times n$ controller transfer function.

2.2. Relative gain array (RGA)

For square systems as $\mathbf{G}_s$, the closed-loop gain is defined as the gain between y_i and u_j when all other outputs are under perfect control, i.e. the gain of u_j when y'_ks are held constant for all $k \neq i$. Generally, only steady-state is considered ($s=0$) in this work, so s is not explicit in the equations. When dynamic scenarios are considered, the Laplace variable will be written. Then, the relative gain can be defined as (Bristol, 1966):

$$\lambda_{ij} = \frac{[(\partial y_i)/(\partial u_j)]_{u_{k,k\neq j}}}{[(\partial y_i)/(\partial u_j)]_{y_{k,k\neq i}}} = \frac{\text{open-loop gain}_{[ij]}}{\text{closed-loop gain}_{[ji]}}. \tag{4}$$

Working mathematically Eq. (4) the expression of the RGA in a reduced form as

$$\Lambda = \mathbf{G}_s \otimes \left(\mathbf{G}_s^{-1}\right)^T \tag{5}$$

where $\otimes$ denotes the element-by-element product.

The elements of the RGA can be numbers that vary from very large negative to very large positive values. If the λ_{ij} is close to 1, means that a slight interaction effect on this specific control loop is performed when the other loops of the multivariable system are closed. Values around 0.5 or lower indicate a certain interaction effect. Negative magnitudes indicate a not desirable situation because the direction of the effect of the manipulated variable on its correspondent controlled variable in the open loop is opposite to the direction of the closed loop. Therefore, if λ_{ij} close to 1 is the best condition to avoid the interaction effect.

However, it has been recognized that the interaction effect not always deteriorates the performance of the controlled process. It depends of the specific plant objectives, if the load rejection is the most important criterion for deciding what variables to pair and what controller structure is the best one (see Luyben and Luyben, 1997, chap. 12), the interaction could be helpful.

2.3. Relative gain array for non-square systems (NRG)

For a non-square system with more outputs than inputs, a certain number of variables will work uncontrolled, hence they will suffer a deviation from their original operating point. In this context, the perfect control can be defined accounting the following two possibilities: Perfect control in least square sense, i.e. the 2-norm of the steady-state error vector is minimized Perfect control over the n variables and minimize the 2-norm steady-state deviation vector of the uncontrolled CV's. Using the first definition it can be proofed (see Chang and Yu, 1990) that steady-state closed-loop relationship, under the least-square perfect control, between $\mathbf{u}$ and $\mathbf{y}^{set}$ becomes

$$\mathbf{u}^{opt} = \mathbf{G}^{\dagger}\mathbf{y}^{set}, \tag{6}$$

where $\mathbf{y}^{set}$ are the reference values of the m outputs, $\mathbf{u}^{opt}$ is the optimal $\mathbf{u}$ under this definition of perfect control and $\mathbf{G}^{\dagger}$ is the Moore-Penrose pseudo-inverse of $\mathbf{G}$. Then, using direct analogy between this case and the case of square system, it is obtained the Relative Gain Array for Non-Square systems (NRG):

$$\Lambda^N = \mathbf{G} \otimes \left(\mathbf{G}^{\dagger}\right)^T. \tag{7}$$

It is important to emphasize that the NRG is output scaling dependent when $m > n$. Since the concept of least-square perfect control is used in the definition of the closed-loop gain, the output scaling is equivalent to the weighted least square. The relative importance of each output is directly related to the magnitude of the output scaling factor weights.

However, generally the second definition is used in the practice because the controller structure is generally implemented via PID modes. In this way, n output variables can be chosen to be perfectly controlled at steady-state. The NRG is helpful for obtaining a possible solution for choosing the variables that have smallest row sum of the NRG (Chang & Yu, 1990). However, this is only a suboptimal solution, to obtain the best solution a more suitable tool is presented in the subsection "Selection of controlled variables", which represents one of the contribution given in this work.

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2.4. Generalized relative disturbance gain (GRDG)

From the controlled process via IMC shown in Fig. 1, it is possible to express the CV's as a function of the disturbances $\mathbf{d}_*$ as

$$\mathbf{y}_s(s) = \left[\mathbf{I} - \left(\tilde{\mathbf{G}}_s(s)\mathbf{G}_c(s)\right)\right] \left[\left(\mathbf{I} + \left(\mathbf{G}_s(s) - \tilde{\mathbf{G}}_s(s)\right)\mathbf{G}_c(s)\right)\right]^{-1} \mathbf{D}_s(s)\mathbf{d}_* \quad (8)$$

From Eq. (8), it becomes clear that a noninteracting control system is not necessary the best control structure from the disturbance rejection point of view, i.e. interaction in some cases, can improve the load rejection. Taking into account this statement, the Generalized Relative Disturbance Gain (GRDG) is proposed in Chang and Yu (1992). This GRDG is defined as

$$\mathrm{GRDG} = \frac{\text{disturbance closed-loop effect}}{\text{disturbance open-loop effect}} \quad (9)$$

Then, the GRDG is a vector for scalar disturbances, i.e. there will be p GRDGs for a process with p disturbances. This GRDG is dependent of $\mathbf{G}$, $\tilde{\mathbf{G}}$, and $\mathbf{D}$. Mathematically it becomes:

$$GRDG\left[\mathbf{G}_s, \tilde{\mathbf{G}}_s, \mathbf{D}_s\right] = \mathbf{D}_s^* \oslash \mathbf{D}_s = \left(\tilde{\mathbf{G}}_s \cdot \mathbf{G}_s^{-1} \cdot \mathbf{D}_s\right) \oslash \mathbf{D}_s \quad (10)$$

where, $\oslash$ is the element-by-element division, $\mathbf{D}_s^*$ denotes the closed-loop effect for a specific structure considering $\mathbf{G}_c = \tilde{\mathbf{G}}^{-1}$, and $\tilde{\mathbf{G}}$ is written as

$$\tilde{\mathbf{G}}_s = \mathbf{G}_s \otimes \Gamma \quad (11)$$

where Γ is

$$\Gamma = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \vdots & \ddots & \vdots \\ \gamma_{n1} & \cdots & \gamma_{nn} \end{bmatrix} \quad (12)$$

with γ_{ij} taking values into the binary code {0;1}. In this way, a 0 in γ_{ij} indicates that the transfer function $g_{s[ij]}$ of the gain matrix $\mathbf{G}_s$ is not considered for the controller sinthesis.

Chang and Yu (1992) demonstrated that the best structure from the disturbance rejection point of view is that which produces the smallest indexes in the GRDGs. However, it becomes very hard to evaluate the best possible structures when several disturbances and CV's are involved in a large scale multivariable process. Furthermore, the net load and interaction effect is not considered for set-point changes. In this work, an improved version of the original GRDG is presented to reduce the net load effect and able to be applied on large scale chemical plants is proposed. It is thought for reducing the interaction effect over the process CV's.

2.5. Relative normalized gain array (RNGA)

When a dynamic model is available, it is possible to use a recently presented control-loop configuration criterion for multivariable processes presented in He et al. (2009). This is an interaction measurement called RNGA which evaluates the control-loop interactions taking into account the dynamics of the process. The RNGA is systematically obtained by analyzing the process characteristics from both steady-state and transient perspectives and derives a feasible solution for the problem.

The calculation of the RNGA is done in the same way that the RGA, but the normalized gain is used instead of steady-state gain. This normalized gain is defined as

$$g_{N,ij} = \frac{g_{ij}}{\tau_{ar,ij}} \quad (13)$$

where $g_{N,ij}$ is the ij-th element of $\mathbf{G}_s$ and $\tau_{ar,ij}$ is the average residence time between the output i and input j, i.e. a measure of how fast the output i is with respect to the input change j. the average residence time. Thus the RNGA is

$$\Lambda_N = \mathbf{G}_N \oslash \left(\mathbf{G}_N^{-1}\right)^T \quad (14)$$

The RNGA presents the same important properties of the RGA, but also it has information about transient of the system. This method is very easy to be implemented and can be a very useful tool for designing decentralized systems.

2.6. Genetic algorithms (GA)

The genetic algorithm is a stochastic global search method that mimics the metaphor of natural biological evolution. Mainly, the GA operates on a population of potential solutions, N_i, applying the principle of survival of the fittest to produce better and better approximation to the solution. In each generation a new set of individuals, $\mathbf{C}_i$, is created based on their level of fitness respect to the specific functional cost for each problem, $FC(\mathbf{C}_i)$, and breeding them together using operators borrowed from natural genetics. Thus the evolution of the population is developed towards the individuals that are better suited to their environment. Specifically, in this work a code version developed in Chipperfield, Fleming, Pohlheim, and Fonseca (1994) for Matlab platform is used.

The individuals, $\mathbf{C}_i = [c_1, \ldots, c_{N_c}]$, are encoded as strings (chromosomes) composed over some alphabet, so that the chromosome values, c_j, are uniquely mapped onto the decision variables domain. Decoding the chromosome representation, the decision variables (individuals) can be evaluated considering some performance or fitness function. This function establishes the basis for pairs selection of individuals that will be mated together during reproduction. In this phase, each individual is assigned to a fitness value derived from this objective function and the selection over the population is made with a determined probability according to their relative fitness. Thus, the recombination process is carried out to produce the next generation.

Typically, the initial population is adjusted between 2 and 20% of the problem dimension for a small combinatorial size (Chipperfield et al., 1994) and about $Nc*100$ to large scale problems (Musulin, Bagajewicz, Nougúes, & Puigjaner, 2004; Musulin, Yelamos, & Puigjaner, 2006). Generally, the GA termination criteria is the maximum generation number, N_g or a level of fitness. The individuals selection is performed by means of the roulette wheel method according to their fitness measure, thus the best $N_{sel} = N_i/2$ individuals are retained. The production of new chromosomes is developed by the crossover operator, in this case double-point method with a probability of P_{co}. Analogously to the natural evolution, the mutation produces a new genetic structure and basically is applied with a low probability, P_m. Mutation generally tends to inhibit the possibility of converging to a local optimum.

The following steps summarize the GA procedure to find the optimal set of individuals along generations,

Initialization: the random initial population, $\mathbf{P}_j = [\mathbf{C}_1^T, \ldots, \mathbf{C}_{N_i}^T]^T$ of dimension $N_i \times N_c$, is defined with $j = 0$. In addition, N_i, N_g, N_{sel}, P_{co} and P_m are selected.
Fitness evaluation: the functional cost is evaluated for each individual, $\mathbf{C}_i$, from the actual population set, $\mathbf{P}_j$. In addition, the individual with the best fitness value is stored in the best population set, $\mathbf{P}_b$.
While: the termination criteria is false do
  Selection: the N_{sel} best individuals are selected using their relative fitness values and stored in $\mathbf{P}_s$. The $N_i - N_{sel}$ remaining individuals are discarded.
  Recombination: the individuals in $\mathbf{P}_s$ are recombining by the crossover operator and stored in, $\mathbf{P}_r$.

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Mutation: the recombined individuals in $\mathbf{P}_r$ suffer the mutation process and new genetic structures are obtained and stored in $\mathbf{P}_m$.
Merging: both selected and mutated populations are merging together to give the next generation of individuals, $\mathbf{P}_j = [\mathbf{P}_s^T, \mathbf{P}_m^T]^T$, with $j = j + 1$ and go to step **B**.

Else: if the termination criteria is true, the set of the best individuals for each generation is obtained from $\mathbf{P}_b$, with a dimension of $j \times N_c$, and the optimization procedure stops.

Thus, the better suited individual to their environment is the best individual in the latest generation.

3. Systematical methodology for plant-wide control structures

The procedure begins by considering that the plant could be driven to its optimal operation point. From this stage it can be found a number of variables that has to be controlled because they are active constrains. Other possibility is considering the plant working at specific point where a set of objectives has to be met, so they are closely related to the selection of a number of controlled variable. The following step selects those controlled variables that are key for stabilizing purposes. Both steps are suggested here for reducing the problem dimensionality even though splitting the synthesis into two phases is suboptimal because assigning certain manipulated variables in the first step can adversely deteriorate the performance of the controllers to be developed in the second stage. Therefore, it is necessary an iteration between the two steps (Arkun and Downs, 1990). The following steps consist on one of the most important issues in plant-wide control problem which is to find the CV's. It lays in the fact that once the CV's are chosen, the steady-state operation of the process is defined (Larsson, Hestemtum, Hovland, & Skogestad, 2001; Skogestad, 2000).

The main steps of the proposed algorithm to find the plant-wide control structure are shown in Fig. 2.

3.1. Optimization and degrees of freedom analysis

The optimization stage is considered here for the beginning of the methodology since the main goal of any chemical plant is to obtain the major economic benefit that is possible. In this way, the operation of the plant has to be optimized taking into account economic goals. Doing this at first, it is possible to define which variables have to be controlled. Therefore, it is recommended controlling those variables that result active constraints such that the inequalities and equalities conditions.

3.2. Plant stabilization

Most of the process are open loop unstable, therefore, the minimal number of loops to achieve the process stability must be selected. These stabilizing loops generally include the levels in vessels for liquid streams since they could turn the process unstable due to they are pure integrator in nature. In this approach it is suggested to select these loops since the beginning of the procedure due to they have an important influence in the plant-wide control. For the stabilizing objective, the method given by McAvoy (1998)is very helpful since it is systematic and profitable. It consists on screening several inventory control strategies using steady-state information and rate of change in integrator variables and redefining the RGA and Niederlinsky Index (Niederlinsky, 1971) for process with pure integrators.

3.3. Selection of controlled variables

Once the process is stabilized the chosen CV's (guests) are considered under perfect control in least-square sense, considering that n variables are perfectly controlled and minimizing the sum of square errors of the remaining $m - n$ output variables. It drives to a controller structure that keeps the output variables of the plant as close as possible to the nominal point when some disturbance

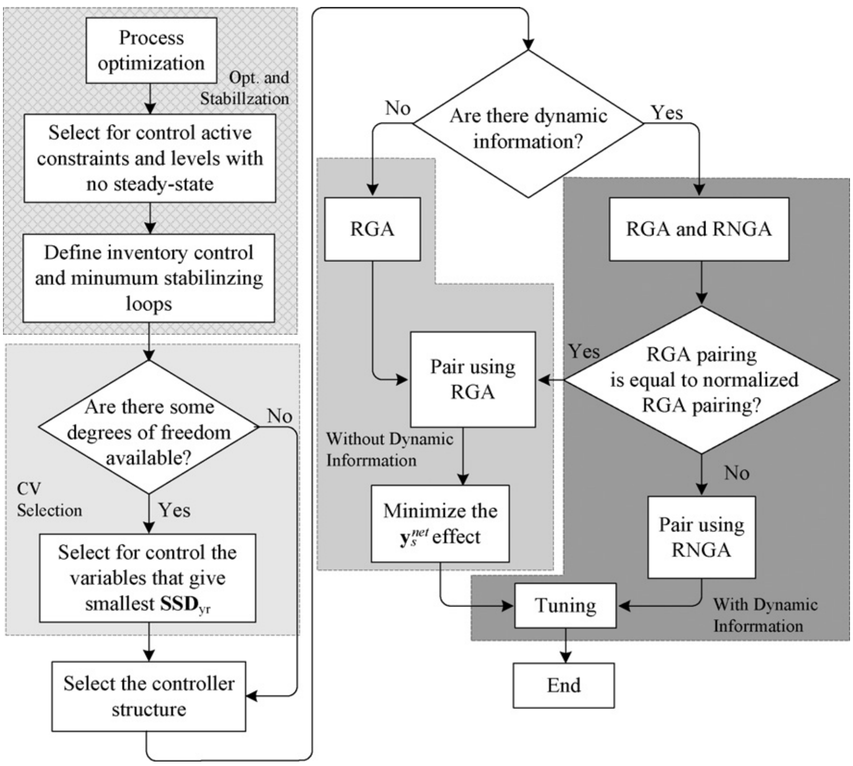

Fig. 2. Methodology for determining the plant-wide control structure.

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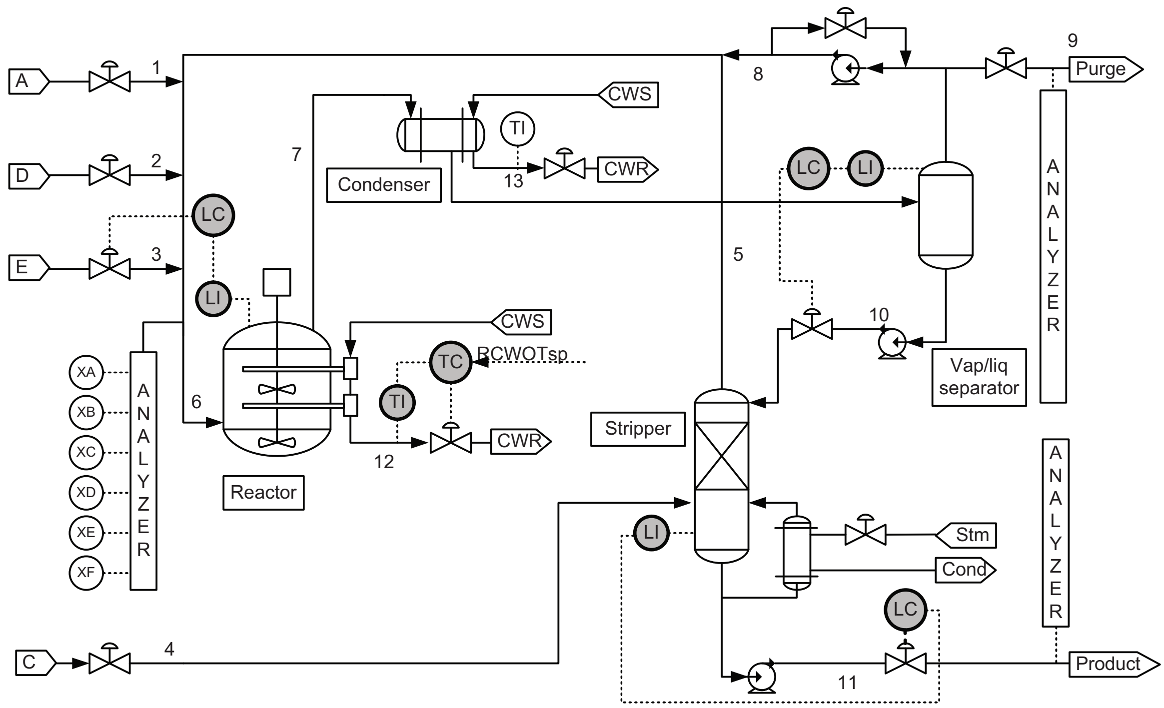

Fig. 3. TE process after stabilization.

and set-point changes are introduced. Using the process shown in Fig. 1, the outputs of the system can be expressed as:

$$\mathbf{y}_s = \mathbf{y}_s^{set} \tag{15}$$

$$\mathbf{y}_r = \mathbf{G}_r\mathbf{G}_s^{-1}\mathbf{y}_s^{set} + \left(\mathbf{D}_r - \mathbf{G}_r\mathbf{G}_s^{-1}\mathbf{D}_s\right)\mathbf{d}_* \tag{16}$$

when $\mathbf{G}_c = \tilde{\mathbf{G}}_s^{-1}$. Then, Eq. (16) can be rewritten as

$$\mathbf{y}_r = \mathbf{S}_{sp}\mathbf{y}_s^{set} + \mathbf{S}_d\mathbf{d}_* \tag{17}$$

where $\mathbf{S}_{sp} = \mathbf{G}_r\mathbf{G}_s^{-1}$ and $\mathbf{S}_d = \mathbf{D}_r - \mathbf{G}_r\mathbf{G}_s^{-1}\mathbf{D}_s$.

As it can be seen from Eqs. (15)–(17), the deviation of those uncontrolled outputs at steady-state when disturbances and set-point changes are present only depends on the selection of the CV's

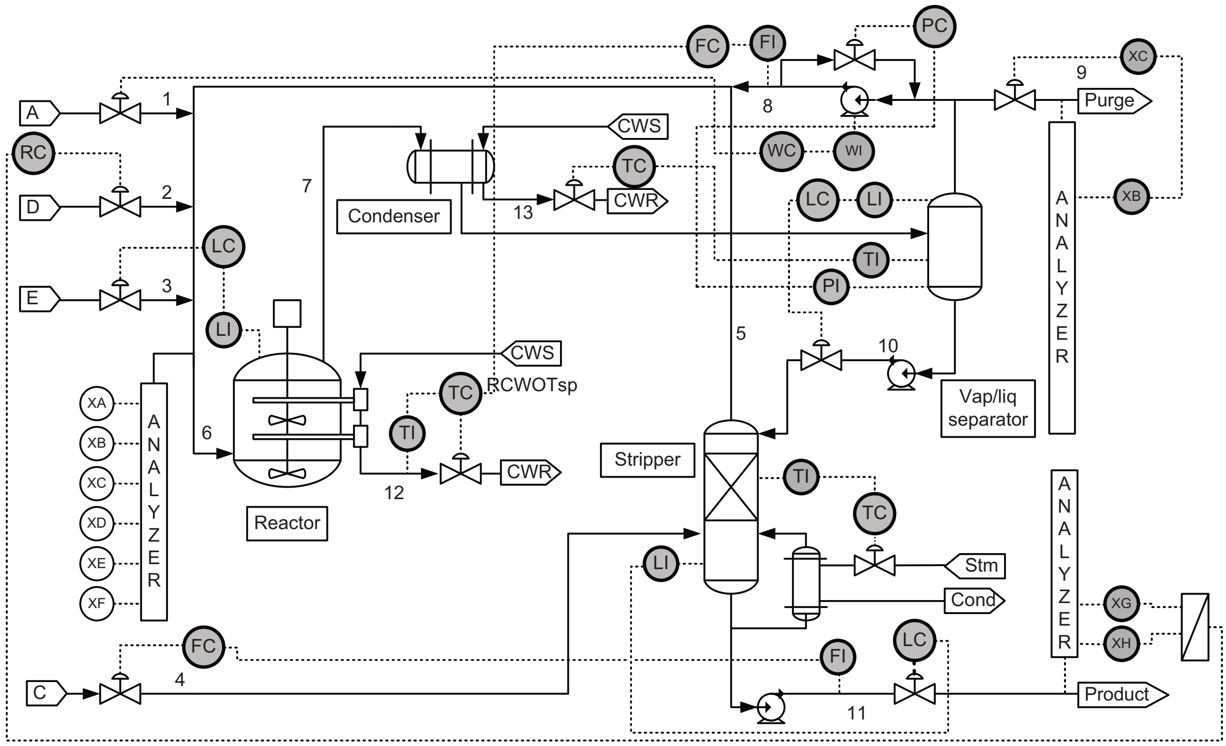

Fig. 4. TE process control structure (base case and RNGA approach).

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of a given plant. The idea is to find the set of CV's that minimize the sum of the deviations in the square sense, expressed as follows:

$$\begin{aligned} SSD_{yr} &= \sum_{i=1}^{n} ||\mathbf{e}_{sp}(i)||_2^2 + \sum_{j=1}^{p} ||\mathbf{e}_d(j)||_2^2 \\ &= \sum_{i=1}^{n} ||\Lambda_2 \mathbf{S}_{sp} \Lambda_1 \mathbf{y}_{set}^n(i)||_2^2 + \sum_{j=1}^{p} ||\Theta_2 \mathbf{S}_d \Theta_1 \mathbf{d}_*^p(j)||_2^2 \end{aligned} \tag{18}$$

where $\mathbf{e}_{sp}(i)$ and $\mathbf{e}_d(j)$ are the vectors of deviations corresponding to the y_r outputs from their nominal operating point values when an unitary change happens in the i set point and j disturbance respectively. On the other hand, the diagonal weighing matrices Λ_1, Λ_2, Θ_1 and Θ_2 allow to include the process control objectives such as set point/disturbance magnitudes and the relative degree of importance among the overall outputs. The vectors $\mathbf{y}_{set}^n(i)$ and $\mathbf{d}_*^p(j)$ of dimension $n \times 1$ and $p \times 1$ have magnitude 1 in the position i and j respectively and 0 in the remaining positions. In Appendix A it is demonstrated that Eq. (18) can be written as

$$SSD_{yr} = \mathrm{trace}\left(\Lambda_1^2 \mathbf{S}_{sp}^T \Lambda_2^2 \mathbf{S}_{sp}\right) + \mathrm{trace}\left(\Theta_1^2 \mathbf{S}_d^T \Theta_2^2 \mathbf{S}_d\right) \tag{19}$$

The expression given in Eq. (19) represents the introduced improvement here which allows doing the calculation independently of the problem dimensionality. Considering that there are n manipulated variables, and m potential CV's or measurements, the combinatorial problem becomes $(m!)/(n!(m-n)!)$. For the example analyzed here, the case of TE process, before the optimization procedure, $m=50$ and $n=12$. Then, the number of possible combination of CV's is 1.2140×10^{11}. Considering that it is a medium scale case, the number of combinations results computationally expensive to do it exhaustively. Most researchers on plant-wide control reduce the problem dimensionality by applying process based experience and engineering judgment. However, this could bring to non-optimal or even, infeasible solutions. In this work, it is proposed the use of GA to do rigorously the optimum selection of CV's. Some preliminary results, applied to lower dimensionality cases, about the optimal CVs selection, which is related to the sensor location accounting control configuration too can be found in Molina et al. (2009); Nieto et al. (2010); Zumoffen and Basualdo (2009); Zumoffen et al. (2009, 2010). Based on those previous successful results the methodology is applied here to more realistic problems such that the TE plant. Basically, the problem is solved encoding each individual $\mathbf{C}_i$ as a chromosome over a binary alphabet, where magnitude 1 indicates that the variable is selected for control and 0 for the opposite situation. In this way, each $\mathbf{C}_i$ will have as many genes as potential CV's. Finally, the minimization of Eq. (19) can be expressed as

$$\begin{aligned} \min_{\mathbf{C}_i} SSD_{yr} &= \min_{\mathbf{C}_i} \left[\mathrm{trace}\left(\Lambda_1^2 \mathbf{S}_{sp}^T(\mathbf{C}_i) \Lambda_2^2 \mathbf{S}_{sp}(\mathbf{C}_i)\right) + \mathrm{trace}\left(\Theta_1^2 \mathbf{S}_d^T(\mathbf{C}_i) \Theta_2^2 \mathbf{S}_d(\mathbf{C}_i)\right)\right] \\ &= \min_{\mathbf{C}_i} \left[||\Lambda_2 \mathbf{S}_{sp}(\mathbf{C}_i) \Lambda_1||_F^2 + ||\Theta_2 \mathbf{S}_d(\mathbf{C}_i) \Theta_1||_F^2\right] \end{aligned} \tag{20}$$

subject to

$$\det(\mathbf{G}_s(\mathbf{C}_i)) \neq 0 \tag{21}$$

where $||\cdot||_F$ is the Frobenius norm for matrices and the constraint in Eq. 21 guarantees that the optimal solution for Eq. 20 is also a feasible one. Can be observed that the optimal solution $\mathbf{C}_i$, parameterizes $\mathbf{G}_s$ and this particular selection of the CVs eventually affects the feasibility of the future IMC design.

From the optimization stated in Eq. 20 the following properties and conclusions can be obtained (see Appendix C),

1. The SSD_{yr} minimization decreases the steady-state effect on $\mathbf{y}_r$ (uncontrolled variables) produced by set points and disturbances modifications. Specifically, reduces the multivariable gains of $\mathbf{S}_{sp}$ and $\mathbf{S}_d$.
2. The properties of the matrix $\mathbf{G}_s$ are improved by increasing its minimum singular value, $\underline{\sigma}(\mathbf{G}_s)$.
   he $\underline{\sigma}(\mathbf{G}_s)$ increase produces that $\mathbf{G}_s$ be away from singularity, so its inverse $\mathbf{G}_s^{-1}$ exists. he condition number of $\mathbf{G}_s$, $\gamma(\mathbf{G}_s) = \bar{\sigma}(\mathbf{G}_s)/\underline{\sigma}(\mathbf{G}_s)$, decreases and tends to the minimum. This occurs because the maximum singular value $\bar{\sigma}(\mathbf{G}_s)$ is bounded. ence this search drives to a very well conditioned matrix $\mathbf{G}_s$ which guarantees that the elements in $RGA = \mathbf{G}_s \otimes (\mathbf{G}_s^{-1})^T$ be inside a good range of values.
3. It represents a quite satisfactory result accounting that the processes with high values in the RGA elements are very difficult or impossible to control (Grosdidier, Morari, & Holt, 1985; Skogetad & Morari, 1987). In addition this kind of processes are very sensitive to both modeling errors and multiplicative input uncertainties.
4. The maximization of $\underline{\sigma}(\mathbf{G}_s)$ does not necessarily lead to a gain minimization of $\mathbf{S}_{sp}$ and $\mathbf{S}_d$.
5. The minimization of $||\mathbf{S}_{sp}||_F^2$ produces that $\underline{\sigma}(\mathbf{G}_s)$ increases and tends to the maximum value allowed by the characteristics of the process, $\underline{\sigma}(\mathbf{G}_s)_{max}$. Eventually it may happen that $\underline{\sigma}(\mathbf{G}_s) = \underline{\sigma}(\mathbf{G}_s)_{max}$.
6. The minimization of $||\mathbf{S}_d||_F^2$ produces that $\underline{\sigma}(\mathbf{G}_s)$ increases, but in this case the maximum value reachable is bounded by the disturbance model characteristics, $\underline{\sigma}(\mathbf{G}_s) \leq \bar{\sigma}(\mathbf{G}_r)\bar{\sigma}(\mathbf{D}_s)/\bar{\sigma}(\mathbf{D}_r)$.

and these properties cannot be ensured by purely heuristic methods, due to they are not explicitly included.

A complete analysis about the parameters setting in GA for large-scale process is given in Zumoffen and Basualdo (2010).

3.4. Plant-wide control structure selection

The previous step defined the optimal selection of CV's based on minimized the SSD_{yr}. In this stage, the loop pairing criterion based on different tools according to the available plant information is proposed for defining the control structure configuration. Two different approaches are considered taking into account the availability or not of the dynamic process model.

3.4.1. Accounting with steady state model only

In case of only the steady state model is available, the performance of the controlled process can be improved by minimizing the impact produced over the CV's when set-point and disturbance changes occur, recognized as the net load effect ($\mathbf{y}_s^{net}$). It can be seen as a good tool for quantification the effect of the external variables because it is directly related to the plant-wide control structure. In this context, it resulted very useful considering the analysis given by Chang and Yu (1992) to evaluate the GRDGs in a simple way, which was previously introduced. However, it was defined for only one disturbance case, and there the set-point changes effect was not considered. In this paper, an extended version of that development is performed so as to obtain a proper function for solving more complex problems than those previously presented by Chang and Yu (1992).

An enhanced approach to minimize the $\mathbf{y}_s^{net}$ is proposed here, defining a new index and using optimization criteria for searching the best control structure.

Let's consider again the $\mathbf{G}_s(s)$ subsystem process shown in Fig. 1. Taking into account that disturbances and plant-model mismatches affect the responses of the controlled process, the controlled output can be expressed as

$$\mathbf{y}_s(s) = \tilde{\mathbf{G}}_s(s)\mathbf{G}_c(s)\mathbf{y}_s^{set}(s) + (\mathbf{I} - \tilde{\mathbf{G}}_s(s)\mathbf{G}_c(s))\mathbf{y}_s^{net}(s), \tag{22}$$

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Table 1
Output variables

Output variable	Mode 1 (base case)	Mode 2 (optimum)
Recycle flow, *xme*5 [kscmh]	26.902	32.18
Reactor feed, *xme*6 [kscmh]	42.339	47.36
Reactor pres., *xme*7 [kPa]	2750.00	2800.00
Reactor level, *xme*8 [%]	75.00	65.00
Reactor temp., *xme*9 [°C]	120.40	123.10
Separator tmp., *xme*11 [°C]	80.109	91.70
Separator pres., *xme*13 [kPa]	2633.7	2706
Stripper pres., *xme*16 [kPa]	3102.2	3326
Production rate, *xme*17 [m^3/h]	22.949	22.89
Stripper temp., *xme*18 [°C]	65.731	66.5
Comp. work, *xme*20 [kW]	341.43	278.90
%A in purge, *xme*29 [mol.%]	32.96	32.73
%B in purge, *xme*30 mol.%]	13.82	21.83
%C in purge, *xme*31 [mol.%]	23.98	13.11
%D in purge, *xme*32 [mol.%]	1.256	0.90
%E in purge, *xme*33 [mol.%]	18.58	16.19
%F in purge, *xme*34 [mol.%]	2.26	5.39
%G in purge, *xme*35 [mol.%]	4.84	6.62
%H in purge, *xme*36 [mol.%]	2.30	3.23
%D in product, *xme*37 [mol.%]	0.02	0.01
%E in product, *xme*38 [mol.%]	0.84	0.58
%F in product, *xme*39 [mol.%]	0.10	0.19
G/H mass ratio, $xme_{G/H}$ [(%kg.mol)/(%kg.mol)]	1	1

Table 2
Input variables

Input variable	Base case	Optimum case
D feed, *xmv*1 [%]	63.05	62.94
E feed, *xmv*2 [%]	53.98	53.15
A feed, *xmv*3 [%]	24.64	26.25
A + C feed, *xmv*4 [%]	61.30	60.56
Comp. rec. valve, *xmv*5 [%]	22.21	1.00
Purge valve, *xmv*6 [%]	40.06	25.77
Separator valve, *xmv*7 [%]	38.10	37.26
Steam valve, *xmv*9 [%]	47.44	1.00
Reactor coolant, *xmv*10 [%]	41.10	35.99
Condenser coolant, *xmv*11 [%]	18.14	23.95
Agitator speed, *xmv*12 [%]	50.00	100.00

Table 3
Operation costs

	Base case	Optimum case
Purge losses [$/h]	114.71	74.31
Product losses [$/h]	30.28	24.93
Compression [$/h]	18.30	14.96
Total cost [$/h]	170.61	114.37

where

$$\mathbf{y}_s^{net}(s) = \mathbf{A}(s)\mathbf{y}_s^{set}(s) + \mathbf{B}(s)\mathbf{d}_*(s) \tag{23}$$

$$\mathbf{A}(s) = [\mathbf{I} + (\mathbf{G}_s(s) - \tilde{\mathbf{G}}_s(s))\mathbf{G}_c(s)]^{-1}(\mathbf{G}_s(s) - \tilde{\mathbf{G}}_s(s))\mathbf{G}_c(s) \tag{24}$$

$$\mathbf{B}(s) = [\mathbf{I} + (\mathbf{G}_s(s) - \tilde{\mathbf{G}}(s))\mathbf{G}_c(s)]^{-1}\mathbf{D}_s(s), \tag{25}$$

$\mathbf{y}_s^{net}$ is introduced over the output and depends of a specific control structure. From Eqs. (22)–(25) it can be noted the effect introduced by the mismatch plant-model $\mathbf{G}_s(s) - \tilde{\mathbf{G}}_s(s)$.

In addition, it is shown that $\mathbf{y}_s^{net}$ effect is null in steady-state when $\mathbf{G}_c = \tilde{\mathbf{G}}_s^{-1}$. Thus the term $(\mathbf{I} - \tilde{\mathbf{G}}_s(s)\mathbf{G}_c(s))$ produces the same effect as an integral mode. Eqs. (24) and (25) show the specific impacts produced by the set-point and disturbance changes respectively in the $\mathbf{y}_s^{net}$. Then, if only steady-state information is used and assuming that $\mathbf{G}_c = \tilde{\mathbf{G}}_s^{-1}$

$$\begin{aligned}\mathbf{A} &= \mathbf{I} - \mathbf{G}_s\tilde{\mathbf{G}}_s^{-1}\\ \mathbf{B} &= \tilde{\mathbf{G}}_s^{-1}\mathbf{G}_s\mathbf{D}_s.\end{aligned} \tag{26}$$

From this equation it can be deduced that the set-point changes are perfectly followed if a perfect model is available (a full controller). However, the situation for the disturbances is different because they are directly introduced to the $\mathbf{y}_s^{net}$ without any rejection. Therefore, the idea is to find the best $\tilde{\mathbf{G}}_s$ that minimizes the disturbance effects and set-point changes over the $\mathbf{y}_s^{net}$.

Then, $\tilde{\mathbf{G}}_s$ can be parameterized as

$$\tilde{\mathbf{G}}_s = \mathbf{G}_s \otimes \Gamma \tag{27}$$

where $\otimes$ denotes the element-by-element product and Γ is a $n \times n$ binary matrix. Then, the objective is to find the most suitable Γ that produces the $\mathbf{y}_s^{net}$ minimization, it can be mathematically written as

$$\mathrm{SSD}_{\mathbf{y}_s^{net}}(\Gamma) = \sum_{i=1}^{n} \left| |\Delta_1\mathbf{A}(\Gamma)\Delta_2\mathbf{y}_{set}^{n}(i)\right| |_2^2 + \sum_{j=1}^{p} \left| |\Xi_1\mathbf{B}(\Gamma)\Xi_2\mathbf{d}_*^{p}(j)\right| |_2^2 \tag{28}$$

or, equivalently (see Appendix A):

$$\mathrm{SSD}_{\mathbf{y}_s^{net}}(\Gamma) = \mathrm{trace}\left(\Delta_1^2\mathbf{A}^T(\Gamma)\Delta_2^2\mathbf{A}(\Gamma)\right) + \mathrm{trace}\left(\Xi_1^2\mathbf{B}^T(\Gamma)\Xi_2^2\mathbf{B}(\Gamma)\right) \tag{29}$$

where Δ_1, Δ_2, Ξ_1 and Ξ_2 are diagonal matrixes that weights the amplitude and importance of the set-point changes and disturbances respectively. Then the following optimization problem has to be solved:

$$\begin{aligned}\min_{\Gamma}\mathrm{SSD}_{\mathbf{y}_s^{net}}(\Gamma) &= \min_{\Gamma}\left[\mathrm{trace}\left(\Delta_2^2\mathbf{A}^T(\Gamma)\Delta_1^2\mathbf{A}(\Gamma)\right) + \mathrm{trace}\left(\Xi_2^2\mathbf{B}^T(\Gamma)\Xi_1^2\mathbf{B}(\Gamma)\right)\right]\\ &= \min_{\Gamma}\left[||\Delta_1\mathbf{A}(\Gamma)\Delta_2||_F^2 + ||\Xi_1\mathbf{B}(\Gamma)\Xi_2||_F^2\right]\end{aligned} \tag{30}$$

subject to

$$\mathrm{Re}[\lambda_i(\mathbf{G}_s\tilde{\mathbf{G}}_s^{-1})] > 0, \quad \text{with} \quad i = 1, \ldots, n \tag{31}$$

where Re[·] is the real parte function, $\lambda_i(\cdot)$ is the *i*-th eigenvalue, and $\tilde{\mathbf{G}}_s$ is the model parametrization/selection. Inequality in Eq. 31 is the stability criterion developed by Garcia and Morari (1985) for

Table 4
GA parameters setting to minimize SSD_{yr} (base case)

N_i	N_c	n	N_g	Mutation probability	Crossver probability
1000	12	8	50	$0.7/N_c$	0.7

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Table 5
Best solutions of SSD_{yr} from GA (base case)

No.	Controlled variables								SSD_{yr}
1	*xme*5	*xme*11	*xme*13	*xme*17	*xme*18	*xme*20	*xme*30	$xme_{G/H}$	3.12
2	*xme*5	*xme*7	*xme*11	*xme*17	*xme*18	*xme*20	*xme*30	$xme_{G/H}$	3.17
3	*xme*6	*xme*11	*xme*13	*xme*17	*xme*18	*xme*20	*xme*30	$xme_{G/H}$	6.03
4	*xme*6	*xme*7	*xme*11	*xme*17	*xme*18	*xme*20	*xme*30	$xme_{G/H}$	6.18
5	*xme*6	*xme*9	*xme*11	*xme*13	*xme*18	*xme*20	*xme*30	$xme_{G/H}$	6.97

control structures designed in the context of IMC theory. Therefore, the multivariable gain reduction of $\mathbf{y}_s^{net}$ is performed on a feasible solutions space.

This is a combinatorial problem which solution can be obtained by applying GA because of its computational advantage for large scale process applications. Therefore, to drive the problem in the context of GA, the matrix Γ is encoded as a chromosome in the following way:

$$\Gamma = \begin{bmatrix} 1 & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & 1 & \cdots & \gamma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & 1 \end{bmatrix} \tag{32}$$

$$\mathbf{C}_{y_s^{net}} = \begin{bmatrix} \gamma_{12}, & \cdots, & \gamma_{ij}, & \cdots \end{bmatrix}, \quad \text{with} \quad i \neq j;\ i, j \leq n; \tag{33}$$

Then, Γ a n-by-n matrix, the problem dimensionality is $2^{(n\times(n-1))}$ due to the diagonal elements that considers the pairing recommended by the RGA procedure so $\gamma_{ii} = 1$, with $i = 1, \ldots, n$.

According to the previous development about the net load evaluation, the following conclusions and comments can be stated,

1. The minimization of $\left[||\Delta_1 \mathbf{A}(\Gamma)\Delta_2||_F^2 + ||\Xi_1 \mathbf{B}(\Gamma)\Xi_2||_F^2 \right]$ decreases the $\mathbf{y}_s^{net}(s)$ effects on the CVs produced in the transient by set point changes and disturbances.
2. A suitable selection of the weighting matrices allows to obtain a trade-off solution between servo and regulator behaviors,

2.1. A full control structure guarantees the best decoupling for the reference changes, $\mathbf{A}(\Gamma) = 0$, but does not introduce disturbance attenuation, $\mathbf{B}(\Gamma) = \mathbf{D}_s$.
2.2. A particular model structure of $\tilde{\mathbf{G}}_s$ could reduce the $\mathbf{B}(\Gamma)$ effect but $\mathbf{A}(\Gamma) \neq 0$ necessarily. A trade-off solution is required.

3. The minimization of $SSD_{\mathbf{y}_s^{net}}(\Gamma)$ only, does not guarantee stability in the resulting control structures, except in the case of $\mathbf{B}(\Gamma) = 0$.

3.1. The IMC stability criterion of Garcia and Morari (1985) should be included.

3.4.2. *A dynamic model is available*

In the proposed procedure, the RNGA is used to find an alternative controller structure to that given by the RGA in case the dynamic information could be obtained. When this RNGA gives a different pairing from the obtained by RGA, it indicates that RNGA paring would be better than that given by the RGA. In this case, it is recommended to use control structure suggested by RNGA.

4. Study case: Tennessee Eastman process

The plant has five principal process unit operations: the reactor, the product condenser, a vapor-liquid separator, a recycle compressor and a product stripper. A more detailed description can be found in Downs and Vogel (1992), and the same nomenclature given there is used in this work. Therefore, the measurements are named as *xme*x, where *x* identifies the corresponding number of measurement. The same is valid for the manipulated variables recognized by *xmv*x. In Fig. 3 can be seen the plant with the inventory and stabilizing control structure. In this case the McAvoy and Ye (1995) stabilizing control structure was used. Two products are generated from four reactants. In addition, an inert and a byproduct are present having eight components named as A, B, C, D, E, F, G, and H. The reactions are:

A(g)+C(g)+D(g) → G(liq),	Product 1,
A(g)+C(g)+E(g) → H(liq),	Product 2,
A(g)+E(g) → F(liq),	Byproduct,
3D(g) → 2F(liq),	Byproduct,

All of them are exothermic, the reaction rates are calculated from Arrhenius expression. The G production needs a higher activation energy so it is more sensitive to the temperature. In addition, first-order kinetics with respect to the reactants concentrations are assumed. This case study was specifically chosen because it is well known benchmark and has been tested before by several researchers. In particular, some of them Ricker (1995) proposed

Table 6
RGA and RNGA for the best solution (base case)

	*xmv*1	*xmv*3	*xmv*4	*xmv*5	*xmv*6	*xmv*9	$xme21_{sp}$	*xme*22
				Relative normalized gain array				
*xme*5	−0.23	0.01	0.03	0.01	0.07	0.00	1.10	0.02
*xme*11	0.02	0.00	0.00	0.01	0.00	0.00	0.07	0.97
*xme*13	0.10	0.20	0.09	0.86	0.04	0.00	−0.30	0.00
*xme*17	−0.01	−0.37	1.35	0.00	0.03	0.00	−0.01	0.00
*xme*18	0.00	0.00	0.00	0.00	0.00	1	0.00	0.00
*xme*20	−0.04	1.28	−0.33	0.11	−0.12	0.00	0.10	0.01
*xme*30	0.16	−0.09	−0.09	0.05	1.01	0.00	−0.03	0.00
$xme_{G/H}$	1.04	−0.01	−0.05	−0.01	−0.04	0.00	0.07	0.00
				Relative gain array				
*xme*5	−0.17	−0.05	0.00	−0.68	0.17	0.02	1.38	0.33
*xme*11	−0.22	−0.73	0.10	0.47	0.31	−1.00	1.45	0.61
*xme*13	0.19	1.07	0.11	0.19	−0.15	−0.04	−0.55	0.17
*xme*17	−0.04	0.10	1.06	0.03	0.01	0.03	−0.20	−0.01
*xme*18	0.06	0.35	−0.11	−0.16	−0.16	1.93	−0.59	−0.32
*xme*20	0.04	0.15	0.00	1.17	0.13	0.03	−0.54	0.02
*xme*30	0.08	0.03	−0.02	−0.02	0.68	0.02	0.05	0.17
$xme_{G/H}$	1.05	0.07	−0.14	0.00	0.00	0.00	0.00	0.01

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an optimized working point different than that presented originally by Downs and Vogel (1992). Therefore, the same process offers two quite different scenarios for testing the methodology presented here. Additionally for both points previous results exist for comparison purposes.

The approach proposed in this work will be applied considering this plant in two different operations points: base case and optimal case. These operation conditions present different degrees of freedom. Thus the resulting controller structures can be different.

Tables 1 and 2 show the base case and optimum values for the main output and input variables respectively. The optimization for the same mode was solved in Ricker (1995). In Table 3 the costs for both operation conditions are shown. It can be seen clearly the economic advantage of operating the process at the optimum even though the control problem there presents some extra difficulties.

5. Results

5.1. Base case operation work

5.1.1. Degrees of freedom analysis

The proposed approach can begin with the process optimization or directly with a specific working point. In this part of the work it is adopted the operation point given in Downs and Vogel (1992) recognized as base case. Thus, the only active constraints are the requirements over production: product quality ($xme_{G/H}$) and production rate ($xme17$).

Furthermore, the separator and stripper levels do not have steady-state effect and have to be selected for control. In this way, four degrees of freedom are used from the available 12.

The open loop process is unstable. Due to this, a minimal number of loops should be selected to achieve process stability. Basically, reactor level and reactor cooling water outlet temperature must be controlled to obtain the desired stability. However, the control of this last one does not suppose a lower number of the degrees of freedom due to it can be used as manipulated variable for an outer controller. The control of this loop is the same as that proposed in McAvoy and Ye (1995).

Here, a similar systematic procedure as that proposed in McAvoy (1998) to achieve the plant stabilization is followed. The main difference with that work is trying to stabilize the plant avoiding the use of inner loops. Hence, the only chosen loop here is the reactor cooling water outlet temperature paired with the reactor cooling water flow.

5.1.2. Selection of controlled variables

Once the plant is stabilized, the CV's are selected minimizing the measure defined in Eq. (19). To solve the problem, the outputs were scaling by considering 0 as a mean value with variance 1. In addition, the matrixes Δ_1, Δ_2, Θ_1 and Θ_2 weights with 1 the set-point changes in production rate and quality product; and with 1 for both of the considered disturbances. The solution is found using GA.

In this case $\Lambda_1 = \Theta_1$, if only set-point changes in production rate and quality product are accounted, then Λ_2 has only 1's for those variables. The amplitude of the disturbance has been equally weighted. In Table 4 the parameters used in the GA are shown, and in Table 5 the five obtained best solutions by this technique are given. A scheme with the resulting controlled variables for the base case process is shown in Fig. 4.

5.1.3. Selection of control structure

Since the dynamic model of the TE process is available, the pairing between manipulated variable and the previously selected CV's can be obtained calculating both, the RGA and the RNGA. These arrays are shown with their best possible pairing marked with gray background in Table 6.

Table 7
$\%IAE_{improv}$ for critical disturbances (base case)

	IDV1	IDV2	IDV8	IDV12 and 15
React. press.	97.59	77.45	77.95	98.36
% B in purge	0.27	−4.21	0.52	0.12

The RNGA recommends different pairings compared to those given by the RGA. It could indicate that the dynamic responses of the system can be improved using the RNGA criterium. In order to check which one gives the best results in the following subsection a deep analisys based on the dynamic simulations and performance indices are presented

5.1.4. Dynamic simulations

For the base case, it was implemented the solution which gives the smallest SSD_{yr} with the pairing given by the RGA and the RNGA. In Fig. 5 is shown the dynamic responses of some of the most important variables when disturbance 1 (a step change in C feed composition) appears. It is shown the great benefit obtained for the reactor pressure responses, which is a very crucial variable because it can produce the plant shut-down. Its peak value and time for returning to its original point are really improved. For the % B in Purge both responses have exactly the same peak, but the structure given by the RNGA has smaller time responses. The variables corresponding to the product composition and production rate, present similar dynamic behavior. Fig. 6 shows the responses for a step change of −15% in the production rate, as it is recommended in Downs and Vogel (1992). Similar conclusions have been found for the most important variables affected by this change. In Table 7 are shown the % of Integral Absolute Error $\%IAE_{improv}$ which quantify the achieved advantages when the paring given by the RNGA is used, i.e.:

$$\%IAE_{improv} = \frac{IAE_{RGA} - IAE_{RNGA}}{IAE_{RGA}} \times 100\%. \tag{34}$$

Thus, positive magnitudes of the $\%IAE_{improv}$ indicate that better results are obtained using the pairing given by the RNGA. From the values given in Table 7 it can be seen that $xme7$ presents an improvement on its dynamic behavior using the pair given by the RNGA. The variable % B in purge ($xme30$), shows slight improvements for disturbances IDV1, IDV8 and IDV12 and 15; and is not advantageous when IDV2 appears. However, the important benefits obtained in the reactor pressure ($xme7$) justifies the use of this pairing.

5.2. Optimal operation work

5.2.1. Process optimization and degrees of freedom analysis

From the data given in Tables 1 and 2, it can be noted that working in the optimal operation point several variables are at their constraint values. This suppose a major difficulties for control purposes. Hence, it represents an interesting challenger testing the methodology with the plant at this working point. As can be seen in Table 3 a reduction of the operating cost at the optimum is of about 33% with respect of the base case.

5.2.2. Selection of controlled variables

The stabilization is done in the same way described for the base case, obtaining the same results.

The TE process at this point practically presents a new problem to be solved. Now, 21 output variables are analyzed. Accounting the process requirements and optimization stage, the production rate ($xme17$), quality product (($xme40$)/($xme41$)) and reactor pres-

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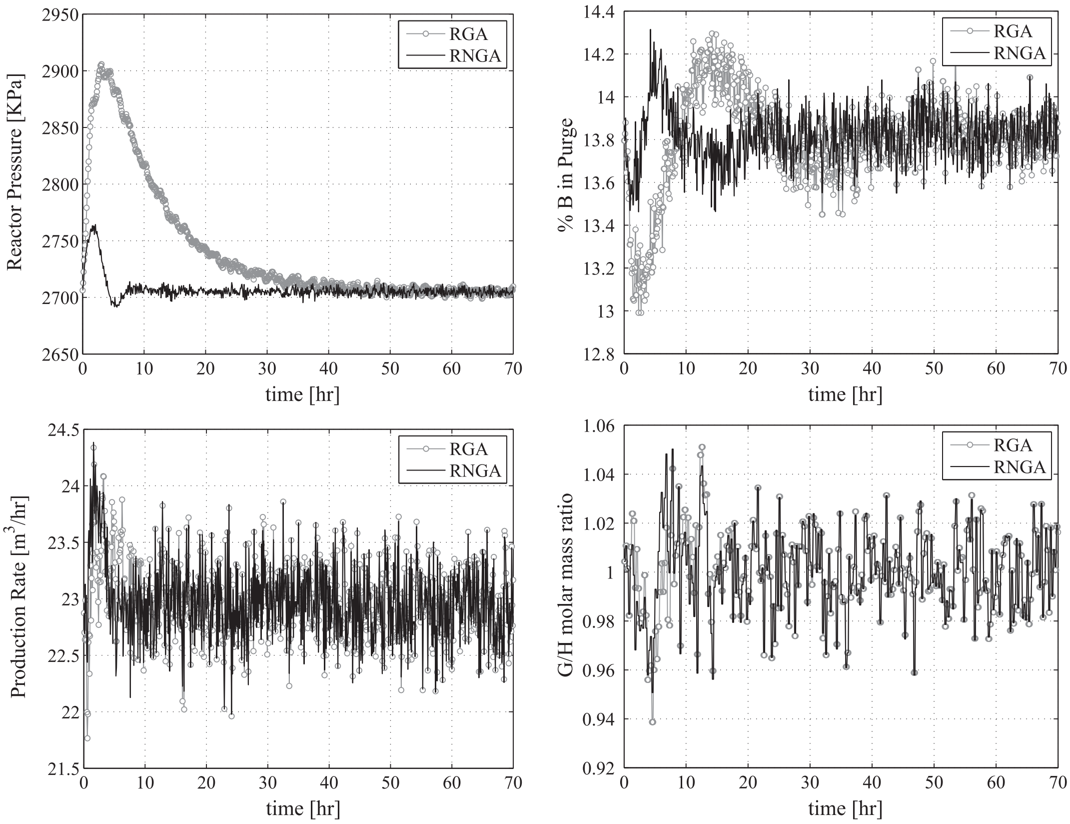

Fig. 5. Responses for disturbance 1 (base case).

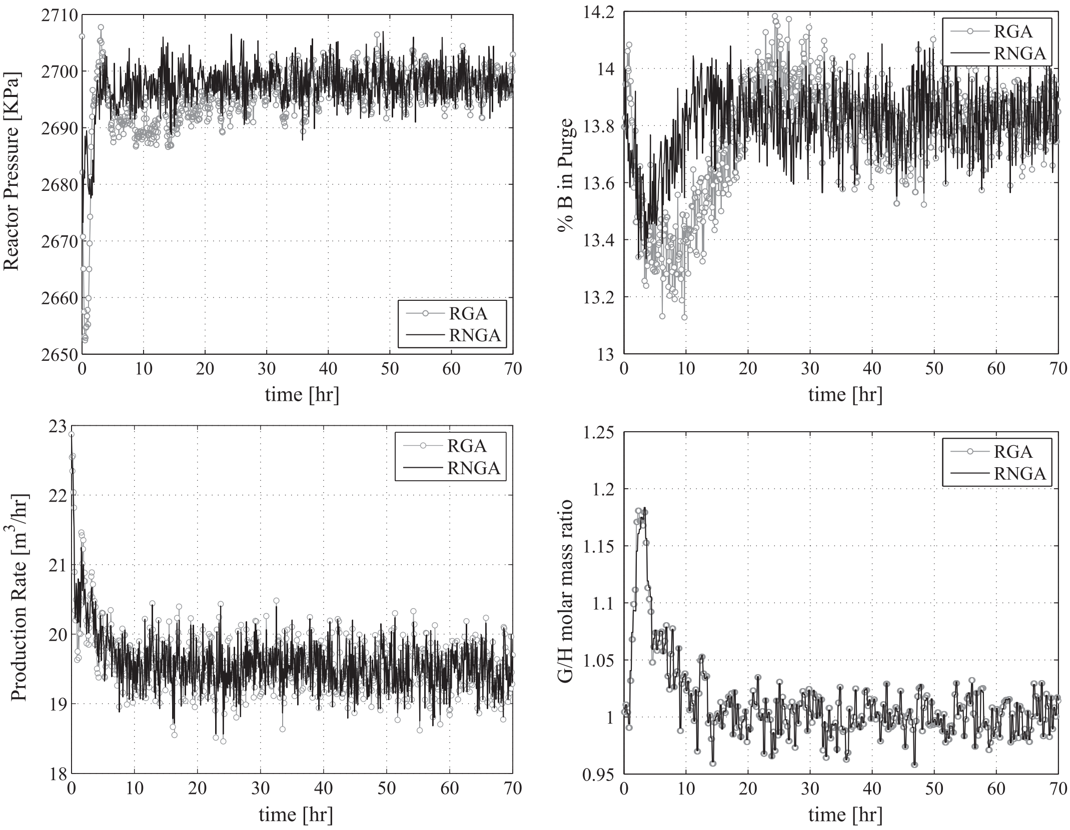

Fig. 6. Responses for production set-point change (base case).

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Table 8
GA parameters setting to minimize SSD_{yr} (optimal case)

N_i	N_c	n	N_g	Mutation probability	Crossver probability
1000	18	3	50	$0.7/N_c$	0.7

Table 9
Best solutions of SSD_{yr} from GA (optimal case)

No.	Controlled variables						SSD_{yr}
1	*xme*7	*xme*17	$xme_{G/H}$	*xme*5	*xme*25	*xme*38	78.7
2	*xme*7	*xme*17	$xme_{G/H}$	*xme*5	*xme*18	*xme*25	78.9
3	*xme*7	*xme*17	$xme_{G/H}$	*xme*6	*xme*18	*xme*25	79.2
4	*xme*7	*xme*17	$xme_{G/H}$	*xme*5	*xme*11	*xme*25	79.5
5	*xme*7	*xme*17	$xme_{G/H}$	*xme*5	*xme*11	*xme*25	79.6

sure (*xme*7) have to be controlled. Furthermore, compressor recycle (*xmv*5), stripper steam (*xmv*9), and agitator speed (*xmv*12) cannot be considered as manipulated variables because they have to be set on their respective constraints. In this way, it has to be controlled 6 variables and 3 of them are known from the optimization step. Thus the total number of possibilities to be evaluated are $(n!)/((n-p)!p!) = (18!)/((18-3)!3!) = 816$.

For this case only the set-point changes in production rate and product quality were considered based on the precise process requirements. In addition, it is done to take into account the effect over $\mathbf{y}_r$.

The results of the evaluation of the SSD_{yr} are presented in Table 9 and were obtained using the GA detailed before with the parameters shown in Table 8.

All the solutions shown in Table 9 have almost the same SSD_{yr}. This means that any of them can be chosen to be implemented. Then, it is considered more convenient using the second one because it does not need composition control. Hence the inherent complexity of this kind of sensors such as hardware costs and the associated times delay are avoided.

5.2.3. Selection of control structure

Following the same procedure that was done in the base case, the RGA and RNGA for the optimum operation case are obtained. For this case both of them recommend the same control structure. So, only the RGA is shown in Table 10.

Therefore, a possible way to improve the dynamic behavior could be trough a proper control structure based on reducing the net load effect $\mathbf{y}_s^{net}$. It is done according the proposed methodology given above.

The resulting scheme of the controlled process is shown in Fig. 7.

5.2.4. Optimal control structure by minimizing the net load effect

The control of the plant is done minimizing $\mathbf{y}_s^{net}$. Then, the problem is formulated through Eq. (29). The optimization problem is solved using GA with the parameters shown in Table 11.

The idea is to reduce the effects of disturbances 1 over the CV's, taking into account the set-point changes too. This is the reason for adopting weight 1 for the disturbances 1, while the set-point on the product quality and production rate are weighted with 0.5. The best results of this optimization are shown in Table 12.

The first solution is implemented using decentralized PI, driven from equivalence with IMC structure accounting the paring given by the RGA and RNGA.

5.2.5. Dynamic simulations

In this section several simulation results are presented so as to show the dynamic behavior of the most important variables under decentralized and with reduced $\mathbf{y}_s^{net}$ control structures. The second best solution is implemented. The benefit of the structures with reduced net load effect is remarkable in time responses in reactor pressure and G/H molar mass ratio which can be seen specially in Fig. 8. For the other variables the differences are not very significant, but at least the responses are very similar to the decentralized control.

For a production set-point change of about −15%, the responses are not better than those obtained with the decentralized option. This is because of the chosen higher weights affecting the disturbances to prevent the plant shut-down if the reactor pressure exceeds its maximum allowable value when disturbances (specially disturbance 1) appears (Fig. 9.).

Finally, to quantify the benefits of using the structure of reduced net load effect to improve the dynamic responses, the IAE_{improv} is redefined in the following way:

$$\%IAE_{improv} = \frac{IAE_{decent} - IAE_{ynet}}{IAE_{decent}} \times 100\%, \tag{35}$$

where IAE_{decent} corresponds to the IAE of the decentralized structure and IAE_{ynet} is the IAE obtained for the structure with reduced net load effect.

In Table 13 is shown the $\%IAE_{improv}$, in which can be concluded that better results can be achieved for the CV's % C in Purge and Reactor Pressure when disturbances appear. It is remarkable the important improvement introduced in the more critic variable that is the reactor pressure for IDV1.

5.3. Hardware requirements

Several authors have applied different methodologies for obtaining acceptable control structures for the TE process that fulfills the main objectives stated in Downs and Vogel (1992). However, there are great differences related to the hardware requirements and to the number of loops employed as can be seen in Table 14. In this table four control alternatives are included for the sake of comparison. The approach given in Ricker (1996) suggested a control structure based on heuristic concepts with 16 measurement points and 19 control loops thinking mainly in multiple modes of operation and implementing a control of production rate and quality product strategy. Meanwhile, Larsson et al. (2001) proposed a methodology based on the ideas of self-optimizing control proposed by Skogestad (2000) obtaining a control structure that is similar to the one proposed by Ricker. On the other hand, in Banerjee and Arkun (1995) a systematic strategy named control configuration design is presented which is based on the condi-

Table 10
RGA for the implemented solution

	*xmv*1	*xmv*3	*xmv*4	*xmv*6	*xmv*11	$xme21_{sp}$
*xme*7	0.098	0.141	−0.152	0.969	−0.022	−0.033
*xme*17	−0.102	−0.008	1.223	−0.042	−0.062	−0.008
$xme_{G/H}$	1.027	0.024	−0.067	0.018	−0.001	−0.001
*xme*5	−0.032	0.029	0.004	−0.046	0.399	0.645
*xme*18	−0.010	0.012	−0.005	0.01	0.612	0.374
*xme*25	0.02	0.801	−0.003	0.091	0.066	0.024

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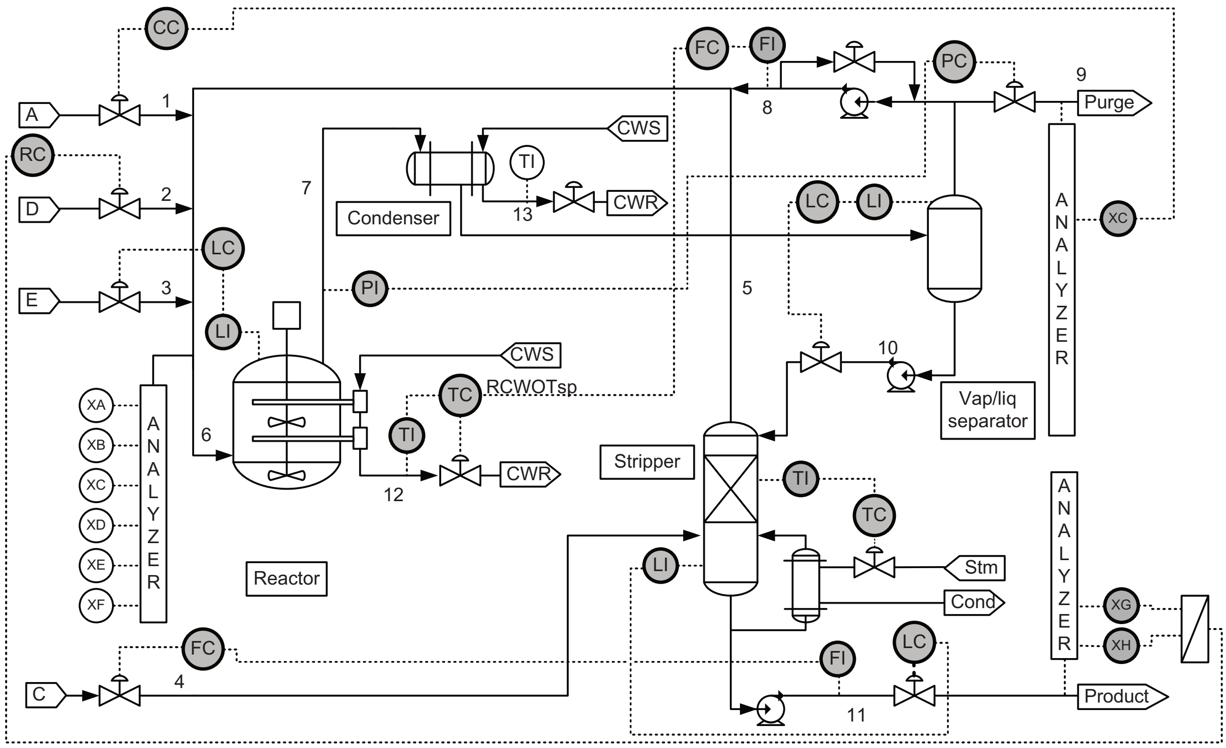

Fig. 7. TE controlled process for optimum operation.

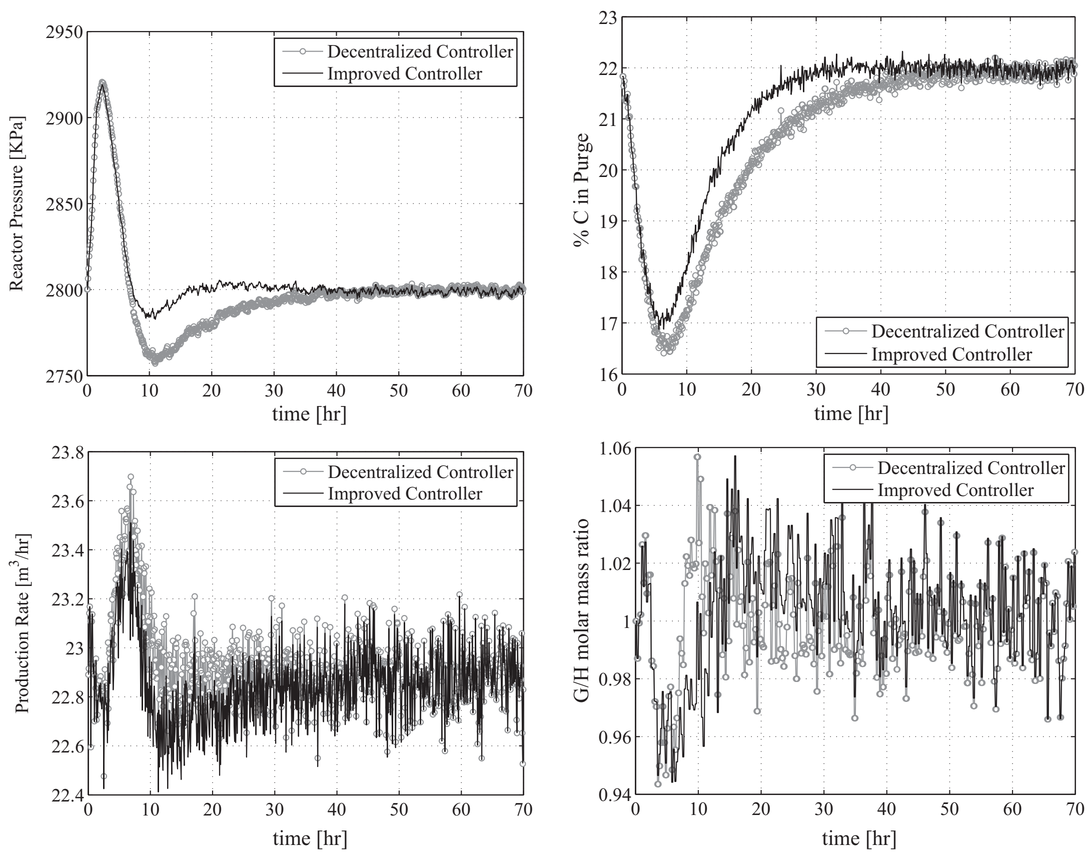

Fig. 8. Responses for disturbance 1 (optimal case).

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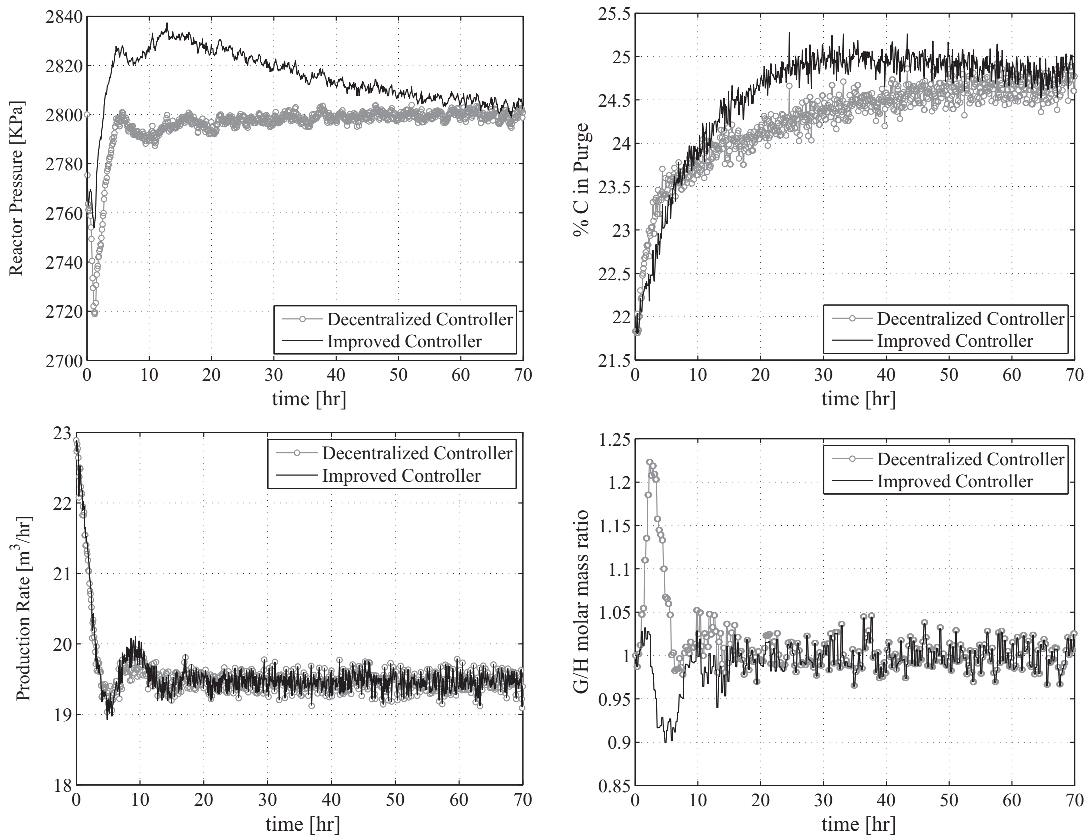

Fig. 9. Responses for production set-point change (optimal case).

Table 11
GA parameters setting for $\mathbf{y}_s^{net}$ (optimal case)

N_i	N_c	N_g	Mutation probability	Crossver probability
1000	30	50	$0.7/N_c$	0.7

Table 12
Controller structure results of $\mathbf{y}_s^{net}$ from GA (optimal case)

	Γ	$SSD_{y_s^{net}}$
1	111101111011111101111111110111	0.894146
2	111111111011110101111111110111	0.894162
3	111111111011100101111111111111	0.894237
4	111101111011111101111111111111	0.894274
5	111111111011111101111111111111	0.894281

Table 13
Improvement in IAE in [%] for disturbances (optimum case)

	IDV1	IDV2	IDV8	IDV12 and 15
React. press.	39.76	1.76	4.52	2.33
% C in purge	11.9	13.55	0.94	0.25

tion number, nominal stability, Niederlinsky Index and cross feed performance degradation.

In the last row in Table 14 the number of composition loops that each one requires to be measured for the specific control structure design is included. Generally, this measurement type has three characteristics: 1, expensive (high investment); 2, delayed (the measurement is obtained with a temporal shift) and 3, infrequent (low samples rates). From the control point of view, delayed and infrequent measurements present serious drawback to guarantee the suitable performance of the control loop. It seems reasonable to avoid excessive use of these measurements. Both structures proposed here have the smallest number of these kind of measurements and represent the main reason for the selection of suboptimal solutions but very close to the optimal one.

6. Conclusions

A systematic approach for defining a preliminary plant-wide control structure, trying to avoid as much as possible the use of heuristic considerations for reducing the problem dimensionality was presented. The defined measure SSD_{yr} was useful to screen the best sets of controlled variables. The minimum SSD_{yr} was successfully obtained using genetic algorithms, which improve the search and require less computational effort. The introduced improvements on the RNG and GRDG concepts demonstrated the great potentiality for multivariable control industrial applications. Furthermore, they gave a way to obtain a very efficient plant-wide control structures with a reduced net load effect using a minimum number of control loops. The method can use only steady-state

Table 14
Hardware requirements

	McAvoy and Ye	Ricker	Larsson et al.	Banerjee and Arkum	Proposed for base case	Proposed for optimal case
Measurements	22	16	22	15	15	15
Control loops	22	19	17	16	15	15
Compositions	4	3	3	5	3	3

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information or can account with dynamic model too which is useful for RNGA application. The method is mathematically consistent as it was demonstrated that it drives to a well conditioned system with a very acceptable RGA. It was tested for the well-known case of the TE process working at two different operating points which presented quite different scenarios. In addition, the TE gave the opportunity of confronting with other control structures available in the literature. Although, all the control structures that fulfill with the main process objectives and possess an acceptable dynamic behavior are almost equally valid control designs, in this case the solution with fewer hardware requirements (sensors, actuators and controllers) is preferred among them. In all the cases seen here, the proposed approach generates a new feasible control structure by minimizing both the heuristic load and the design complexity. The result is a control policy that has the lowest hardware requirement compared with the other proposals found in the literature. The dynamic simulations demonstrated the clear advantages achieved specially in the most critical process variables.

Moreover, as future works, the proposed strategy is under testing on the bigger control problem that up today exists in the process control community "the pulp mill benchmark" (Castro and Doyle (2004), Journal of Process Control) which represents a control problem with, 8200 states, 142 inputs and 114 outputs. The main idea of the authors is to make a generalized test of the proposal in as many cases as possible, from small-scale to large-scale problems. It allows to discover new aspects to be considered that are very common in large scale plants.

Appendix A.

In this appendix it is proofed the first term of the final expression of Eq. (18) is equal to the first of Eq. 19, i.e.:

$$\sum_{i=1}^{n} ||\mathbf{e}_{sp}(i)||^2 = \sum_{i=1}^{n} ||\Lambda_2 \mathbf{S}_{sp} \Lambda_1 \mathbf{y}_{set}^{n}(i)||_2^2 \qquad (36)$$

Working with this equation, we get:

$$\begin{aligned}\sum_{i=1}^{n} ||\mathbf{e}_{sp}(i)||_2^2 &= \sum_{i}^{n} ||\Lambda_2 \mathbf{S}_{sp} \Lambda_1 \mathbf{y}_{set}^{n}(i)||_2^2 \\ &= ||\Lambda_2 \mathbf{S}_{sp} \Lambda_1 \mathbf{y}_{set}^{n}(1)||_2^2 + \ldots + ||\Lambda_2 \mathbf{S}_{sp} \Lambda_1 \mathbf{y}_{set}^{n}(n)||_2^2 \\ &= \lambda_1^{11} ||\lambda_2^{11} s_{sp}^{11}, \lambda_2^{22} s_{sp}^{21}, \ldots, \lambda_2^{m'm'} s_{sp}^{m'1}||_2^2 + \ldots + \qquad (37) \\ &+ \lambda_1^{nn} ||\lambda_2^{11} s_{sp}^{1m'}, \lambda_2^{22} s_{sp}^{2m'}, \ldots, \lambda_2^{m'm'} s_{sp}^{m'm'}||_2^2 \\ &= \sum_{i=1}^{n} \lambda_1^{ii} \left[\mathrm{col}_i^T(\mathbf{S}_{sp})\right] \Lambda_2^2 \left[\mathrm{col}_i(\mathbf{S}_{sp})\right]\end{aligned}$$

Finally, it is obtained

$$\sum_{i=1}^{n} ||\mathbf{e}_{sp}(i)||^2 = \mathrm{trace}\left(\Lambda_1^2 \mathbf{S}_{sp}^T \Lambda_2^2 \mathbf{S}_{sp}\right) = ||\Lambda_2 \mathbf{S}_{yr} \Lambda_1||_F^2 \qquad (38)$$

where s_{sp}^{ij} is the element placed in the row i and column j of $\mathbf{S}_{sp}$; λ_1^{ii} and λ_2^{ii} are the elements of the diagonal matrices Λ_1 and Λ_2 respectively; $\mathrm{col}_i(_{Ssp})$ denotes the column i of matrix $\mathbf{S}_{sp}$. The operation trace is the sum of the diagonal elements of the matrix.

Analogously, it is obtained the second term of equation (19) and both terms of (29).

Appendix B.

The tuning parameters of each PI was accomplish using Internal Model Control (IMC) rules presented in Rivera, Morari, and Skogestad (1986). In Table B.1 are shown the final tunings of the stabilizing loops. These loops are the same for the two operations cases of the TE process. Tables B.2 and B.3 shows the tuning parameters for the base case operation using the paring obtained by the RGA and RNGA respectively. In Table B.4 are shown the tuning parameters for the control structure propose for the optimum case.

Table B.1
PI-controller parameters. Stabilizing loops for both cases

cv	mv	K_C	τ_I
*xme*12	*xmv*7	−0.1	2
*xme*15	*xmv*8	−0.12	7.5
*xme*8	*xmv*2	9	1.7
*xme*21	*xmv*10	−1.7	0.03

Table B.2
PI-controller parameters. Base case with RGA

cv	mv	K_C	τ_I
$xme_{G/H}$	*xmv*1	9.5	1
*xme*30	*xmv*6	−2.7	2
*xme*20	*xmv*5	0.35	0.6
*xme*18	*xmv*9	7.9	0.2
*xme*17	*xmv*4	1.5	1
*xme*13	*xmv*3	−0.2	10
*xme*11	$xme21_{sp}$	−0.7	1
*xme*5	*xmv*12	−0.5	0.3

Table B.3
PI-controller parameters. Base case with RNGA

cv	mv	K_C	τ_I
$xme_{G/H}$	*xmv*1	9.5	1
*xme*30	*xmv*6	−8.3	2
*xme*20	*xmv*3	0.8	10
*xme*18	*xmv*9	7.9	0.2
*xme*17	*xmv*4	1.5	1
*xme*13	*xmv*5	−0.065	0.5
*xme*11	*xmv*12	−1.6	0.2
*xme*5	$xme21_{sp}$	−1.9	0.2

Table B.4
PI-controller parameters. Optimum case

cv	mv	K_C	τ_I
$xme_{G/H}$	*xmv*1	10	1
*xme*31	*xmv*3	−3.5	15
*xme*17	*xmv*4	1.3	1
*xme*7	*xmv*6	−0.35	20
*xme*18	*xmv*11	−3.4	0.2
*xme*5	$xme21_{sp}$	−1.9	0.2

Appendix C.

Some matrix properties are presented here according to the functional cost used in the optimization problem stated in Eq. 20. Remembering that it was proposed from steady-state analysis to reduce the operating point deviations for the uncontrolled outputs (Eq. 20). This functional has some additional/indirect properties.

C.1. Minimization of $||\Lambda_2 \mathbf{S}_{sp} \Lambda_1||_F^2$*- setpoint effect*

The first contribution of the functional cost of Eq. 20 is $||\Lambda_2 \mathbf{S}_{sp} \Lambda_1||_F^2$. Without loss of generality, the term $||\mathbf{S}_{sp}||_F$ will be analyzed first. Remembering the Frobenius norm definition (Skogestad and Postlethwaite, 2005), and the relationships stated in section 3.3,

$$||\mathbf{S}_{sp}||_F = ||\mathbf{G}_r \mathbf{G}_s^{-1}||_F = \sqrt{\sum_{i=1}^{k} \sigma_i^2 \left(\mathbf{G}_r \mathbf{G}_s^{-1}\right)}, \quad \text{with } k = \min(m-n, n) \qquad (39)$$

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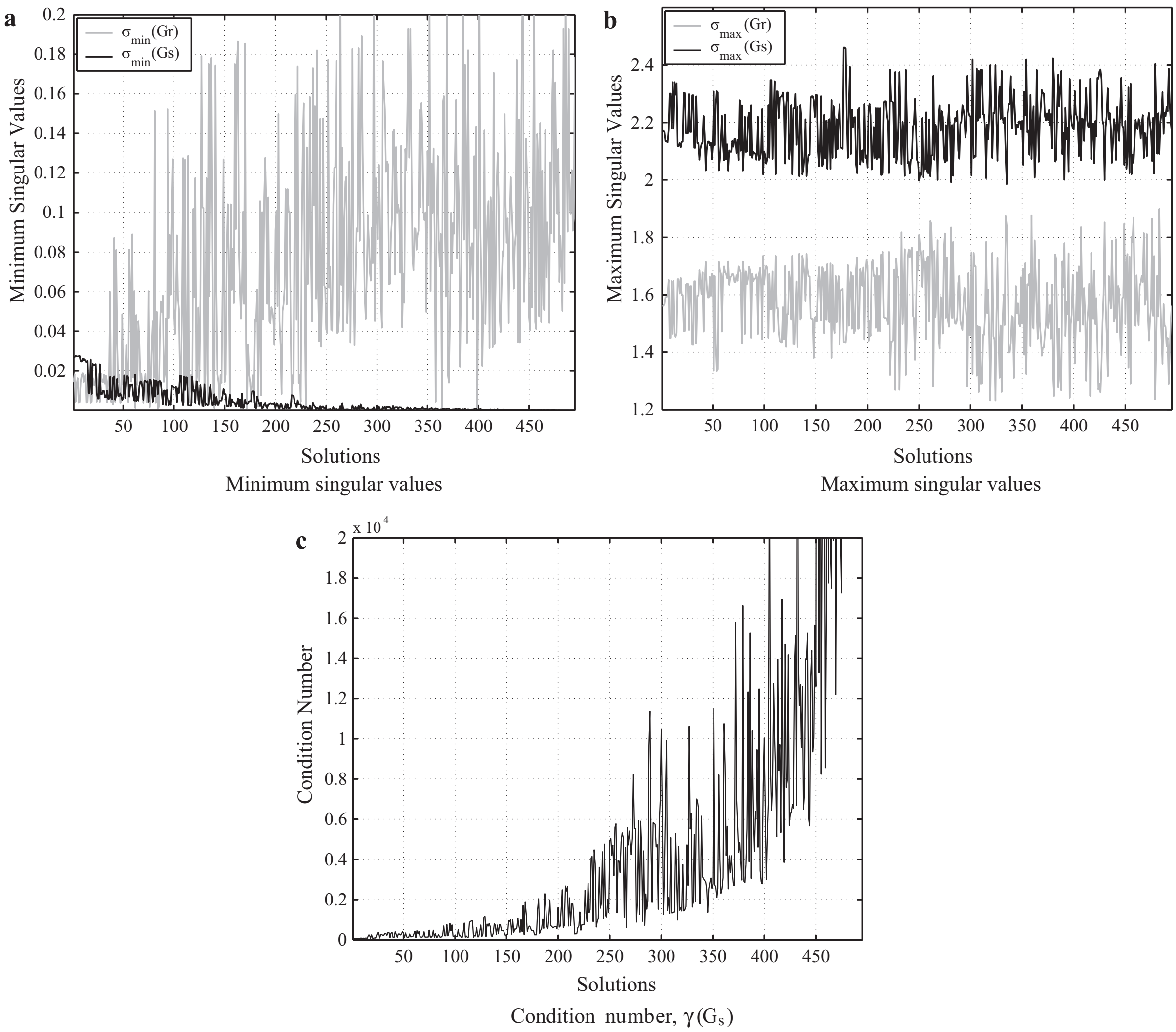

Fig. C.1. Matrix properties evolution.

being $\sigma_i\left(\mathbf{G}_r\mathbf{G}_s^{-1}\right)$ the i-th singular value of the matrix $\mathbf{G}_r\mathbf{G}_s^{-1}$. In this context and taking into account some singular value properties (Horn & Johnson, 1990; Golub & Van Loan, 1996) the following relationships can be written,

$$\underline{\sigma}(\mathbf{G}_r)\bar{\sigma}(\mathbf{G}_s^{-1}) = \frac{\underline{\sigma}(\mathbf{G}_r)}{\underline{\sigma}(\mathbf{G}_s)} \le \bar{\sigma}(\mathbf{G}_r\mathbf{G}_s^{-1}) \le ||\mathbf{G}_r\mathbf{G}_s^{-1}||_F \quad (40)$$

$$\bar{\sigma}(\mathbf{G}_r)\underline{\sigma}(\mathbf{G}_s^{-1}) = \frac{\bar{\sigma}(\mathbf{G}_r)}{\bar{\sigma}(\mathbf{G}_s)} \le \bar{\sigma}(\mathbf{G}_r\mathbf{G}_s^{-1}) \le ||\mathbf{G}_r\mathbf{G}_s^{-1}||_F \quad (41)$$

these expressions shown a clear link between the functional cost and the matrix properties of $\mathbf{G}_s$. Here, $\underline{\sigma}$ (or σ_{min}) and $\bar{\sigma}$ (or σ_{max}) represent the minimum and maximum singular values. Note that the functional cost reduction requires the minimization of the left-hand ratios in Eqs. 40 and 41, so the matrix properties will be modified. Particularly the $\mathbf{G}_s$ properties are important, because this is the final plant to be controlled.

The $\underline{\sigma}(\mathbf{G}_s)$ value measures how close is the matrix $\mathbf{G}_s$ of the singularity and ill-conditioning. Another important index is the matrix condition number $\gamma(\mathbf{G}_s) = \bar{\sigma}(\mathbf{G}_s)/\underline{\sigma}(\mathbf{G}_s)$, high values in $\gamma(\mathbf{G}_s)$ indicates ill-conditioning and eventually a poor Λ. All this behaviors indicate that $\mathbf{G}_s$ is very difficult to control and with high sensitivity to modeling errors (Grosdidier et al., 1985; Skogetad and Morari, 1987; Skogestad and Postlethwaite, 2005).

Note that the original problem was stated for to select $\mathbf{G}_s$ from the original process $\mathbf{G} = [\mathbf{G}_s^T \;\; \mathbf{G}_s^T]^T$ so that the cost function is minimized, in other words a rows permutation problem. In this context, and considering the submatrices or interlacing property (corollary 3.1.3 of Horn and Johnson (1990)) the following instances for Eq. 41 can be stated,

$$0 \le \bar{\sigma}(\mathbf{G}_r)/\bar{\sigma}(\mathbf{G}_s) \le 1 \quad (42)$$

$$1 \le \bar{\sigma}(\mathbf{G}_r)/\bar{\sigma}(\mathbf{G}_s) \le ||\mathbf{G}_r\mathbf{G}_s^{-1}||_F \quad (43)$$

for $n > m/2$ and $n < m/2$ cases respectively. Eqs. 42 and 43 show that the functional cost minimization almost has no effect on $\bar{\sigma}(\mathbf{G}_s)$. In the former the ratio is perfectly bounded regardless the functional cost profile, and in the latter case the functional cost produces that $\bar{\sigma}(\mathbf{G}_s)$ and $\bar{\sigma}(\mathbf{G}_r)$ approaching each other. In other words, $\bar{\sigma}(\mathbf{G}_s)$ is bounded and with minimal modifications. This shows that the real impact of the minimization is performed on the ratio shown in Eq. 40. This is true because changes in $\underline{\sigma}(\mathbf{G}_s)$ have stronger impact in the functional cost than those in $\bar{\sigma}(\mathbf{G}_s)$.

Summarizing, this minimization selects the best rows of $\mathbf{G}$ into $\mathbf{G}_s$ such that the multivariable transference gain of $\mathbf{G}_r\mathbf{G}_s^{-1}$ is minimized and consequently the matrix properties of $\mathbf{G}_s$ are improved by increasing $\underline{\sigma}(\mathbf{G}_s)$ (better conditioning and RGA).

As an example, Fig. C.1 can be observed the evolution of the matrix properties along the solutions. In this case the TE process in base scenario was used to test the previously discussed properties of minimizing $||\mathbf{S}_{sp}||_F$. Fig. C.1(a) displays the profile of the minimum singular value for the matrices $\mathbf{G}_s$ and $\mathbf{G}_r$ when

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the solutions of the optimization algorithm was ordered from the best (left) to the worst (right). Can be observed as the $\underline{\sigma}(\mathbf{G}_s)$ increases towards the optimal solution. Analogously, Fig. C.1(b) shows the evolution profile of the maximum singular values in this case. Eq. 42 is fulfill here and there is no apparent influences on $\bar{\sigma}(\mathbf{G}_s)$ and $\bar{\sigma}(\mathbf{G}_r)$ when $||\mathbf{S}_{sp}||_F$ is minimized. Due to this is expected to have a decreasing condition number of $\mathbf{G}_s$ as is shown in Fig. C.1(c).

C.2. Minimization of $||\Theta_2 \mathbf{S}_d \Theta_1||_F^2$- disturbance effect

Again here without loss of generality the term $||\mathbf{S}_d||_F$ can be analyzed instead $||\Theta_2 \mathbf{S}_d \Theta_1||_F^2$. Remembering that $\mathbf{S}_d = [\mathbf{D}_r - \mathbf{G}_r\mathbf{G}_s^{-1}\mathbf{D}_s]$ and take into account some singular and eigenvalues properties (Horn & Johnson, 1990; Golub & Van Loan, 1996), the following inequality can be stated,

$$\left|\bar{\sigma}(\mathbf{D}_r) - \bar{\sigma}(\mathbf{G}_r\mathbf{G}_s^{-1}\mathbf{D}_s)\right| \leq \bar{\sigma}(\mathbf{D}_r - \mathbf{G}_r\mathbf{G}_s^{-1}\mathbf{D}_s) \leq ||\mathbf{D}_r - \mathbf{G}_r\mathbf{G}_s^{-1}\mathbf{D}_s||_F \tag{44}$$

by following the methodology presented in the previous section an upper bound in the minimum singular value of $\mathbf{G}_s$ can be obtained,

$$\underline{\sigma}(\mathbf{G}_s) \leq \frac{\bar{\sigma}(\mathbf{G}_r)\,\bar{\sigma}(\mathbf{D}_s)}{\bar{\sigma}(\mathbf{D}_r)\,.} \tag{45}$$

this result shows that in this case, minimization of $||\mathbf{S}_d||_F$, the $\underline{\sigma}(\mathbf{G}_s)$ cannot be increased freely. In fact, it is bounded above by the process and disturbance transference matrix characteristics. Anyway, in this case also the matrix property $\underline{\sigma}(\mathbf{G}_s)$ tends to be improved, since it is the main reason of singularity in the left-hand component for Eq. 44. Note that, if $||\mathbf{D}_r - \mathbf{G}_r\mathbf{G}_s^{-1}\mathbf{D}_s||_F \to 0$ then $\bar{\sigma}(\mathbf{G}_r\mathbf{G}_s^{-1}\mathbf{D}_s) \to \bar{\sigma}(\mathbf{D}_r)$ and $\mathbf{G}_s$ cannot be ill-conditioned or singular.

So, the minimization of $||\mathbf{S}_d||_F$ can suggests a different solution to the opportunely stated by $||\mathbf{S}_{sp}||_F$, basically due to the constraint in Eq. 45. Obviously, again here appear the classic trade-off control problem between servo and regulator behaviors. In this context, the complemented and weighted functional cost stated in Eq. 20 was proposed.

If the weighting matrices Λ_1, Λ_2, Θ_1 and Θ_2 are used, then the partial control objective from synthesis process and non normalized models can be accounted. The previous analysis is still valid in this case but now the new matrices become $\mathbf{S}_{sp}^* = \Lambda_2 \mathbf{S}_{sp} \Lambda_1$ and $\mathbf{S}_d^* = \Theta_2 \mathbf{S}_d \Theta_1$, and of course the new optimal solution may differ from the unweighted case.

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