Powder Technology 225 (2012) 214–220

Contents lists available at SciVerse ScienceDirect

Powder Technology

journal homepage: www.elsevier.com/locate/powtec

Experimental study of discharge rate fluctuations in a silo with different hopper geometries

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ARTICLE INFO

Article history:
Received 11 May 2011
Received in revised form 3 April 2012
Accepted 4 April 2012
Available online 10 April 2012

Keywords:
Flow
Silo
Fluctuations

ABSTRACT

An experimental study of the flow of quartz particles with two different mean size was performed on a 3D silo at the scale of the laboratory. Two kinds of hopper geometry were used for the cylindrical silo: flat-bottomed and conical. The diameter of the outlet was varied. The dependence of the flow rate and its fluctuations on the silo geometry and outlet size was studied. The Beverloo equation was satisfied in all cases studied and the dependence of its parameters on the silo geometry and on the particle size was analyzed. Using a Fourier analysis, we found a characteristic frequency which varied with the outlet diameter and was virtually independent of the hopper geometry.

1. Introduction

The flow of granular matter through hoppers, extensively used for storing or distributing a wide variety of raw materials, is a long-standing problem. Typically, one of the main objectives from an application perspective is to obtain a constant flow. The difficulty in achieving this goal is due to the complex flow patterns that spontaneously occur inside a hopper. In this sense, the pioneering work of Jenike [1] was fundamental to identify the key flow patterns of the so-called mass flow and funnel flow inside a silo.

Experiments performed using raw granular materials are of great interest due to their direct industrial application. In these experiments it is not possible to control the geometry and physical properties of individual particles. On the other hand, they offer a valuable test to validate models that are based on theoretical studies and/or on experimental results obtained in the case of uniform particles.

Numerous studies have investigated the flow of granular materials (such as glass beads, aggregates, or minerals, among others) through hoppers of various geometries. In general, a given height of a silo is initially filled with the granular material. Then, the outlet of the silo (often a conical or flat hopper) is opened and the mass flow rate is recorded as a function of time. These experiments provide useful information on the relation between the flow rate and different geometrical and physical properties of the system, such as the size and shape of the particles and the outlet, the presence of friction forces and density fluctuations, and others.

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doi:10.1016/j.powtec.2012.04.013

The purpose of this work is to study the discharge rate of quartz particles as a function of the aperture of the hopper and to relate fluctuations in the flow rate to jamming events that take place due to the formation of arches at the outlet. The results presented here have potential applications to industrial processes.

The flow rate of particles through a hopper is expected to increase with the size of the aperture. Beverloo and co-workers studied a variety of sands with different mean particles sizes and of seeds with different shapes and sizes and proposed a relation (here given by Eq. (1)) that will be discussed in detail in the next section [2]. Earlier work by Fowler and Glastonbury [3] and by Brown and Richards [4] predicted a discharge rate that decreases with increasing particle size.

Ulissi et al. [5] characterized the velocity profile corresponding to the discharge of quartz particles flowing out of a quasi-2D hopper; in the continuous regime, the mass flow was found to follow the Beverloo-like behavior expected for quasi 2D silos [6].

More recently, simulations have provided details about the granular flow that are not accessible to experiments such as the influence of frictional parameters between particles, particle shape and measurement of stress chains [7–10]. For example, Ristow applied a molecular dynamics technique to the simulation of the discharge process in a two-dimensional hopper [8]. He measured the dependence of the mass flow rate on the geometrical parameters and the materials used in the experiments and concluded that flow rate did not depend on the restitution coefficient and that the transition from funnel to mass-flow is clearly observable when the hopper angle increases. On the other hand, Cleary and Sawley performed discrete element method (DEM) simulations of granular flows in 2D and 3D silos to demonstrate how particle shape affects the characteristics of the flow [9]. Using the same simulation method, Anand et al. [10], investigated the parameters influencing the discharge rate from hoppers, such as particle size, size distribution, and hopper geometry.

When the size of the flowing particles is comparable to the width of the aperture, jamming can take place, and the particles stop flowing unless additional energy is provided to the system. Many practical problems are caused by jamming at the outlet of hoppers used in production lines, where it is necessary to maintain a constant flow of material. Analogous problems occur in the storage of raw materials in silos, especially after a certain period of accumulation. Frequently, when it is necessary to empty the silo, the material does not flow, either due to the presence of unwanted moisture or because of the compaction of grains sealing the outlet [11–15]. These problems are caused by arches of particles that form at the outlet, which lead to fluctuations and even the interruption of the flow. In Ref. [16] the authors report data that suggest temporal oscillations in the packing fraction near the outlet with a frequency around 2 Hz. This result is important because it is directly related to the likelihood of jamming in the silo [8,17]. Other authors performed flow experiments using metal disks in a two dimensional hopper in order to study the statistical properties of the arches forming at the outlet and found that the average number of disks in an arch increased with the diameter of the aperture [17]. They noted that flow of granular particles in a hopper near the jamming region is a complicated dissipative process and proposed a stochastic approach to describe it. Yang and Hsiau [18] performed simulations and experiments investigating the flow pattern during the filling and discharge of two-dimensional plane silos and they observed fluctuations in the flow due to the formation of arches at the opening. On the other hand, Muite et al. related the sound resonance produced by the vibrations of a silo to the pulsating motion of the granular material inside it [19]. Janda and coworkers [20] studied experimentally the discharge of a two-dimensional flat bottomed silo. In this paper, they found that the power spectrum of the flow-rate oscillations did not display any peak frequency.

Despite many theoretical and experimental studies concerning the discharge flow rate in silos and the relationship between the parameters involved in the process, the presence of fluctuations in the flow has not been explicitly investigated so far, especially in the case of raw materials [20]. In this paper, we focus our effort on experiments performed with an industrial material of widespread use, namely quartz. Specifically, we study the fluctuations of the discharge rate and their relation to geometric characteristics of the silo and to the mean size of the particles.

In the next section we present a brief review of the Beverloo correlation relating the particle discharge rate to the diameter of the outlet. In the second section we describe the experimental set up and the raw material used in this work. In the third section we analyze and discuss the results and compare them to those obtained by other authors. Finally, in the last section, we discuss the most important conclusions of the present work.

2. The Beverloo's correlation

One of the most widely used equations relating the discharge rate of particles to the diameter of the outlet in a flat-bottomed cylindrical silo is the Beverloo's equation [2]:

$$W_{\text{flat}} = C\rho_a g^{1/2}(D-kd)^{5/2} \tag{1}$$

where D is the diameter of the outlet orifice and d is the mean particle size. The constant k is called the Beverloo constant and it is expected to depend on depends on the size and shape of the particles. The constant C is found empirically and is usually in the range $0.55<C<0.65$ and commonly associated with the geometry of the hopper. ρ_a represents the bulk density of the packing after filling the silo and g is the acceleration of gravity. In their work, Beverloo et al. plotted $W^{2/5}$ versus D and obtained a straight line with an intercept Z on the abscissa. The value of Z is proportional to the average particle diameter d with $Z=kd$.

Beverloo's equation has been adjusted many times to take into account effects not included in the original model. For example, in order to take into account the effect of the hopper angle, some authors have modified analytically the correlation developed for a flat bottomed silo [4,8,21]. Other modifications account for the influence of the geometry of the outlet orifice, the size distribution of the particles and for the behavior of the discharge rate at small apertures [10,11,14,22,23].

Nevertheless, Eq. (1) alone predicts very well experimental and numerical experiments, both in two and three dimensional set ups. Understanding the dependence of k and C on experimental parameters is still a challenge today and has motivated a great deal of work [14,24–26].

In the present paper, only the classical version of the Beverloo equation is used [Eq. (1)]. The influence of the particle size and of the geometry of the silo will be taken into account by analyzing the behavior of parameters k and C in the context of Eq. (1). This choice is based on two reasons. On the one hand, previous studies have demonstrated that Beverloo equation is still a very good and simple description for flow discharge in most of the experimental situations, provided appropriate values of k and C are given [10,14,23,25,27]. On the other hand, to take into account the hopper angle one has to modify Eq. (1) multiplying it by a function of the angle. The later analysis of the flow data would also give as a result the values for k and C that best fit the whole results. This latter procedure ends up being the same that the one before, not contributing nothing relevant to our subsequent analysis. These two points have to be taken into account when comparing the values of k and C obtained in this work with those given by other authors for which the influence of the material properties and the hopper geometry are considered to be separated. Moreover, given our experimental conditions (hoppers with different outlet diameters and angles at the same time), we were not able to state here the dependence of C on the angle of inclination of the hoppers at a given D, as other authors have done.

3. Experimental procedure and material

The flow of quartz particles was studied in a cylindrical silo made of a zinc sheet, of diameter 33 cm and height 100 cm. The hoppers fitted at the bottom of the silo were either conical or had a flat bottom. The diameter of the orifice at the outlet was varied by means of an interchangeable set of hoppers. A sketch of the silo and of the interchangeable hoppers is shown in Fig. 1. The mean diameter D of the different outlets used in the experiments ranged approximately from 2 cm to 11 cm, as shown in Table 1. Orifice diameters were measured using a digital caliper and each value in Table 1 was the average of five measurements taken on different equidistant directions in the orifice. All the interchangeable

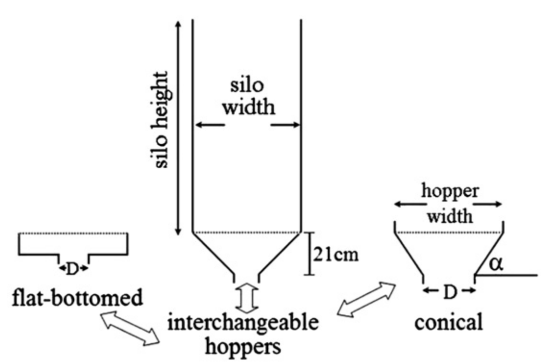

Fig. 1. Sketch of the silo and interchangeable hoppers used in experiments. The widths of the silo and hopper are both 33 cm; silo height is 100 cm.

Table 1
Values for the mean diameter D of the hoppers and angles for hopper walls.

Conical D (cm)	Angle α (°)	Flat D (cm)
1.9	53.3	1.9
2.4	53.7	2.4
3.1	54.3	2.9
3.4	54.6	3.4
3.9	55.1	3.9
4.5	55.6	4.4
4.9	56.0	4.8
5.4	56.5	5.3
6.0	57.0	5.9
6.4	57.5	6.3
6.9	58.0	6.9
8.9	60.0	9.0
11.0	62.2	11.0

conical hoppers had the same height $h = 21$ cm but a different angle α which determined the diameter of the outlet.

Crushed quartz coming from a local quarry was the granular material used in the experiments. Two sets of particles with different sizes were sieved: (i) 5 mm $<\varphi<$ 10 mm (large particles, L) and (ii) 2 mm $<\varphi<$ 5 mm (small particles, S) with mean diameters 7.5 mm and 3.5 mm, respectively. Digital image processing was used to measure the particle size distribution and their shape. The cumulative size distribution around the mean value was similar for both sets of particles. Both had the same roundness (around 0.8) and their mean aspect ratio was around 0.7 for set (i) and 0.65 for set (ii), i.e., set (ii) was slightly more elongated. The density of quartz was determined experimentally and is $\rho = (2.65 \pm 0.01)\,g/cm^3$.

The bulk densities for the two different sets of particles were measured by pouring the granular material inside a container of known volume. The material was poured at a constant flow rate and at a constant height above the surface of the filled container (constant absolute height). The container was filled up to its rim and the material was leveled. The density was computed as the ratio between the mass of material inside the container and its volume. The measured densities were $(1.41 \pm 0.01)\,g/cm^3$ and $(1.35 \pm 0.01)\,g/cm^3$, for sets (i) and (ii), respectively.

We also determined the angle of repose or tilt angle of the surface of the stagnant zone during the flow of particles in a flat-bottomed hopper. The results were $(40.4 \pm 0.6)^{\circ}$ and $(40.4 \pm 0.4)^{\circ}$ for sets (i) and (ii), respectively. These values were smaller than angle α for all the conical hoppers used in the experiments; thus, no stagnant zone was expected for those hoppers.

Before each experiment, the same amount of material (around 30 kg) was loaded into the silo using the same filling procedure to obtain the same degree of compaction. Nevertheless, the effects of the initial packing on the discharge rate have been proven to be of little importance [12]. The filling height (around 17 cm measured from the base of the silo) was greater than the critical value needed to ensure that the flow rate be independent of the height, i.e., of the order of the outlet orifice diameter [10,22]. Besides, the diameter of the silo was large enough to avoid boundary effects during the discharge [4].

In order to prevent contamination between the two set of particles, all the experiments using a given granulometry were performed consecutively. The flow rate was measured using an electronic balance with a maximum capacity of 30 kg and an accuracy of 10 g. The balance was linked by an RS 232 interface to a PC allowing one to measure the weight as a function of time with a maximum temporal resolution of 0.2 seconds. This enabled one to record dynamically the long term variation of the discharge flow rate and its fluctuations as a function of time. For particle size distributions (i) and (ii), ten experiments were performed under the same conditions for each hopper geometry and each outlet size.

We verified that inertial effects were negligible for these experiments and for the kind of electronic balance which we used. This was shown by the fact that neither the measured flow rate nor its fluctuations varied as the vertical path length of the free falling grains changed. Since the grains were discharged from a fixed outlet, the height of the free fall (for the grains) became shorter as the receiving container got filled. This means that the inertial force would decrease between the beginning and the end of the experiment. If this were the case, a departure from a linear variation of the mass as a function of time would have been noted. However, this was not the case at any time.

In order to characterize the frequency response of the balance, the following experiment was performed. A solid body of known volume partly immersed in a liquid filling up a container was forced to oscillate at a known frequency; the variations of the apparent weight of the container due to the varying buoyancy force resulting from Archimedes principle were measured by the electronic balance used in the experiments and recorded as a function of time at a sampling interval of 0.2 seconds. The frequencies used for the forced oscillations were equal to 0.6 and 0.95 Hz, both below the value of the Nyquist frequency related to the sampling interval. The Fourier transform of the time series showed a periodic component of the mass fluctuations exactly equal to the frequency of the external forced oscillations. Moreover, no characteristic frequency due to the balance or to the method of measurement was detectable. This showed that the balance could successfully record the mass fluctuations in the range of frequencies studied.

4. Results and discussion

The results can be classified into two groups: conical hopper flow (CHF) and flat bottomed hopper flow (FHF). Fig. 2 displays the mass

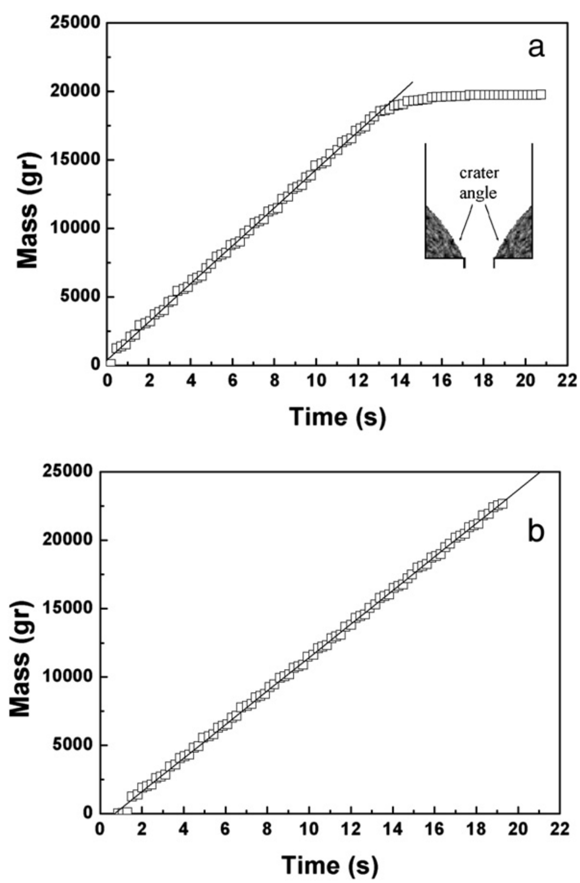

Fig. 2. Time variation of two typical mass discharge rates obtained using small particles. To compare both cases, we show a similar range for the values of mass discharge for both types of hoppers. (a) Flat-bottomed hopper with $D = 5.9$ cm; mass flow rate = 1391 gr/s. The inset indicates the stagnant zone. (b) Conical hopper with $D = 5.9$ cm; mass flow rate = 1226 gr/s. In both cases the line indicates a linear fit.

discharged from the hopper as a function of time for two typical discharge profiles using small particles. Part (a) of the figure corresponds to an FHF with $D = 5.9$ cm, while part (b) corresponds to a CHF with $D = 5.9$ cm. Fig. 2(a) clearly shows a linear variation region. There is also a transition region in which the slope decreases to zero until the flow stops. This is a typical behavior in a flat bottomed hopper. On the other hand, Fig. 2(b) also displays a linear behavior during the whole duration of the discharge, as expected for a conical hopper. For each of these plots, the corresponding mass flow rate is determined as the slope of the linear fit of the curve in the region of constant slope.

In the case of flat bottomed hoppers, the residual mass of particles in the stagnant zone was measured as a function of the outlet size for both sets of particles. These data allowed to determine, indirectly, the angle of repose of the crater. The results were in good agreement with those obtained from direct measurements, within experimental errors.

Fig. 3 shows the mean mass flow rate, W, as a function of the outlet size D for the two different particle size distributions and hopper geometries.

Mean flow rate values for flat bottomed hoppers are slightly higher than those for conical hoppers for both size distributions and for all outlet sizes, as seen in Fig. 3. This effect could be understood given that in the flat bottom hopper experiments the flow channel angle (created by the flowing particles) was likely steeper than with the conical hopper (flow channel "walls" were more frictional) and this explains why the measured flat bottom hopper flow rates were always higher than the conical hopper flow rates [21]. In addition, for all outlet sizes, W is a little greater for smaller particles. This is consistent with the prediction of Beverloo's equation that the prefactor kd is lower for smaller particles [2].

In order to test the validity of the theoretical predictions, Fig. 4 displays the variation of $W^{2/5}$ with D for the data in Fig. 3. It is clear that $W^{2/5}$ varies linearly with D in all cases. Parameters k and C can be obtained from the slope of the curve and from the intercept of the linear regression with the abscissa using Eq. (1) and $Z = kd$. In all cases, excellent linear correlations with coefficients in excess of 0.99 were obtained.

Table 2 shows the results obtained in the four cases. It is worthy to mention that the values of k and C show a dependence on both silo geometry and particle properties. Nevertheless, k seems to depend more strongly on the properties of the particles, while C appears to be more influenced by the geometry of the hopper.

The values of k for larger particles were smaller than for smaller ones. It is noteworthy that experiments carried out by other authors

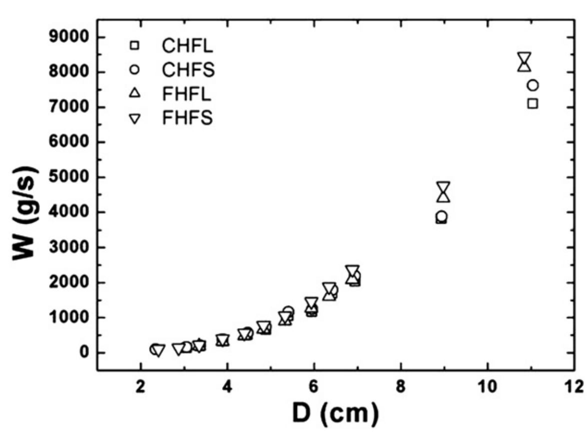

Fig. 3. Average mass flow rate, W, for the different particle sizes and hopper geometries, as a function of the outlet size D. Flow configurations: CHFS (conical hopper flow for small particles), CHFL (conical hopper flow for large particles), FHFS (flat bottomed hopper flow for small particles) and FHFL (flat bottomed hopper flow for large particles). Error bars would be of the order of the height of the symbols.

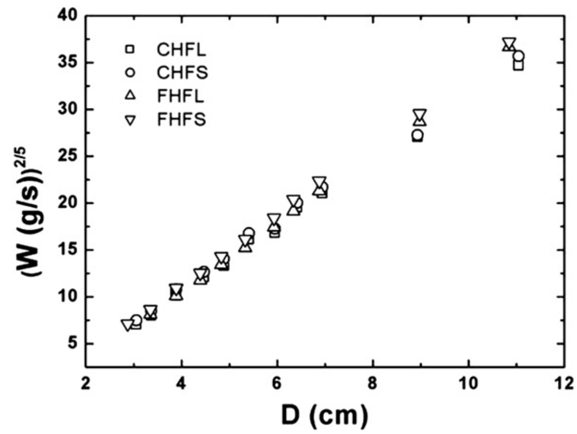

Fig. 4. Average mass flow rate, $W^{2/5}$, for the two different particle sizes and hopper geometries, as a function of the outlet size D. Experimental data are the same as in Fig. 3.

with spherical particles predicted a value for $k = 1.5 \pm 0.1$ [24]. On the other hand, a value of $1.3 < k < 2.9$ was found in the pioneering work of Beverloo [2] and, for different particle shapes, values of $1 < k < 2$ are found and discussed in the literature [11,14,20]. The value of k is influenced by the variations of the value of W associated with different particle–particle friction coefficients, shapes and size dispersion; thus, factor k is the one that subsumes this effect [24]. For example, in the case of sand, values of $k = 2.9$ were found in Ref. [2], while a value of 1.5 is reported in [25]. Thus, no definite conclusion can be said about the correct value for k expected in a given experimental situation. In the present study, we do not have a comprehensive conclusion on this point, but we suspect that the presence of a size distribution in the grains could be quite relevant and the use of just a representative mean value in Eq. (1) might be decisive for the value of k obtained. It is not our present interest to analyze all the possible factors influencing the value of k. Besides, k values were slightly larger for flat-bottomed hoppers than for conical ones.

C is smaller for conical hoppers than for flat bottomed ones. The values for C are influenced not only by the geometry of the silo, but also by the choice in the value of the bulk density [11]. In our case, the values for the bulk densities employed are those for the static material and may not accurately represent the bulk density of the flowing material within the hopper [25]. It is highly probable that the density for conical hoppers could be slightly lower than that for the static material, explaining the lower value for C in the conical hoppers. Nevertheless, the values obtained are close to the limits predicted by Beverloo [2]. Moreover, for each hopper geometry, C is practically the same as predicted in Ref. [25] for coarse sand and being slightly larger for smaller particles.

As mentioned before, in each experiment, the mass discharge rate was obtained by differentiating the variation of the mass with time, and a typical plot of the fluctuations observed is shown in Fig. 5(a), for a conical hopper with $D = 5.9$ cm and for particles belonging to set (i). The horizontal line indicates the value for the mean discharge rate. It is interesting to mention that other authors have measured flow fluctuations during the discharge of rounded particles from a model silo through DEM simulations [28]. In that paper the authors

Table 2
Values for parameters k and C obtained from linear fits on experimental data.

Hoppers	Particle sizes $\bar{d}$ (mm)	C	k
CHFL	7.5	0.50	1.27
CHFS	3.5	0.53	2.34
FHFL	7.5	0.63	1.68
FHFS	3.5	0.65	2.92

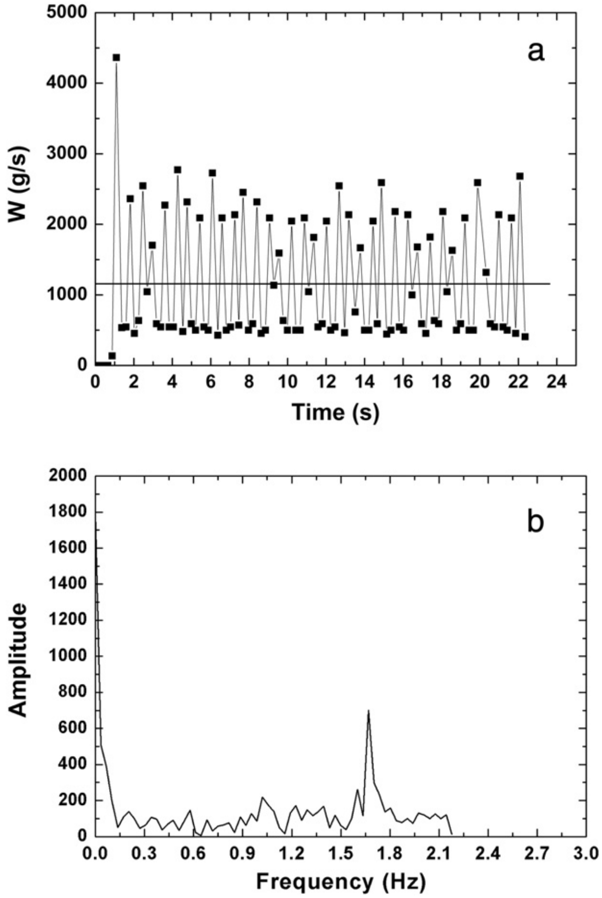

Fig. 5. (a) Typical variation of the granular mass flow rate as a function of time (conical hopper with $D=5.9$ cm for particles from set (i)). The horizontal line indicates the value for the mean discharge rate. (b) Fourier analysis of part (a).

found an upper bound of 1.2W and a lower bound of 0.75W for flow fluctuations with a mean value W. In our case, we find larger fluctuations than those reported in Ref. [28] for comparable outlet diameter/particle diameter ratios. It is expected that for irregular particles such as quartz, the fluctuations will be higher than those for rounded particles since the likelihood of arching is higher, given the increased friction and the irregular shape. In fact, we have performed experiments with rounded seeds (soya beads with similar size like quartz particles of set (i)) and found that fluctuations using the same hopper were systematically smaller for soya beads than for quartz particles. A more extensive discussion regarding the fluctuations found in our study is presented below.

Fig. 5(b) shows the Fourier analysis of the fluctuations with time corresponding to part (a). A clear peak is visible at a frequency on the order of 1.67 Hz. One has to be careful in analyzing the data because, as is well known, spectral aliasing problems may arise. We discuss this point further below.

In all experiments, the power spectrum of the flow rate fluctuations at the outlet of the hopper was analyzed, in order to detect periodic oscillations.

We found indeed that a well defined peak frequency reflecting a periodic oscillation is always present and that its value depends on the aperture of the outlet of the hopper while being almost independent of its geometry or of the particle size. In Fig. 6 we show the variation of this frequency with the size of the outlet for all the cases investigated. In this figure, the results correspond to the mean values obtained from ten runs performed under the same conditions as above.

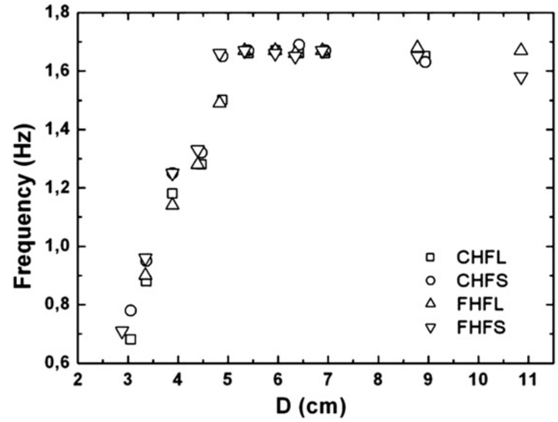

Fig. 6. Typical variations of the flow rate fluctuation frequencies determined from a Fourier analysis as a function of the outlet size for the four configurations studied.

In Fig. 6, a nearly linear dependence on D can be seen up to approximately $D=5.0$ cm. In this linear region, the frequency increases with the size of the outlet; for small particles, it is slightly higher than that corresponding to large particles. For $D>5.0$ cm, the highest value of the frequency is 1.7 Hz; this is still far enough from the Nyquist frequency equal to 2.17 Hz. (corresponding to the sampling interval $t_0=0.23$ seconds). Moreover, we can assume that there are no aliasing problems since the frequency increases continuously in the whole range of values of D below 5.0 cm.

Flow fluctuations may be interpreted as due to particle arching near the outlet of the hopper. These arches may cause transient jams that decrease the outflow. The breaking of the arches increases the flow so that the dynamical process of arch formation and destruction causes the fluctuations observed here. The period of the oscillations is thus an increasing function of the duration of these arches. This accounts for the increase of the frequency with the diameter of the outlet of the hopper. Indeed, the wider the outlet, the larger the number of particles required for the formation of the arches; then, the probability of building up an arch is smaller and, besides, the arch is more unstable and the time lapse for its destruction is shorter.

Additional information is provided by the distribution of the values of the flow rate in Fig. 5(a). The first feature is that this distribution is asymmetrical with respect to the mean value; the lower limit is well defined while there is a broad distribution of the maximum values. The second feature is the existence of two characteristic times during the discharge process; this will be explained now. Fig. 7 displays the typical pattern of the fluctuations of the mass flow rate around its mean value W_{mean}. On the one hand, there is a time

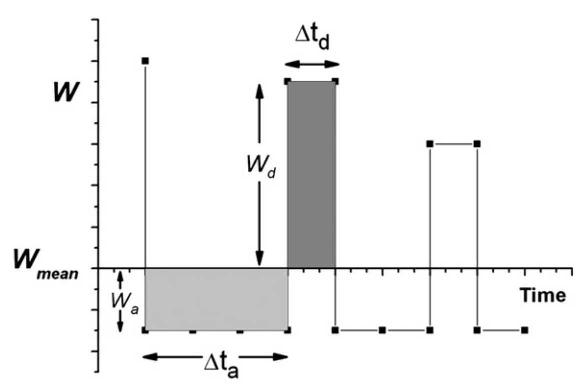

Fig. 7. Typical variations with time of the deviation of the instantaneous mass flow rate W with respect to its mean value W_{mean}. The gray areas correspond to the delayed mass (light gray) and mass gained (dark gray) during a complete fluctuation cycle. Note that $W_{\mathrm{d}}\,\Delta t_{\mathrm{d}}=W_{\mathrm{a}}\,\Delta t_{\mathrm{a}}$.

interval Δt_a during which the flow rate decreases down to a value W_a (below W_{mean}); on the other hand, during a subsequent time interval Δt_d, the flow rate increases up to a value W_d (above W_{mean}). This former decreasing trend is likely related to the partial transient blockage of the grains and will end by an avalanche resulting in a new increase of the flow rate. This means that the discharge process presents pulses with a period $T = \Delta t_d + \Delta t_a$; one has the relation $\Delta t_a / \Delta t_d = W_d / W_a$ between the average values of the four variables. This is equivalent to a mean pulse frequency given by:

$$f = \frac{1}{T} = \frac{1}{\Delta t_d \left(1 + \frac{W_d}{W_a}\right)} \quad (2)$$

From the experimental time series like that shown in Fig. 5(a), one can obtain the corresponding average values for Δt_d, W_d and W_a, allowing one to compute f.

The frequencies calculated from Eq. (2) have been compared to the values obtained from the Fourier analysis as a function of the outlet size. As an example, Fig. 8 shows the comparison for the case of flat bottomed hoppers and small particles. A good agreement is obtained in this case, as can be appreciated from the figure, and in all other cases not shown in the figure.

The discussion above suggests a physical explanation of the periodic pulses observed; they likely result from a dynamic process of transient blockage decreasing the mass flow rate with a characteristic time of accumulation of material prior to the occurrence of an internal avalanche near the outlet. This accounts for the asymmetry in the distribution of the fluctuations around the average flow.

Results from two other previous studies provide an additional input to our description. On the one hand, authors in Ref. [29] found experimentally a linear variation for the number of particles forming an arch with the outlet size in a 2D silo. On the other hand, other authors reported a decaying behavior for the stability of an arch as a function of the number of particles forming it, during the discharge of a 2D silo [17]. This stability is characterized by the average duration of an arch. The duration of an arch is directly proportional to the oscillation period of the flow at the outlet and it is also expected to be valid for 3D systems. Following the trend for stability above, it is expected that the period of the fluctuations will decrease monotonically with the increase in the number of particles forming an arch, or, equivalently, with the increase of the outlet size. This might explain, at least qualitatively, the increase of the frequency with the outlet size found in the present experimental results. In this sense, it is interesting to know how does the amplitude of the discharge rate fluctuations vary as D increases. It is expected that this amplitude

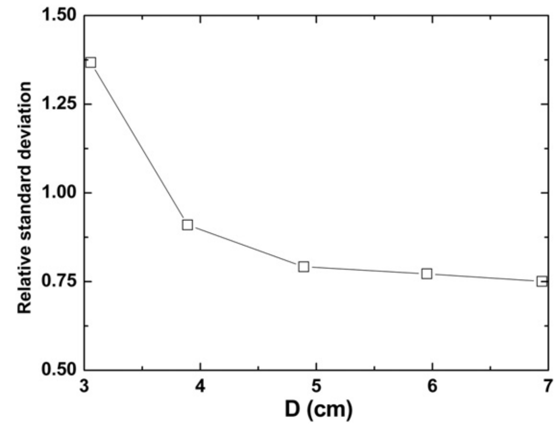

Fig. 9. Standard deviation of the discharge rate fluctuations relative to the mean discharge rate as a function of D for conical hoppers and large particles (CHFL).

decreases with D, given that a smaller stability for arching is present. To illustrate the case, Fig. 9 shows the standard deviation of the discharge rate fluctuations relative to the mean discharge rate as a function of D for conical hoppers with particles from set (i) (CHFL). As can be observed, the relative amplitude of the fluctuations decreases as the outlet size increases, confirming the expectations. The behavior for the other hoppers or particle size is similar.

Another mechanism might be the presence of density waves at the outlet of the silo [19,30,31]; this may influence the dynamics of arching, thus accounting for the periodic component of flow fluctuations.

Finally, the variation of the characteristic frequency f as a function of the dimensionless size of the outlet aperture, $R = D/\bar{d}$, is plotted in Fig. 10 for both sets of sizes, where $\bar{d}$ is the mean particle size. It is important to remember here that R plays the role of a mean dimensionless parameter for each of these sets. It is clear from the figure that the curves are shifted relative to each other and that their slopes are different due to the fact that the results for flat and conical hoppers coincide while the mean sizes of the particles differ. For a given value of R, the frequency corresponding to the larger particles is greater than for smaller ones. Although the ratio between the size of the outlet and the average particle diameter is fixed (the number of particles to form an arch is fixed for a given R), the ratio of the height and diameter of the silo to the size of the particles is not constant. This means that a calculation of the number of particles available in the volume near the outlet gives a greater number of smaller particles than of larger ones. If one thinks that the probability of jamming the system is proportional to both, the number of particles needed to form an arch and the availability of particles near the outlet, it results

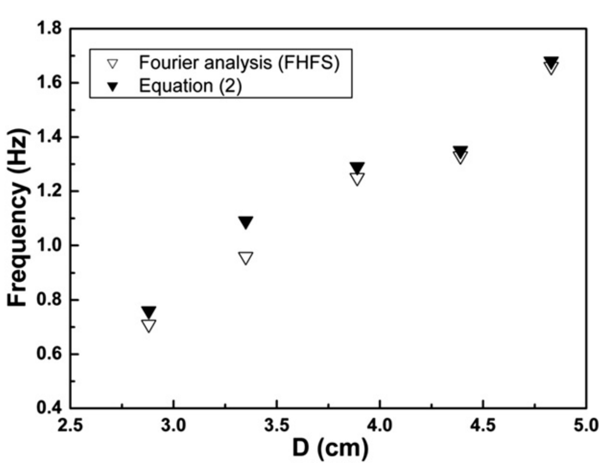

Fig. 8. Comparison of frequencies computed using Eq. (2) and those obtained from Fourier analysis.

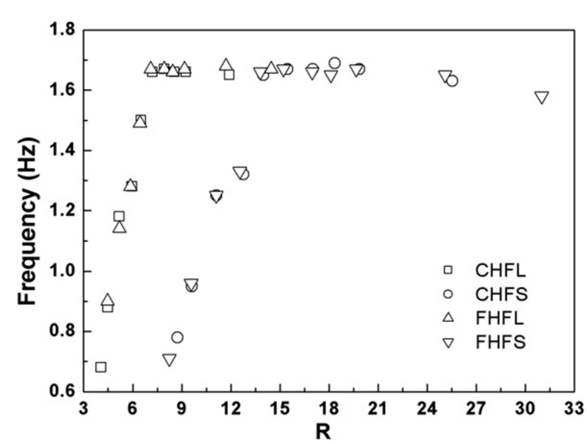

Fig. 10. Typical frequencies as a function of the relative orifice outlet $R = D/\bar{d}$ for both family sizes and hopper geometries.

that the likelihood of arching is higher for the smaller particles, this leading to a greater lifetime and a lower frequency. In other words, the probability that two arches formed by the same number of particles be destroyed is the same for small and large particles when *R* is the same in both cases, but, it is easier to quickly reconstitute the arch that has at its disposal more particles to form. The net effect of this dynamic is as if the arch duration for smaller particles were greater, resulting in a lower frequency for the flow fluctuation of those particles.

5. Conclusion

We reported experimental results for the flow of dry quartz grains through a laboratory-scale silo. The material was classified into two sets of grains with different size distributions. The hoppers used at the outlet of the silo were of two types: conical and flat-bottomed.

In all the experiments and given our experimental conditions (hoppers with different outlet diameters and angles at the same time), we found that Beverloo's equation [Eq. (1)] was adequate enough to describe the dependence of the flow on the size of the hopper outlet, provided appropriate values of the parameters *k* and *C* were chosen. In this sense, one can get rid of the modifications of the classical version of Eq. (1) and simply include the influence of the particle size and silo geometry into the values of *k* and *C*, at least for our present scope.

The values of *k* and *C* show a dependence on both silo geometry and particle properties but, *k* seems to depend more strongly on the properties of the particles, while *C* appears to be more influenced by the geometry of the hopper. The values of *k* for large particles were smaller than those for small particles. The values calculated for *C* showed to be lower for the case of conical hoppers. This may be due to the choice of the value of the bulk density for conical hoppers. Nevertheless, all the values of *C* were close to the limits predicted by Beverloo.

In all cases, flow fluctuations presented a characteristic frequency of oscillation related to accumulation and discharge cycles. These cycles were used to calculate the frequency of the flow pulses in the discharge of the silo.

The variation of the frequency as a function of the outlet aperture was almost linear up to a diameter of 5 cm. These results were qualitatively consistent with those of previous studies on the stability of arches as a function of the number of particles forming them [17]. Besides, the behavior of the relative amplitude of the discharge rate fluctuations shows a decrease with the size of the outlet aperture, confirming that a smaller stability for arching is present.

On the other hand, regarding the observed saturation zone, it should be noted that the same work showed that arches larger than 5 particles (in two dimensions) had a probability of survival practically constant and very low. Therefore, the period of flow fluctuations in this region would not substantially change with the size of the arches or, equivalently, with the diameter of the outlet. To elucidate this point, the sampling interval for the measurement of flow rates should be reduced in order to increase the upper measurable frequency. The above analysis implied that periodic fluctuations in flow are closely related to the dynamics of formation and destruction of arches near the outlet of the silo.

Other authors' measurements gave closely related results; they have pointed out the presence of a transition from a plug flow to a converging flow as the material approached the orifice [19]. These authors performed pressure measurements close to the transition from the bin to the hopper; they indicated that there was also a stress discontinuity and that there could be large pulsating stresses, which corresponded to the cyclical formation and rupture of the dynamic arch.

Future efforts will be directed to study the relation between the frequency of oscillations in the discharge rate for a given silo-material system and the jamming problems frequently present in industry.

Acknowledgments

This work was supported by CONICET (Argentina) through Grant PIP No. 1022 and by the Secretary of Science and Technology of Universidad Nacional de San Luis. Authors want to thank Dr. J.P. Hulin (FAST–CNRS, Université Paris Sud 11, France) for fruitful discussions of the paper.

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