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Inflation in Inhomogeneous Spacetimes: Bubble Evolution

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Abstract. The evolution of a vacuum bubble embedded in an inhomogeneous spacetime is relevant for the modelling of inflation in the presence of inhomogeneities. We developed a numerical scheme based on Israel's matching conditions to solve the evolution of the bubble for spherically symmetric inhomogeneous backgrounds. Particular attention is paid to spacetimes with perfect fluid with non-zero pressure as a source, which are described by Lemaître solution of Einstein's equations. As a previous step that serves as a check of the numerical scheme, we present here the case of a bubble evolving in a FLRW metric with radiation as a source.

1. Motivations

Although inflation is specifically designed to solve some of the problems of the standard cosmological model, it is not free of its own problems. Among these, perhaps the most relevant ones are those related to the beginning of inflation in the presence of inhomogeneities, and the evolution of an inflating region in an inhomogeneous ambient. Regarding the first problem, numerical (Goldwirth & Piran, 1989) and analytical (Perez & Pinto Neto, 2011) studies show that spacetime has to be homogeneous and isotropic to a high degree for inflation to start. Regarding the second problem, an inflating region may stop inflating in particular ambient spacetimes, as discussed in (Fischler *e*t al., 2009; Rakic *e*t al., 2009) for the FLRW and LTB cases.

The question of whether an inflating region can continue to inflate if the ambient region is inhomogeneous has been explored by several authors for different backgrounds (Goldwirth & Piran, 1989; Fischler *e*t al., 2009; Rakic *e*t al., 2009). In all these cases, the inflating region was modelled by a vacuum bubble, separated from the exterior by a thin wall. The results show that the presence of matter slows down (and may even stop) the expansion of the bubble. An inhomogeneous outer spacetime was only studied for spherically symmetric dust (Fischler *e*t al., 2009; Rakic *e*t al., 2009), which is a first step aiming at a more realistic description in terms of ultrarelativistic matter. We intend here to undertake this latter task.

2. General Framework

A propagating bubble divides the space-time into three regions:

1. The inner region, which is described by a de Sitter spacetime in the case at hand.
2. The bubble, which is assumed to be a thin-shell, with a perfect fluid energy-momentum tensor with an equation of state (EoS) given by $P = w\sigma$. It is described by the hypersurface Σ with geometry

$$\mathrm{d}s^2|_\Sigma = \mathrm{d}\tau^2 - \rho^2(\tau)\mathrm{d}\Omega^2. \tag{1}$$

3. The background, which will be assumed to be spherically symmetric and filled with a perfect fluid with non-zero pressure ($T_{\mu\nu} = (\epsilon+p)u_\mu u_\nu - pg_{\mu\nu}$). This back is described by Lemaître's solution of Einstein's equations, which in comoving coordinates takes the form

$$\mathrm{d}s^2 = \mathrm{e}^{A(t,r)}\mathrm{d}t^2 - \mathrm{e}^{B(t,r)}\mathrm{d}r^2 - R(t,r)^2\mathrm{d}\Omega^2. \tag{2}$$

Einstein's equations for this geometry are given by

$$\kappa R^2 R_{,r}\epsilon = 2M_{,r} \tag{3}$$
$$\kappa R^2 R_{,t} p = -2M_{,t}, \tag{4}$$

where $M(t,r)$ is defined by

$$2M = R + R\mathrm{e}^{-A}R_{,t}^2 - \mathrm{e}^{-B}R_{,r}^2 R - \Lambda R^3/3. \tag{5}$$

Using the conservation of $T_{\mu\nu}$, we obtain

$$A_{,r} = -2p_{,r}/(\epsilon+p) \tag{6}$$
$$\mathrm{e}^B = \frac{R_{,r}^2}{1+2E}\exp\left(\int_{t_0}^{t}\frac{2R_{,t}}{[\epsilon+p]R_{,r}}p_{,r}\mathrm{d}\tilde{t}\right), \tag{7}$$

where $E(r)$ is an arbitrary function related to the local curvature (Bolejko *et* al., 2006). Note that in the case of dust, the above equations reproduce the LTB model, where $\mathrm{e}^A = 1$ and $\mathrm{e}^B = R_{,r}^2/(1+2E)$. The FLRW limit is obtained when $R(t,r) \to a(t)r$, $M \to M_0 r^3$ and $E \to E_0 r^2$.

3. Numerical Scheme

In order to describe the evolution of the bubble we use the general thin-wall formalism based on the Israel's junction conditions (Israel, 1966; Berezin & Kuzmin, 1987; Visser, 1996). They relate the discontinuity in the extrinsic curvature to the energy-momentum tensor of the bubble across the hypersurface Σ which separates two given spacetimes. The outer coordinates are related to the comoving time of the bubble by $\mathrm{d}\tau^2 = \mathrm{e}^A\mathrm{d}t^2 - \mathrm{e}^B\mathrm{d}r^2$. Since the coordinates r and t are both functions of τ on the shell, the evolution of the radius of the bubble $\rho(\tau)$ can be parameterized by the function $\tilde{r} = r(t)|_{shell}$, with $\rho(\tau) = R(t, \tilde{r}(t))$.

The equations that govern the evolution of the radius and the density of the bubble are

$$\left(\frac{\mathrm{d}\tilde{r}}{\mathrm{d}t}\right)^2 (R_{,\tilde{r}^2} + e^B(R^2C^2 - 1)) + 2R_{,t}R_{,\tilde{r}}\left(\frac{\mathrm{d}\tilde{r}}{\mathrm{d}t}\right) + R_{,t}^2 - e^B(R^2C^2 - 1) = 0, \quad (8)$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} + \frac{2R_{,t}}{R}\sigma(1+w) - \left[e^{A/2}e^{B/2}(\epsilon + 3p)\left(\frac{\mathrm{d}\tilde{r}}{\mathrm{d}t}\right)\right] \Big/ \sqrt{e^A - e^B(\mathrm{d}\tilde{r}/\mathrm{d}t)^2} = 0, \quad (9)$$

where

$$C^2 = \frac{\Lambda_{in}}{3} + \left[\frac{\sigma}{4} + \frac{1}{\sigma}\left(\frac{\Lambda_{out} - \Lambda_{in}}{3} + \frac{2M}{R^3}\right)\right]^2.$$

These expressions depend on the background geometry and are general in the sense that they can be used for FLRW, LTB or Lemaître backgrounds by calculating the corresponding metric functions A and B, and using a proper EoS for the perfect fluid on the shell.

The evolution of a bubble of vacuum on a background with a non-zero pressure fluid and non-zero Λ will substantially depend on the pressure balance and on the relation of the surface tension to the difference in inner and outer Λ, and dust density (Rakic *et* al., 2009). This can be seen from the equation for the acceleration of the radius of an initially comoving bubble, which in the FLRW case is given by

$$\ddot{r}|_{\dot{r}=0} = \frac{1}{a}\left(\frac{\Lambda_{out} - \Lambda_{in}}{24\pi\sigma} - 2\pi\sigma - \frac{2\rho + \alpha p}{3\sigma}\right), \quad (10)$$

where α is a numerical coefficient. It follows from this equation that both the density and the pressure of the external fluid tend to slow down the expansion of the bubble.

4. Results and Discussion

We obtained a system of ordinary differential equations which determines the evolution of the shell based in the Israel's junction conditions. These general expressions allow us to analyze several combinations of background metrics and/or matter content, and different EoS for the matter on the shell.

A numerical scheme was developed to solve the problem for different choices of the initial conditions and the parameters which describe the shell and the background. The goal of the analysis is to determine if the outer non-zero pressure fluid can slow down the bubble and eventually stop the inflation of the vacuum region in inhomogeneous backgrounds. We will also investigate if the external inhomogeneities leave any traces in the region contained in the bubble.

The case of an homogeneous background with radiation fluid as a source is presented as an example. The outer FLRW metric is described by $\Lambda = 3 \times 10^{-5}$, curvature function $E(r) = -kr^2$ with $k = 4 \times 10^{-6}$, $\rho_{rad} = A/a^4(t)$ with $A = 1 \times 10^{-3}$, and the initial condition $a_0 = 1$. The evolution of the radius of the bubble for several initial sizes and different choices of the EoS is shown in Fig. 1.

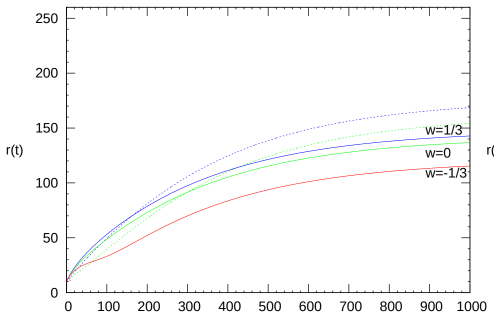

Figure 1. A closed FLRW background with perfect fluid as a source that will eventually asymptote to a de Sitter spacetime is considered. Solid and dashed lines indicate, respectively, the pure radiation and the pure dust cases. The two panels show the evolution for different initial radius of the bubble ($\tilde{r}_0 = 10, 100$, respectively). The initial density of the shell is $\sigma_0 = 1 \times 10^{-3}$ and several values of the parameter w of the EoS are considered. Qualitatively, the asymptotic behaviour of the comoving bubble coordinate $\tilde{r}(t)$ does not depend on the initial conditions or on the EoS of the bubble.

As a test for our code, and for comparison with the radiation case, the curves for the dust case presented in Fischler *et* al. (2009) are also displayed in the Figure. The plots show that the evolution of the bubble when pure radiation is considered is qualitatively the same as in the case of pure dust.

We are presently exploring the evolution of the bubble for different curvature profiles for inhomogeneous backgrounds described by the Lemaître metric (i.e. spherically symmetry and non-zero pressure perfect fluid as a source). We are also deducing a theoretical expresion for the acceleration of the bubble, that will generalize Eq. (10) in order to gain insight on the role of the density and pressure in the evolution equations.

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