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Robust model predictive controller with output feedback and target tracking

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Abstract: A model predictive controller (MPC) is proposed, which is robustly stable for some classes of model uncertainty and to unknown disturbances. It is considered as the case of open-loop stable systems, where only the inputs and controlled outputs are measured. It is assumed that the controller will work in a scenario where target tracking is also required. Here, it is extended to the nominal infinite horizon MPC with output feedback. The method considers an extended cost function that can be made globally convergent for any finite input horizon considered for the uncertain system. The method is based on the explicit inclusion of cost contracting constraints in the control problem. The controller considers the output feedback case through a non-minimal state-space model that is built using past output measurements and past input increments. The application of the robust output feedback MPC is illustrated through the simulation of a low-order multivariable system.

1 Introduction

The lack of guaranteed robust stability is still one of the weaknesses of the available model predictive controller (MPC) commercial packages that are based on linear models [1]. A robust controller is supposed to provide closed-loop stability for some classes of model uncertainty or unknown disturbances. As many process systems are non-linear, and usually operate in the vicinity of distinct operating points, different linear models should be used to represent the process across the operating region. The design of the MPC controller is usually based on a nominal linear model of the process and no explicit consideration of model uncertainty or unmeasured disturbances is included. So, even when a nominally stable controller is used, stability is still an issue when the real plant model may be significantly different from the nominal model. This subject has been extensively treated in the control literature for the case where the system is represented by a minimal order state-space model where the state is measured. Some of these works have addressed robustness with respect to model uncertainty [2–6], whereas other references have considered robustness with respect to disturbances [7–9]. For the case of state feedback, the existing solutions to the robust MPC problem seem to be already in an acceptable stage for practical implementation.

An interesting approach to the problem of robust control was proposed by Badgwell [4] that developed a robust MPC for the regulator operation of stable systems assuming the multi-plant uncertainty. The method was extended to the case of output tracking of systems with unmeasured disturbances [10, 11], and to the output tracking of systems with stable and integrating modes [6] for the same sort of model uncertainty. Other recent MPC formulation to solve the robust control problem is based on the development of a control Lyapunov function, which is independent of the control cost function. Mhaskar [12] applied this approach to the case of model uncertainty and control actuator fault.

All the developments listed above were also based on the assumption that the system state is perfectly known at each sampling instant of the MPC control cycle. In practice, the state-space model is usually built based on experimental data, and the state of the model is not measured. To estimate the model state that is used in the MPC control problem, it is necessary to include a state observer. The separation principle guarantees that a stable state feedback linear controller, associated with an asymptotic stable observer, stabilises the closed-loop system. However, in the constrained MPC, the controller becomes non-linear when the constraints become active, and the separation principle cannot be applied.

An alternative is to develop an MPC, which is based on a state-space model where the state is always known. Maciejowski [13] presented one of such models and Wang and Young [14] discussed the advantages of the MPC based on that model. One disadvantage of the model presented by Maciejowski [13] is that, it is not of minimal order and this property complicates the application of the infinite horizon MPC (IHMPC) with robust stability proposed by Odloak [11]. Thus, the main scope here is to extend the methods of Odloak [11] and Badgwell [4] to the case where the system is represented by a non-minimal order model in which the state at the present time is composed of the output and input measurements at the present and past sampling instants. Here, it is extended from the previous work [15] where an IHMPC with output feedback was developed for the nominal system. This controller can be characterised as an MPC with output feedback and robust stability to the multi-model uncertainty, as well as to unmeasured persistent disturbances in the target tracking and regulatory operations. The paper is organised as follows: in Section 2, the non-minimal model utilised for the proposed MPC strategy is presented; in Section 3, the nominal IHMPC formulation is developed, together with a simulation example; in Section 4, the nominal MPC is extended to the multi-model robust case, and a simulation example is presented to illustrate the controller performance. Finally, in Section 5, the paper is concluded.

2 Non-minimal incremental state-space model

The state-space model considered in this work is based on the following ARX model

$$y(k) = -\sum_{i=1}^{na} A_i y(k-i) + \sum_{i=1}^{nb} B_i u(k-i) \tag{1}$$

where $y \in \Re^{ny}$ and $u \in \Re^{nu}$. Several authors [14, 16] have considered the model defined in (1) in a non-minimal state-space form where the state is measured. The incremental form in the input of this non-minimal model can be written as follows

$$\begin{aligned} x(k+1) &= Ax(k) + B\Delta u(k) \\ y(k) &= Cx(k) \end{aligned} \tag{2}$$

where

$$A = \begin{bmatrix} A_y & A_{\Delta u} \\ 0 & \underline{I} \end{bmatrix}, \quad B = \begin{bmatrix} B_{\Delta u} \\ \bar{I} \end{bmatrix}, \quad C = \begin{bmatrix} C_y & C_{\Delta u} \end{bmatrix}$$

$$A_y = \begin{bmatrix} I_{ny} - A_1 & A_1 - A_2 & \cdots & A_{na-1} - A_{na} & A_{na} \\ I_{ny} & 0 & \cdots & 0 & 0 \\ 0 & I_{ny} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_{ny} & 0 \end{bmatrix}$$
$$\times \in \Re^{(na+1)ny \times (na+1)ny}$$

$$A_{\Delta u} = \begin{bmatrix} B_2 & \cdots & B_{nb-1} & B_{nb} \\ 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & \cdots & 0 & 0 \end{bmatrix} \in \Re^{(na+1)ny \times (nb-1)nu}$$

$$\underline{I} = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ I_{nu} & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & I_{nu} & 0 \end{bmatrix} \in \Re^{(nb-1)nu \times (nb-1)nu}$$

$$B_{\Delta u} = \begin{bmatrix} B_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \bar{I} = \begin{bmatrix} I_{nu} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$C_y = \begin{bmatrix} I_{ny} & 0 & \cdots & 0 & 0 \end{bmatrix}, \quad C_{\Delta u} = \begin{bmatrix} 0 & \cdots & 0 & 0 \end{bmatrix}$$

I_{ny} and I_{nu} are identity matrices of dimensions $ny \times ny$ and $nu \times nu$, respectively.

Furthermore, state x is given by

$$x(k) = \begin{bmatrix} x_y(k) \\ x_{\Delta u}(k) \end{bmatrix} \in \Re^{nx} \tag{3}$$

with

$$x_y(k) = \begin{bmatrix} y(k)^{\mathrm{T}} & y(k-1)^{\mathrm{T}} & \cdots & y(k-na+1) & y(k-na)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \Re^{(na+1)ny}$$

$$x_{\Delta u}(k) = \begin{bmatrix} \Delta u(k-1)^{\mathrm{T}} & \Delta u(k-2)^{\mathrm{T}} & \cdots & \Delta u(k-nb+1)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \Re^{(nb-1)nu}$$

$$nx = (na+1)ny + (nb-1)nu$$

The state partition defined in (3) is convenient in order to separate the state components related to the system output at past sampling steps, from the state components related to the input at past sampling steps. Also, since the model

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is written in terms of the input increment (velocity model), model (2) contains the modes of model (1) plus ny integrating modes that results from the incremental form of the model. The incremental model is interesting as it precludes the need to compute the system steady state in order to prevent output offset. Here, we assume that the system represented in (1) has only stable modes. Also, the model defined in (2) has the following useful properties:

1. It is an incremental model in the input and state, and consequently, there is no need to include integrating disturbance models and target calculation in order to estimate the steady state x_s and u_s of the system. For the model defined in (2), if the steady state defined by set point y^{sp} is reachable, the state at this steady state is always defined by $x_{y,s} = \begin{bmatrix} y_{sp}^T & y_{sp}^T & \cdots & y_{sp}^T \end{bmatrix}^T$, $x_{\Delta u,s} = \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}^T$ and $\Delta u_s = \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}^T$. So, this model is adequate for output target tracking strategies.

2. The model defined in (2) is detectable and stabilisable. These properties can be easily proven by verifying that $\text{rank}\begin{bmatrix} \lambda I - A \\ C \end{bmatrix} = nx$ and $\text{rank}\begin{bmatrix} \lambda I - A, & B \end{bmatrix} = nx$ $\forall \lambda \in \mathbb{C}/|\lambda| \geq 1$. So, the proposed model can be used in IHMPC control strategies, contrary to the conclusion of Pannocchia and Rawlings [17] who considered a velocity model that cannot be used in IHMPC.

3. Since x_y is a vector of past measured outputs and x_u is a vector of implemented inputs, the state $x(k)$ is known at any time step k. So, a filter can be used to remove the output measurement noise and there is no need to include a state estimator in the MPC algorithm. Then, using the model defined in (2), any stable MPC based on state feedback can be easily transformed into a stable output feedback MPC.

The main disadvantage of the model defined in (2) is that matrix A is rank deficient, which complicates the application of the IHMPC to this model. The rank of A can be easily obtained if we observe that

$$A^j = \begin{bmatrix} A_y^j & \sum_{i=1}^{j} (A_y^{j-i} A_{\Delta u} \underline{I}^{i-1}) \\ 0 & \underline{I}^j \end{bmatrix} \quad \text{for any } j \geq 1$$

and $\underline{I}^j = 0$ for $j \geq nb - 1$. So, it is clear that $\text{rank}(A) = \text{rank}(A_y) = (na + 1)ny < nx$.

To gain some insight into the application of the above model to the IHMPC, one should keep in mind that, in the MPC strategy, at any time step k, a sequence of control moving along a control horizon m is computed by solving an open-loop optimisation problem that minimises a cost function subject to constraints. Let this control sequence be defined as follows

$$\Delta u_k = \begin{bmatrix} \Delta u(k|k)^T & \Delta u(k+1|k)^T & \cdots & \Delta u(k+m-1|k)^T \\ 0 & \cdots \end{bmatrix}^T$$

For this control sequence, based on the above considerations about matrix A, the state prediction beyond time step $k + n$ where $n = m + nb - 1$ can be obtained as follows

$$x_y(k + n + j|k) = (A_y)^j x_y(k + n|k)$$

and

$$x_{\Delta u}(k + n + j/k) = 0$$

Now, assuming that matrix A_y is full rank, one can perform the following eigenvalue–eigenvector Jordan decomposition

$$A_y V = V A_d \tag{4}$$

where

$$A_d = \begin{bmatrix} I_{ny} & 0 \\ 0 & F^{st} \end{bmatrix} \tag{5}$$

is a block diagonal matrix (Jordan canonical form) that makes explicit the different modes of the system. The ny integrating modes results from the incremental form of the model and it is here assumed that F^{st} has eigenvalues strictly inside the unit circle. The columns of V are the eigenvectors of A_y.

3 Infinite horizon MPC for the non-minimal state-space model

The control cost of the IHMPC for output tracking based on the non-minimal incremental model defined in (2) can be written as follows

$$V_{1,k} = \sum_{j=0}^{\infty} (Cx(k+j|k) - y^{sp})^T Q (Cx(k+j|k) - y^{sp}) + \sum_{j=0}^{m-1} \Delta u(k+j|k)^T R \Delta u(k+j|k) \tag{6}$$

where $x(k+j|k)$ is the predicted state at time $k+j$ computed at the present time k and y^{sp} is the output target that usually does not correspond to the origin.

To deal with the infinite summation term of the cost defined in (6), two approaches are usually adopted in the MPC literature:

1. The dual paradigm is used [18, 19], where a finite number (m) of constrained control moves is used to steer the state into an admissible target set that is invariant under a state feedback law, which in the case of the incremental model

would take the form $\Delta u = Fx$, and the input constraints are satisfied. This means that in the invariant target set the input constraints will not become active. In this case, the dual-mode MPC provides the optimal solution to the problem of minimising the cost defined in (6). However, this condition may be too restrictive when the system is expected to maximise economic objectives, which correspond to operating conditions where one or more inputs will lie on constraints. Input saturation may also occur at steady state when a persistent disturbance forces the input to saturate for enough time to reach steady state. In these cases, the control law produced by the dual-mode MPC is not guaranteed to be stabilising. This problem has been dealt in the literature for the cases where it is previously known whose inputs will saturate at the predicted steady state [20, 21], but a general method has not been found yet.

2. The null controller is used ($\Delta u = 0$) after the m control moves that are applied to the system [2, 10, 22]. In this case, a suboptimal control law is obtained where all the unstable modes of the system have to be zeroed at the end of the control horizon, which corresponds to additional constraints to the MPC problem that may become infeasible and loose robustness when the system is subject to persistent disturbances or the output reference target is changed significantly. To overcome this problem, Rodrigues and Odloak [10] proposed a modified cost function that includes additional slack variables that enlarge the feasible set of the MPC problem.

In this work, the approach where the null controller is used to deal with the IHMPC is followed. The method will be extended to the case of output feedback through the model defined in (2). Considering the case of open-loop stable systems, which is the most common case when the controller is designed to track optimising targets, the m control moves that are the degrees of freedom of MPC have to cancel the ny integrating modes depicted in (5). To make these integrating modes explicit, it is convenient to define the following state transformation

$$x_y = V z$$

and consequently

$$z = (V)^{-1} x_y$$

At this point, it is convenient to decompose the transformed state as follows

$$x_y = \begin{bmatrix} V^{\rm i} & V^{\rm st} \end{bmatrix} \begin{bmatrix} z^{\rm i} \\ z^{\rm st} \end{bmatrix} \tag{7}$$

and

$$\begin{bmatrix} z^{\rm i} \\ z^{\rm st} \end{bmatrix} = \begin{bmatrix} V^{\rm i}_{\rm in} \\ V^{\rm st}_{\rm in} \end{bmatrix} x_y \tag{8}$$

It can be shown that for $n = m + nb - 1$, the transformed state satisfies the following equation

$$\begin{bmatrix} z^{\rm i}(k+n+j) \\ z^{\rm st}(k+n+j) \end{bmatrix} = \left(\begin{bmatrix} I_{ny} & 0 \\ 0 & F^{\rm st} \end{bmatrix} \right)^j \begin{bmatrix} z^{\rm i}(k+n) \\ z^{\rm st}(k+n) \end{bmatrix} \tag{9}$$

Now, the cost defined in (6) can be developed as follows

$$\begin{aligned} V_{1,k} = {} & \sum_{j=0}^{n} (Cx(k+j|k) - y^{\rm sp})^{\rm T} Q (Cx(k+j|k) - y^{\rm sp}) \\ & + \sum_{j=1}^{\infty} (C_y x_y(k+n+j|k) - y^{\rm sp})^{\rm T} Q (C_y x_y(k \\ & + n + j|k) - y^{\rm sp}) + \sum_{j=0}^{m-1} \Delta u(k+j|k)^{\rm T} R \Delta u(k+j|k) \end{aligned} \tag{10}$$

Using (8) and (9), the infinite sum on the right-hand side of (10) can be written as follows

$$\begin{aligned} & \sum_{j=1}^{\infty} (C_y V^{\rm i} z^{\rm i}(k+n|k) - y^{\rm sp} + C_y V^{\rm st} (F^{\rm st})^j z^{\rm st}(k+n|k))^{\rm T} \\ & \quad Q(C_y V^{\rm i} z^{\rm i}(k+n|k) - y^{\rm sp} + C_y V^{\rm st} (F^{\rm st})^j z^{\rm st}(k+n|k)) \end{aligned}$$

Then, when adopting the null controller at time steps beyond the control horizon, the integrating modes have to be zeroed at time n, which is the time where the last control move $\Delta u(k + m - 1/k)$ still affects the state of the model defined in (2). This condition, in the output tracking case, corresponds to including in the MPC control problem the following constraint, which also guarantees that the control cost will be bounded

$$C_y V^{\rm i} z^{\rm i}(k+n|k) - y^{\rm sp} = 0 \tag{11}$$

Considering (11), the infinite sum term can be replaced by $z^{\rm st}(k+n|k)^{\rm T} \bar{Q} z^{\rm st}(k+n|k)$ where $\bar{Q}$ can be computed through equation

$$F^{\rm stT} \bar{Q} F^{\rm st} - \bar{Q} = F^{\rm stT} V^{\rm stT} C_y^{\rm T} Q C_y V^{\rm st} F^{\rm st}$$

Finally, the control cost defined in (6) takes the following form

$$\begin{aligned} V_{1,k} = {} & \sum_{j=0}^{n} (Cx(k+j|k) - y^{\rm sp})^{\rm T} Q (Cx(k+j|k) - y^{\rm sp}) \\ & + x(k+n|k)^{\rm T} \tilde{V}_{\rm in}^{\rm stT} \bar{Q} \tilde{V}_{\rm in}^{\rm st} x(k+n|k) \\ & + \sum_{j=0}^{m-1} \Delta u(k+j|k)^{\rm T} R \Delta u(k+j|k) \end{aligned} \tag{12}$$

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where

$$\tilde{V}_{\text{in}}^{\text{st}} = \begin{bmatrix} 0 & V_{\text{in}}^{\text{st}} \end{bmatrix}$$

To express the transformed state, which appears in (11) in terms of the original state, one can use the following relations

$$z^{\text{i}}(k+n|k) = V_{\text{in}}^{\text{i}} x_y(k+n|k) = \begin{bmatrix} V_{\text{in}}^{\text{i}} & 0 \end{bmatrix} x(k+n|k)$$
$$= \tilde{V}_{\text{in}}^{\text{i}} x(k+n|k)$$

Then, (11) can be written as follows

$$C_y V^{\text{i}} \tilde{V}_{\text{in}}^{\text{i}} (A^n x(k) + B_{\text{aug}} \Delta u_k) - y^{\text{sp}} = 0 \qquad (13)$$

where $B_{\text{aug}} = \begin{bmatrix} A^{n-1}B & A^{n-2}B & \cdots & B \end{bmatrix}$.

Then, the IHMPC for output tracking of open-loop stable systems can be obtained from the solution of the following problem

$$\min_{\Delta u_k} V_{1,k} \qquad (14)$$

subject to (13) and

$$\Delta u(k+j|k) \in U, \quad j = 1, \ldots, m-1 \qquad (15)$$

where

$$U = \left\{ \Delta u(k+j) \left| \begin{array}{c} -\Delta u^{\max} \le \Delta u(k+j) \le \Delta u^{\max} \\ u^{\min} \le u(k-1) + \sum_{i=0}^{j} \Delta u(k+i) \le u^{\max} \end{array} \right. \right\}$$

Theorem 1: In the output tracking of the undisturbed system represented in (2), if the MPC defined through problem (14) is feasible at time step k and y^{sp} is reachable, then the controller will remain feasible at any subsequent time step. Also, the system output will converge to the target and the input increment will converge to zero.

Proof: It is easy to show that if at time k the optimal solution to problem (14) is represented by $\Delta u_k^* = \begin{bmatrix} \Delta u^*(k|k)^{\text{T}} & \Delta u^*(k+1|k)^{\text{T}} & \cdots & \Delta u^*(k+m-1|k)^{\text{T}} \end{bmatrix}^{\text{T}}$ and the corresponding optimal cost is represented by $V_{1,k}^*$, then, at time $k+1$, $\Delta \tilde{u}_{k+1} = \begin{bmatrix} \Delta u^*(k+1|k)^{\text{T}} & \cdots & \Delta u^*(k+m-1|k)^{\text{T}} 0 \end{bmatrix}^{\text{T}}$ is feasible and corresponds to the cost

$$\tilde{V}_{1,k+1} = V_{1,k}^* - (y(k) - y^{\text{sp}})^{\text{T}} Q (y(k) - y^{\text{sp}})$$
$$- \Delta u(k|k)^{\text{T}} R \Delta u(k|k)$$

Then, $V_{1,k}$ converges to zero and the output converges to the target. □

Remark 1: The IHMPC with output feedback defined through the solution of problem (14) is not robust to large persistent disturbances and to large changes in the output target. This is so because in these cases, even if the target is reachable, there may be a conflict between constraints (13) and (15) that turns the control problem infeasible.

To make the IHMPC resulting from the solution to the problem defined in (14) feasible, one can follow two equivalent approaches. In the first approach an artificial output target is introduced in the MPC problem as a new decision variable [21]. The distance between the artificial target and the real target is then penalised in the cost function of the new MPC. When the incremental model is used, the method proposed by Limon *et al.* [23] leads to the following problem

$$\min_{\Delta u_k, \hat{y}^{\text{sp}}} V_{2,k} = \sum_{j=0}^{n} (Cx(k+j|k) - \hat{y}^{\text{sp}})^{\text{T}} Q (Cx(k+j|k) - \hat{y}^{\text{sp}})$$
$$+ x(k+n|k)^{\text{T}} \tilde{V}_{\text{in}}^{\text{stT}} \bar{Q} \tilde{V}_{\text{in}}^{\text{st}} x(k+n|k)$$
$$+ \sum_{j=0}^{m-1} \Delta u(k+j|k)^{\text{T}} R \Delta u(k+j|k)$$
$$+ (\hat{y}^{\text{sp}} - y^{\text{sp}})^{\text{T}} Q_{y\text{sp}} (\hat{y}^{\text{sp}} - y^{\text{sp}}) \qquad (16)$$

subject to (15) and

$$C_y V^{\text{i}} \tilde{V}_{\text{in}}^{\text{i}} (A^n x(k) + B_{\text{aug}} \Delta u_k) - \hat{y}^{\text{sp}} = 0$$

where $\hat{y}^{\text{sp}}$ is the artificial target for the system output.

In the second approach [10], instead of the artificial target a slack variable is introduced in the output predictions that are used in the computation of the control cost. In this case the control problem becomes

$$\min_{\Delta u_k, \delta} V'_{2,k} = \sum_{j=0}^{n} (Cx(k+j|k) - y^{\text{sp}} - \delta)^{\text{T}} Q (Cx(k+j|k)$$
$$- y^{\text{sp}} - \delta) + x(k+n|k)^{\text{T}} \tilde{V}_{\text{in}}^{\text{stT}} \bar{Q} \tilde{V}_{\text{in}}^{\text{st}} x(k+n|k)$$
$$+ \sum_{j=0}^{m-1} \Delta u(k+j|k)^{\text{T}} R \Delta u(k+j|k) + \delta^{\text{T}} Q_\delta \delta$$
$$(17)$$

subject to (15) and

$$C_y V^{\text{i}} \tilde{V}_{\text{in}}^{\text{i}} (A^n x(k) + B_{\text{aug}} \Delta u_k) - y^{\text{sp}} - \delta = 0$$

where δ is a slack variable that makes the above constraint always feasible.

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Observe that the same control law is obtained by solving either problem (16) or (17) with $Q_{ysp} = Q_{\delta}$

Theorem 2: The IHMPC with output feedback that results from the solution to the problem defined in (17) is robust to track piecewise constant output targets. Also, as long as the output target is reachable and Q_{δ} is large enough, the control cost of problem (17) is converging to zero and the output converges to the target.

Proof: Since the slack variable δ is unbounded, problem (17) is always feasible. To prove the convergence of $V'_{2,k}$ to zero, one may follow the same steps as Theorem 1 in Odloak [11]. □

Remark 2: The controller obtained from the solution to problem (16) shares the same properties related to the robustness to the tracking of piecewise constant targets as the controller defined in (17). Also, as the two controllers are always feasible, they are robust to the presence of any persistence disturbance. These controllers can also be considered robust even when the output target is unreachable. In this case, the control cost cannot be reduced to zero and the controller will only minimise the distance between the steady state that can be reached and the desired target.

Example 1: To evaluate the performance of the infinite horizon controller defined through the solution to problem (17), we consider the ill-conditioned distillation column presented by Skogestad *et al.* [24]. In this system the distillate composition y_D and the bottom composition x_B are controlled by manipulating reflux L and boilup V. The transfer function model representing this system is the following

$$\begin{bmatrix} y_D(s) \\ x_B(s) \end{bmatrix} = \frac{1}{75s+1}\begin{bmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{bmatrix}\begin{bmatrix} L(s) \\ V(s) \end{bmatrix}$$

For a sampling period $\Delta t = 15$, the model defined in (2) takes the following form

$$\begin{bmatrix} y_D(k+1) \\ x_B(k+1) \\ y_D(k) \\ x_B(k) \end{bmatrix} = \begin{bmatrix} 0.1813 & 0 & 0.8187 & 0 \\ 0 & 0.1813 & 0 & 0.8187 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} y_D(k) \\ x_B(k) \\ y_D(k-1) \\ x_B(k-1) \end{bmatrix} + \begin{bmatrix} 0.1592 & -0.1566 \\ 0.1961 & -0.1987 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} \Delta L(k) \\ \Delta V(k) \end{bmatrix}$$

For the simulations considered here, the following parameters were considered

$$m = 3; \quad Q = \text{diag}(1, 1);$$
$$R = \text{diag}(0.01, 0.01); \quad S = 10^3 \times \text{diag}(1, 1)$$
$$u_{max} = \begin{bmatrix} 20 \\ 20 \end{bmatrix}; \quad u_{min} = \begin{bmatrix} -40 \\ -35 \end{bmatrix}; \quad \Delta u_{max} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The system starts from the origin and the output target is changed to $y^{sp} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, which is a reachable target. Fig. 1 shows that the system output converges quite easily to this target, whereas Fig. 2 shows that the system inputs remain inside their bounds. In this case, Fig. 3 shows that the control cost converges asymptotically to zero. In Fig. 1, it is also represented by the values of $y^{sp} + \delta$, which is here defined as the artificial set point for the system output. It is clear that when the target is feasible, the artificial set point tends to the target, which means that the slack variable tends to zero. At time step $k = 100$, the target is changed to $y^{sp} = \begin{bmatrix} -1 \\ 0.1 \end{bmatrix}$. This target is not reachable because it corresponds to the following input at steady state $u_{ss} = \begin{bmatrix} -43.09 \\ -42.62 \end{bmatrix}$, which does not satisfy the constraints defined above. Figs. 1–3 also shows the responses of the

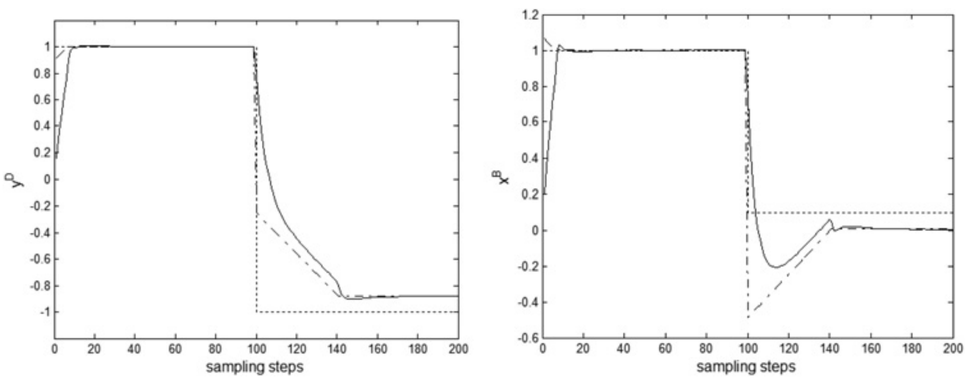

Figure 1 *Outputs for target tracking (——) and artificial set points (–· – ·–)*

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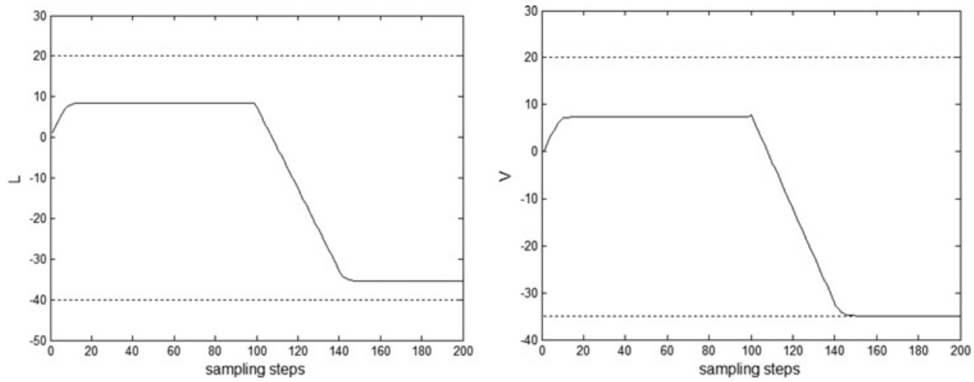

Figure 2 *Inputs of the distillation system for target tracking*

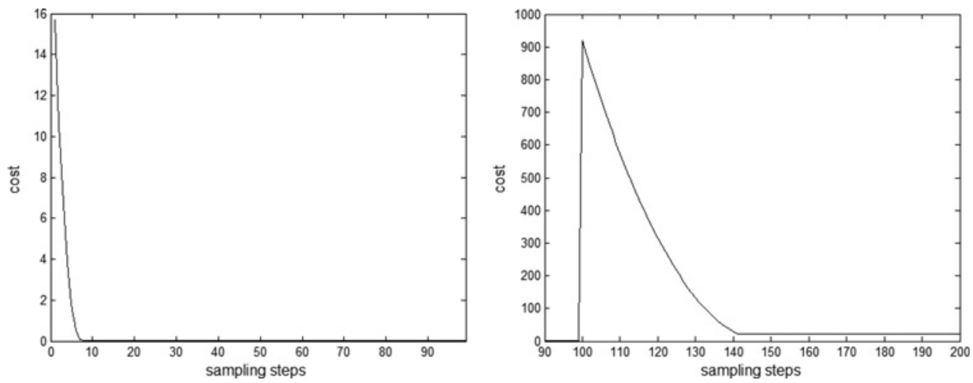

Figure 3 *Control cost of the distillation system for output tracking*

distillation system with the infinite horizon proposed here. Fig. 1 shows that the output targets are not reached and there is a permanent offset in the two outputs. Observe that the artificial set points also do not tend to the targets, which means that the slacks are not zeroed at steady state and the control cost does not converge to zero as shown in Fig. 3. Fig. 2 shows that input V saturates at its minimum bound while input L remains inside its allowable range. These results show that, when the output target is unreachable, the proposed IHMPC remains feasible and leads the system to the nearest point to the target in terms of the control cost.

In the second simulation experiment, the distillation system starts from the origin and at time $k = 10$, a persistent unmeasured disturbance corresponding to a step in the system input $du = \begin{bmatrix} 10 \\ -20 \end{bmatrix}$ is introduced into the system, while the output target remains at the origin. Observe that with this disturbance and the input constraints adopted in this problem, the origin is a target that is still reachable since the new input steady state corresponds to $u_{ss} = \begin{bmatrix} -10 \\ 20 \end{bmatrix}$, which lies on the boundary of the input definition set. Fig. 4 shows that the controller is robust to this disturbance as the controlled outputs converge to the origin and the inputs converge to the expected steady state as shown in Fig. 5. It can also be verified that the cost function also converges to zero. At time step $k = 80$, a new persistent step disturbance corresponding to $\Delta u = \begin{bmatrix} 10 \\ -2 \end{bmatrix}$ is introduced in the distillation system. With this new disturbance the origin becomes unreachable by the MPC controller as the input steady state corresponding to the origin is $u_{ss} = \begin{bmatrix} -20 \\ 22 \end{bmatrix}$ that lies outside the definition set of the system input as input V is larger than its maximum value. However, the proposed controller is robust to this disturbance as it remains feasible and uses the available degrees of freedom of the distillation system to minimise the distance between the system outputs and the target that is the origin. Fig. 4 shows that for time steps larger than 100 both outputs stabilise close to zero, but the target is not reached. This is so because, as shown in Fig. 5, input V remains saturated as it cannot be increased any further. Consequently, in this case, the control cost will converge to a value larger than zero.

4 MPC for target tracking with model uncertainty

A robust model predictive controller is a controller which explicitly accounts for modelling errors in the control design procedure. With the model structure presented in (2), model uncertainty is related to uncertainty in matrices A and B. The most common way to represent model uncertainty in model predictive control is to consider a

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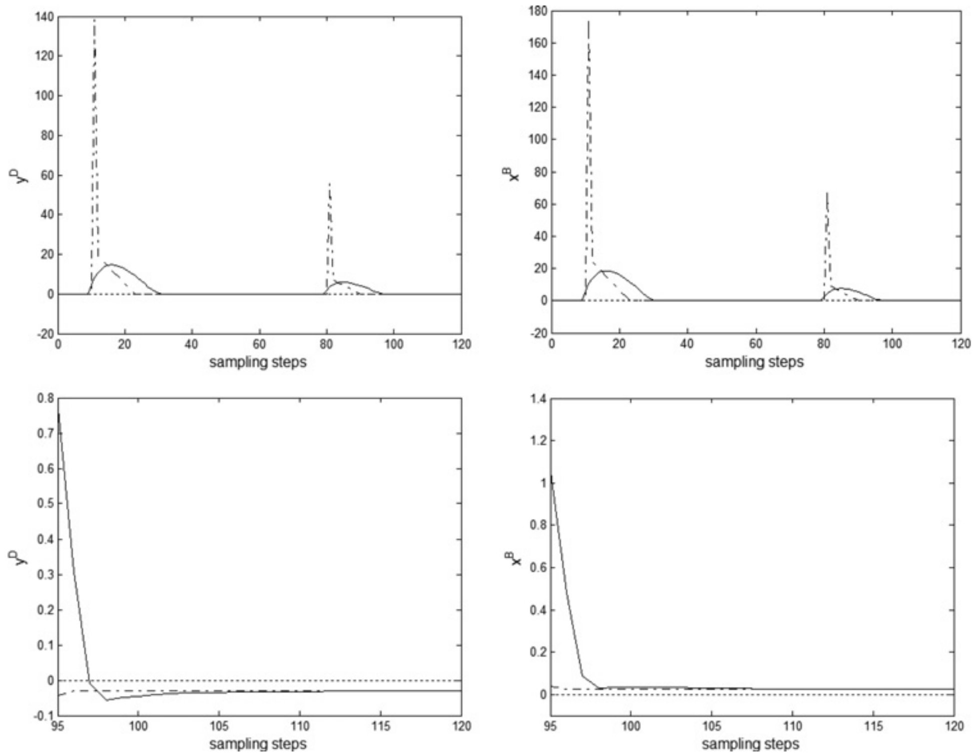

Figure 4 *Outputs (——) and artificial set points (−· − ·−) for persistent disturbances*

discrete set of models $\Omega = \{\theta_1, \ldots, \theta_L\}$, where each θ_i corresponds to a particular set of parameters that define the model. We may consider that Ω corresponds to several operating points of the process system along its operating region, and these models may be considered as a discrete linear approximation of the true non-linear model. In this case, the uncertain model representation is designated as multi-plant uncertainty [4]. Some authors [3, 25, 26] extended the multi-plant uncertainty by assuming that these models constitute the vertices of a polytopic set that characterises model uncertainty. In this case, all convex combinations of the vertices of the polytope $\left(\theta = \sum_{i=1}^{L} \alpha_i \theta_i,\ \sum_{i=1}^{L} \alpha_i = 1,\ \alpha_i \geq 0\right)$ are considered as possible realisations of the true model. In the model defined in (2), uncertainty lies in matrices A and B and the multi-plant or polytopic uncertainty can be described through the set of models $(A(\theta_p), B(\theta_p))$, $p = 1, \ldots, L$. Also, for each model θ_p the Jordan decomposition (4) can be written as follows

$$A_y(\theta_p)V(\theta_p) = V(\theta_p)A_{\rm d}(\theta_p)$$

where

$$A_{\rm d}(\theta_p) = \begin{bmatrix} I_{ny} & 0 \\ 0 & F^{\rm st}(\theta_p) \end{bmatrix}$$

For this model, one can also define a transformed state as follows

$$x_y = V(\theta_p)\, z_{\theta_p}$$

and consequently

$$z_{\theta_p} = (V(\theta_p))^{-1} x_y, \quad p = 1, \ldots, L$$

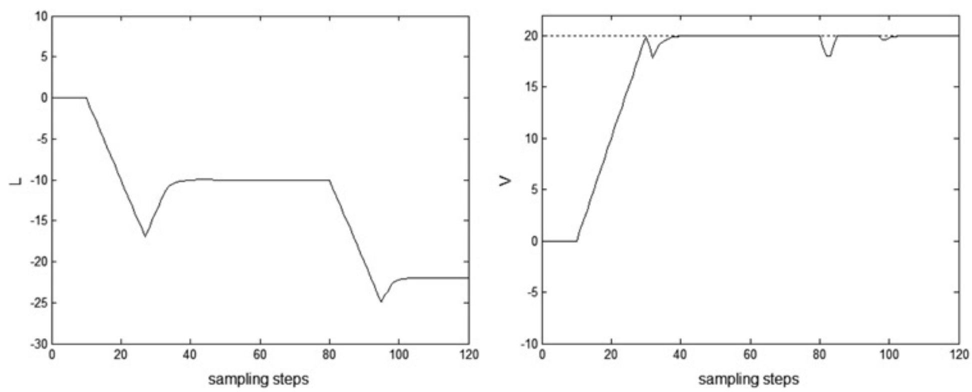

Figure 5 *Inputs of the distillation system for persistent disturbances*

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As in (7) one can decompose the transformed state as follows

$$x_y = \begin{bmatrix} V^{\mathrm{i}}(\theta_p) & V^{\mathrm{st}}(\theta_p) \end{bmatrix} \begin{bmatrix} z^{\mathrm{i}}_{\theta_p} \\ z^{\mathrm{st}}_{\theta_p} \end{bmatrix}$$

and from (8)

$$\begin{bmatrix} z^{\mathrm{i}}_{\theta_p} \\ z^{\mathrm{st}}_{\theta_p} \end{bmatrix} = \begin{bmatrix} V^{\mathrm{i}}_{\mathrm{in}}(\theta_p) \\ V^{\mathrm{st}}_{\mathrm{in}}(\theta_p) \end{bmatrix} x_y, \quad p = 1, \ldots, L$$

Analogous to the nominal case, for the uncertain transformed state, one has

$$\begin{bmatrix} z^{\mathrm{i}}_{\theta_p}(k+n+j) \\ z^{\mathrm{st}}_{\theta_p}(k+n+j) \end{bmatrix} = \left(\begin{bmatrix} I_{ny} & 0 \\ 0 & F^{\mathrm{st}}(\theta_p) \end{bmatrix} \right)^j \begin{bmatrix} z^{\mathrm{i}}_{\theta_p}(k+n) \\ z^{\mathrm{st}}_{\theta_p}(k+n) \end{bmatrix},$$
$$p = 1, \ldots, L$$

There are several methods to develop an MPC that is robust to model uncertainty. Kothare *et al.* [3] propose the use of linear matrix inequalities (LMIs) to find a Lyapunov function $x^{\mathrm{T}}Qx$ and a state feedback control law $u = Kx$ that minimises an upper bound for the cost functions corresponding to each vertex of the polytopic set that defines model uncertainty. The dual paradigm was used to extend the above method by including in the state feedback control law additional degrees of freedom [16, 21, 27]. The resulting control law becomes $u = Kx + v$, where v is calculated following a min–max approach in which the cost of the worst model combination is minimised from the present state until the invariant set under the feedback control is reached. Badgwell [4] proposed to achieve robust stability for the regulator case with multi-plant uncertainty by including a contraction constraint for the cost functions corresponding to the vertices of set Ω. The method was extended to the output target tracking of stable systems [10, 11] and integrating systems [6] by considering a minimal state-space model in the velocity form. Here, the approach will be applied to the non-minimal state-space model presented in the introduction section of this work.

For the development that follows, it is assumed that the true plant, which lies within the set Ω is designated as θ_{T}, and there is a most likely a nominal plant that also lies in Ω and designated as θ_{N}.

For any model lying in Ω, the cost function considered in (17) can be written as follows

$$\begin{aligned} V_{3,k}(\Delta u_k, \delta_k(\theta_p), \theta_p) &= \sum_{j=0}^{\infty} (Cx_{\theta_p}(k+j|k) - y^{\mathrm{sp}} - \delta_k(\theta_p))^{\mathrm{T}} Q (Cx_{\theta_p}(k+j|k) \\ &\quad - y^{\mathrm{sp}} - \delta_k(\theta_p)) + \sum_{j=0}^{m-1} \Delta u(k+j|k)^{\mathrm{T}} R \Delta u(k \\ &\quad + j/k) + \delta_k(\theta_p)^{\mathrm{T}} S \delta_k(\theta_p) \end{aligned} \tag{18}$$

Following the same steps as in the nominal system presented in the previous section, the tail of infinite sum on the right-hand side of (18) becomes

$$\begin{aligned} &\sum_{j=1}^{\infty} (C_y V^i(\theta_p) z^{\mathrm{i}}_{\theta_p}(k+n|k) - y^{\mathrm{sp}} - \delta_k(\theta_p) \\ &\quad + C_y V^{\mathrm{st}}(\theta_p)(F^{\mathrm{st}}(\theta_p))^j z^{\mathrm{st}}_{\theta_p}(k+n|k))^{\mathrm{T}} \\ &Q(C_y V^{\mathrm{i}}(\theta_p) z^{\mathrm{i}}_{\theta_p}(k+n|k) - y^{\mathrm{sp}} - \delta_k(\theta_p) \\ &\quad + C_y V^{\mathrm{st}}(\theta_p)(F^{\mathrm{st}}(\theta_p))^j z^{\mathrm{st}}_{\theta_p}(k+n|k)) \end{aligned}$$

Then, in order to the infinite sum on the right-hand side of (18) to be bounded, for all the models in Ω, it is necessary to impose the following constraint

$$C_y V^{\mathrm{i}}(\theta_p) z^{\mathrm{i}}_{\theta_p}(k+n|k) - y^{\mathrm{sp}} - \delta_k(\theta_p) = 0,$$
$$p = 1, \ldots, L \tag{19}$$

Considering (19), the infinite sum term can be replaced by $z^{\mathrm{st}}_{\theta_p}(k+n|k)^{\mathrm{T}} \bar{Q}(\theta_p)\, z^{\mathrm{st}}_{\theta_p}(k+n|k)$ where $\bar{Q}(\theta_p)$ can be computed through the following equation

$$(F^{\mathrm{st}}(\theta_p))^{\mathrm{T}} \bar{Q}(\theta_p) F^{\mathrm{st}}(\theta_p) - \bar{Q}(\theta_p) = (F^{\mathrm{st}}(\theta_p))^{\mathrm{T}} V^{\mathrm{st}}(\theta_p)^{\mathrm{T}} \times C_y^{\mathrm{T}} Q C_y V^{\mathrm{st}}(\theta_p) F^{\mathrm{st}}(\theta_p)$$

Now, as for the nominal system, the transformed states that depend on the particular model that is considered can be represented in terms of the original state as follows

$$\begin{aligned} z^{\mathrm{i}}_{\theta_p}(k+n|k) &= V^{\mathrm{i}}_{\mathrm{in}}(\theta_p) x_y(k+n|k) = \begin{bmatrix} V^{\mathrm{i}}_{\mathrm{in}}(\theta_p) & 0 \end{bmatrix} x(k+n|k) \\ &= \tilde{V}^{\mathrm{i}}_{\mathrm{in}}(\theta_p) x(k+n|k) \\ z^{\mathrm{st}}_{\theta_p}(k+n|k) &= V^{\mathrm{st}}_{\mathrm{in}}(\theta_p) x_y(k+n|k) \end{aligned}$$

The last equation can also be written as follows

$$z^{\mathrm{st}}_{\theta_p}(k+n|k) = \begin{bmatrix} 0 & V^{\mathrm{st}}_{\mathrm{in}}(\theta_p) \end{bmatrix} x(k+n|k)$$

or

$$z^{\mathrm{st}}_{\theta_p}(k+n|k) = \tilde{V}^{\mathrm{st}}_{\mathrm{in}}(\theta_p) x(k+n|k)$$

Then, (19) can be written as follows

$$C_y V^{\mathrm{i}}(\theta_p) \tilde{V}^{\mathrm{i}}_{\mathrm{in}}(\theta_p)(A^n(\theta_p) x(k) + B_{\mathrm{aug}}(\theta_p) \Delta u_k) - y^{\mathrm{sp}}$$
$$- \delta_k(\theta_p) = 0, \quad p = 1, \ldots, L$$

where $B_{\mathrm{aug}}(\theta_p) = \begin{bmatrix} A(\theta_p)^{n-1} B(\theta_p) & A(\theta_p)^{n-2} B(\theta_p) & \cdots & B(\theta_p) \end{bmatrix}$.

Thus, with the above considerations, the control cost defined in (18) can be written as follows

$$V_{3,k}(\Delta u_k, \delta_k(\theta_p), \theta_p) = \sum_{j=0}^{n} (Cx_{\theta_p}(k+j|k) - y^{\text{sp}} - \delta_k(\theta_p))^{\text{T}} Q(Cx_{\theta_p}(k+j|k) - y^{\text{sp}} - \delta_k(\theta_p)) + x(k+n|k)^{\text{T}} \tilde{V}_{\text{in}}^{\text{st}}(\theta_p)^{\text{T}} \bar{Q}(\theta_p) \tilde{V}_{\text{in}}^{\text{st}}(\theta_p) x(k+n|k) + \sum_{j=0}^{m-1} \Delta u(k+j|k)^{\text{T}} R \Delta u(k+j|k) + \delta_k(\theta_p)^{\text{T}} S \delta_k(\theta_p) \quad (20)$$

The IHMPC, which is robust to the multi-plant model uncertainty, as well as to unmeasured persistent disturbances and target tracking, is obtained from the solution to the following problem

$$\min_{\Delta u_k, \delta_k^i(\theta_1), \ldots, \delta_k^i(\theta_L)} V_{3,k}(\Delta u_k, \delta_k(\theta_{\text{N}}), \theta_{\text{N}}) \quad (21)$$

subject to (15), (19) and

$$C_y V^{\text{i}}(\theta_p) \tilde{V}_{\text{in}}^{\text{i}}(\theta_p)(A^n(\theta_p)x(k) + B_{\text{aug}}(\theta_p)\Delta u_k) - y^{\text{sp}} - \delta_k(\theta_p) = 0, \quad p = 1, \ldots, L$$
$$V_{3,k}(\Delta u_k, \delta_k(\theta_p), \theta_p) \le V_{3,k}(\Delta \tilde{u}_k, \tilde{\delta}_k(\theta_p), \theta_p), \quad p = 1, \ldots, L \quad (22)$$

where $\Delta \tilde{u}_k = [\, \Delta u^*(k|k-1)^{\text{T}} \quad \cdots \quad \Delta u^*(k+m-2|k-1) 0]^{\text{T}}$ and $\tilde{\delta}_k(\theta_p)$ is such that

$$C_y V^{\text{i}}(\theta_p) \tilde{V}_{\text{in}}^{\text{i}}(\theta_p)(A^n(\theta_p)x(k) + B_{\text{aug}}(\theta_p)\Delta \tilde{u}_k) - y^{\text{sp}} - \tilde{\delta}_k(\theta_p) = 0, \quad p = 1, \ldots, L \quad (23)$$

Remark 3: Notice that the current state $x(k)$ does not depend on the model parameters, since it is composed of measured outputs and inputs. The cost, which is minimised in the control problem, is based on the nominal model θ_{N}, while the constraints are written for each of the models that define set Ω. Equation (23) is necessary to the computation of the pseudo slack $\tilde{\delta}_k(\theta_p)$, which is associated to the optimal solution of problem (21) at the previous time step and to the current state $x(k)$.

Remark 4: The MPC controller generated by the solution to the problem defined in (21) is robust to persistent unmeasured disturbances and to changes in the output target. This is so because the problem defined in (21) is always feasible as it is easy to show that at time step k, the following solution $\Delta u_k = \Delta \tilde{u}_k$ and $\delta_k(\theta_p) = \tilde{\delta}_k(\theta_p)$ is feasible.

Also, observe that constraint (23) makes the control problem defined in (22) a non-linear programming, which can be solved by any available solver as the ones based on the sequential quadratic programming algorithm. However, it can be shown that constraint (23) and the control cost can be converted into LMIs, and so, the robust controller problem can be formulated as an LMI problem.

Theorem 3: Consider a stable system whose true model is unknown but it is known to lie within the set Ω, and the desired output target is reachable. Then, the control law obtained from the sequential solution to the problem defined in (21) is stable and drives the true system to the reference value.

Proof: Assume that at time k we inject the optimal control action $\Delta u^*(k|k)$ into the true system and we move to time step $k+1$. As $(\Delta \tilde{u}_{k+1}, \tilde{\delta}_{k+1}(\theta_1), \ldots, \tilde{\delta}_{k+1}(\theta_L))$ is a feasible solution to the problem defined in (21) at $k+1$ and, in addition, for the undisturbed system one has $\tilde{\delta}_{k+1}(\theta_{\text{T}}) = \delta_k^*(\theta_{\text{T}})$, then, from (20) the value of the cost for the true plant with this feasible solution is given by

$$V_{3,k+1}(\Delta \tilde{u}_{k+1}, \tilde{\delta}_{k+1}(\theta_{\text{T}}), \theta_{\text{T}}) = V_{3,k}(\Delta u_k^*, \delta_k^*(\theta_{\text{T}}), \theta_{\text{T}}) - (Cx(k) - y^{\text{sp}} - \delta_k^*(\theta_{\text{T}}))^{\text{T}} \times Q(Cx(k) - y^{\text{sp}} - \delta_k^*(\theta_{\text{T}})) - \Delta u^*(k|k)^{\text{T}} R \Delta u^*(k|k)$$

Therefore

$$V_{3,k+1}(\Delta u_{k+1}^*, \delta_{k+1}^*(\theta_{\text{T}}), \theta_{\text{T}}) \le V_{3,k}(\Delta u_k^*, \delta_k^*(\theta_{\text{T}}), \theta_{\text{T}})$$

which means that the closed-loop corresponding to the true model is stable and for a time step $\bar{k}$ large enough converges to a stationary point in which

$$Cx(\bar{k}) - y^{\text{sp}} = \delta_{\bar{k}}^*(\theta_{\text{T}}) \quad (24)$$

In addition, since $x(\bar{k})$ corresponds to the measured state (past values of inputs and outputs) and y^{sp} is fixed, then

$$\delta_{\bar{k}}^*(\theta_{\text{T}}) = \delta_{\bar{k}}^*(\theta_1) = \cdots = \delta_{\bar{k}}^*(\theta_L)$$

It can be shown [2] that if weight S is large enough, (24) will hold only if $Cx(\bar{k}) - y^{\text{sp}} = 0$ and $\Delta u(\bar{k}|\bar{k}) = 0$. This shows that the sequence of optimal costs for the true plant is decreasing and converges to zero although $V_{3,k}(\theta_i)$ is not necessarily decreasing for $\theta_i \neq \theta_{\text{T}}$. □

Example 2: To evaluate the performance and stability of the controller produced by the solution to the problem (21), the ill-conditioned distillation column presented in Example 1 is considered again, but now the system may

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have two different models as represented below

$$G_1(s) = \frac{1}{75s+1}\begin{bmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{bmatrix}$$

$$G_2(s) = \frac{1}{75s+1}\begin{bmatrix} 1.319 & -0.864 \\ 1.082 & -1.096 \end{bmatrix}$$

The sampling period of the controller is $\Delta t = 15$ and in this case, the tuning parameters are the following

$$Q = \text{diag}(0.1 \quad 0.1), \quad R = \text{diag}(1 \quad 1), \quad S = \text{diag}(1 \quad 1) \times 10^6, \quad u_{\max} = [10 \quad 10], \quad u_{\min} = [-10 \quad -10], \quad \Delta u_{\max} = [1 \quad 1], \quad m = 3 \tag{25}$$

In the simulations performed here, model 2 is the nominal model used in the cost function that is minimised by the robust controller. The simulation starts with the plant being also represented by model 2 and the output target is changed from the origin to $y^{sp} = [1 \quad 1]^T$, which is reachable and corresponds to the input steady state $u_{ss} = [0.454 \quad -0.464]^T$. Fig. 6 shows that the output targets are reached nicely, whereas Fig. 7 shows that the inputs tend to their expected steady states. From Fig. 8*a*, we can see that during the initial simulation period (before time step 50), the cost computed with model 2 that is the true plant model decreases asymptotically and reaches zero as the desired target is feasible, while the cost corresponding to model 1 does not decrease asymptotical although it eventually reaches zero after the true system has converged (see Fig. 9*a*). At time step 50, the model that represents the true plant is switched from model 2 to model 1, while the output target remains the same. Corresponding to the required output target, the new input steady state becomes $u_{ss} = [8.455 \quad 7.434]^T$. Figs. 6 and 7 show that from time step 50 until time 100, the proposed controller is robust to this disturbance and the outputs converge to the target while the inputs are led to the new steady state, although input L reaches temporarily its maximum bound. We can also observe from Fig. 9*b* that during this period of time the cost computed with model 1 is decreasing and converges again to zero as the target remains reachable, while Fig. 8*b* shows that the cost computed with model 2 is no longer asymptotically decreasing. Finally, at time 100 the output target is changed to $y^{sp} = [-1 \quad 0.1]^T$ which for model 1 (true plant) would correspond to $u_{ss} = [-43.09 \quad -42.63]^T$ that is unreachable because of the input constraints. In this case, the proposed controller is still able to stabilise the distillation column but the outputs do not reach the targets and permanent offsets are observed. This is so because input V saturates at the minimum bound (−10), while input L remains inside its definition range. Fig. 8*b* shows that the cost computed with

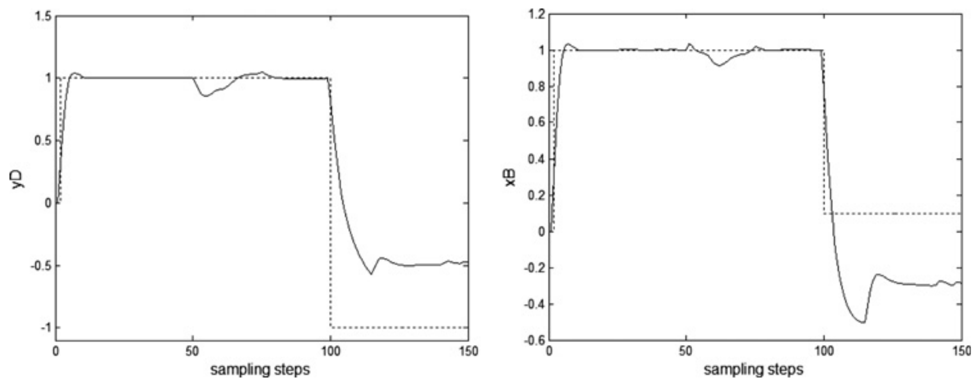

Figure 6 *Outputs for output tracking and model uncertainty*

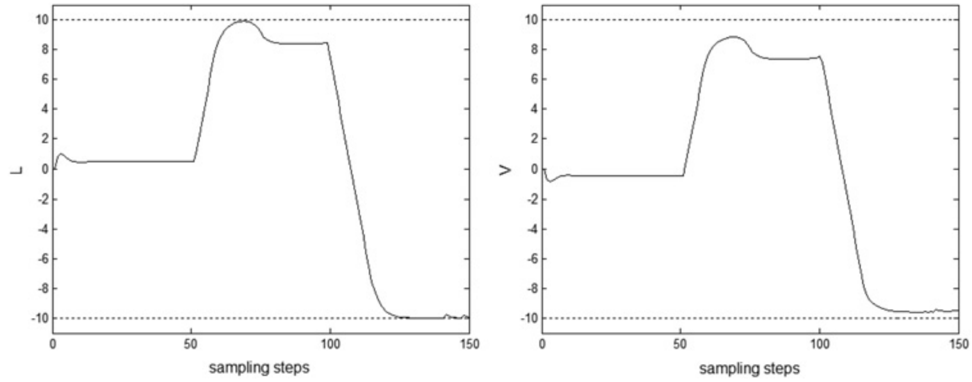

Figure 7 *Inputs for output tracking and model uncertainty*

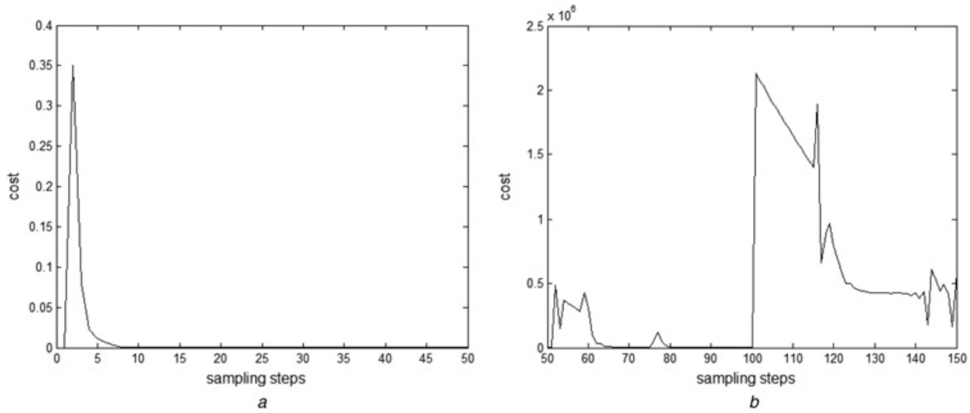

Figure 8 *Cost function for model $G_2(s)$*
a When the true model is $G_2(s)$
b When the true model is $G_1(s)$

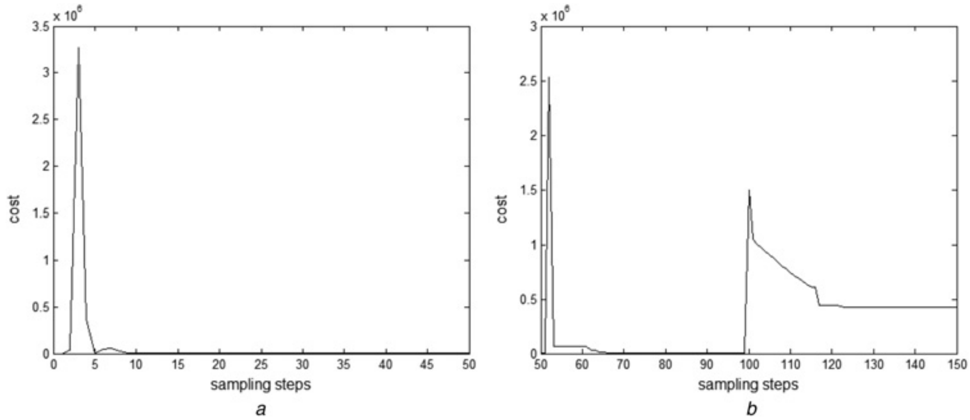

Figure 9 *Cost function for model $G_1(s)$*
a When the true model is $G_2(s)$
b When the true model is $G_1(s)$

model 2 is not decreasing, and Fig. 9*b* shows that the cost of the true plant is decreasing but does not converge to zero.

To emphasise the advantage of the proposed robust controller, which guarantees stability in the presence of model uncertainty, over the nominal stable MPC where stability is only assured when the model is perfect, the two controllers are compared for the output tracking case represented in Figs. 10 and 11. The true plant model is represented by model 1 and the nominal controller is based only on model 2, while the robust controller uses the two models. The same tuning parameters defined in (25) are used in the two controllers. It is clear that the robust controller has a much better performance than the nominal controller, mainly when the system is far from input saturation. The nominal controller is almost unstable,

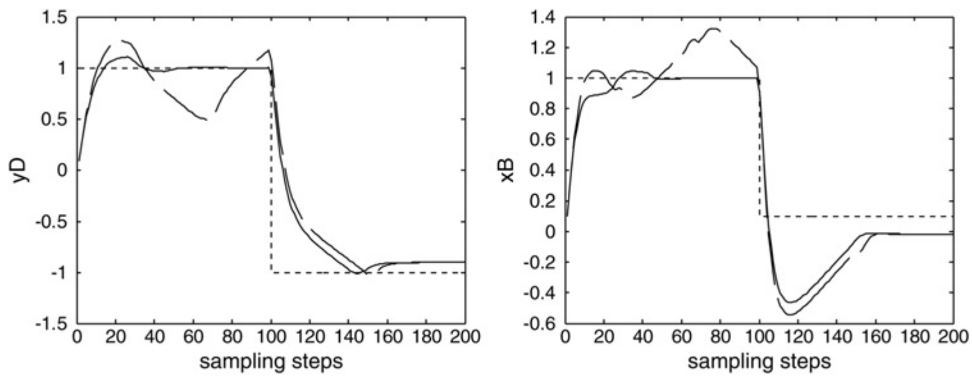

Figure 10 *Outputs with model uncertainty: robust (——) and nominal (- - -)*

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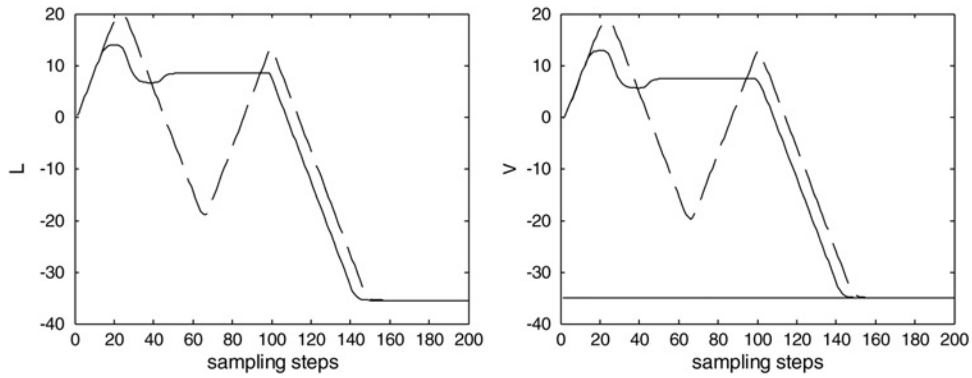

Figure 11 *Inputs with model uncertainty: robust (——) and nominal (- - -)*

whereas the robust controller performs quite well. When the system input reaches saturation, the responses of the two controllers tend to be close to the open-loop response and the same steady state is reached.

5 Conclusion

In this work, an IHMPC with output feedback that, for open-loop stable systems, is robustly stable to persistent disturbances and model uncertainty was presented. The proposed controller is based on a non-minimal state-space model in the velocity form in which the state is always known, as the state is constituted by the output at the present and past time steps and the input increments at past time steps. The consequence is that the controller does not need a state observer and robust stability can be obtained by extending to the non-minimal model the conventional approach of considering an infinite prediction horizon and including constraints that force the control cost to be bounded. The main difficulty of implementing the IHMPC to the non-minimal incremental model that is to guarantee that the cost function is bounded was solved by forcing the zeroing of the integrating modes at a suitable time instant beyond the control horizon. This idea could be conveyed to the case where model uncertainty can be approximated by a finite set of models. Although the robust controller proposed here has not yet been extended to unstable systems, the practical application of the proposed strategy to a number of systems seems to be quite promising as the computation cost is acceptable.

6 Acknowledgments

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