



An analytical method for selecting the optimal nozzle external geometry for fluid dynamic gauging

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ABSTRACT

Fluid dynamic gauging (FDG) was developed to measure, *in situ* and in real time, the thickness of a soft deposit layer immersed in a liquid without contacting the surface of the layer. An analysis based on the lubrication assumption for the flow patterns in the space between the nozzle and the surface being gauged yielded analytical expressions for the relationships between the main flow variables and system parameters. Nozzle shapes for particular pressure, pressure gradient and shear stress profiles could then be identified. The effect of flow rate, nozzle geometry and nozzle position on the pressure beneath the nozzle and shear stress on the gauged surface showed very good agreement with computational fluid dynamics (CFD) simulations. Case studies presented include nozzle shapes for uniform pressure and shear stress profiles, which are useful for measuring the strength of soft deposit layers.

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1. Introduction

The deposition of films on process surfaces from a flowing liquid occurs in many industrial manufacturing processes. Some of these films are desired (e.g. chocolate coatings on biscuits) and some are unwanted (e.g. fouling layers in heat exchangers). In both cases, it is often important to measure the thickness and strength of these films. In the biotechnology and food sectors, these films are frequently soft and/or fragile and their state is strongly dependent on the presence of liquid, so that measurements of thickness and strength should be determined *in situ* and in real time particularly where the deposit is evolving. Portable, rapid, non-contact and precise techniques are needed.

Fluid dynamic gauging (FDG) is a relatively novel technique that was developed by Tuladhar et al. (2000) to estimate *in situ* and in real time the thickness of a soft deposit layer immersed in liquid without touching the surface of this layer. The concept consists of a nozzle that withdraws liquid from the region near the deposit surface, as shown in Fig. 1. For a certain range of clearances between the nozzle and the surface (h), the flow rate (\dot{m}) through the nozzle is usefully sensitive to h (Fig. 1(b)). The

thickness of the deposit, δ , can be calculated from the difference in clearance between the nozzle and the deposit layer, h (inferred from the flow rate), and the position of the nozzle relative to the substrate, h_{init} (established either by calibration or independent sensing). The technique provides high accuracy thickness measurement with a resolution of $\pm 5 \mu\text{m}$ with a sensing time of $\leq 5 \text{ s}$ (Gordon et al., 2010).

Studies such as that by Chew et al. (2004a) have shown that FDG could also be used to quantify the strength of soft deposits. The gauge employs flows in the laminar regime, allowing computational fluid dynamics (CFD) to be used to give reliable estimates of the flow field and stress distribution in the flow. Chew et al. quantified the strength of different tomato paste layers by measuring the deformation of the film following gauging at a known shear stress exerted by the gauge on the film. Thickness measurements were made at high clearance (low shear stress) and following exposure of the film to higher shear stresses induced by moving the nozzle closer to the film.

The stresses induced on the film by the gauging flow are determined by the liquid flow rate, clearance and also the external geometry of the nozzle. Peralta et al. (submitted for publication) demonstrated how the external shape of the nozzle could affect, very noticeably, the shear stress (τ_w) and pressure (p) profiles on a gauged surface. They identified geometries that produced interesting τ_w and p profiles such as linear, peaked or bimodal distributions. These types of profiles are attractive to FDG operation as they offer the opportunity to manipulate the forces exerted on a film by simply changing

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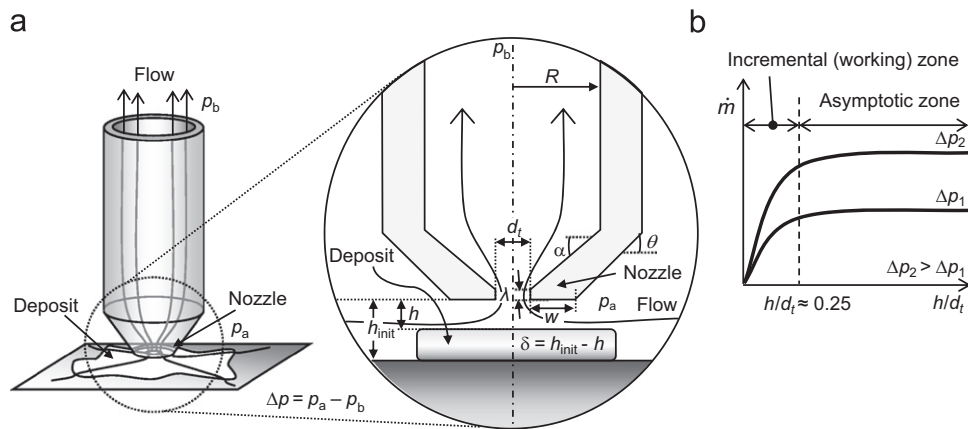


Fig. 1. Fluid dynamic gauging principles. (a) Schematic of nozzle, with inset showing key dimensions and (b) calibration curves showing relationship between mass flow rate, \dot{m} , and dimensionless clearance, h/d_t .

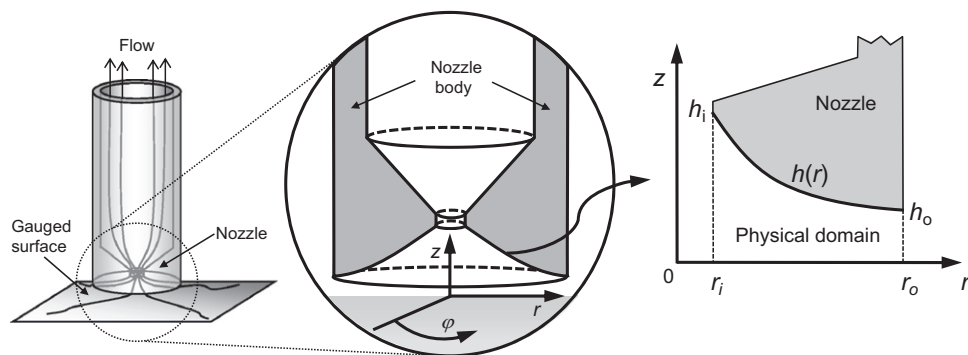


Fig. 2. Schematic of the physical domain investigated in this work.

the nozzle. For example, an approximately constant erosion of the film can be produced under the nozzle rim when an even shear stress profile is used. Alternately, another shape could yield high sensitivity to clearance (measurement precision) while minimising fluid shear.

Previous workers have employed computational fluid dynamics (CFD) simulations to estimate how the operating variables affect the important flow variables such as the profiles of shear stress and pressure exerted on the deposit (Chew et al., 2004b; Gu et al., 2009, 2011; Lister et al., 2011). The definition of the geometry and meshing is straightforward and simulations take a few minutes to converge on modern PCs (Peralta et al., submitted for publication). Exploring nozzle geometries is time consuming, however, as the mesh must be re-defined for each simulation. This paper therefore explores the scope for analytically based approaches to investigate the impact of nozzle shape on FDG performance, to be used to identify likely configurations for fine tuning by simulation.

A key feature of FDG operation (Fig. 1(b)) is that the nozzle must be close to the film surface during measurements, i.e. $h/d_t \leq 0.25$. The width of the nozzle rim is much larger than the gap between the nozzle and the film which, when combined with the laminar nature of the flow, suggests that analyses based on the lubrication approximation should yield useful results. This approach was employed by Chew et al. (2005) and later by Gu et al. (2009) to estimate the shear stress imposed on the surface. In both cases shear stress distributions obtained from CFD simulations showed good agreement with the analytical solution for a radial flow between two parallel disks obtained by Middleman (1998) using lubrication theory. This paper extends the approach to consider the effect of nozzle geometry.

2. Theoretical approach

2.1. Equations of change

The physical domain of interest is the gap between the nozzle and the gauged surface (Fig. 2). The equations of change, adapted to include the lubrication approximation, are written in cylindrical co-ordinates following the procedure presented by Denn (1980). Only the principal steps of the analysis are presented. The following assumptions are made: (i) steady state; (ii) Newtonian fluid; (iii) the velocity component in the φ direction is negligibly small; (iv) axisymmetry in φ ; and (v) external forces are due to gravity and act only in the z direction. These assumptions simplify the continuity (Eq. (1)) and Navier-Stokes (Eqs. (2) and (3)) equations to

$$\frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{\partial v_z}{\partial z} = 0 \quad (1)$$

$$\rho \left(v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} \right) = -\frac{\partial p}{\partial r} + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (rv_r) \right) + \frac{\partial^2 v_r}{\partial z^2} \right] \quad (2)$$

$$\rho \left(v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{\partial^2 v_z}{\partial z^2} \right] + \rho g_z \quad (3)$$

Eqs. (1)–(3) can be made non-dimensional using the following identities:

$$\tilde{r} = \frac{r}{r_o - r_i} \quad (4)$$

$$\tilde{z} = \frac{z}{h_o} \quad (5)$$

$$\tilde{v}_r = \frac{v_r}{U_o} \quad (6)$$

$$\tilde{v}_z = \frac{v_z}{V} \quad (7)$$

$$\tilde{p} = \frac{p - \rho g_z z}{\Pi} \quad (8)$$

where r_i and r_o are the internal and external radius of the nozzle, respectively; h_o is the clearance between the nozzle and the gauged surface at r_o , U_o is the mean velocity at the external radius calculated from the flow rate, and V and Π are the characteristic values for z -velocity and pressure that will be defined later. The use of these dimensionless variables assists the order of magnitude analysis discussed below and allows the results to be condensed into general equations and a small number of figures.

Introducing Eqs. (4)–(8) into Eqs. (1)–(3) yields

$$\frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r} \tilde{v}_r) + \frac{V(r_o - r_i)}{U_o h_o} \frac{\partial \tilde{v}_z}{\partial \tilde{z}} = 0 \quad (9)$$

$$\begin{aligned} \frac{\rho U_o^2}{(r_o - r_i)} \left[\tilde{v}_r \frac{\partial \tilde{v}_r}{\partial \tilde{r}} + \frac{V(r_o - r_i)}{U_o h_o} \tilde{v}_z \frac{\partial \tilde{v}_r}{\partial \tilde{z}} \right] = - \frac{\Pi}{(r_o - r_i)} \frac{\partial \tilde{p}}{\partial \tilde{r}} \\ + \frac{\mu U}{h_o^2} \left[\left(\frac{h_o}{(r_o - r_i)} \right)^2 \frac{\partial}{\partial \tilde{r}} \left(\frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r} \tilde{v}_r) \right) + \frac{\partial^2 \tilde{v}_r}{\partial \tilde{z}^2} \right] \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{\rho U_o V}{(r_o - r_i)} \left[\tilde{v}_r \frac{\partial \tilde{v}_z}{\partial \tilde{r}} + \frac{V(r_o - r_i)}{U_o h_o} \tilde{v}_z \frac{\partial \tilde{v}_z}{\partial \tilde{z}} \right] = - \frac{\Pi}{h_o} \frac{\partial \tilde{p}}{\partial \tilde{z}} \\ + \frac{\mu V}{h_o^2} \left[\left(\frac{h_o}{(r_o - r_i)} \right)^2 \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \left(\tilde{r} \frac{\partial \tilde{v}_z}{\partial \tilde{r}} \right) + \frac{\partial^2 \tilde{v}_z}{\partial \tilde{z}^2} \right] \end{aligned} \quad (11)$$

Denn (1980) presented a dimensional analysis of Eq. (9), which showed that the magnitude of the factor multiplying the derivative in the second term is $O[V(r_o - r_i)/(U_o h_o)] \cong 1$. Denoting $\varepsilon = h_o/(r_o - r_i)$, this gives $O(V) = O(\varepsilon U_o)$. This relation, based on the continuity equation and a dimensional analysis of the problem, shows a natural way to define V . Therefore, the definition of V used here will be $V = \varepsilon U_o$. Eq. (7) can then be written as $\tilde{v}_z = v_z/(\varepsilon U_o)$. Eq. (10) becomes

$$\text{Re} \left(\tilde{v}_r \frac{\partial \tilde{v}_r}{\partial \tilde{r}} + \tilde{v}_z \frac{\partial \tilde{v}_r}{\partial \tilde{z}} \right) = - \frac{\Pi h_o \varepsilon}{\mu U_o} \frac{\partial \tilde{p}}{\partial \tilde{r}} + \varepsilon^2 \frac{\partial}{\partial \tilde{r}} \left[\frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r} \tilde{v}_r) \right] + \frac{\partial^2 \tilde{v}_r}{\partial \tilde{z}^2} \quad (12)$$

where the Reynolds number is defined as $\text{Re} = \rho U_o h_o / \mu$

The operating mode of the gauge requires the nozzle to be very close to the gauged surface such that ε is small, i.e. $\varepsilon^2 \ll 1$. The flow is in the laminar regime (discussed in Section 2.3), so that $\text{Re} \varepsilon \ll 1$, and Eq. (12) becomes:

$$0 \cong - \frac{\Pi h_o \varepsilon}{\mu U_o} \frac{\partial \tilde{p}}{\partial \tilde{r}} + \frac{\partial^2 \tilde{v}_r}{\partial \tilde{z}^2} \quad (13)$$

From the order of magnitude of the terms in Eq. (13) (Denn, 1980), the parameter Π can be defined as $\Pi = \mu U_o / (h_o \varepsilon)$. Rearranging Eq. (11) gives

$$\text{Re} \varepsilon^3 \left(\tilde{v}_r \frac{\partial \tilde{v}_z}{\partial \tilde{r}} + \tilde{v}_z \frac{\partial \tilde{v}_z}{\partial \tilde{z}} \right) = - \frac{\partial \tilde{p}}{\partial \tilde{z}} + \varepsilon^4 \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \left(\tilde{r} \frac{\partial \tilde{v}_z}{\partial \tilde{r}} \right) + \varepsilon^2 \frac{\partial^2 \tilde{v}_z}{\partial \tilde{z}^2} \quad (14)$$

Incorporating the relationships $\text{Re} \varepsilon \ll 1$, $\varepsilon^2 \ll 1$ and $\varepsilon^4 \ll 1$, gives the equations of change as

$$\frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} (\tilde{r} \tilde{v}_r) + \frac{\partial \tilde{v}_z}{\partial \tilde{z}} = 0 \quad (15)$$

$$0 \cong - \frac{\partial \tilde{p}}{\partial \tilde{r}} + \frac{\partial^2 \tilde{v}_r}{\partial \tilde{z}^2} \quad (16)$$

$$0 \cong - \frac{\partial \tilde{p}}{\partial \tilde{z}} \quad (17)$$

These equations, which are the typical set of expressions proposed for a lubrication flow in a fluid system (Denn, 1980), will be used to describe the flow in the domain between the nozzle and the gauged surface.

2.2. Useful expressions derived from the equations of change

2.2.1. Velocity distributions

The dimensionless velocity component in the r direction, \tilde{v}_r , can be determined from Eq. (16). Given that the pressure is a function of r alone (Eq. (17), i.e. $\partial \tilde{p} / \partial \tilde{r} \cong f(\tilde{r})$), and imposing boundary conditions such as $\tilde{v}_r = 0$ at $\tilde{z} = 0$ (gauged surface) and $\tilde{v}_r = 0$ at $\tilde{z} = h(\tilde{r})/h_o = \tilde{h}(\tilde{r})$ (nozzle surface), integration of Eq. (16) yields:

$$\tilde{v}_r(\tilde{r}, \tilde{z}) \cong \frac{d\tilde{p}}{d\tilde{r}} \frac{\tilde{h}^2}{2} \left[\left(\frac{\tilde{z}}{\tilde{h}} \right)^2 - \frac{\tilde{z}}{\tilde{h}} \right] \quad (18)$$

Note that $\tilde{z}/\tilde{h} < 1$ so \tilde{v}_r is negative. In FDG operation the flow rate through the nozzle is set and/or known. Therefore in order to find the dimensionless pressure gradient in Eq. (18), the following expression for the dimensionless flow rate will be used:

$$\tilde{Q} = \tilde{r} \int_0^{\tilde{h}(\tilde{r})} |\tilde{v}_r| d\tilde{z} \quad (19)$$

where $\tilde{Q} = Q/[2\pi(r_o - r_i)h_o U_o]$ and $|\tilde{v}_r|$ is the absolute value of \tilde{v}_r . Incorporating Eq. (18) into Eq. (19) and integrating gives the dimensionless pressure gradient as

$$\frac{d\tilde{p}}{d\tilde{r}} \cong \frac{12\tilde{Q}}{\tilde{r}\tilde{h}^3} \quad (20)$$

Eq. (18) then becomes:

$$\tilde{v}_r(\tilde{r}, \tilde{z}) \cong \frac{6\tilde{Q}}{\tilde{r}\tilde{h}} \left[\left(\frac{\tilde{z}}{\tilde{h}} \right)^2 - \frac{\tilde{z}}{\tilde{h}} \right] \quad (21)$$

The corresponding expression for the z component of velocity is obtained by incorporating Eq. (21) into the dimensionless continuity equation (Eq. (15)):

$$\tilde{v}_z(\tilde{r}, \tilde{z}) \cong \frac{6\tilde{Q}}{\tilde{r}\tilde{h}} \left(\frac{d\tilde{h}}{d\tilde{r}} \right) \left[\left(\frac{\tilde{z}}{\tilde{h}} \right)^3 - \left(\frac{\tilde{z}}{\tilde{h}} \right)^2 \right] \quad (22)$$

An interesting relationship is obtained by dividing Eqs. (22) and (21):

$$\frac{\tilde{v}_z(\tilde{r}, \tilde{z})}{\tilde{v}_r(\tilde{r}, \tilde{z})} \cong \frac{\tilde{z}}{\tilde{h}} \frac{d\tilde{h}}{d\tilde{r}} \quad (23)$$

Eq. (23) shows that there will be a non-zero component of velocity in the z direction if the nozzle profile is not horizontal.

2.2.2. Shear stress on gauged surface

For a Newtonian fluid, the component of the shear stress relevant to the stress on the gauged surface τ_{rz} can be expressed as (Bird et al., 2007):

$$\tau_{rz} = -\mu \left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right) \quad (24)$$

Incorporating the above results, namely (i) $V = \varepsilon U_o$ and (ii) $\varepsilon = h_o/(r_o - r_i)$ gives the dimensionless form:

$$\tilde{\tau}_{rz} = - \left(\frac{\partial \tilde{v}_r}{\partial \tilde{z}} + \varepsilon^2 \frac{\partial \tilde{v}_z}{\partial \tilde{r}} \right) \quad (25)$$

where $\tilde{\tau}_{rz} = \tau_{rz}/[\mu(U_o/h_o)]$.

Applying the condition $\varepsilon^2 \ll 1$, Eqs. (21) and (22) in Eq. (25), yields the expression for the dimensionless shear stress evaluated

at the gauged surface ($\tilde{z} = 0$):

$$\left. \tilde{\tau}_{rz} \right|_{\tilde{z}=0} = \tilde{\tau}_w \cong \frac{6\tilde{Q}}{\tilde{r}\tilde{h}^2} \quad (26)$$

It is helpful to introduce a new normalised radial co-ordinate in order to simplify the above results. Defining

$$\tilde{r}' = \frac{\tilde{r}}{\tilde{r}_o} = \frac{r}{r_o} \quad (27)$$

scales the radial dimension of the domain from 0 to 1. Similarly, the following expressions for the dimensionless shear stress and pressure gradient are introduced so that the variables are only dependent of the geometrical parameters:

$$\tilde{\tau}_w^* = \frac{\tilde{\tau}_w(r_o - r_i)}{6r_o\tilde{Q}} \quad (28)$$

$$\frac{d\tilde{p}^*}{d\tilde{r}'} = \frac{(d\tilde{p}/d\tilde{r}')}{12\tilde{Q}} \quad (29)$$

Eqs. (20) and (26) can now be expressed as

$$\frac{d\tilde{p}^*}{d\tilde{r}'} \cong \frac{1}{\tilde{r}'\tilde{h}^3} \quad (30)$$

$$\tilde{\tau}_w^* \cong \frac{1}{\tilde{r}'\tilde{h}^2} \quad (31)$$

Combining Eqs. (30) and (31) gives a useful expression for calculating the wall shear stress:

$$\tilde{\tau}_w^* \cong \tilde{h} \left(\frac{d\tilde{p}^*}{d\tilde{r}'} \right) \quad (32)$$

Measuring local shear stresses is challenging but it is relatively straightforward to measure local pressure values under the nozzle and therefore the pressure gradient, as reported by Chew et al. (2004a) and Peralta et al. (submitted for publication).

Eqs. (30)–(32) are important in considering nozzle geometry because they relate the local shear stress and the pressure gradient to \tilde{h} . By specifying the profile of one of \tilde{h} , $\tilde{\tau}_w^*$ or $d\tilde{p}^*/d\tilde{r}'$, the expressions for the rest of the main parameters (i.e. shear stress, pressure gradient, external nozzle geometry, velocity components, etc.) can be determined.

2.3. Range of theoretical validity of the approach

An important consideration is the ability to verify the range of validity of the solutions. Verification is based on checking whether the main assumptions of the model are met. These assumptions are

$$(i) \text{ Re}_d = \frac{\rho U_o h_o^2}{\mu(r_o - r_i)} = \frac{\dot{m} h_o}{2\pi\mu(r_o - r_i)r_o} = \frac{d\text{Re}_d h_o}{8(r_o - r_i)r_o} \ll 1 \quad (33)$$

or, simply

$$\text{Re}_d \ll \frac{8r_o}{\varepsilon d} \quad (34)$$

where $\text{Re}_d = 4\dot{m}/\pi\mu d$ and

$$(ii) \varepsilon^2 = \frac{h_o^2}{(r_o - r_i)^2} \ll 1 \quad (35)$$

It is noteworthy that the above statements represent a sufficient set for checking, because these are less restrictive conditions than those appearing in Eqs. (12) and (14), such as those relating to $\text{Re}_d^m (m > 1)$ and $\varepsilon^n (n > 2)$.

A typical set of parameters from FDG measurements using water as the gauging fluid reported by Chew et al. (2004a) are used as an

illustration of the approach. The dimensions are $h_o = 0.25$ mm, $r_o = 12.5$ mm, $r_i = 2.5$ mm, $d = 0.02$ m, flow rate ≤ 5 g s⁻¹, giving $\text{Re}_d \ll 200$ and $\varepsilon^2 = 0.000625$. These conditions guarantee that for low flow rates, the approach presented in this work can be used to describe the FDG system.

3. Materials and methods

3.1. Case studies

The methodology was tested by proposing simple expressions for one of $\tilde{\tau}_w^*$, $d\tilde{p}^*/d\tilde{r}'$ or \tilde{h} and calculating the solution for the remaining two parameters. The expressions thus obtained were compared with numerical results obtained using CFD simulations. A general type of expression was assigned for each parameter specified:

$$\tilde{f}(\tilde{r}') = \frac{f(\tilde{r}')}{f(1)} \quad (36)$$

where $f(\tilde{r}')$ is a generic expression for either $\tilde{\tau}_w^*$, $d\tilde{p}^*/d\tilde{r}'$ or \tilde{h} , and $f(1)$ is $f(\tilde{r}')$ evaluated at $\tilde{r}' = \tilde{r}'_o = 1$ (outer radius).

3.2. CFD simulations

Some of the case study scenarios were evaluated using a commercial CFD code, in a similar manner to previous studies reported by Peralta et al. (submitted for publication). Briefly, a 2D-axisymmetric computational domain based on that employed by Chew et al. (2004a) for modelling quasi-static FDG systems was used. Fig. 3 shows a representative computational domain employing cylindrical co-ordinates and the associated boundary conditions.

The simulations were performed using different values of Re_d . Unless otherwise specified all simulations employed a clearance (closest point between the nozzle and the gauged surface) set at $h/d = 0.05$, which lies within the incremental or working zone depicted in Fig. 1(b).

The flow at the exit of the domain was assumed to be laminar and fully developed owing to the range of Re_d values used. The governing equations were the Navier-Stokes and continuity equations. The fluid properties were taken as those of water at 20 °C.

The domain was discretised using a triangular mesh. In zones where it was important to estimate the velocity gradients accurately (e.g. the gap between the nozzle and the gauged surface) a higher density mesh was used (Fig. 3(b)). The commercial finite-element-based software COMSOL Multiphysics 3.5a and 4 (COMSOL Ltd., Hatfield, United Kingdom) was used to perform the CFD simulations on a 2.4 GHz desktop PC equipped with 5 processors and 4 GB of RAM. Each simulation took about 5 min to converge.

The CFD simulations were validated in the previous study (Peralta et al., submitted for publication). In brief, the validation consisted of a test of the independency of the studied variables on the mesh configuration and agreement between predicted values with experimental observations. The mesh independency test was carried out using different mesh densities over the entire domain. Monotonic convergence was observed for meshes with > 3000 elements. The second step was carried out using: (i) experimental values of pressure on the gauged surface, and (ii) theoretical values of shear stress for selected operating conditions based on the assumption that the flow can be approximated as radial between two parallel disks.

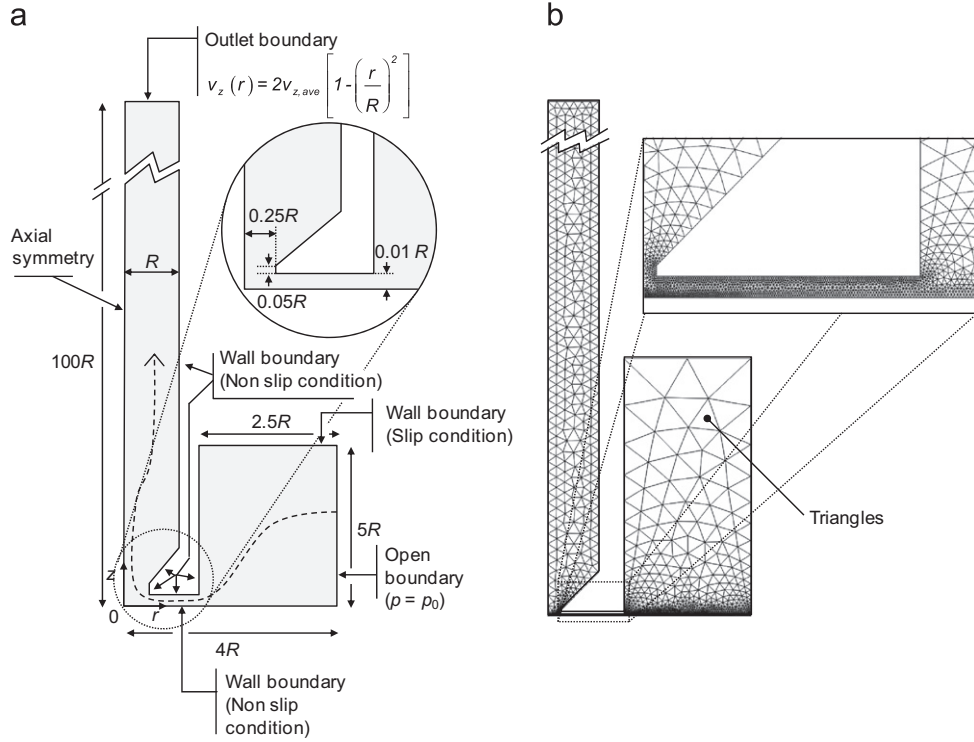


Fig. 3. Simulation geometry (a) showing boundary conditions and domain dimensions, and (b) mesh used (a high mesh density is employed under the nozzle rim, in the throat and along the base).

4. Results and discussion

Table 1 summarises the expressions for the external nozzle geometry, pressure gradient under the nozzle and shear stress on the gauged surface, based on the use of an arbitrary function $\tilde{f}(\tilde{r}')$ for one of these variables. The relationships are easy to manipulate. These expressions will be used in the following sections to show some examples for specified nozzle geometry, pressure gradient and shear stress.

4.1. Specified nozzle geometry

4.1.1. Linear nozzle profile

A linear profile is one of the simplest external nozzle geometries to manufacture and analyse. The nozzle shape can be either normal (i.e. parallel to the gauged surface), convergent or divergent. For this case, the dimensionless nozzle profile is given by

$$\tilde{f}_h(\tilde{r}') = \tilde{h} = a_h(\tilde{r}' - 1) + 1 \quad (37)$$

where a_h is the slope of the linear profile of the external nozzle geometry. Working from the first row in Table 1, expressions for the pressure gradient and shear stress are

$$\frac{d\tilde{p}^*}{d\tilde{r}'} \cong \frac{1}{\tilde{r}'[a_h(\tilde{r}' - 1) + 1]^3} \quad (38)$$

$$\tilde{\tau}_w^* \cong \frac{1}{\tilde{r}'[a_h(\tilde{r}' - 1) + 1]^2} \quad (39)$$

Fig. 4 shows the functionality obtained for a linear profile of \tilde{h} (Fig. 4(a)) for different and representative values of a_h . The $\tilde{\tau}_w^*$ profiles in (Fig. 4(c)) show a marked decrease as \tilde{r}' increases for positive values of a_h which is accentuated at higher a_h . With negative a_h , the $\tilde{\tau}_w^*$ profiles still present a concave shape but with a minimum within the nozzle region. This feature is discussed in detail in Section 4.1.1.1.

Table 1

Expressions for \tilde{h} , $d\tilde{p}^*/d\tilde{r}'$ and $\tilde{\tau}_w^*$ using a general expression of the type $\tilde{f}(\tilde{r}')$ (within each row of the table) for either \tilde{h} , $d\tilde{p}^*/d\tilde{r}'$ or $\tilde{\tau}_w^*$.

\tilde{h}	$\frac{d\tilde{p}^*}{d\tilde{r}'}$	$\tilde{\tau}_w^*$
$\tilde{f}_h(\tilde{r}')$	$\frac{1}{\tilde{r}'[\tilde{f}_h(\tilde{r}')]^3}$	$\frac{1}{\tilde{r}'[\tilde{f}_h(\tilde{r}')]^2}$
$\frac{1}{\sqrt[3]{\tilde{r}'\tilde{f}_{pg}(\tilde{r}')}}}$	$\tilde{f}_{pg}(\tilde{r}')$	$\sqrt[3]{\frac{[\tilde{f}_{pg}(\tilde{r}')]^2}{\tilde{r}'}}$
$\frac{1}{\sqrt[3]{\tilde{r}'\tilde{f}_s(\tilde{r}')}}}$	$\sqrt[3]{\tilde{r}'[\tilde{f}_s(\tilde{r}')]^3}$	$\tilde{f}_s(\tilde{r}')$

The pressure profiles in Fig. 4(d) were obtained by integrating the gradients in Fig. 4(b):

$$-\tilde{p}^* \cong -\frac{a_h(\tilde{r}' - 1)\{a_h - 1 + (a_h - 3)[a_h(\tilde{r}' - 1) + 1]\}}{2[a_h(\tilde{r}' - 1) + 1]^2(a_h - 1)^2} - \frac{1}{(a_h - 1)^3} \ln \left[\frac{a_h(\tilde{r}' - 1) + 1}{\tilde{r}'} \right] - \tilde{p}_o^* \quad (40)$$

Fig. 4(d) shows a change in the profiles from convex to concave as a_h changes from positive to negative values. This follows the change in the nozzle shape from a converging to a diverging one. As a_h becomes more positive, the external part of the nozzle becomes more convergent, concentrating the pressure drop near the inner radius of the nozzle. Conversely, highly negative a_h values concentrate the pressure loss near the outer rim of the nozzle. The same information can be extracted from the pressure gradient (Fig. 4(b)).

Finally, it is noteworthy that when $a_h = 0$, Eq. (40) reduces to the expression for the radial flow between two parallel disks

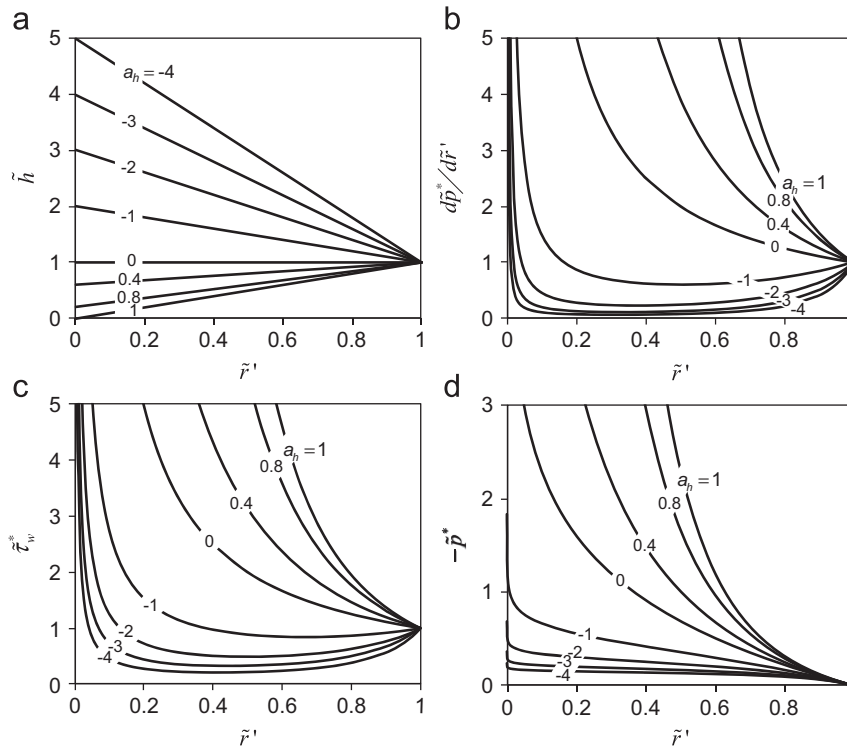


Fig. 4. Dimensionless profiles of: (a) nozzle external geometry (\hat{h}), (b) pressure gradient ($d\tilde{p}^*/d\tilde{r}'$), (c) shear stress ($\tilde{\tau}_w^*$) and (d) pressure ($-\tilde{p}^*$) as a function of the dimensionless radial position (\tilde{r}') for different values of a_h , for the linear nozzle shape.

(Middleman, 1998):

$$-\tilde{p}^* \cong -\ln(\tilde{r}') - \tilde{p}_o^* \quad (41)$$

An important parameter to quantify is the area-averaged shear stress on the gauged surface, $\langle \tilde{\tau}_w^* \rangle$, as this can be related to material properties when the nozzle deforms the film. This is defined as

$$\langle \tilde{\tau}_w^* \rangle = \frac{2}{(1-\tilde{r}_i'^2)} \int_{\tilde{r}_i'}^1 \tilde{\tau}_w^* \tilde{r}' d\tilde{r}' \quad (42)$$

where $\langle \tilde{\tau}_w^* \rangle = \langle \tau_w \rangle / \tau_o$ and $\tau_o = 3\mu\dot{m}/(\pi\rho h_o^2 r_o)$: τ_o is the shear stress calculated for radial flow between parallel disks with separation h_o , evaluated at r_o (Middleman, 1998). Introducing Eq. (39) into Eq. (42), gives

$$\langle \tilde{\tau}_w^* \rangle \cong \frac{2}{(1+\tilde{r}_i') [a_h(\tilde{r}_i'-1)+1]} \quad (43)$$

Fig. 5(a) shows that the average shear stress $\langle \tilde{\tau}_w^* \rangle$ increases with Re_d : it should be noted that these results were all calculated for a fixed clearance, as varying the clearance for a given flow rate will also change Re_d and $\langle \tilde{\tau}_w^* \rangle$. The effect of nozzle shape on the averaged shear stress is stronger when $a_h > 0$, i.e. convergent nozzles, as the position where the nozzle approaches the film most closely is where the circumferential area is also a minimum.

Fig. 5(b) summarises the effect of nozzle width in terms of $\langle \tilde{\tau}_w^* \rangle$, the area-averaged shear stress divided by τ_o . When $\tilde{r}_i' = 1$, the width is negligible. The plots show that $\langle \tilde{\tau}_w^* \rangle$ increases with nozzle width when a_h is positive. When $a_h < 0$, $\langle \tilde{\tau}_w^* \rangle \leq 1$ and at certain values of \tilde{r}_i' there is a minimum in $\langle \tilde{\tau}_w^* \rangle$, located at

$$\tilde{r}_i' \cong -\frac{1}{2a_h} \quad (44)$$

Minima are obtained for $a_h \leq -0.5$. The existence of minimum values of $\langle \tilde{\tau}_w^* \rangle$ is of interest for measuring deposit thicknesses, where a low shear stress is advantageous in order to avoid

deforming the deposit. When comparing the reduction in $\langle \tilde{\tau}_w^* \rangle$ for a nozzle with \tilde{r}_i' given by Eq. (44), a suitable reference value is that evaluated at $\tilde{r}_i' = 0$ (denoted $\langle \tilde{\tau}_w^* \rangle_{\tilde{r}_i'=0}$). Introducing Eq. (44) into Eq. (43) and dividing by $\langle \tilde{\tau}_w^* \rangle_{\tilde{r}_i'=0}$ gives

$$\frac{\langle \tilde{\tau}_w^* \rangle_{\min}}{\langle \tilde{\tau}_w^* \rangle_{\tilde{r}_i'=0}} \cong -\frac{4a_h(1-a_h)}{(2a_h-1)^2} \quad (45)$$

Fig. 6 shows the results computed for $a_h \leq -0.5$. The reduction in shear stress by using the minimum area-averaged shear stress exhibits a maximum at $\tilde{r}_i' = 0.5$ (Fig. 6(b)). The plots show that the reduction in $\langle \tilde{\tau}_w^* \rangle$ can be in the order of 10% if the inner radius of the nozzle is made as small as possible.

4.1.1.1. The 'saddle profile' case. Peralta et al. (submitted for publication) demonstrated that a mildly divergent nozzle with a linear external surface profile and angle of approximately -5° gave a shear stress profile with two peaks of the same magnitude located at the positions of the inner and outer rim. This is termed a 'saddle profile' and is considered further using the approach presented above. For this case, (a_h) can be related to the angle (θ) by

$$a_h = \frac{r_o}{h_o} \tan\left(\frac{\pi\theta}{180}\right) \quad (46)$$

and the shear stress profile on the gauged surface is estimated as

$$\tilde{\tau}_w^* \cong \frac{1}{\left(\frac{r}{r_o}\right) \left[\frac{r_o}{h_o} \tan\left(\frac{\pi\theta}{180}\right) \left(\frac{r}{r_o} - 1\right) + 1 \right]^2} \quad (47)$$

Evaluating Eq. (47) at r_i and assuming that $\tilde{\tau}_w^* \cong 1$ (i.e. a peak at r_i), gives the following expression for the angle that will produce a

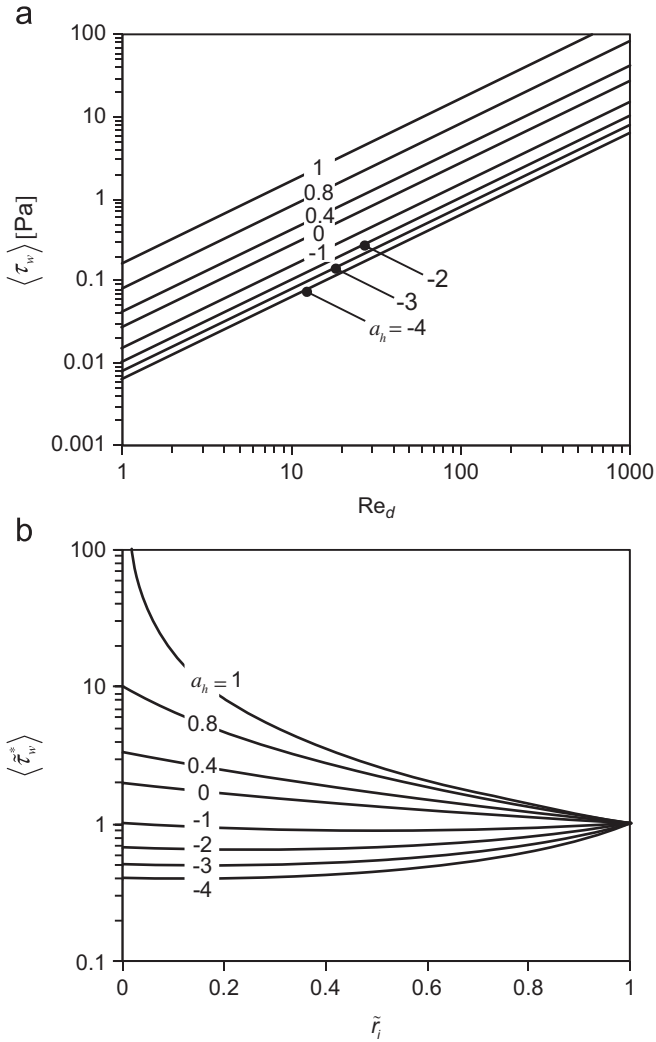


Fig. 5. (a) Effect of Re_d on area-averaged shear stress $\langle \tau_w \rangle$ and (b) dimensionless area-averaged shear stress $\langle \tau_w^* \rangle$ as a function of \tilde{r}_i' , for different values of a_h for the case of linear nozzle shape.

saddle profile, θ_{sp} :

$$\theta_{sp} \cong \frac{180}{\pi} \arctan \left[\frac{h_o}{(r_o - r_i)} \left(1 - \sqrt{\frac{r_o}{r_i}} \right) \right] \quad (48)$$

Expressing the \arctan function as an infinite series (Bronshtein et al., 2007) and recalling that its argument is small (because $\varepsilon \ll 1$), gives:

$$\theta_{sp} \cong \frac{180}{\pi} \frac{h_o}{(r_o - r_i)} \left(1 - \sqrt{\frac{r_o}{r_i}} \right) \quad (49)$$

Eq. (49) indicates that θ_{sp} is (i) negative; (ii) proportional to h_o ; (iii) independent of the flow conditions, and (iv) dependent on r_o and r_i , that is, the size of the nozzle.

The distribution of shear stress on the gauged surface for selected values θ_{sp} is compared with CFD simulations in Fig. 7(a). The profiles exhibit two peaks, as expected, with very good agreement in magnitudes and locations of the peaks.

Fig. 7(b) shows the dimensionless shear stress values evaluated at \tilde{r}_i' (the inner radius of the nozzle) as a function of the slope angle of the nozzle rim, for different values of clearance, h_o . θ_{sp} corresponds to the angle where $\tilde{\tau}_w^*|_{\tilde{r}_i'} = 1$, and becomes more negative as h_o increases. The $\tilde{\tau}_w^*|_{\tilde{r}_i'}$ profiles show a strong dependency on h_o ,

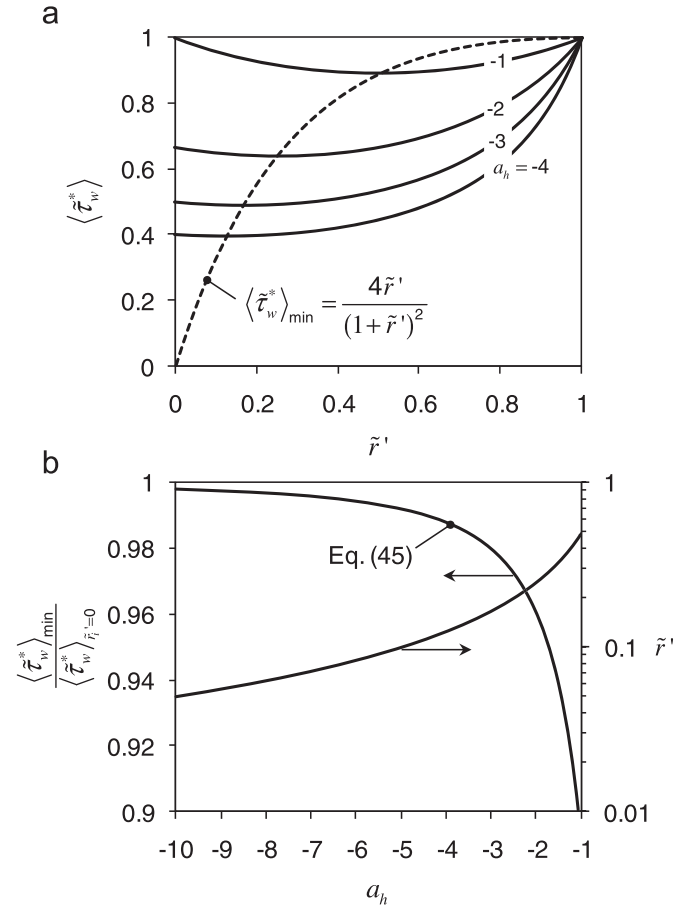


Fig. 6. (a) Dimensionless area-averaged shear stress $\langle \tau_w^* \rangle$ as a function of \tilde{r}_i' for selected negative values of a_h showing the existence of a minimum $\langle \tau_w^* \rangle_{\min}$; loci of minima denoted by the dashed line. (b) Evaluation of Eq. (45) showing the effect of a_h on loci position and magnitude.

which varies with the nozzle angle. The strong dependence of $\tilde{\tau}_w^*|_{\tilde{r}_i'}$ on θ in the range close to θ_{sp} was also observed by Peralta et al. (submitted for publication). These results indicate that the desired effect, of a shear stress distribution close to uniform, is only achievable with a narrow range of operating conditions for a given nozzle, so that alternative geometries should be investigated.

4.1.2. Non-linear nozzle profiles

The flexibility of modern fabrication techniques means that there are few limits on the shapes available for FDG nozzles. Some geometries are easier to manufacture than others, two examples being toroidal and parabolic-toroidal profiles (Fig. 8). The non-dimensional expressions for these profiles are

$$\tilde{h} = 1 - \sqrt{(\tilde{r}' - \tilde{r}_i') (1 - \tilde{r}')} \quad (\text{toroidal}) \quad (50)$$

$$\tilde{h} = \frac{4(1 - \tilde{h}_m)}{(1 - \tilde{r}_i')^2} \left[\tilde{r}'^2 - (1 + \tilde{r}_i')\tilde{r}' + \tilde{r}_i'^2 \right] + 1 \quad (\text{parabolic-toroidal}) \quad (51)$$

where \tilde{h}_m is the value of \tilde{h} at the position of the minimum distance between the nozzle and the gauged surface. The shear stress profiles evaluated for the above geometries are compared with CFD simulations in Fig. 8 for a representative set of operating conditions. Good agreement is obtained, supporting the use of the analytical approach.

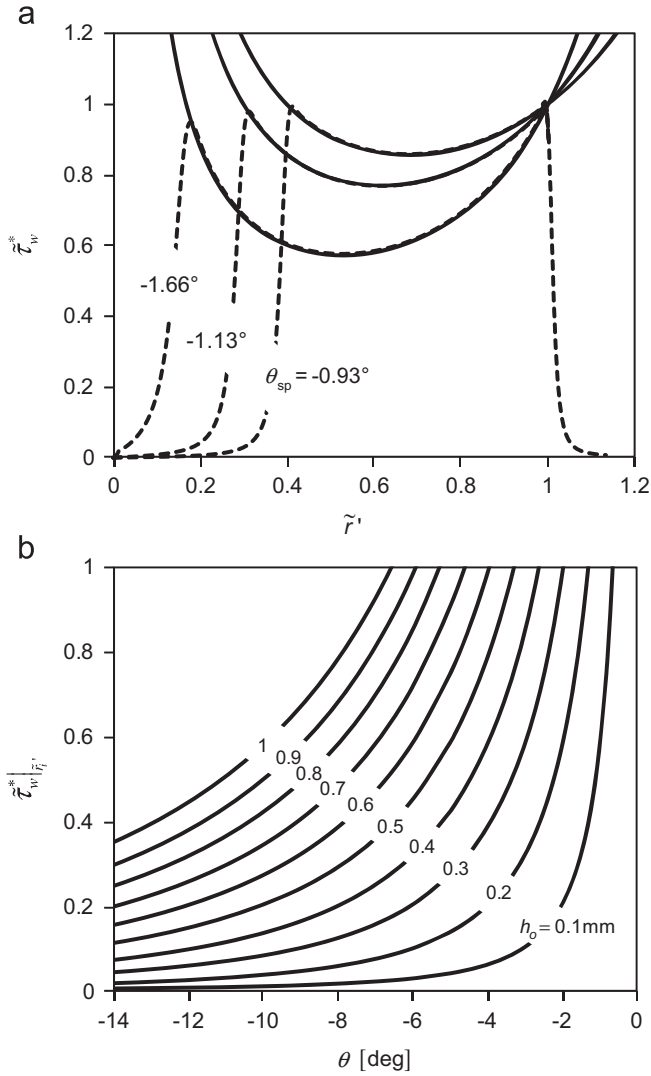


Fig. 7. (a) Comparison of theoretical (Eq. (47), solid loci) and CFD simulated (dashed loci) dimensionless shear stress τ_w^* profiles for selected values of θ_{sp} . (b) Dimensionless shear stress evaluated at the inner radius of the nozzle (\tilde{r}_i') as a function of the angle of the nozzle rim for different values of h_o .

4.2. Specified pressure gradient

The simplest profile that represents also an interesting scenario is that of a linear variation in pressure gradient. The pressure gradient is written as

$$\tilde{f}_{pg}(\tilde{r}') = \frac{d\tilde{p}^*}{d\tilde{r}'} = a_{pg}(\tilde{r}' - 1) + 1 \quad (52)$$

where a_{pg} is the slope. From Table 1, the following expressions for the external nozzle profile and the shear stress on the gauged surface are obtained:

$$\tilde{h} \cong \frac{1}{\sqrt[3]{\tilde{r}'[a_{pg}(\tilde{r}' - 1) + 1]}} \quad (53)$$

$$\tau_w^* \cong \sqrt[3]{\frac{[a_{pg}(\tilde{r}' - 1) + 1]^2}{\tilde{r}'}} \quad (54)$$

Fig. 9 shows the profiles for each parameter for different and representative values of a_{pg} . In general, the uniform pressure gradient requires \tilde{h} to decrease as \tilde{r}' increases for $a_{pg} \leq 0$, and this

effect is accentuated as a_{pg} becomes more negative. This behaviour persists even for $a_{pg} > 0$. For some positive values of a_{pg} , a minimum is observed in \tilde{h} . When $a_{pg} \cong 1$, the profile changes from convex to concave with a minimum near $\tilde{r}' = 0$.

The corresponding shear stress profiles in Fig. 9(b) show increasing τ_w^* values as \tilde{r}' decreases for $a_{pg} \geq 0$, and a minimum when $a_{pg} < 0$. The presence of the minimum indicates that a saddle profile (in terms of shear stress on the gauged surface) can be obtained in this case. The figure also shows that a convex type of profile can be expected for both positive and negative values of a_{pg} .

The pressure profiles in Fig. 9(c) were evaluated using

$$-\tilde{p}^* \cong \frac{a_{pg}}{2}(1 - \tilde{r}'^2) + (1 - a_{pg})(1 - \tilde{r}') - \tilde{p}_o^* \quad (55)$$

The plots show a change in the profiles from concave to convex when a_{pg} changes from positive to negative. The pressure values in this case are lower than those in the previous section because the gradients are linear.

The area-averaged shear stress can be calculated, as in the case for linear \tilde{h} , using Eqs. (42) and (54), viz.

$$\begin{aligned} \langle \tau_w^* \rangle \cong & \frac{2}{(1 - \tilde{r}_i'^2)} \left\{ \frac{3}{14a_{pg}} \left[1 + a_{pg} - (1 - a_{pg}) {}_2F_1 \left(1, \frac{4}{3}; \frac{5}{3}; \frac{a_{pg}}{a_{pg} - 1} \right) \right] \right. \\ & - \frac{3}{14a_{pg}} \left\{ \tilde{r}_i' [a_{pg}(\tilde{r}_i' - 1) + 1] \right\}^{2/3} \\ & \left. \left[1 + a_{pg}(2\tilde{r}_i' - 1) + (1 - a_{pg}) {}_2F_1 \left(1, \frac{4}{3}; \frac{5}{3}; \frac{a_{pg}}{a_{pg} - 1} \tilde{r}_i' \right) \right] \right\} \quad (56) \end{aligned}$$

where ${}_2F_1[a, b; c; x]$ is the Gauss hypergeometric function (Gasper and Rahman, 2004). The dependence of $\langle \tau_w^* \rangle$ on Re_d and \tilde{r}_i' is plotted in Fig. 10. The profiles exhibit a stronger effect of Re_d on $\langle \tau_w^* \rangle$ compared with Fig. 5 (\tilde{h} linear) for a particular \tilde{r}_i' value and the reduced sensitivity of $\langle \tau_w^* \rangle$ to the magnitude of a_{pg} values as \tilde{r}_i' is decreased.

4.2.1. Linear pressure profile: $d\tilde{p}^*/d\tilde{r}' = 1$, $a_{pg} = 0$

A second case of interest is where the pressure varies linearly across the nozzle rim, i.e. $a_{pg} = 0$ and Eq. (52) gives $d\tilde{p}^*/d\tilde{r}' = 1$. The dimensional pressure profile evaluated at the gauged surface ($z = 0$) is then given by

$$-p \cong \frac{3\mu^2 dRe_d}{2\rho h_o^3} \left(1 - \frac{r}{r_o} \right) - p_o \quad (57)$$

where p_o is the pressure at r_o . Comparisons between Eq. (57) and CFD simulations for different values of Re_d in Fig. 11 show very good agreement up to $Re_d \cong 40$. At the highest Re_d value considered, $Re_d = 60$, the mean percentage error was around 6.64%.

4.3. Specified shear stress profile

The scenario where the shear stress on the gauged surface varies linearly is now considered. The basis function is

$$\tilde{f}_s(\tilde{r}') = \tau_w^* = a_s(\tilde{r}' - 1) + 1 \quad (58)$$

where a_s is the gradient in the shear stress profile. The corresponding forms for the nozzle shape and pressure gradient are

$$\tilde{h} \cong \frac{1}{\sqrt{\tilde{r}'[a_s(\tilde{r}' - 1) + 1]}} \quad (59)$$

$$\frac{d\tilde{p}^*}{d\tilde{r}'} \cong \sqrt{\tilde{r}'[a_s(\tilde{r}' - 1) + 1]^3} \quad (60)$$

The loci for each parameter evaluated for different and representative values of a_s are presented in Fig. 12. Fig. 12(c) shows that \tilde{h} decreases as \tilde{r}' increases for $a_s > 0$, corresponding to a recessed nozzle with the point of closest approach to the layer

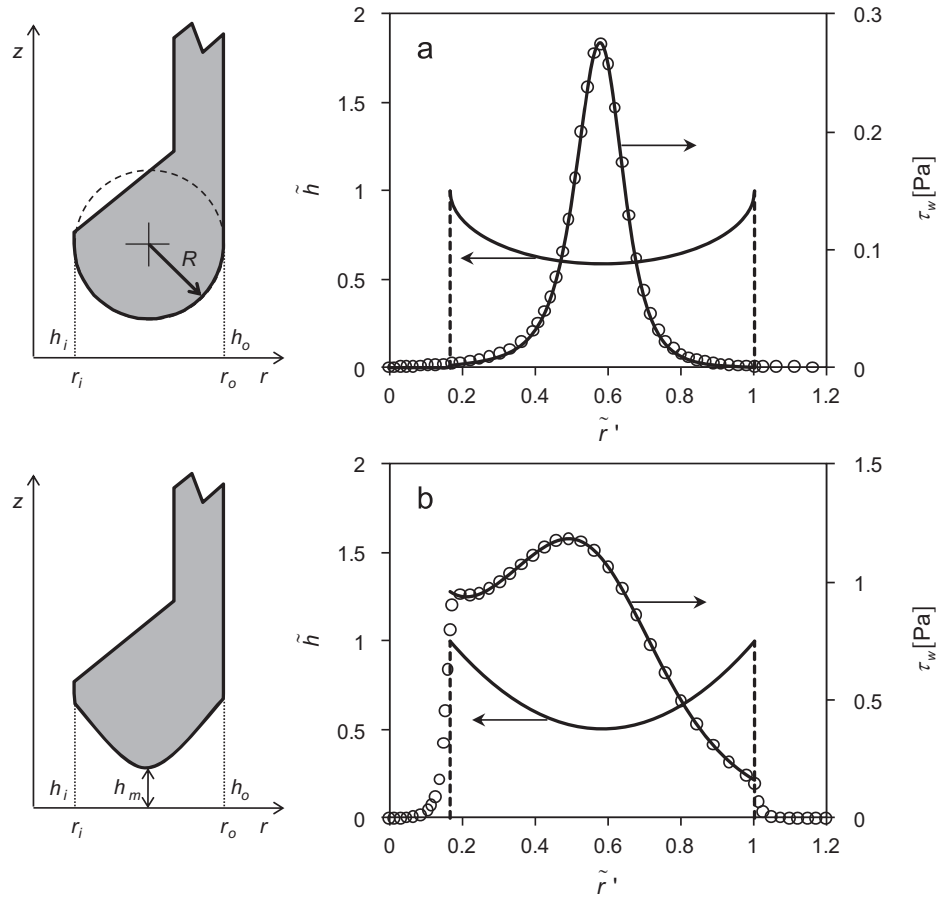


Fig. 8. Comparison between shear stress profiles obtained analytically (solid loci) and from CFD simulations (circles) for (a) toroidal nozzle (Eq. (50)) and (b) parabolic-toroidal profile of \tilde{h} (Eq. (51)). CFD conditions: $Re_d=10$, $h_o=0.25$ mm, $r_i=2.5$ mm and $r_o=15$ mm.

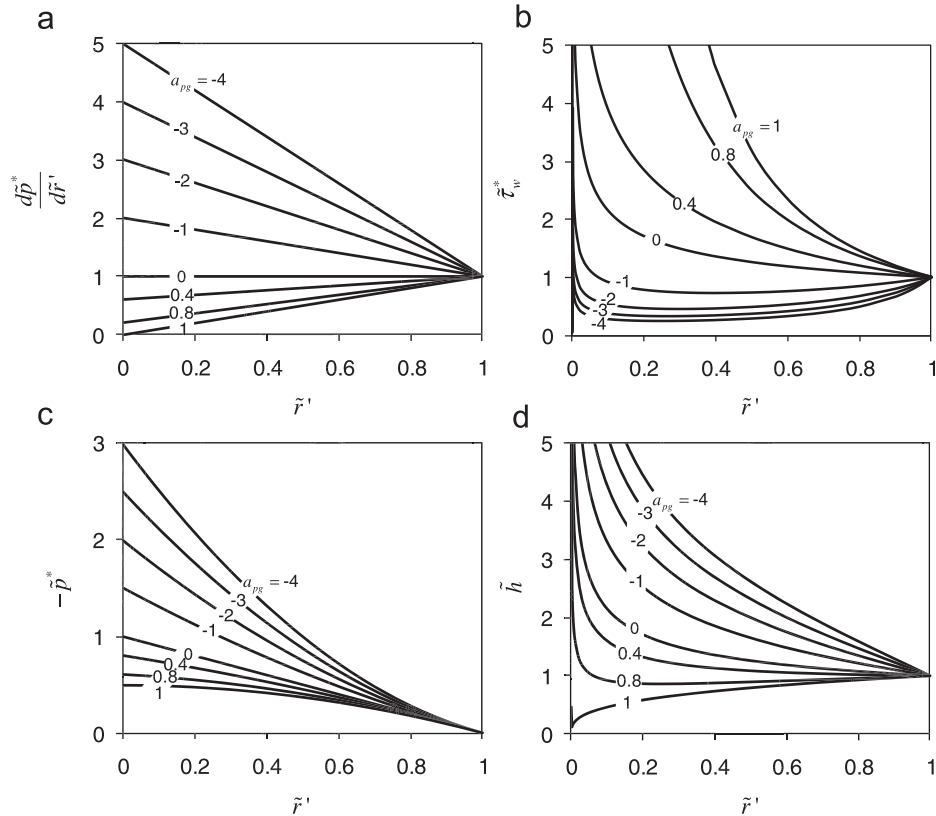


Fig. 9. Dimensionless profiles of: (a) pressure gradient ($d\tilde{p}^*/d\tilde{r}'$), (b) shear stress ($\tilde{\tau}_w^*$), (c) pressure ($-\tilde{p}^*$) and (d) nozzle external geometry (\tilde{h}) as a function of the dimensionless radial position (\tilde{r}') for different values of a_{pg} , for the case of linear pressure gradient profile.

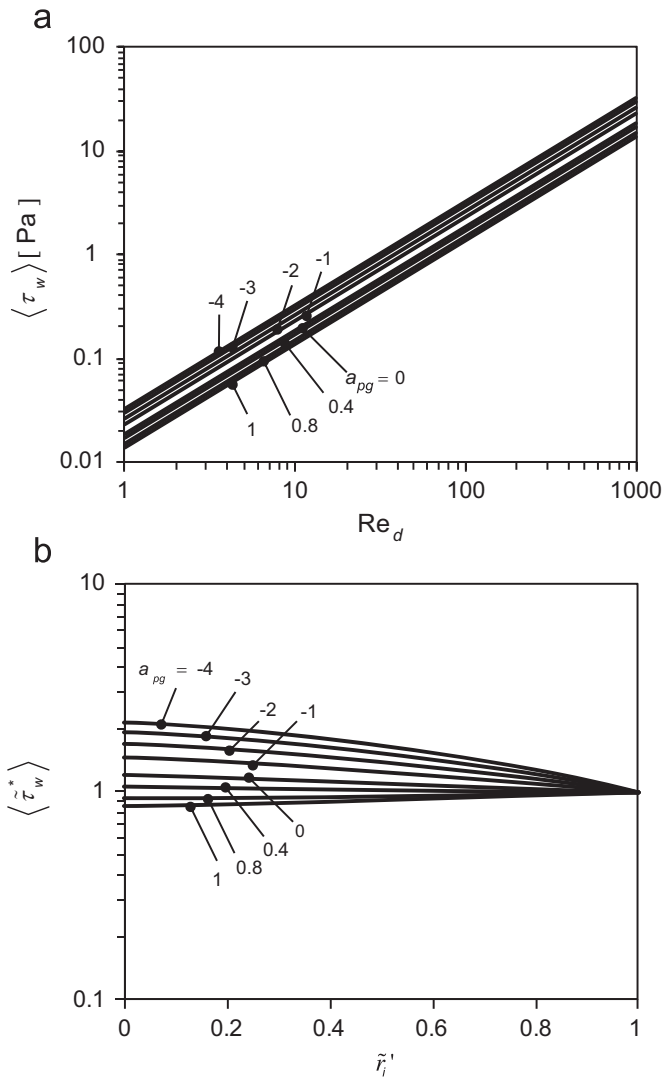


Fig. 10. (a) Effect of Re_d on area-averaged shear stress $\langle \tau_w \rangle$ and (b) dimensionless area-averaged shear stress $\langle \tilde{\tau}_w^* \rangle$ as a function of \tilde{r}_i' , for different values of a_{pg} for the linear pressure gradient profile case.

located at the outer rim. When $a_s < 0$, there is a minimum in \tilde{h} as \tilde{r}' increases. The pressure gradient profiles in Fig. 12(d) show strongly non-linear behaviour and are more sensitive to a_s . The corresponding pressure profiles are all monotonic in a_s , given by

$$-\tilde{p}^* \cong \frac{2}{3}(1-a_s)^{3/2} \left[{}_2F_1 \left(-\frac{3}{2}, \frac{3}{2}; \frac{5}{2}; \frac{a_s}{a_s-1} \right) - \tilde{r}'^{3/2} {}_2F_1 \left(-\frac{3}{2}, \frac{3}{2}; \frac{5}{2}; \frac{a_s \tilde{r}'}{a_s-1} \right) \right] - \tilde{p}_0^* \quad (61)$$

where ${}_2F_1[a, b; c; x]$ is the Gauss hypergeometric function (Gasper and Rahman, 2004).

4.3.1. Constant $\tilde{\tau}_w^*$

A special case of Eq. (61) arises when $a_s = 0$ and the shear stress on the surface being studied is uniform. This scenario is desirable for FDG measurements of deposit strength or deformation. A number of scenarios were evaluated and compared with CFD simulations. Fig. 13 shows a comparison with CFD and analytical predictions of pressure and shear stress on the gauged surface for different values of Re_d , where the local pressure is given by

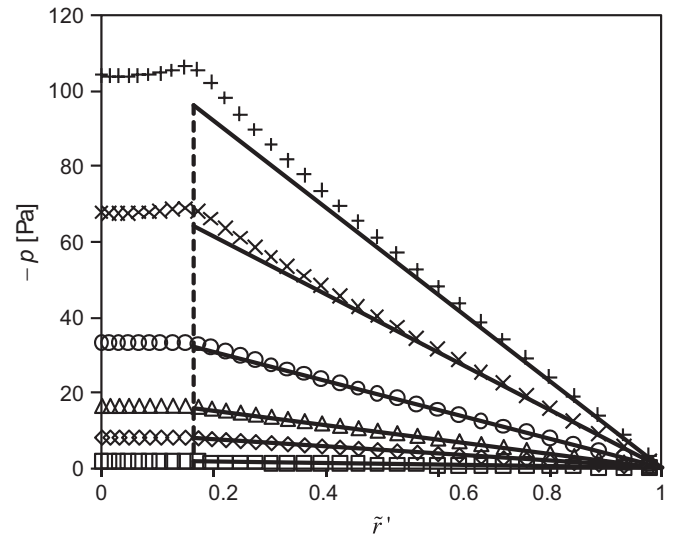


Fig. 11. Comparison between theoretical (Eq. (57), solid loci) and CFD simulation (symbols) of pressure as a function of \tilde{r}' for different Re_d or the case of a linear pressure profile. Symbols: (\square) $Re_d = 1$, (\diamond) $Re_d = 5$, (\triangle) $Re_d = 10$, (\circ) $Re_d = 20$, (\times) $Re_d = 40$ and (+) $Re_d = 60$. For each simulation: $h_o = 0.25$ mm, $r_i = 2.5$ mm and $r_o = 15$ mm.

Eq. (61), in dimensional form:

$$-p \cong \frac{\mu^2 d Re_d}{\rho h_o^3} \left[1 - \left(\frac{r}{r_o} \right)^{3/2} \right] - p_o \quad (62)$$

Fig. 13(a) shows that the agreement was good in all cases, with a mean percentage error $< 6\%$. Absolute pressures are considered here as these can be measured with reasonable accuracy for verification purposes. Fig. 13(b)–(d) shows the corresponding shear stress profiles: the simulations agree with Eq. (58) well underneath the nozzle lip, with a peak at the inner and outer rim locations.

Finally, the area-averaged shear stress is given by

$$\langle \tilde{\tau}_w^* \rangle \cong \frac{2a_s(1-\tilde{r}_i'^3)}{3(1-\tilde{r}_i'^2)} - a_s + 1 \quad (63)$$

The effect of Re_d and \tilde{r}_i' on $\langle \tilde{\tau}_w^* \rangle$ summarised in Fig. 14 shows similar behaviour to the results obtained for linear $d\tilde{p}^*/d\tilde{r}'$.

4.4. Composite nozzles

The analytical expressions developed here can be used to investigate potential combinations of features. For example, the nozzle shape could be specified to exploit two aspects of FDG action, one of which is sensitive to nozzle shape near the inner rim and a second which is sensitive to shape near the outer rim. By way of example, Fig. 15 shows a nozzle geometry with an outer zone of constant \tilde{h} and an inner zone with constant $\tilde{\tau}_w^*$. The figure also shows good agreement between the dimensional shear stress calculated for a particular Re_d and the composite, analytical model. This nozzle affords better sensitivity to clearance (data not shown), important in locating the surface in thickness measurements, and a reasonably uniform shear stress. The scope for designing nozzles for particular applications is therefore demonstrated. In these combinations, it is important to note that Eqs. (34) and (35) must hold for the individual simpler systems.

5. Conclusions

An analysis of the flow pattern under the gauging nozzle has been developed using the lubrication approximation to obtain

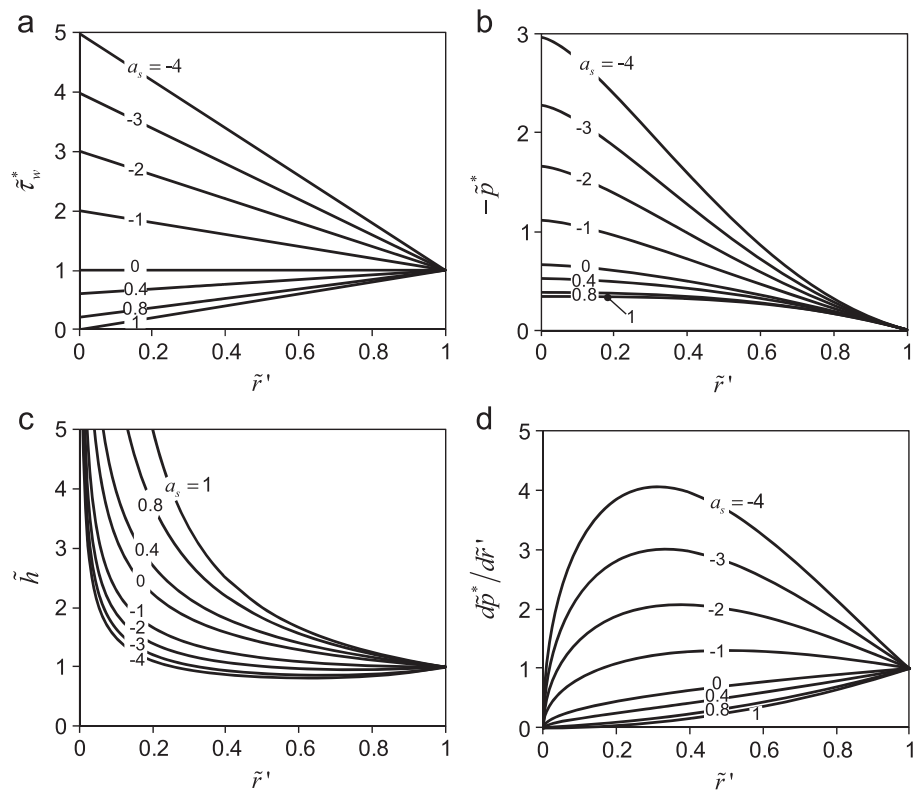


Fig. 12. Dimensionless profiles of (a) shear stress ($\tilde{\tau}_w^*$), (b) pressure ($-\tilde{p}^*$), (c) nozzle external geometry (\tilde{h}) and (d) pressure gradient ($d\tilde{p}^*/d\tilde{r}'$) as a function of the dimensionless radial position (\tilde{r}') for different values of a_s , for the case of linear shear stress profile.

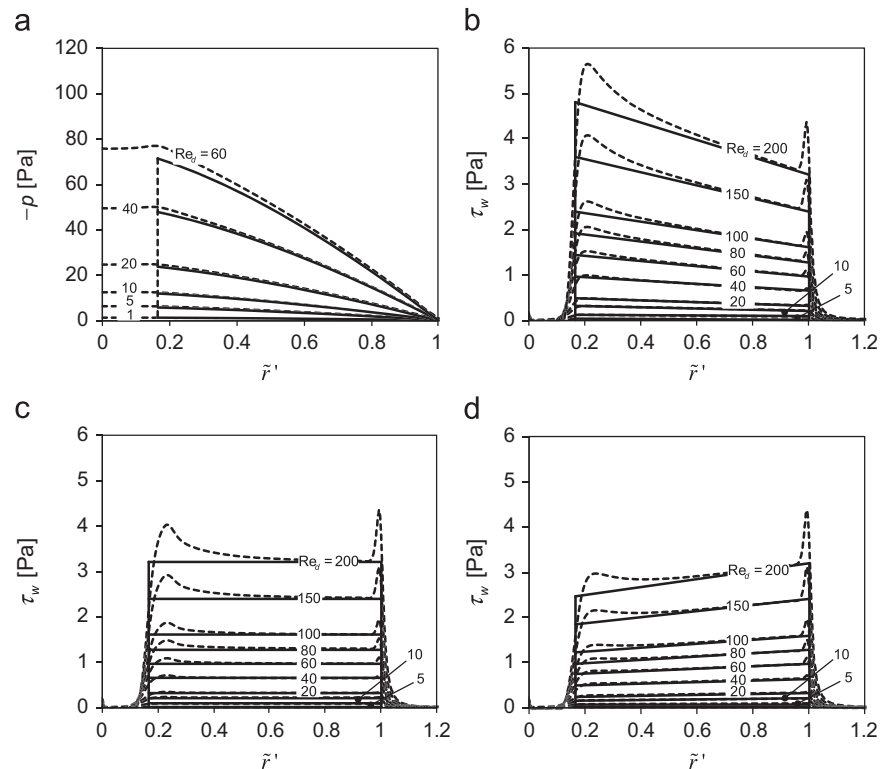


Fig. 13. Comparison between CFD simulation (dashed loci) and analytical predictions (solid loci) of (a) pressure with $a_s = 0$ (Eq. (62)) and shear stress values (Eq. (58)), for (b) $a_s = -0.7$, (c), $a_s = 0$ and (d) $a_s = 0.25$, as a function of \tilde{r}' for different Re_d . Conditions: $h_o = 0.25$ mm, $r_i = 2.5$ mm and $r_o = 15$ mm.

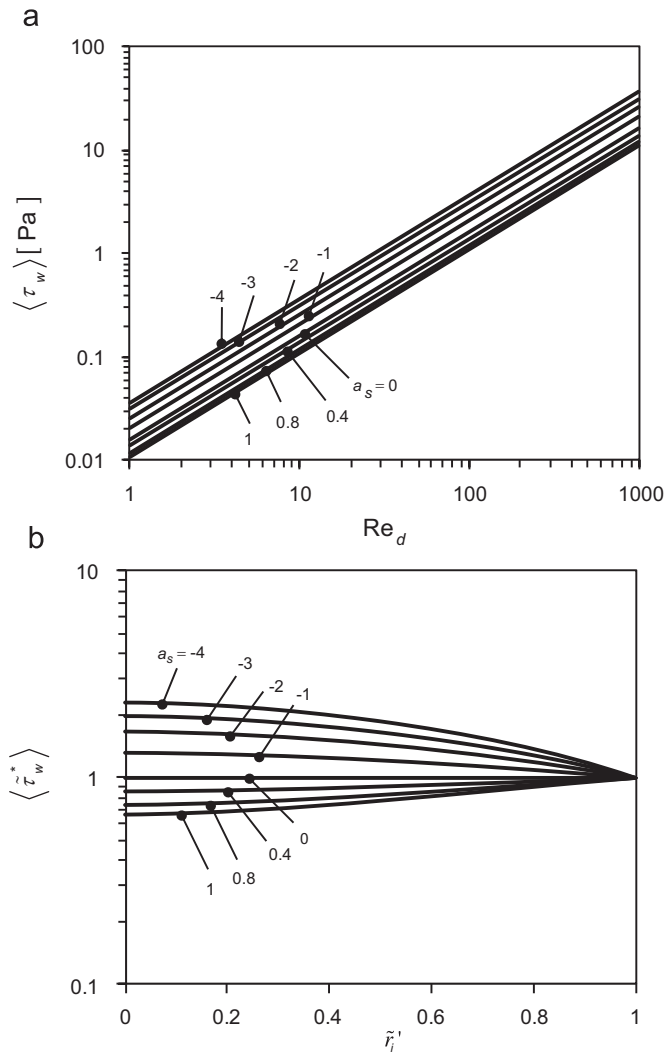


Fig. 14. (a) Effect of Re_d on area-averaged shear stress $\langle \tau_w \rangle$ and (b) dimensionless area-averaged shear stress $\langle \tilde{\tau}_w^* \rangle$ as a function of \tilde{r}_i' , for different values of a_s for the linear shear stress profile case.

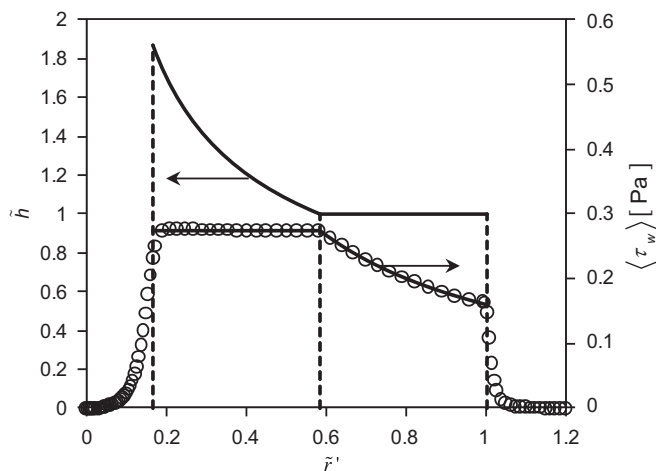


Fig. 15. Shear stress values as a function of \tilde{r}' for the example of a composite nozzle with constant \tilde{h} at the outer part of the nozzle and constant $\tilde{\tau}_w^*$ at the inner part ($Re_d=10$, $h_o=0.25$ mm). Loci show the analytical model predictions and circles the values obtained from CFD simulation.

expressions for the main flow variables in an FDG experiment. A set of general equations for the external nozzle geometry, pressure gradient under the nozzle and shear stress on the gauged surface was obtained. Case studies were presented considering the simple or common scenarios (i.e. linear profiles of all variables studied). Computational fluid dynamics simulations were used to validate the model using representative cases and showed good agreement. The range of applicability of the expressions lay in the range estimated by the model.

These results not only provide a tool to investigate the effect of the external nozzle geometry on parameters affecting the surface being studied by the FDG technique, in *design*, but also allow the effect of changes on operating variables on the performance of an existing FDG nozzle to be evaluated, i.e. assessing *operability*. The tools allow initial configurations of nozzle shape for specified operating conditions to be identified, for optimisation by CFD simulations and eventually *in vivo* by experiments.

Further developments could include extension of the analysis for systems with higher Reynolds numbers (i.e. significant inertial effects), non-Newtonian fluids, and porous surfaces (e.g. membranes).

Nomenclature

a_i	parameter of Eqs. (37), (52) and (58) ($i=h, pg, s$)
d	diameter of the tube (m)
d_t	nozzle throat diameter (m)
f_i	radial profile function for specified variable ($i=h, pg, s$)
\tilde{f}	dimensionless radial profile function defined in Eq. (36)
${}_2F_1[a,b;c;x]$	Gauss hypergeometric function
g_z	standard gravity in z direction (-9.81) ($m\ s^{-2}$)
h	clearance between the nozzle and the gauged surface (m)
h_{init}	position of the nozzle relative to the substrate (m)
\tilde{h}	dimensionless clearance (h/h_o)
\tilde{h}_m	minimum dimensionless clearance
\dot{m}	mass flow rate ($kg\ s^{-1}$)
p	pressure (Pa)
\tilde{p}	dimensionless pressure defined by Eq. (8)
\tilde{p}^*	dimensionless pressure defined in Eq. (29)
Q	flow rate ($m^3\ s^{-1}$)
\tilde{Q}	dimensionless flow rate ($Q/[2\pi(r_o-r_i)h_oU_o]$)
r	radial position (m)
\tilde{r}	dimensionless radial position defined by Eq. (4)
\tilde{r}'	dimensionless radial position defined in Eq. (27)
R	radius of the tube (m)
Re	Reynolds number ($\rho U_o h_o / \mu$)
Re_d	Reynolds number based on d
U_o	mean fluid velocity at the external radius calculated from the flow rate ($m\ s^{-1}$)
v_r	velocity component in r direction ($m\ s^{-1}$)
v_z	velocity component in z direction ($m\ s^{-1}$)
\tilde{v}_r	dimensionless velocity component in r direction defined by Eq. (6)
\tilde{v}_z	dimensionless velocity component in z direction defined by Eq. (7)
$v_{z,ave}$	average velocity at the exit of the tube ($m\ s^{-1}$)
V	characteristic velocity in z direction ($m\ s^{-1}$)
w	length of the nozzle rim (m)
z	axial position (m)
\tilde{z}	dimensionless axial position defined by Eq. (5)

Greek Letters

α	angle of the internal divergent zone of the nozzle (deg)
δ	thickness of the deposit (m)

ε	dimensional relation ($h_o/(r_o-r_i)$)
φ	azimuthal co-ordinate (dimensionless)
θ	angle of the external surface of the nozzle (dimensionless)
θ_{sp}	angle of the external surface of the nozzle that gives a saddle shear stress profile (dimensionless)
λ	length of nozzle exit (m)
μ	dynamic viscosity (Pa s)
Π	characteristic pressure, defined as $\Pi = \mu U_o/(h\varepsilon_o)$ (Pa)
ρ	density (kg m^{-3})
τ	shear stress (Pa)
$\tilde{\tau}$	dimensionless shear stress ($\tau/[\mu(U_o/h_o)]$)
$\tilde{\tau}^*$	dimensionless shear stress defined in Eq. (28)

Subscripts

o	at the outer extreme of the nozzle
i	at the inner extreme of the nozzle
w	at the gauged surface
h	clearance
pg	pressure gradient
s	shear stress

Special symbol

$\langle \cdot \rangle$	area-averaged quantity
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