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Jesús Martín Romero<sup>2,a</sup> and Mauricio Bellini<sup>1,2,b</sup>

<sup>1</sup> Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad Nacional de Mar del Plata, Funes 3350, (7600) Mar del Plata, Argentina

<sup>2</sup> Instituto de Investigaciones Físicas de Mar del Plata (IFIMAR), Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Mar del Plata, Argentina

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**Abstract.** We study a traversable wormhole originated by a transformation over the 4D Dymnikova metric, which describes analytic black holes (BH). By using a transformation of coordinates which is adapted from the one used in the Einstein-Rosen bridge, we study a specific family of geodesics in which a test particle with non-zero electric charge induces an effective magnetic monopole, that is perceived by observers outside the wormhole. Because the Riemannian geometry cannot explain the presence of magnetic monopoles, we propose a torsional geometry in order to explore the possibility that magnetic monopoles can be geometrically induced. We obtain an expression that relates torsion and magnetic fields jointly with a Dirac-like expression for magnetic and electric charges, such that torsion makes it possible to define a fundamental length that provides a magnetic field and a spacetime discretization.

## 1 Introduction and motivation

Up to now, General Relativity (GR) has passed every experimental tests. The more recent was the prediction of the waveforms of gravitational waves coming from the merger of two black holes (BH) [1], and two neutron stars [2], which were detected in 2015 by the LIGO-VIRGO Collaboration. Since the first detection, many other BH-BH mergers have been detected, in agreement with expectations of GR [3–6]. The problem, today not resolved is that at very little scales the theory is plagued by singularities [7–9], that indicate the breakdown of its predictivity at very small distances. It is manifested at the center of black holes. The more known case is the Schwarzschild metric, but it is expected that quantum gravity can solve this important problem.

In order to solve this problem, there were many attempts. The first author to propose a non-singular black-hole solution was Bardeen [10]. Later, some authors developed other proposals [11–20]. Their scheme was generalized by Dymnikova [21] for spherical and static solutions, and in the frame of nonlinear electrodynamics coupled to gravity [22]. Further properties can be found in [23]. Wormholes were proposed by Flamm [24] in 1916, after the introduction of GR. Weyl [25] also speculated about the wormhole existence in 1928, but the first serious calculations were done by Einstein and Rosen [26] in 1935, in order to obtain an atomistic theory of matter and electricity. This was named the Einstein-Rosen bridge. The term wormhole was introduced by Wheeler [27] some years later. The Einstein-Rosen bridge was constructed to connect two Schwarzschild black holes, but the presence of a singularity in the bridge implies that this is not-traversable. Recent progress has shown that traversable wormholes may exist, being real physical objects [28–32]. Such advances open a very interesting research area which deserves to be explored. Traversable wormholes produce violations of the null energy condition, which is only possible quantum mechanically. Then, one may expect that such wormholes must be of quantum size or must be surrounded by exotic matter. An alternative to that picture is to consider the presence of torsion as a geometrical entity which “can provide how exotic the wormhole is” [33–35]. In this work, we develop a wormhole without singularity and we provide the link to a first approximation to quantization of magnetic and electric charges, once a torsional geometry associated to the wormhole is assumed in the sense of Weitzenböck geometry [36]. Furthermore, we study the Dymnikova proposal for a black hole in order to study the wormhole associated to this metric and the possibility to geometrically induce a magnetic charge using the torsional structure coefficients.

<sup>a</sup> e-mail: [jesusromero@conicet.gov.ar](mailto:jesusromero@conicet.gov.ar)

<sup>b</sup> e-mail: [mbellini@mdp.edu.ar](mailto:mbellini@mdp.edu.ar)

This paper is organized as follows: In sect. 2 we present some basic elements about the Dymnikova spacetime, which describes an analytic black hole. We include some basic considerations about what is a regular black hole, and we mention the meaning, units and value of the parameters which are present in Dymnikova metric. In sect. 3 we develop a transformation which is based on the Einstein-Rosen bridge, but adapted to the Dymnikova black hole. We study the transformed metric and obtain an induced analytic wormhole with finite throat. Furthermore, we introduce a special family of geodesics that represent orbits along the throat of the wormhole, which are of special interest. In sect. 4 we study the magnetic field originated by an electric charge that orbits the wormhole's throat and is observed by an exterior observer. The magnetic field agrees qualitatively with that of a magnetic monopole with an effective magnetic charge, that is related to electric charge by a Dirac-like discretization. In sect. 5 we study a torsional geometry which is compatible with the induced magnetic monopole. Because the Riemann geometry cannot explain the presence of magnetic monopoles without a singularity, then we introduce a geometry capable to explain the observed monopole, for which, after using a Weitzenböck geometry, we obtain a new connection which is defined in terms of vierbein. These vierbein are related to the trajectory of the test charge, and the torsion related to the Weitzenböck connections enables us to discretize magnetic and electric charges. Finally, in sect. 6 we present final considerations and remarks.

## 2 Some elements about Dymnikova metric

We started from the Dymnikova metric [13], which describes an analytic black hole with a line element given by

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 (d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

with

$$f(r) = 1 - \frac{ab}{r} \left( 1 - e^{-\frac{r^3}{a^3}} \right), \quad (2)$$

in which  $a$  and  $b$  are parameters. The parameter  $a$  has dimensions of length and  $b$  is dimensionless. This line element is a de Sitter-like spacetime near the center and resembles the Schwarzschild metric with total mass  $m = \frac{ab}{2}$  at large distances. Null radial curves in which  $ds^2 = 0$ ,  $d\theta = 0$  and  $d\phi = 0$  defines what is called a *coordinate speed of light* [37], according to

$$c(r) = \left| \frac{dr}{dt} \right| = |f(r)|. \quad (3)$$

An effective refractive index is defined by  $\eta(r) = \frac{1}{|f(r)|}$ . Such index remains finite when  $c(r) \neq 0$ . The Dymnikova metric guarantees that for  $b < b_{crit} = 1.456$ , the function  $f(r)$  does not have zeros [38, 39]. Therefore, it is easy to see from eq. (3), that  $c(r) \neq 0$  represents a regular black hole. We are going to impose condition  $b = 1 < b_{crit}$  for economy reasons, without loss of generality.

## 3 Dymnikova wormhole

The usual transformation to obtain the Einstein-Rosen bridge [26, 40] is defined by

$$u^2 = r - r_s, \quad (4)$$

where  $u$  is the new “radial” coordinate and  $r_s$  is the minimum radius in the new metric. Such transformation mathematically collapses the interior of the BH to a single point at the zero of the new coordinate. When we are dealing with a Schwarzschild BH, then  $r_s$  is the Schwarzschild radius, but the Dymnikova metric is well defined in all the domain  $r_s = 0 \leq r < \infty$ . Therefore the transformation given in eq. (4) is reduced to

$$u^2 = r. \quad (5)$$

We can define the throat area for a wormhole by

$$A(u) = 4\pi r^2(u), \quad (6)$$

which, once we use the transformation defined in eq. (5), results in the fact that the minimum throat area is  $A(0) = 0$ , breaking with the effective traversability of the obtained wormhole.

We must modify eq. (5) to be of the shape of eq. (4), in order to obtain a finite minimum throat area, for that we need to introduce a length  $l_0$ ,

$$u^2 = r - l_0, \tag{7}$$

which guarantees that  $A(u) = 4\pi(u^2 + l_0)^2$  and then the minimum throat area remains finite

$$A_{min} = A(0) = 4\pi l_0^2. \tag{8}$$

The wormhole metric obtained from the Dymnikova metric is described by the line element

$$ds^2 = f(u)dt^2 - \frac{1}{f(u)}du^2 - (u^2 + l_0)^2 (d\theta^2 + \sin^2 \theta d\phi^2), \tag{9}$$

where

$$f(u) = \left[ 1 - \frac{a}{u^2 + l_0} \left( 1 - e^{-\left(\frac{u^2+l_0}{a}\right)^3} \right) \right], \tag{10}$$

such that  $u = (-\infty, +\infty)$ . Here, one obtains  $u(r) = 0$  for  $r = l_0$ . The metric of (9) excludes a part of the manifold which is inside the ball of radius  $r = l_0$ . Therefore, we need  $l_0$  to be a small quantity. In order to propose a value for  $l_0$ , we must notice that for a big  $l_0$ , most of the manifold will remain out of our mathematical description, which is not desirable. Therefore, there are physical arguments to chose a small and finite  $l_0$ , in order to introduce a fundamental length in the theory (maybe Plankian length). We must notice that  $l_0$  plays an analogous role to the Schwarzschild radius in the Einstein-Rosen bridge, but there are not singularities in the Dymnikova metric. Of course,  $l_0$  must be of the order of Plank length, when we want describes quantum wormholes. The throat of traversable wormholes has been extensively studied by Visser and Hochberg in [41], where they analyze the energy conditions and qualitative properties. On the other hand, the parameter  $a$  is related to the mass of a Schwarzschild BH that could be very significant. Then, we assume  $l_0 \ll a$ . The geodesics are defined by the equations

$$\begin{aligned} \frac{\partial^2 t}{\partial \lambda^2} + \frac{12u^6 e^{-u^6} + e^{-u^6} - 1}{u(\sqrt{u} - 1 + e^{-u^6})} \frac{\partial t}{\partial \lambda} \frac{\partial u}{\partial \lambda} &= 0, \\ \frac{\partial^2 u}{\partial \lambda^2} + \frac{(\sqrt{u} - 1 + e^{-u^6})(12u^6 e^{-u^6} + e^{-u^6} - 1)}{16u^4} \left( \frac{\partial t}{\partial \lambda} \right)^2 \\ + \frac{12u^7 e^{-u^6} + 4u^{\frac{3}{2}} + 5ue^{-u^6} - 5u}{(\sqrt{u} - 1 + e^{-u^6})u^2} \left( \frac{\partial u}{\partial \lambda} \right)^2 - \frac{1}{2} (u - \sqrt{u} + \sqrt{u}e^{-u^6}) \left( \frac{\partial \theta}{\partial \lambda} \right)^2 \\ - \frac{1}{2} (u - \sqrt{u} + \sqrt{u}e^{-u^6}) \sin(\theta)^2 \left( \frac{\partial \phi}{\partial \lambda} \right)^2 &= 0, \\ \frac{\partial^2 \phi}{\partial \lambda^2} + \frac{4}{u} \frac{\partial u}{\partial \lambda} \frac{\partial \phi}{\partial \lambda} + \frac{2 \cos(\theta)}{\sin(\theta)} \frac{\partial \theta}{\partial \lambda} \frac{\partial \phi}{\partial \lambda} &= 0, \\ \frac{\partial^2 \theta}{\partial \lambda^2} + \frac{4}{u} \frac{\partial u}{\partial \lambda} \frac{\partial \theta}{\partial \lambda} - \sin(\theta) \cos(\theta) \left( \frac{\partial \phi}{\partial \lambda} \right)^2 &= 0, \end{aligned} \tag{11}$$

in which  $t \equiv t(\lambda)$ ,  $u \equiv u(\lambda)$ ,  $\theta \equiv \theta(\lambda)$ , and  $\phi \equiv \phi(\lambda)$ . The metric of (9) admits a special family of geodesics in which the affine parameter is  $\lambda = t$  and the coordinate  $u$  is constant  $u = 0$ . Under such assumptions the system of differential equations (11) is very simplified and admits at least some special solutions. Two of the equations become trivial and the other two are very simple. The pair of remaining equations describes the behaviour of  $\theta$  and  $\phi$ . These geodesics are described by curves situated over some kind of casket spire located at the throat of the wormhole according with

$$\begin{aligned} 0 &= \frac{\partial^2 \phi}{\partial \lambda^2} + 2 \left( \frac{\cos(\theta)}{\sin(\theta)} \right) \frac{\partial \theta}{\partial \lambda} \frac{\partial \phi}{\partial \lambda}, \\ 0 &= \frac{\partial^2 \theta}{\partial \lambda^2} - \sin(\theta) \cos(\theta) \frac{\partial^2 \phi}{\partial \lambda^2}. \end{aligned} \tag{12}$$

A particular solution with constant angular velocity is admitted:  $\theta, \phi \sim \lambda$ , by taking into account that such coordinates are compact and cyclical (we use natural units in the present paper).

### 4 Effective magnetic monopole

Magnetic monopoles are the magnetic charges which originate divergent magnetic fields, and therefore play an analogous role to that of the electric charge for the electric field. A magnetic monopole must be the center of a magnetic

radial field which is decaying with the square of the distance and proportional to the magnetic charge magnitude. The presence of magnetic monopoles preserves the symmetry of the Maxwell equations by duality transformations between electric and magnetic fields. The first modern mention of a magnetic monopole was made in 1931 by Dirac who demonstrated that the mere existence of magnetic charges grants that both electric and magnetic charges are quantized [42]. In 1968, Wu-Yang showed that when a central monopole is obtained from a defined potential field, it must obey some conditions under gauge transformations, which grants charge quantization [43]. In 1974 't Hooft proved that great unification theories imply the presence of massive magnetic monopoles [44]. Magnetic monopoles are very interesting elements that were studied and are still investigated. They are present in condensed-matter theories and in string theory. However, our approach is different to those, and has a geometrical nature.

The charged particle which is following geodesic of eq. (12), moves on a trajectory  $l$ , which is restricted to  $u = 0$  and enclose the interior of the ball with radius  $r = l_0$ , that is the part of the original manifold that was neglected by metric (9), in the throat of the wormhole. Accomplishing with

$$dl = \underbrace{l_u du}_{=0} + l_\theta d\theta + l_\phi d\phi, \quad (13)$$

the particle moves with velocity  $\mathbf{v} = v_\theta d\theta + v_\phi d\phi$ . We can take a look in a fixed moment over the surface given by  $u = 0, t = t_0$ . We consider a metric tensor associated to the line element (9). We can specialize on the surface described by  $u = 0, t = t_0$ , in order to obtain a two-dimensional metric. We shall refer  $\mathbf{g}_2$  to the metric tensor that describes this surface, and we shall denote its determinant with  $g_2$ . In this framework, the magnetic field generated by some unitary magnetic charge is

$$\mathbf{B}^{(u)} = |v| \oint \left[ \frac{v_\theta d\theta + v_\phi d\phi}{S^2(l, u)} \wedge dl \right] \sqrt{g_2} = \oint \left[ \frac{v_\theta l_\phi - v_\phi l_\theta}{S^2(l, u)} \right] \boldsymbol{\varepsilon}^{\theta\phi(u)}. \quad (14)$$

The 2-form  $d\theta \wedge d\phi$  can be understood as parallel to  $du$  at a fixed time. Then  $\boldsymbol{\varepsilon}^{\theta\phi}(u)du = \sqrt{g_2}d\theta \wedge d\phi|_l = \sqrt{g_2}(d\theta \wedge d\phi|_l)^{(u)}du$  will be the length form over the casket surface. Outside the wormhole, when  $|u| \gg l_0$ , we obtain that  $S^2(l, u)$  is given by

$$S^2(l, u) = |u|^2, \quad (15)$$

such that  $|u|$  is the distance from the source to the point in which we are measuring the field. Obviously the obtained magnetic field is a 1-form, and must point in the  $u$ -direction. Then, the components of the magnetic field due to the magnetic monopole, will be

$$\mathbf{B}^u = \frac{|v|}{|u|^2} \oint (v_\theta l_\phi - v_\phi l_\theta) \boldsymbol{\varepsilon}^{u\theta\phi}. \quad (16)$$

The term between parenthesis in (16) is the angular momentum of the field source particle with respect to the point in which the particle is earlier located. The spacetime discretization [45] is necessary as an approach to spacetime quantization. It could be made by linking torsion, which provides a geometric description, and using the Plank length. To discretize the spacetime, we must discretize the amount between parenthesis in (16). Supported by previous considerations we could assume the spacetime discretization. Therefore, the effective magnetic charge can be quantized (we shall return over spacetime discretization in sect. 5):

$$|\mathbf{B}| \sim \frac{n}{|u|^2} = \frac{q_m}{|u|^2}, \quad (17)$$

where  $\mathbf{n}^{(u)} = \oint (v_\theta l_\phi - v_\phi l_\theta) \boldsymbol{\varepsilon}^{\theta\phi(u)}$ ,  $\mathbf{n} = n\tilde{u}$ . For a source given by an electric charge  $q_e$ , we obtain that

$$q_m = nq_e, \quad (18)$$

which describes a Dirac-like discretization between a true electrical charge  $q_e$  and the effective magnetic charge  $q_m$ , which can be observed from outside the wormhole. Therefore, outside of the wormhole, an  $u$ -dial magnetic field which is decaying with  $\frac{1}{|u|^2}$  can be detected. Of course this field is not decaying as  $\frac{1}{r^2}$ , and then can be related to the magnetic monopole only in the transformed spacetime outside the wormhole. The Riemann geometry does not support magnetic monopoles when the system of coordinates is coordinated. However, we have showed that a magnetic monopole can be induced as an effective entity by a transformation which excludes a part of the original manifold, creating a topological defect. Such exclusion looks to be not physical but only a mathematical trick, because there is no singularity to hide. Then, there is not physical reasons to interpret  $l_0$  as a true event horizon. Therefore, the magnetic monopole will be accessible because is located on the throat of a traversable wormhole. However, in the proximity of the throat (see eq. (15)), an observer notes that we are dealing with an electric current spire. Notice that, in the present example, no magnetic monopoles are located outside the wormhole.

## 5 On torsional geometry and quantization

In this section, we shall provide an interpretation of the geometrical nature of the effective magnetic monopoles. As we shall see, they can be linked to a torsional geometry, which is the simplest one that can be given by a construction, and is similar to those of Weitzenböck geometry [36]. The Weitzenböck geometry is characterized by a connection which is defined by the vierbein, such that its covariant derivative is null and the torsion can be nonzero. The Weitzenböck geometry is a paradigmatic example of a geometry which is characterized by a connection defined in terms of the vierbein, which relate two distinct basis. In most of cases, one of the basis is chosen to be normalized but it can be non-coordinate, and therefore, the Weitzenböck geometry will describe torsion, which is really a consequence of the structure that appears under a transformation of coordinates. This fact motivates us to build a torsional geometry over the casket  $u = 0$ . We are going to interpret (17) as an integral of a specific torsion, in which the elements  $l_\theta, l_\phi, v_\theta, v_\phi$  can be associated to a vierbein and their derivatives.

### 5.1 Vierbein, 2-dimensional part of the transformation

The 2-dimensional part of the transformation and the vierbein interpretation can be obtained from the way in which the test particle is moving over the geodesics restricted to the casket. We must start by defining a convenient basis:  $\{\underline{E}_\alpha\} = \{d\theta, d\phi\}$ , and a basis of the cotangent space (of the casket with  $u = 0$  in any given point). We can define a new basis of the cotangent space at the same point:  $\{\underline{e}^i\} = \{e^3, e^4\}$ , according to

$$e^3 = \underbrace{l_\theta}_{e_\theta^3} d\theta + \underbrace{l_\phi}_{e_\phi^3} d\phi, \tag{19}$$

$$e^4 = e_\theta^4 d\theta + e_\phi^4 d\phi, \tag{20}$$

in which we selected  $e^3 = dl$  as the direction pointing along curve  $l$ , and  $e^4$  is determined up a factor, by  $\mathbf{g}_2(e^3, e^4) = [\mathbf{g}_2]_{\theta\theta} e_\theta^3 e_\theta^4 + [\mathbf{g}_2]_{\phi\phi} e_\phi^3 e_\phi^4 = 0$ . Equations (19) and (20), can be expressed as

$$\underline{e}^i = e_\alpha^i \underline{E}^\alpha, \tag{21}$$

where objects  $e_\alpha^i$  are called vierbein [46]<sup>1</sup>. The analog relation to eq. (21) for the corresponding dual basis of the tangent space, is given by

$$\vec{e}_i = e_i^\alpha \vec{E}_\alpha, \tag{22}$$

where we must remember that duality is expressed by  $\underline{e}^i(\vec{e}_j) = \delta_j^i$ , and  $\underline{E}^\alpha(\vec{E}_\beta) = \delta_\beta^\alpha$ . Therefore, we can say that vierbein relating basis of casket's tangent space are determined by  $e_i^\alpha e_\alpha^j = \delta_i^j$ ,  $e_i^\alpha e_\beta^i = \delta_\alpha^\beta$ . The speed of the source particle is defined as a cotangent vector by

$$\mathbf{v} = \underbrace{\frac{dl_\theta}{dt}}_{v_\theta = \frac{\partial e_\theta^3}{\partial t}} d\theta + \underbrace{\frac{dl_\phi}{dt}}_{v_\phi = \frac{\partial e_\phi^3}{\partial t}} d\phi. \tag{23}$$

Of course,  $\mathbf{v}$  is a vector which is co-tangent to the 1-dimensional manifold given by curve  $l$ .

### 5.2 Vierbein, 4-dimensional extension

It is easy to extend previous construction to both, 4-dimensional tangent and cotangent spaces, which consider the casket as part (trivially embedded) of the 4-dimensional spacetime. Therefore, we obtain

$$\{\widetilde{\underline{e}}\} = \{dt, du, e^3, e^4\}, \tag{24}$$

$$\{\widetilde{\underline{E}}\} = \{dt, du, d\theta, d\phi\}. \tag{25}$$

<sup>1</sup> Latin indexes  $i$  and  $j$  are taking values  $i, j = 3, 4$  (the chosen notation is going to be clear soon) and Greek indexes  $\alpha$  and  $\beta$  are running over values  $\alpha, \beta = \theta, \phi$ .

We must consider a few more vierbein:  $e_t^1 = e_u^2 = 1$ . The set of vierbein can be written as  $\{e_\nu^n\}$  and  $\{e_m^\mu\}$ . The bases and vierbein are related by

$$\begin{aligned} \widetilde{e}_\nu^n &= e_\nu^n \widetilde{E}_\nu^\nu, \\ \widetilde{e}_{\nu n} &= e_n^\nu \widetilde{E}_\nu^\nu, \\ \delta_m^n &= e_\mu^n e_m^\mu, \\ \delta_\nu^\mu &= e_\nu^m e_m^\mu. \end{aligned} \tag{26}$$

### 5.3 Connections

The Weitzenböck connections can be expressed as

$${}^{(W)}\Gamma_{\mu\nu}^\rho = e_n^\rho \frac{\partial e_\mu^n}{\partial x^\nu}, \tag{27}$$

that are related with a Weitzenböck torsion

$${}^{(W)}T_{\nu\mu}^\rho = e_n^\rho \frac{\partial e_\mu^n}{\partial x^\nu} - e_n^\rho \frac{\partial e_\nu^n}{\partial x^\mu}. \tag{28}$$

The field of (16) cannot be explained in terms of such torsion, but keeps a great resemblance. This is the reason by which we choose a vierbein-based connection definition. In order to provide a geometric interpretation to eq. (16), is necessary to propose a more simple (but still torsional) geometry, in which connections are still given by the vierbein, but according to

$$\Gamma_{bc}^a = T_{bc}^a + K_{bc}^a. \tag{29}$$

Here,  $K_{bc}^a = \frac{\partial e_a^3}{\partial t} e_b^3 h^a$  are the contortion tensor components, with  $h^a|_{a=u} = 1$  and  $h^a|_{a=t,\theta,\phi} = 0$ . The relevant components are

$$\begin{aligned} K_{\phi\theta}^u &= \frac{\partial e_\phi^3}{\partial t} e_\theta^3, \\ K_{\theta\phi}^u &= \frac{\partial e_\theta^3}{\partial t} e_\phi^3. \end{aligned} \tag{30}$$

Notice that the connections that differs from the Riemannian ones, are  $\Gamma_{\theta\phi}^u = \{\theta_\phi^u\} + K_{\theta\phi}^u$ , and  $\Gamma_{\phi\theta}^u = T_{\phi\theta}^u + K_{\phi\theta}^u$ . All other connections match with the Riemannian ones. Therefore, the geometry is torsional, with

$$T_{\phi\theta}^u = \frac{\partial e_\theta^3}{\partial t} e_\phi^3 - \frac{\partial e_\phi^3}{\partial t} e_\theta^3 = -T_{\theta\phi}^u, \tag{31}$$

which are the only non-vanishing torsion terms. The torsion is present, but geodesics in eq. (12) remains unchanged. From eq. (16), after some algebraic manipulation, we obtain

$$\begin{aligned} \mathbf{B}^u &= \frac{|v|}{|u|^2} \oint (v_\theta l_\phi - v_\phi l_\theta) \boldsymbol{\varepsilon}^{u\theta\phi} \\ &= \frac{|v|}{|u|^2} \oint T_{\phi\theta}^u \boldsymbol{\varepsilon}^{\theta\phi}, \end{aligned} \tag{32}$$

which is the only nonzero component. The last integral could be understood as the Burgers vector [47]. In material science, when mapping an ideal crystal into a crystal containing a fracture, Burgers vector represents the magnitude and direction of a lattice dislocation. Burgers vector can be interpreted as originated in torsional geometry and defined by

$$\oint T_{\phi\theta}^u \boldsymbol{\varepsilon}^{\theta\phi} = b^u. \tag{33}$$

de Sabbata and Kumar Datta [45] have studied the physical meaning of such defect term, by including the interpretation of torsion as a magnetic spin, which is of course quantized. Therefore, it will be reasonable to suppose that the components of the Burgers vector could be an integer number of Planck lengths, that is  $b_m = n_m l_p$ , in which  $n_m$  is an integer number (we use natural units). The former expression can be adapted to the context of the present paper, in which the magnetic field is an entity related to the torsion. Therefore, we obtain  $|b| = |b_u| = n q_e$ , in which  $n$  is zero or a positive integer. This argument is supporting discretization that we have used in sect. 4, in order to obtain eq. (17) and the quantization in eq. (18).

## 6 Final remarks

We have introduced and studied the case of a traversable wormhole obtained from Dymnikova spacetime, in which there is a charged test particle in a specific kind of geodesics located in the throat of the wormhole. The Riemannian geometry is not sufficient to explain the presence of monopoles in a spacetime which is free of singularities. Therefore, by using a Weintzenböck geometric representation, we have obtained that it induces an effective interior magnetic monopole for any observer outside the wormhole, so that both charges meet Dirac's quantization condition.

In sect. 5.3 we have mentioned that the Burgers vectors can be associated with Planck length, in order to give a geometrical nature to the constant. This is not convenient in the present work, in which the obtained torsion is related to the magnetic field. However, it can be fixed by the choice  $l_0 = n_l l_p$ , that could be an optional manner to introduce the Planck length to the theory as a magnitude originated by the geometry. In this manner the Planck length would be introduced from a transformation described in eq. (7), and by restricting the wormhole to the quantum domain.

We have studied an effective magnetic monopole which is obtained as the manifestation of an electric charge in the throat of the traversable wormhole. An inverse picture is obtained from the same scenario, with the only difference that we supposed a magnetic monopole test particle, and therefore from the outside of the wormhole it is viewed as an electric charge. The last assertion can be easily obtained from sect. 3, by exchanging the electric with the magnetic charge. This is consistent with the results obtained by us in a previous work in which we described a wormhole linking two effective 4D induced black holes. We induced it from 5D spacetime by doing a specific set of foliations obtaining that a not accessible magnetic (electric) charge cited in the interior of the wormhole can be intended as an effective electric (magnetic) charge outside the induced 4D black hole [40].

The Riemannian geometry (that we have employed in this work), does not support the magnetic monopoles existence without a topological defect, due to the fact  $d(dA) = 0$ . However, in our present case we have obtained an effective magnetic monopole with one unreal singularity, because the Dymnikova metric is analytic. That can be explained because the used transformation removes a part of the original manifold in a such manner that it is restricted to the throat of the wormhole, that includes an effective torsional defect at the origin, for the observer located outside the wormhole.

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