Higher derivative terms in the $\pi\Delta N$ interaction: Some phenomenological consequences

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In this paper, we implement the use of a $\pi N\Delta(1232)$ vertex interaction containing both first- and second-order derivative terms, as required by renormalization and power-counting considerations. As was previously shown, both interactions present quantization shortcomings but can be used in a pertubative calculation. Our results indicate that the usual π derivative plus the spin-3/2 gauge invariant (derivative also in the Δ field) should be included in amplitude calculations, as also all higher derivative interactions respecting chiral invariance. We show that both interactions make essentially the same resonant contribution to the elastic π^+p cross section, so changing the ratio between both coupling constants amounts to a correction of the background. The elastic π^+p cross section up to 300 MeV changes only mildly when that ratio is changed, but the total π^-p scattering, which has poor fit within both interactions separately, can be much improved in the same energy range by tuning the ratio between both coupling constants.

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I. INTRODUCTION

In order to treat the Δ (1232 MeV) within an effective Lagrangian approach, it is rendered as the quantum of a Rarita-Schwinger (RS) field and generic effective interactions are proposed that respect relevant symmetries: Lorentz, electromagnetic gauge, and, in the case of strong interactions, chiral symmetries.

The simplest interaction term for describing pion-nucleon-Δ interactions is one derivative only in the pion field (named from now on I_1), which was proposed in Refs. [1,2] several decades ago and used intensely although that interaction was shown to lead to a nondefinite Fock space in the presence of a nonuniform background pion field [3,4] and the RS was already known to present signature problems in background fields even for nonminimal electromagnetic coupling [5]. Nevertheless, perturbative series have perfect sense in amplitude calculations. This interaction is known as conventional or inconsistent coupling for the mentioned shortcomings.

In an effort to overcome these difficulties, at the turn of the century, a new interaction was proposed [6–8] which is derivative both in the Δ and π fields (named from now on I_2) and known as 3/2 gauge invariant (with the same structure as the free Lagrangian with Δ mass $m = 0$, see below) or consistent coupling. The formal advantage of I_2 over I_1 is that preserves it the degree of freedom counting when $m \to 0$ independently of the interaction term, as can be seen from the constraint analysis [9]. Also, in πN elastic scattering at tree level, it decouples the spin-1/2 background contributing off shell in the Δ propagator. Nevertheless, in radiative scattering, the spin-1/2 background no longer decouples and one-loop radiative corrections force us to introduce also the interaction I_1 to act as counterterm to avoid divergences [10]. It was believed that this interaction leads to positive definite norm states, but as we showed recently [9] this is not the case: When background fields are present, some physical negative norm states arise.

From another point of view, these two interaction terms could be worked under the scheme of chiral perturbation theory. This is an expansion tightly around the resonance peak, and when doing so both interactions can be considered of the same order. Indeed, each pion momentum is assigned a contribution to the power counting in the expansion parameter $\delta \sim (m - m_N)/\Lambda_{\chi PT}$ ($\Lambda_{\chi PT} = 1$ GeV) or $\delta^2 \sim m_\pi/\Lambda_{\chi PT}$, depending on its value, and both interactions are of the same order since momentum coming from $\partial_\mu \Psi_\nu$ behaves as order 1 at the threshold [10].

So, in a perturbative calculation, one should consider both I_1 and I_2 (the Lagrangians $\mathcal{L}_{I_k}$ depending on a coupling constant g_k together with the kinematical one $\mathcal{L}_{\text{free}}$, will be defined in the next section). It is true that we have shown that separately for the elastic π^+p scattering channel, I_1 fits the data better than I_2 in the region of the resonance [11], and contrary to what is claimed in Ref. [12], we have shown that it is not possible to get fits of the same quality as the ones obtained using I_2 by changing the σ meson parameters. (Recall that ρ parameters are fixed from the other low-energy processes through vector dominance). Nevertheless, as we will see below, it is not possible to reproduce the $\pi^- p$ channel with either I_1 or I_2 alone.

The fact that one can fit properly with an isobar model one determined channel reaction but not others is not unique to πN scattering. We find the same problem with the hadronic current $W^+N \to \pi N'$ in weak pion production, where the $W^+p \to \pi^+ p$ hadronic channel can be reproduced properly but the $W^+n \to \pi^+ n$ one cannot, similar to the $\pi^+ p$ and $\pi^- p$ problem in elastic scattering, where both $W^+n \to \pi^+ n$ and $\pi^- p \to \pi^- p$ reactions have the same isospin coefficients for the pole and cross-Δ amplitudes at least up to a sign. To get an improvement, other authors have invoked the equivalence $\mathcal{L}_{\rm free} + \mathcal{L}_{I_1} \to \mathcal{L}_{\rm free} + \mathcal{L}_{I_2}(g_2 = -\frac{g_1}{m}) + \mathcal{L}_C(\frac{g_1^2}{m^2})$, with $\mathcal{L}_C$ describing contact terms without the Δ field, through the transformation $\Psi_\nu \to \Psi_\nu - g_1/m(\partial_\nu \Phi)\Psi$ (Ψ_ν, Φ, Ψ are Δ, pion, and nucleon fields isospin omitted, respectively).[1] In Ref. [13], the replacement $\mathcal{L}_{I_1} + C\mathcal{L}_C \to \mathcal{L}_{I_2} + (1+C)\mathcal{L}_C$ is proposed, with adjusting the low-energy constant C to get a better fitting. This procedure has two limitations and/or arbitrariness. First, the coupling of I_2 results in $g_2 = -\frac{g_1}{m}$ fixed by the transformation, and second, the coupling constant of the contact term is $(1+C)\frac{g_1^2}{m^2}$ and not $\frac{g_1^2}{m^2}$, as should come from the field transformation. The addition of $C\mathcal{L}_C$ is based on the argumentation that within the ChPT framework, the $\mathcal{L}_C$ appearing in the $\mathcal{L}_{I_1}, \mathcal{L}_{I_2}$ equivalence can be added with an arbitrary (to be adjusted) coefficient C [12].

We will approach the problem in another way, since ChBPT expansion makes sense only around the resonance peak and it is not the only criterion to assign an order to an interaction term. We could consider I_2 to be of higher order in consideration of the dimension of the coupling constant and the number of derivatives in the term, in line with Ref. [14]. In this sense, the interaction I_2 could be easily seen as the next order in derivatives from the conventional coupling [10], and the addition of each term I_k implies the fixing of the corresponding coupling constant g_k. This criterion makes the idea of considering both interactions together reasonable, and we explore here some phenomenological consequences of doing so. We will calculate the total $\pi^- p$ and elastic (for which background contributions are far more relevant than in the $\pi^+ p$ due to isospin coefficients) scattering cross sections. We will show that each interaction term fits the data poorly, while a judicious mixture of both interactions leads to a better description of this channel while keeping a good description of the $\pi^+ p$ one, without *ad hoc* manipulations of the background. Thus, from our point of view, the procedure followed in Ref. [13] appears naturally in our scenario of $\mathcal{L}_{I_1} + \mathcal{L}_{I_2}$ where $\mathcal{L}_{I_1} + \mathcal{L}_{I_2} \to \mathcal{L}_{I_1}(g_1, g_2) + \mathcal{L}_C(g_1, g_2)$ by $\Psi_\nu \to \Psi_\nu - g_2 \Psi \partial \Phi$.

II. LAGRANGIANS

The free RS field is described by the Lagrangian

$$\mathcal{L}_{\rm free} = \overline{\psi}_\mu(x)\mathcal{K}(\partial, A)^{\mu\nu}\Psi_\nu(x) \tag{1}$$

where

$$\mathcal{K}(\partial, A)^{\mu\nu} = R\left(-\frac{1}{2}(1+A)\right)^{\mu\mu'}\mathcal{K}(\partial, -1)_{\mu'\nu'} \times R\left(-\frac{1}{2}(1+A)\right)^{\nu'\nu} \tag{2}$$

and

$$\mathcal{K}(\partial, -1)_{\mu'\nu'} = \epsilon_{\mu'\nu'\alpha\beta}\partial^\alpha\gamma^\beta\gamma_5 + im\sigma_{\mu'\nu'}, \tag{3}$$

where $\sigma_{\mu,\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] = i\gamma_{\mu\nu}$, $\epsilon_{0123} = 1$, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and $R_{\mu\nu}(a) = g_{\mu\nu} + a\gamma_\mu\gamma_\nu$.[2] Note that the matrices $R(-\frac{1}{2}(1+A))$ appear because by construction the field Ψ_μ has a spurious spin-1/2 component and as a consequence the Lagrangians $\mathcal{L}_{\rm free}$ are connected by the contact transformation $\Psi^\mu \to R^{\mu\nu}\Psi_\nu$, $A \to \frac{A-2a}{1+4a}(A \neq -\frac{1}{2})$, which change the proportion of the 1/2 states while leaving the equations of motion invariant.

Using the properties of R matrices in $\mathcal{L}_{\rm free}$ at Eq. (2), we can write the general propagator in terms of the propagator for $A = -1$ (which renders the calculations simpler) as

$$G(p, A)^{\mu\nu} = R^{-1}\left(-\frac{1}{2}(1+A)\right)^\mu_{\ \alpha} G(p, -1)^{\alpha\beta} \times R^{-1}\left(-\frac{1}{2}(1+A)\right)^\nu_{\ \beta}, \tag{4}$$

where $G(p, -1)_{\mu\nu}$ can be put in terms of the well-known projectors $P^{3/2}$, $P^{1/2}_{11,22}$, $P^{1/2}_{21}$, and $P^{1/2}_{12}$ (see Appendix A) as

$$G(p, -1)_{\mu\nu} = -\left[\frac{\not{p} + m}{p^2 - m^2}P^{3/2}_{\mu\nu} - \frac{2}{3m^2}(\not{p} + m)\left(P^{1/2}_{22}\right)_{\mu\nu} + \frac{1}{\sqrt{3}m}\left(P^{1/2}_{12} + P^{1/2}_{21}\right)_{\mu\nu}\right]. \tag{5}$$

The derivative first-order (in k_π) interaction term involving the Δ, nucleon, and pion fields respecting chiral invariance and which dominates at small energies [1] can be written as ($\mathbf{T}$ are the $N \to \Delta$ isospin excitation operators)

$$\mathcal{L}_{I_1} = g_1\bar{\psi}\partial_\mu\phi^\dagger \cdot \mathbf{T}R\left(\frac{1}{2}(1+4Z_1)A + Z_1\right)^{\mu\nu}\Psi_\nu + \text{c.c.}, \tag{6}$$

and it is also invariant (at the level of the equations of motion and the amplitudes) under contact transformations, since it can be shown that $R(\frac{1}{2}(1+4Z_1)A + Z_1)^{\mu\nu} = R(-\frac{1}{2}(1+A))^{\mu\alpha}R(-\frac{1}{2}(1+2Z_1))^\nu_{\ \alpha}$. It is clear that the A dependence cancels in any physical amplitude and that the factor $R(-\frac{1}{2}(1+A))$ can be dropped, but dependence on Z_1 persists. Z_1 is thus a free parameter of the interaction [11]. As already stated, this interaction was shown to lead to negative probabilities [3]. However, this term can be used to get low-energy amplitudes since it is the most general first-derivative Lagrangian which respects covariance and chiral symmetry and admits a nonproblematic perturbative order-by-order approach for the amplitude [15]. Another concern about Δ interaction amplitudes, the existence of the so-called spin-1/2 background, has been proved baseless since lowest

[1] In the same way, we would make the transformation $\Psi_\nu \to \Psi_\nu - g_2\Psi\partial\Phi$ to eliminate I_2 and get I_1 if I_2 were the starting interaction and we would make $\mathcal{L}_{I_2} \to \mathcal{L}_{I_1} + C\mathcal{L}_C$.

[2] These are Bjorken and Drell conventions.

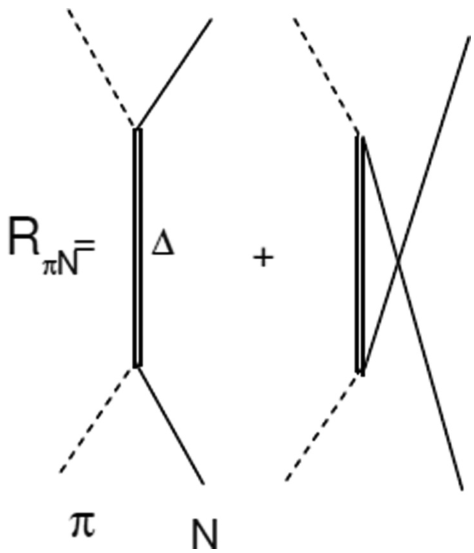

FIG. 1. Amplitude of pion (π)–nucleon (N) scattering split in s or pole (left) and u or cross (right) contributions for Δ resonance.

spin representation contributions are present in other cases [16].

The next interaction in number of derivatives and chiral invariant is

$$\mathcal{L}_{I_2} = -\, g_2\, \bar{\Psi}\partial_\mu\phi^\dagger \cdot \mathbf{T}\epsilon^{\mu\nu\rho\beta}\gamma_\beta\gamma_5 R\left(\frac{1}{2}(1+4Z_2)A + Z_2\right)_\nu^{\ \eta} \times \partial_\rho\Psi_\eta + \text{c.c.}, \tag{7}$$

and it is important to note that this interaction term is the most general second-order interaction containing a derivative on the pion field, which is necessary for chiral invariance, and a free parameter Z_2. Provided that all free parameters Z_k are set such that Lagrange multiplier fields of the free theory do not acquire dynamics due to the interaction, as explained in Appendix A and Ref. [16], is used to fix $Z_1 = 1/2$ for I_1 as in Ref. [1], and dropping $R(-\frac{1}{2}(1+A))$ or equivalently putting $A=-1$ everywhere gives

$$\mathcal{L}_{I_1} = g_1\bar{\psi}\partial_\mu\phi^\dagger \cdot \mathbf{T}R(-1)^{\mu\nu}\Psi_\nu + \text{c.c.}. \tag{8}$$

For I_2, it leads to $Z_2 = -1/2$, which for $A=-1$ corresponds to the interaction

$$\mathcal{L}_{I_2} = -g_2\partial_\mu\phi^\dagger \cdot \mathbf{T}\epsilon^{\mu\nu\rho\beta}\gamma_\beta\gamma_5\partial_\rho\Psi_\nu + \text{c.c.}, \tag{9}$$

originally proposed in Ref. [6]. This is called a spin-3/2 gauge-invariant interaction since it remains unchanged under the transformation $\Psi_\mu \to \Psi_\mu + \partial_\mu\chi$, where χ is an arbitrary spinor that leaves invariant $\mathcal{L}_{\text{free}}(m=0)$.

III. PION-NUCLEON AMPLITUDE

We first calculate the tree-level pion-nucleon amplitude involving the Δ from the Lagrangian in Eqs. (1) with $A=-1$, (8), and (9) as already adopted in Refs. [1] and [6]:

$$\begin{aligned}\mathcal{L}_\Delta =\ & \overline{\psi}_\mu\{\epsilon^{\mu\nu\alpha\beta}\partial_\alpha\gamma_\beta\gamma_5 + [im\sigma^{\mu\nu} \equiv mR(-1)^{\mu\nu}]\}\Psi_\nu \\ & + g_1\bar{\psi}\partial_\mu\phi^\dagger \cdot \mathbf{T}R(-1)^{\mu\nu}\Psi_\nu + g_1\bar{\Psi}_\mu R(-1)^{\mu\nu}\partial_\nu\phi \cdot \mathbf{T}^\dagger\psi \\ & - g_2\,\bar{\psi}\partial_\mu\phi^\dagger \cdot \mathbf{T}\epsilon^{\mu\nu\alpha\beta}\partial_\alpha\gamma_\beta\gamma_5\Psi_\nu + g_2\partial_\alpha\overline{\psi}_\mu\epsilon^{\mu\nu\alpha\beta}\gamma_\beta\gamma_5\partial_\nu\phi \cdot \mathbf{T}^\dagger\psi,\end{aligned} \tag{10}$$

and with the propagator in Eq. (5), we can calculate the resonant amplitude contribution($R_{\pi N}$) as shown in Fig. 1. By omitting the nucleon spinors and isospin factors and letting the incoming and outgoing pions momentum be noted as k and k' respectively and the Δ momentum as p, the s amplitude reads

$$\begin{aligned}R^s_{\pi N} =\ & g_1^2k'_\mu R^{\mu\alpha}G_{\alpha\beta}(p)R^{\beta\nu}k_\nu + g_2^2k'_\mu(-i)\Gamma^{\mu\alpha}(p)G_{\alpha\beta}(p)(-i)\Gamma^{\beta\nu}(p)k_\nu \\ & - g_1g_2k'_\mu R^{\mu\alpha}G_{\alpha\beta}(p)(-i)\Gamma^{\beta\nu}(p)k_\nu - g_1g_2k'_\mu(-i)\Gamma(p)^{\mu\alpha}G_{\alpha\beta}(p)R^{\beta\nu}k_\nu,\end{aligned} \tag{11}$$

where $\Gamma^{\mu\nu}(p) = \epsilon^{\mu\nu\alpha\beta}\gamma_\beta\gamma_5 p_\alpha$, $R \equiv R(-1)$, and where we have used $\partial_\mu\psi, \phi, \Psi_\nu \sim iq_\mu\psi, \phi, \Psi_\nu$. The u-channel contribution is obtained by simply replacing p by $p-k-k'$ and k by k' in the former expression.

Observe that the first two terms correspond to the first- and second-order (in derivatives) contributions to the Lagrangian, respectively, while the last two can be construed as interference terms between them. Let us analyze first the amplitude for I_1. If $g_2 = 0$, we get the s amplitude

$$R^{s,g_2=0}_{\pi N} = -(g_1)^2\frac{\not{p}+m}{p^2-m^2}P^{3/2}_{\mu\nu}k_\mu k'_\nu - \frac{(g_1)^2}{m^2}\left[2(\not{p}+m)P^{1/2}_{11\,\mu\nu} + m\sqrt{3}\left(P^{1/2}_{12}+P^{1/2}_{21}\right)_{\mu\nu}\right]k_\mu k'_\nu, \tag{12}$$

where the relations

$$\begin{aligned}R^{\mu\nu} &= P^{\mu\nu}_{3/2} - \sqrt{3}\left(P^{1/2}_{12}+P^{1/2}_{21}\right)^{\mu\nu} - 2\left(P^{1/2}_{11}\right)^{\mu\nu}, \\ (p^\mu\gamma^\nu - \gamma^\mu p^\nu) &= \sqrt{3}\ \not{p}\left(P^{1/2}_{12}+P^{1/2}_{21}\right)^{\mu\nu} = -\sqrt{3}\left(P^{1/2}_{12}+P^{1/2}_{21}\right)^{\mu\nu}\ \not{p}, \\ \left[P^{3/2}_{\mu\nu}, \not{p}\right] &= \gamma^\mu P^{3/2}_{\mu\nu} = P^{3/2}_{\mu\nu}\gamma^\nu = 0\end{aligned} \tag{13}$$

were used. The second term in Eq. (12) is the so-called spin-1/2 background, but what is relevant for the asymptotic behavior of the amplitude is that it represents a nonpole (without a pole) contribution which grows with p, since the projectors go as p^0

(see Appendix B). On the other hand, if $g_1 = 0$, the I_2 amplitude can be expressed (in the s-channel $p^2 \neq 0$) also as a pole and well-behaved term at $p^2 = m^2$ as

$$\begin{aligned} R_{\pi N}^{s,g_1=0} &= -(g_2)^2 m^2 \frac{\not p + m}{p^2 - m^2} P^{3/2}_{\mu\nu} k_\mu k'_\nu \\ &\quad -(g_2)^2(\not p + m) P^{3/2}_{\mu\nu} k_\mu k'_\nu, \end{aligned} \tag{14}$$

where we have used Eqs. (13) and that $-i\Gamma(p)^{\mu\nu} = -\not p R^{\mu\nu} - (p^\mu\gamma^\nu - \gamma^\mu p^\nu) = -R^{\mu\nu}\not p + (p^\mu\gamma^\nu - \gamma^\nu p^\nu)$. Observe that, except for the dimensions of the coupling constants, the pole terms for both amplitudes are identical in form. Additionally, there is also a term as poorly behaved asymptotically as the nonpole term in Eq. (12). This term should be treated together with the pole one in the u channel since p^2 could be zero in $P^{3/2}_{\mu\nu}$.

When we put both interactions together, the amplitude reads

$$R_{\pi N} = \left(\frac{g_1 + m g_2}{m}\right)^2 k'_\mu \left\{-p^2 \frac{\not p + m}{p^2 - m^2} P^{\mu\nu}_{3/2}\right\} k_\nu + \left[\left(\frac{g_1 + m g_2}{m}\right)^2 - g_2^2\right] k'_\mu \{(\not p + m)R^{\mu\nu} + (p^\mu\gamma^\nu - \gamma^\mu p^\nu)\} k_\nu - 2g_1 g_2 k'_\mu R^{\mu\nu} k_\nu \tag{15}$$

$$\begin{aligned} &= -(g_1 + m g_2)^2 k'_\mu \left\{\frac{\not p + m}{p^2 - m^2} P^{\mu\nu}_{3/2}\right\} k_\nu - \left(\frac{g_1 + m g_2}{m}\right)^2 k'_\mu \big\{2(\not p + m)\big(P^{1/2}_{11}\big)^{\mu\nu} + m\sqrt{3}\big(P^{1/2}_{12} + P^{1/2}_{21}\big)^{\mu\nu}\big\} k_\nu \\ &\quad + g_2^2 k'_\mu \big\{2(\not p + m)\big(P^{1/2}_{11}\big)^{\mu\nu} + m\sqrt{3}\big(P^{1/2}_{12} + P^{1/2}_{21}\big)^{\mu\nu}\big\} k_\nu - g_2^2 k'_\mu (\not p + m) P^{\mu\nu}_{3/2} k_\nu \\ &\quad - 2g_1 g_2 k'_\mu \big(P^{\mu\nu}_{3/2} - 2\big(P^{1/2}_{11}\big)^{\mu\nu} - \sqrt{3}\big(P^{1/2}_{12} + P^{1/2}_{21}\big)^{\mu\nu}\big) k_\nu. \\ &= -\left(\frac{g_1 + m g_2}{m}\right)^2 k'_\mu \left\{\frac{\not p + m}{p^2 - m^2} P^{\mu\nu}_{3/2}\right\} k_\nu - \frac{g_1^2}{m^2} k'_\mu \big\{2(\not p + m)\big(P^{1/2}_{11}\big)^{\mu\nu} + m\sqrt{3}\big(P^{1/2}_{12} + P^{1/2}_{21}\big)^{\mu\nu}\big\} k_\nu - g_2^2 k'_\mu (\not p + m) P^{\mu\nu}_{3/2} k_\nu \\ &\quad - \frac{2g_1 g_2}{m} k'_\mu \big(2\not p\big(P^{1/2}_{11}\big)^{\mu\nu} + m P^{\mu\nu}_{3/2}\big) k_\nu, \end{aligned} \tag{16}$$

where we have assumed that the projectors are defined for each p^2. That is the case for the s-channel contribution for which $p^2 > 0$ always, while for the u-channel one, since p^2 could be arbitrarily small, it is preferable to express the amplitude without separating pole from nonpole terms, as in Eq. (15). As can be seen in Eq. (16), the first term corresponds to the pole contribution both in (12) with $g_2 = 0$ and (14) with $g_1 = 0$, but now with a coupling constant $g = (g_1 + m g_2)$. The third and fourth terms are the corresponding backgrounds from Eqs. (12) and (14) respectively, with the last term being a background contribution coming from the interference of both vertices. Let us rewrite the couplings in order to separate pole and background amplitudes, and let us introduce the parameter κ so that $g_1 + m g_2 = g$ and $g_2 = \kappa g/m$. Thus, we get

$$g_1 = (1 - \kappa) g, \tag{17}$$

$$g_2 = \kappa \frac{g}{m}, \tag{18}$$

and consider g and κ as the phenomenologically relevant coupling constants. Observe that for $\kappa = 0$ we obtain the amplitude for pure leading first-derivative interaction, while in the limit $\kappa = 1$ we get pure second-derivative interaction. g gives the resonant contribution $g = (g_1 + m g_2)$, but κ amounts to a change in background contributions depending on the mixture of I_1 and I_2 interaction. We would thus expect that in a channel where background contributions are unimportant (as is the case with the elastic $\pi^+ p$ dispersion due to the isospin coefficients and values for $p^2 \sim m^2$) the fit to the data is relatively insensitive to changes in κ. Instead, in channels where background contributions are relevant, the value of κ might become critical. In terms of g and κ, the amplitude reads [from Eq. (15)]

$$\begin{aligned} R_{\pi N} = -g^2 k'_\mu \Bigg\{ &\frac{p^2}{m^2}\frac{\not p + m}{p^2 - m^2} P^{\mu\nu}_{3/2} - \frac{1}{m^2}[(\not p + m)R^{\mu\nu} + (p^\mu\gamma^\nu - \gamma^\mu p^\nu)] \\ &+ \kappa^2 \frac{1}{m^2}[(\not p + m)R^{\mu\nu} + (p^\mu\gamma^\nu - \gamma^\mu p^\nu)] + \frac{2(1-\kappa)\kappa}{m} R^{\mu\nu} \Bigg\} k_\nu, \end{aligned} \tag{19}$$

or, putting the backgrounds in terms of projectors as [from Eq. (16)],

$$\begin{aligned} R_{\pi N} = -g^2 k'_\mu \Bigg\{ &\frac{\not p + m}{p^2 - m^2} P^{\mu\nu}_{3/2} + \frac{(1-\kappa)^2}{m^2} g^2 k'_\mu \big[2(\not p + m)\big(P^{1/2}_{11}\big)^{\mu\nu} + m\sqrt{3}\big(P^{1/2}_{12} + P^{1/2}_{21}\big)^{\mu\nu}\big] \\ &+ \frac{\kappa^2}{m^2}(\not p + m) P^{\mu\nu}_{3/2} + \frac{2(1-\kappa)\kappa}{m^2}\big(2\not p P^{1/2}_{11} + m P_{3/2}\big)^{\mu\nu} \Bigg\} k_\nu. \end{aligned} \tag{20}$$

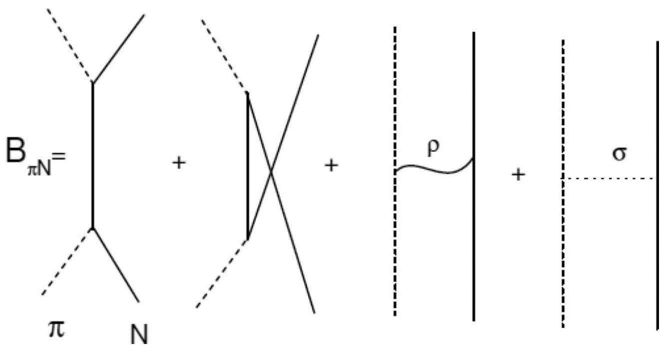

FIG. 2. Background non-resonant contributions to the πN amplitude.

Observe that we get the same peak contribution if we assume the same value of $g \equiv \frac{f_{\pi N\Delta}}{m_\pi}$ used when $g_2 = 0, g_1 = g$ but as $\kappa, (1-\kappa) < 1$ the separate backgrounds are reduced by a smaller factor $\kappa^2, (1-\kappa)^2$, and with an interference background (last term) coming from the last term in Eqs. (19) and (20) of the same order since it goes as $(1-\kappa)\kappa$.

IV. RESULTS

We implemented our theoretical proposal to calculate $\pi^+ p$ and $\pi^- p$ total cross-section data. To do so, we used a minimal realistic model for the nonresonant background including the ρ and σ fields (Fig. 2), while the unstable character of the Δ has been taken into account under the complex mass scheme (CMS), where we make the replacement $m \to m + i\Gamma$ in the *full* propagator, with Γ being the Δ width, which is supposed to be constant (see Ref. [17] for details).[3] The contribution of the s channel to the cross section is dominant for the $\pi^+ p$ case (the isospin factor being equal to 1, while it is 1/3 for the u channel), so the parameters of the Δ (g, m, Γ) are usually fitted to this channel data [11,17], while (due to the exchange isospin factors between u and s channels and $p^2 \neq m^2$) the u channel becomes important for the $\pi^- p$ elastic case and is so far more sensitive to the background content of the model. Nevertheless, we analyze data for the total $\pi^- p$ cross section since a model appropriate to reproduce the elastic and charge-exchange contributions should include final-state charge-exchange interactions to couple final $\pi^- p$ and $\pi^0 n$ states. The strategy we follow is to use the $\pi^+ p$ data first to fit the Δ mass, width Γ, and coupling g. This was done previously in Refs. [17,19,20], and this model was considered precise enough by the Particle Data Group to publish our determination of the Δ magnetic moment [21] as the most recent one. Here, we repeat the fit since we have enlarged the number of data points, but we conserve the same level of precision. Then, keeping these fitted parameters, we use the $\pi^- p$ data to fix the background-related κ parameter [recall the definitions of g and κ in Eqs. (17) and (18)]. Then, a refitting of g, m, Γ is done until we get the same levels of precision in both channels. This should be equivalent to fix g_1 and g_2. The experimental data are taken from Ref. [22].

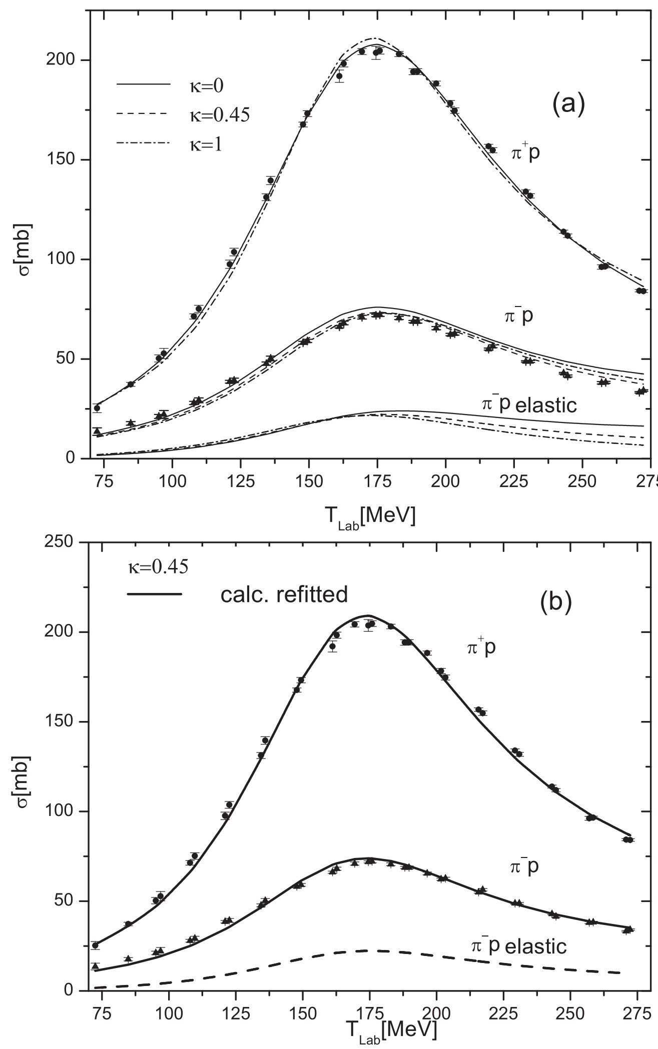

FIG. 3. Elastic $\pi^+ p$ and $\pi^- p$ total and elastic cross sections. (a) Fittings obtained with the interaction I_1 $(\kappa = 0)$ and I_2 $(\kappa = 1)$ for $\pi^+ p$ and results for $\pi^- p$ (total and elastic), keeping the $\kappa = 0$ fitted parameters with other values of κ to improve the description. (b) Refitting obtained with $I_1 + I_2$ $(\kappa = 0.45)$ for $\pi^+ p$ and reproduction of $\pi^- p$ with the obtained parameters.

We show the results in Fig. 3. First, we get $g = 0.32$, $m = 1211.41$, $\Gamma = 88.00$ for I_1 $(\kappa = 0)$ with $\chi \equiv \chi^2/\text{dof} = 4.4$ as before in Refs. [17] and [21]. It would be seem that this χ value is big, but nevertheless consider that the errors are of the order of 1%, which is very good for any model. In addition, the value of $\chi = 4.5$ (considering dof $= n^0$ data) is obtained with the WI08 solution of Ref. [23], which confirms the reliability of our model. When we repeat the fitting with I_2 $(\kappa = 1)$, we get $g = 0.30$, $m = 1210.14$, $\Gamma = 80.77$, $\chi = 16.30$ and it is not possible to improve the fit by manipulating the σ parameters, as we concluded in Ref. [11]. Then, the fit with I_1 is much better, and we adopt the Δ parameters obtained with I_1.

While we intend to reproduce the total $\pi^- p$ data, we get for I_1 $(\kappa = 0)$ a value $\chi = 80.80$ that is not satisfactory, as

[3] A more accurate procedure would be the use of the energy-dependent Δ self-energy, taking into account the Δ mixing with πN states at one or higher loop bubbles to all orders [18]. Nevertheless, since we are interested in the qualitative behavior of the different components of the $\pi N\Delta$ interaction and not in a full description of the πN scattering data, we use the simpler CMS scheme.

shown in Fig. 3. We get a similar result when use I_2 ($\kappa = 1$) with $\chi = 40.40$. In order to test how $I_1 + I_2$ ($\kappa \neq 0, 1$) can accommodate to both data sets, we try to fix κ to improve the total $\pi^- p$ description and we see that for $\kappa = 0.45$ results improve to $\chi = 18$. In Fig. 3, we can see how κ controls the values for the elastic cross section where the Δ essentially contributes with the u background term. Finally, we refit the $\pi^+ p$ channel with $I_1 + I_2$ ($\kappa = 0.45$) and get $g = 0.31$, $m = 1210.67$, $\Gamma = 84.60$ with $\chi = 7.1$ and reproduce the total $\pi^- p$ data with $\chi = 7.2$. The WI08 solutions gives results for this channel of $\chi = 3.5$. Then, we conclude that this combination $I_1 + I_2$ ($\kappa = 0.45$) enables us reproduce both channels with the same level of precision and not appreciably less than in the initial fitting, as shown in Fig. 3.

It is important to mention that Eq. (19) with $\kappa = 1$ is roughly equivalent to Eq. (32) of Ref. [13] with their fitted value $c = 1.1$. There, the authors try to improve the $\nu n \rightarrow \mu^- n \pi^+$ description where the hadronic current $W^+ n \rightarrow n\pi^+$ current is analogous to the elastic $\pi^- p \rightarrow \pi^- p$ channel (but enlarging and not diminishing the cross section) but with a combination of weak and strong vertices. The difference with our point of view is that in place of adding arbitrarily a $c\delta P$ background to cancel the undesirable contributions, we expand the πN interaction to the second derivative order, and this cancellation mechanism appears naturally. In that case, the result $c = -1.1$ seems equivalent to using only the I_2 interaction in our formalism. Nevertheless, the errors in that reaction are of the order of 20–30%, which makes it difficult to definitively conclude which c value fits both the $\nu p \rightarrow \mu^- p \pi^+$ and $\nu n \rightarrow \mu^- n \pi^+$ channels at the same time.

V. CONCLUSIONS

To date, publications devoted to the description of pion-production reactions describe the $\pi N \Delta$ vertex by either using the conventional first derivative pion field Lagrangian [Eq. (6)] or the more recent second-order derivative Lagrangian [Eq. (7)], as if they were mutually exclusive possibilities. As mentioned earlier, both interaction vertexes can be used in a perturbative approach. Within the spirit of effective Lagrangian theories, we have developed the interaction to a second derivative order, and in addition, both interactions should be present when radiative corrections are introduced. At the threshold, i.e., in the resonance region, both interactions contribute with the same power counting in $\delta \sim (m - m_N)/\Lambda_{\chi PT}$ and $\delta^2 \sim m_\pi/\Lambda_{\chi PT}$.

As we can see in Fig. 3, using both interactions together we can get good fits to the data in cases where the background contributions are important, without *ad hoc* manipulations. This scheme can be applied to other interactions like pion photo or weak production.

This is just a first approach. The procedure could be iterated to optimize the fit, the breaking of isospin symmetry (leading to slight changes in masses and widths for Γ^{++}, Γ^+, Γ^0, and Γ^-) could be taken into account, or an approach beyond tree level (like solving the Bethe-Salpeter equation for the self-energy as in Ref. [24]) could be used; however, our results are of the same order as those obtained with the model WI08. It is clear that using I_1 and I_2 together is not only theoretically correct but also phenomenologically sound.

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APPENDIX A: FREE PARAMETERS IN THE I_2 INTERACTION

To show the generality of the form of $\mathcal{L}_{I_2}$, observe that the general covariant form of the coefficients to contract with pion and a second derivative is [taking for simplicity $A = -1$ (see Sec. II) and omitting isospin factors and integrating by parts]

$$\mathcal{L}_{I_2} = \bar{\Psi}^\mu R\left(-\tfrac{1}{2} - Z_2\right)_{\mu,\sigma} \mathcal{M}^{\sigma\alpha\nu}(\partial_\alpha \psi)(\partial_\nu \phi), \tag{A1}$$

where the most general tensor structure for $\mathcal{M}$ is $\mathcal{M}^{\mu\alpha\nu} = z_1\gamma^\mu\gamma^\alpha\gamma^\nu + z_2 g^{\mu\alpha}\gamma^\nu + z_2 g^{\mu\nu}\gamma^\alpha + z_3 g^{\alpha\nu}\gamma^\mu$. Observe that in the free RS Lagrangian in (1) and (2), there is no term containing $\dot{\Psi}^0$ for $A = -1$. So, the equation of motion for it is a true constraint, and Ψ^0 has no dynamics. It is necessary then that interactions do not change that (see Ref. [9], Appendix A). Since $\mathcal{L}_{I_2}$ contributes a $\dot{\Psi}^0$ in the equation of motion for ψ via $R(-\frac{1}{2} - Z_2)^0_\sigma \mathcal{M}^{\sigma\alpha\nu}$, this condition is used for $\dot{\Psi}^0$ not appearing in the equations of motion is that this contribution contains no time derivative of any of the other fields of the theory. This can be realized if R is diagonally achieved with $Z_2 = -1/2$ and if $\mathcal{M}^{0\alpha 0} = \mathcal{M}^{00\nu} = 0$. This leads to

$$\mathcal{M}_{\mu\alpha\nu} = [\gamma_\mu\gamma_\alpha\gamma_\nu + g_{\mu\nu}\gamma_\alpha - g_{\mu\alpha}\gamma_\nu - g_{\alpha\nu}\gamma_\mu] = i\epsilon_{\mu\nu\alpha\rho}\gamma_\rho\gamma 5, \tag{A2}$$

where we have used a property of γ matrices. Finally, if we replace Eq. (A2) in (A1), we get Eq. (7) for $A = -1$.

APPENDIX B: SPIN PROJECTORS

We have introduced P^k_{ij}, which projects on the $k = 3/2, 1/2$ sectors of the representation space, with $i, j = 1, 2$ indicating the subsectors of the $1/2$ subspace, defined as

$$\begin{aligned}
(P^{3/2})_{\mu\nu} &= g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}[\not{p}\gamma_\mu p_\nu + p_\mu\gamma_\nu \not{p}],\\
\left(P^{1/2}_{22}\right)_{\mu\nu} &= \frac{p_\mu p_\nu}{p^2},\\
\left(P^{1/2}_{11}\right)_{\mu\nu} &= g_{\mu\nu} - P^{3/2}_{\mu\nu} - \left(P^{1/2}_{22}\right)_{\mu\nu}\\
&= \left(g_{\mu\alpha} - \frac{p_\mu p_\alpha}{p^2}\right)(1/3\gamma^\alpha\gamma^\beta)\left(g_{\beta\nu} - \frac{p_\beta p_\nu}{p^2}\right),\\
\left(P^{1/2}_{12}\right)_{\mu\nu} &= \frac{1}{\sqrt{3}p^2}(p_\mu p_\nu - \not{p}\gamma_\mu p_\nu),\\
\left(P^{1/2}_{21}\right)_{\mu\nu} &= \frac{1}{\sqrt{3}p^2}(-p_\mu p_\nu + \not{p} p_\mu \gamma_\nu).
\end{aligned} \tag{B1}$$

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