A New Information Theoretic Test of the Markov Property of Block Errors in Fading Channels

Mario R. Hueda and Carmen E. Rodríguez

Abstract—**This paper introduces a new and effective information theoretic test of the accuracy of the Markov property of block errors in fading channels. We apply the test to verify the validity of a first-order Markov model for block error processes on Rayleigh and Ricean fading channels. Unlike previous work, we address the effects of maximal ratio diversity-combining at the receiver. As a second application, we investigate the behavior of the block success/failure process in transmission systems that incorporate closed-loop power control. This topic is of great interest in the design of next-generation wireless networks. Numerical results show that our approach provides significant improvements in accuracy over previously proposed methods.**

Index Terms—**Block errors, closed-loop power control, fading channels, Markov models, performance analysis.**

I. INTRODUCTION

RECENT years have witnessed a growing interest in the development of simple Markov models of wireless fading channels. It has become increasingly clear that certain nonlinear functions of the fading envelope are more significant descriptors of the performance of the communication system than the envelope itself [1]. Of particular significance is the success/failure process of block transmissions. This process can be seen as the result of quantizing the fading envelope with respect to a threshold [1].

Several authors have used a first-order Markov model (FOMM) to represent the block success/failure process in transmissions over radio channels with the motivation of making performance analysis more tractable [1]–[4]. This helps the design of complex protocols over wireless channels [e.g., automatic repeat request (ARQ) [1], [3]]. First-order Markov models of the block success/failure process are the simplest. However, it was found in [5] that they could result in an oversimplified description of the process. Therefore, further analysis of their accuracy is required.

Previous literature [1], [3] adopted an information theoretic metric developed by Wang and Chang [6] as a measure of the accuracy of FOMM for the block error process. However, Tan and Beaulieu [7] showed that, even when Wang and Chang's criterion is satisfied, it cannot be concluded that higher order Markov models may not perform significantly better than an FOMM. Other tests of the accuracy of the Markovian approximation have been used [2], [5], [7]. For example, statistical tests were used in [2] to validate a reduction of order of a given high-order Markov chain. However, one cannot infer from these tests that the quantized fading process can be modeled as Markov of a given order [2]. A method to obtain Markov models of arbitrary order in transmission over narrow-band fading channels was investigated in [5]. Although this approach yields valuable information for medium/fast fading channels, it is not suitable for slow fading channels, since excessively long training sequences are required [5]. A stochastic analysis based on comparing autocorrelation coefficients (ACCs) was adopted in [7] to test FOMMs in Rayleigh channels. Their goodness can be assessed at any fading rate using this approach. However, this method does not seem practical to determine the memory required by the Markov chain.

From all of the above, we conclude that a general technique to test the accuracy of the Markovian approximation for block errors in any environment would be very useful. Even more importantly, an indicator of the goodness of Markov modeling similar to Wang and Chang's metric not only would simplify the analysis or provide an alternative method of verification but also would explain the behavior of block error processes on the basis of simple physical arguments.

In the rest of this section we discuss the *mutual information criterion,* proposed in previous literature to test the Markov property of block errors. Let β_i be a binary process such that $\beta_i = 1$ if the received data block i is in error and 0 otherwise. Mutual information was used in [1] to show that the success/failure of the transmission in the previous block or frame (i.e., β_{i-1}) summarizes almost all the information contained in the past. Let $I(\beta_i;\beta_{i-1}\beta_{i-2})$ be the average mutual information between the random variable β_i and the two past transmissions β_{i-1} and β_{i-2} [8]. We can write

$$I(\beta_i;\beta_{i-1}\beta_{i-2}) = I(\beta_i;\beta_{i-1}) + I(\beta_i;\beta_{i-2}|\beta_{i-1}) \quad (1)$$

where $I(\beta_i;\beta_{i-1})$ is the information on β_i contained in β_{i-1}, and $I(\beta_i;\beta_{i-2}|\beta_{i-1})$ is the residual information on β_i contained in β_{i-2}, once β_{i-1} is known. In [6], a measure of the goodness of the first-order Markov approximation has been given in terms of

$$\zeta = \frac{I(\beta_i;\beta_{i-2}|\beta_{i-1})}{I(\beta_i;\beta_{i-1})}. \quad (2)$$

If $\zeta \ll 1$, the relative importance of the numerator is small with respect to the denominator meaning that, after β_{i-1} is known, the additional information on β_i carried by β_{i-2} is negligible. Therefore, the pattern of packet errors could be assumed to

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follow an FOMM [1]. Tan and Beaulieu demonstrated, however, that even when $\zeta = 0$, it cannot be concluded that higher order Markov models may not perform significantly better than a first-order Markov chain [7]. This is because Wang's criterion is based on (1) rather than

$$I(\beta_i;\beta_{i-1}\beta_{i-2}\cdots\beta_{i-\infty}) = I(\beta_i;\beta_{i-1}) + I(\beta_i;\beta_{i-2}|\beta_{i-1}) + I(\beta_i;\beta_{i-3}|\beta_{i-1}\beta_{i-2}) + \cdots. \quad (3)$$

Based on Tan and Beaulieu's observation (3), this paper introduces a new information theoretic criterion for the accuracy of the Markovian approximation for block error processes. Unlike Wang's metric, which compares $I(\beta_i;\beta_{i-1})$ and $I(\beta_i;\beta_{i-1}\beta_{i-2})$ [see (1) and (2)], our approach compares $I(\beta_i;\beta_{i-1})$ with the information about β_i carried by all past samples β_{i-k} $k = 1,\cdots,\infty$, and also takes into account the amount of information about β_i carried by all past samples. The latter is important in several situations such as in transmission systems with low block error rates. As we shall show later, the proposed approach outperforms Wang's criterion.

Although our method is general (i.e., it can be used to test higher order models), in this paper we focus our study on FOMM. The new theory is used to test the validity of first-order Markov modeling of block errors in transmission over 1) Rayleigh channels with maximal ratio diversity combining and 2) Ricean fading channels. It is also used to investigate the behavior of the block success/failure process in transmissions over code-division multiple-access (CDMA) systems with closed-loop power control. This is of great interest in the design of next-generation wireless networks. In all cases, our results show the excellent behavior of the proposed metrics. Unlike previous methods, our approach is based on simple information theoretic arguments, and it can be efficiently used not only to reliably evaluate the goodness of the first-order Markovian approximation but also to explore the accuracy of higher order Markov chains.

This paper is organized as follows. Section II introduces the new information theoretic criterion. Examples of application of the theory are presented and discussed in Section III, and concluding remarks are given in Section IV. To facilitate the reading of this paper, parameters, variables, and abbreviations most frequently used are listed in Table I.

TABLE I
LIST OF PARAMETERS, VARIABLES, AND ABBREVIATIONS MOST USED

ζ	Wang's metric (2)
λ	Fraction of total information on β_i carried by all past samples that is contained in β_{i-1} (5)
ψ	New metric to verify the accuracy of FOMM (6)
δ	New metric to verify the accuracy of the memoryless model (7)
ε	Block error rate
β_i	Block error process (β_i=1 if the received data block i is in error, β_i=0 otherwise)
γ_i	SNR at the combiner output at instant/block i
γ_{Ref}	SINR required at the channel decoder input to achieve a given block error rate in CLPC
σ_{step}	Step-size to adjust the transmission power in CLPC schemes
ACC	Autocorrelation coefficient
CDMA	Code division multiple access
CLPC	Closed-loop power control
CMMM	Channel multi-state Markov model
F	Fade margin
f_D	Maximum Doppler frequency
f_DT	Fading rate
FOMM	First-order Markov model
L	Total number of MRC branches
MAE	Maximum absolute error
MRC	Maximal ratio combiner
$N_{(1/F)}$	Level crossing rate
$\boldsymbol{S}_i$	State vector of CMMM
s_i	Component of the state vector $\boldsymbol{S}_i$
SINR	Signal-to-interference-plus-noise ratio
SNR	Signal-to-noise ratio
T	Block duration
$T_{(1/F)}$	Mean time that the SNR is below $1/F$

II. A NEW INFORMATION THEORETIC APPROACH

We express (3) as

$$I(\beta_i;\beta_{i-1}\beta_{i-2}\cdots\beta_{i-\infty}) = H(\beta_i) - H(\beta_i|\beta_{i-1}\beta_{i-2}\cdots\beta_{i-\infty}) \quad (4)$$

where $H(.)$ and $H(.|.)$ denote entropy and conditional entropy, respectively [8]. Then, we define

$$\lambda = \frac{I(\beta_i;\beta_{i-1})}{I(\beta_i;\beta_{i-1}\cdots\beta_{i-\infty})} = \frac{H(\beta_i) - H(\beta_i|\beta_{i-1})}{H(\beta_i) - H(\beta_i|\beta_{i-1}\cdots\beta_{i-\infty})}. \quad (5)$$

Parameter λ compares the information about β_i carried by β_{i-1} with the information about β_i carried by all past samples β_{i-k} $k = 1,\ldots,\infty$. Note that $0 \le \lambda \le 1$ and it grows as the information about β_i carried by $\beta_{i-2},\ldots,\beta_{i-\infty}$ decreases. In particular, $\lambda = 0$ if and only if β_i and β_{i-1} are independent with $I(\beta_i;\beta_{i-1}\cdots\beta_{i-\infty}) > 0$, while $\lambda = 1$ if and only if β_i is an FOMM [8] (i.e., the information about β_i carried by all past samples is contained in β_{i-1}).

When $\lambda < 1$ and $\lambda \to 1$, however, it cannot be concluded that the process β_i follows an FOMM. This is because λ also tends to unity if β_i and β_{i-1} are highly correlated. Thus, based on (5) and taking into account that $I(\beta_i;\beta_{i-1}\cdots\beta_{i-\infty}) \le 1$ (β_i is binary) and gets smaller as the correlation of β_i with the past samples decreases, we propose the following criterion to assess if an FOMM is adequate for modeling β_i:

$$\psi = \frac{\lambda}{I(\beta_i;\beta_{i-1}\cdots\beta_{i-\infty})} \gg 1, \quad \lambda < 1. \quad (6)$$

If $\lambda < 1$ and ψ is not "sufficiently large," the criterion does not discriminate whether β_i can be well modeled by an FOMM or not, even when $\lambda \to 1$. As we shall show in Section III, the new measure (6) outperforms that based on Wang's criterion, used in [1]–[3]. This is because Wang's metric only compares $I(\beta_i;\beta_{i-1})$ with $I(\beta_i;\beta_{i-1}\beta_{i-2})$, while metric ψ compares $I(\beta_i;\beta_{i-1})$ with $I(\beta_i;\beta_{i-1}\cdots\beta_{i-\infty})$ through λ. Furthermore, note that criterion (6) takes into account the amount of information about β_i carried by all past samples through the denominator of ψ, which is important in several situations such as in transmission systems with low block error rates. In this way, note that if (6) is satisfied, an FOMM can be adopted *even* when λ does not tend to unity. In this case, the information about β_i carried by all past samples is negligible; thus we infer that the accuracy of an FOMM will be reasonable. It is important to realize that (6) can be easily modified to explore the accuracy of higher order Markov chains. For example, a second-order Markov model only requires evaluating (5) with $I(\beta_i;\beta_{i-1})$ replaced by $I(\beta_i;\beta_{i-1}\beta_{i-2})$.

An intrinsic limitation of the criterion based on an information theoretic metric is the unavailability of a *confidence level* (i.e., the probability to reject the hypothesis that β_i is Markovian when this hypothesis itself is true), given the statistic [9]. Therefore, an additional measure could be required to determine when a value of an information theoretic metric is "sufficiently large" (ψ) or "small" (ζ) to accept a Markov modeling. For example, using simulations of an ARQ protocol, it has been verified in [1] that an FOMM can be adopted if $\zeta < 0.01$.

A. Measure of the Goodness of the Memoryless Model

In transmissions over very fast fading channels, block errors can be satisfactorily modeled as a memoryless process [1]. Although (6) can be used to explore the accuracy of the memoryless model for β_i, in very fast fading environments both terms $I(\beta_i;\beta_{i-1})$ and $I(\beta_i;\beta_{i-1}\cdots\beta_{i-\infty})$ tend to zero. Thus, when λ and/or ψ are numerically estimated, roundoff errors may give rise to problems. To avoid this difficulty, in the following we propose an alternative metric to explore the accuracy of the memoryless model for β_i.

In order to provide a measure of the goodness of a memoryless model of β_i, Zorzi *et al.* [1] compared the mutual information $I(\beta_i;\beta_{i-1})$ with the entropy $H(\beta_i)$. It was stated in [1] that if $I(\beta_i;\beta_{i-1}) \ll H(\beta_i)$, a memoryless model can be adopted to approximate the block success/failure process. However, similar to Wang's metric, this measure may be inaccurate since it does not consider the dependence between β_i and the past samples β_{i-k}, $k > 1$ (e.g., see Section III-A in [7]). To overcome this limitation (and possible problems caused by roundoff errors in the estimation of ψ), we propose to use the following metric:

$$\delta = \frac{I(\beta_i;\beta_{i-1}\cdots\beta_{i-\infty})}{H(\beta_i)} = \frac{H(\beta_i) - H(\beta_i|\beta_{i-1}\cdots\beta_{i-\infty})}{H(\beta_i)}. \tag{7}$$

Note that $\delta = 0$ if and only if the random variables β_i and β_{i-k} are independent $\forall i, k, k \neq 0$ [8]. Therefore, we conclude that β_i can be well described by a memoryless process if

$$\delta \to 0. \tag{8}$$

B. Bounds on the New Metrics

Since λ, ψ, and δ cannot be calculated because $H(\beta_i|\beta_{i-1}\cdots\beta_{i-\infty})$ is not known, in this section we derive bounds that allow us to characterize the behavior of these parameters. These bounds are derived using the *channel multistate Markov model* (CMMM), which is described in the following. Let γ be the signal-to-noise ratio (SNR) at the channel decoder input. This is partitioned into a finite set of ranges Φ. The ranges are specified by the end points $0 = \hat{\gamma}_1 < \hat{\gamma}_2 < \cdots < \hat{\gamma}_{\Phi+1} = \infty$. If the SNR value at instant i, γ_i, is in the range $\bar{s}_n = [\hat{\gamma}_n, \hat{\gamma}_{n+1})$, the channel is mapped into state $s_i = \bar{s}_n$. Assuming that the probability of state s_i depends only on the m_s past samples [5], [10], it is possible to show that the fading channel can be accurately approximated by a stationary Markov chain (or CMMM) with states $\boldsymbol{S}_i = (s_i, s_{i-1}, \cdots, s_{i-m_s+1})$, that is

$$\Pr\{\boldsymbol{S}_i|\boldsymbol{S}_{i-1}\boldsymbol{S}_{i-2}\cdots\} = \Pr\{\boldsymbol{S}_i|\boldsymbol{S}_{i-1}\}. \tag{9}$$

Since the block error process β_i is a binary function of the corresponding state in the Markov chain, $\beta_i = \phi(\boldsymbol{S}_i)$, it can be shown from [8, 4-58] that

$$\begin{aligned} H\left(\phi(\boldsymbol{S}_i)|\phi(\boldsymbol{S}_{i-1})\cdots\phi(\boldsymbol{S}_{i-n+1})\boldsymbol{S}_{i-n}\right) &\leq H\left(\phi(\boldsymbol{S}_i)|\phi(\boldsymbol{S}_{i-1})\cdots\phi(\boldsymbol{S}_{i-\infty})\right) \\ &\leq H\left(\phi(\boldsymbol{S}_i)|\phi(\boldsymbol{S}_{i-1})\cdots\phi(\boldsymbol{S}_{i-n})\right) \end{aligned} \tag{10}$$

or simply

$$\begin{aligned} H(\beta_i|\beta_{i-1}\cdots\beta_{i-n+1}\boldsymbol{S}_{i-n}) &\leq H(\beta_i|\beta_{i-1}\cdots\beta_{i-\infty}) \\ &\leq H(\beta_i|\beta_{i-1}\cdots\beta_{i-n}) \end{aligned} \tag{11}$$

where n is a finite number and $\boldsymbol{S}_{i-n}$ is the state of the CMMM at instant $i-n$. Thus, from (5) through (11), we verify

$$\lambda \geq \lambda_{\min} = \frac{I(\beta_i;\beta_{i-1})}{H(\beta_i) - H(\beta_i|\beta_{i-1}\cdots\beta_{i-n+1}\boldsymbol{S}_{i-n})} \tag{12}$$

$$\psi \geq \psi_{\min} = \frac{\lambda_{\min}}{H(\beta_i) - H(\beta_i|\beta_{i-1}\cdots\beta_{i-n+1}\boldsymbol{S}_{i-n})} \tag{13}$$

$$\delta \leq \delta_{\max} = \frac{H(\beta_i) - H(\beta_i|\beta_{i-1}\cdots\beta_{i-n+1}\boldsymbol{S}_{i-n})}{H(\beta_i)}. \tag{14}$$

Note that probability mass functions of various consecutive (quantized) fading symbols (e.g., $\Pr\{\beta_i\beta_{i-1}\cdots\beta_{i-n+1}\boldsymbol{S}_{i-n}\}$) are required to compute $H(\beta_i|\beta_{i-1}\cdots\beta_{i-n+1}\boldsymbol{S}_{i-n})$. Although this could be achieved for Rayleigh fading using the series expansion technique adopted in [1], it was found in [2] that this approach may not be convenient for Ricean fading because the large number of terms in the summation leads to roundoff errors (in this situation, the probability mass functions in [2] were obtained numerically via a random number generator). Furthermore, the difficulty of the analytical evaluation grows significantly when diversity schemes such as maximal ratio combining are incorporated. Therefore, similar to [2], in this paper the probability mass functions required to compute metrics (12)–(14) are obtained via numerical evaluation.

III. Examples of Application

As an application of the above theory, in this section we investigate the accuracy of an FOMM to approximate the behavior of the block error process in transmissions over Rayleigh and Ricean fading channels. In addition, we explore the accuracy of a first-order Markov model for block errors in transmission systems with closed-loop power control and diversity.

To show the accuracy of our metrics in analyzing the validity of first-order Markovian modeling, in this section we use both 1) the proposed approach based on the new metrics and 2) the test based on comparing the ACCs suggested in [7]. Since $\beta_i \in \{0, 1\}$, the ACCs are given by

$$\mathrm{ACC}(m) = \frac{E\{\beta_i\beta_{i+m}\} - E\{\beta_i\}E\{\beta_{i+m}\}}{E\left\{\beta_i^2\right\} - E\{\beta_i\}^2} = \frac{v(m) - \varepsilon}{1 - \varepsilon} \tag{15}$$

where $E\{\cdot\}$ denotes expectation, ε is the block error probability, and $v(m)$ is the probability that the transmission in slot $i+m$ is unsuccessful, given that the transmission in slot i was unsuccessful.

A. Analysis of Block Errors in Transmissions Over Rayleigh Channels

In the following, numerical results are derived adopting the *threshold model* for the block error process [1]. When the number of symbols in the block is large, this model approximates very well the block success/failure process [2]. Diversity with maximal ratio combining (MRC) and identical average SNR at all branches are considered in this paper. In this way, the threshold model is defined as

$$\beta_i = \begin{cases} 0 & \gamma_i = \sum_{k=1}^{L} \gamma_i^k > \frac{1}{F} \\ 1 & \gamma_i = \sum_{k=1}^{L} \gamma_i^k \leq \frac{1}{F} \end{cases} \tag{16}$$

where γ_i is the SNR at the combiner output at instant i, γ_i^k is the SNR of the kth branch, L is the total number of MRC branches, and F is the *fade margin* [1]. Rayleigh fading has been generated according to the filtered method presented in [11][1] with $E\{\gamma_i\} = 1$. Let f_D and T be the maximum Doppler frequency and the block duration, respectively. The fading rate is defined as $f_D T$. The interval of the fading rate analyzed in this paper is $0.001 \leq f_D T \leq 1$. We consider $L = 1$ and 3, CMMM with $\Phi = 22$ states, and 500 000 simulated blocks. The ranges of CMMM are specified by the end points $\{-\infty, -10, -9, \ldots, 10, \infty\}$ dB. The range of interest for the block error is $0.01 \leq \varepsilon \leq 0.65$. The fade margin interval is chosen to achieve this range of ε. For example, $0 \leq F \leq 20$ dB and $-0.5 \leq F \leq 8.5$ dB are required for $L = 1$ and $L = 3$, respectively.

Note that a numerical evaluation of (12)–(14) is difficult since the number of the states $\boldsymbol{S}_i$ given by Φ^{m_s} is large [see (9)]. However, if we 1) take into account that β_i depends essentially on s_i rather than $\boldsymbol{S}_i$ [see (16)] and 2) select a value of n large enough (e.g., $n \geq m_s$), it can be verified that

$$H(\beta_i | \beta_{i-1} \cdots \beta_{i-n+1} \boldsymbol{S}_{i-n}) \approx H(\beta_i | \beta_{i-1} \cdots \beta_{i-n+1} s_{i-n}) \tag{17}$$

and the numerical evaluation of (12)–(14) can be significantly simplified. In this paper, we use (17) with $n = 5$ [5], [9].

1) Analysis of FOMM in Rayleigh Fading Channels With No Diversity: Fig. 1 presents results for transmission with no diversity ($L = 1$). Fig. 1(a) depicts $\delta_{\max}$. From (8) note that β_i is described better by a memoryless process as the color of the region tends to white (i.e., $\delta_{\max} \to 0$). In particular, β_i can be well modeled as a memoryless process if $\delta_{\max} < 0.005$ [1]. Based on this result, Fig. 1(b) and (c) shows results for the new metric $\psi_{\min}$ and Wang's metric ζ, respectively. Fig. 1(d) depicts the maximum absolute error (MAE) between the ACCs of the process β_i and the FOMM. Assuming the isotropic scattering, omnidirectional receiving antenna (ISORA) model for the fading channel [7], the ACC of β_i is given by (15) with

$$v(m) = \begin{cases} 1 - \frac{Q(\theta(m), \rho(m)\theta(m)) - Q((\rho(m)\theta(m), \theta(m))}{e^{\frac{1}{F}} - 1} & m \neq 0 \\ 1 & m = 0 \end{cases} \tag{18}$$

where $Q(.,.)$ is the Marcum Q-function, $\theta(m) = \sqrt{2/F/(1-\rho^2(m))}$, $\rho(m) = |J_0(2\pi f_D m T)|$, and $J_0(\cdot)$ is the zero-order Bessel function of the first kind. For the Markov approximation, the ACC is given by (15) with $v(m)$ obtained from the transition matrix (see [1, (27)]). Details regarding the estimation of Markov parameters appear in [1].

Comparisons of both information theoretic criteria ($\psi \gg 1$ and $\zeta \ll 1$) with MAE of ACCs obtained from Fig. 1 reveal the excellent behavior of our metric and the limitations of the traditional Wang's metric used in [1]–[3]. Note that MAE is smaller than 0.05 for $\psi_{\min} > 30$. For $f_D T > 0.01$, this result agrees with that obtained in [5] based on context tree models. However, unlike [5], results derived from our metric suggest that an FOMM may not be useful for $f_D T \leq 0.01$.

When fading is very slow, $\zeta \to 0$ since β_i and β_{i-1} are highly correlated. On the other hand, in very slow fading channels, it is verified that $\lambda_{\min} \to 1$ and $I(\beta_i; \beta_{i-1} \cdots \beta_{i-\infty}) \to H(\beta_i)$; therefore condition $\psi_{\min} \gg 1$ is not satisfied for the range of ε considered in this paper (e.g., $\zeta < 10^{-4}$ and $\psi_{\min} \approx H(\beta_i)^{-1} < 3$ for $\varepsilon \approx 0.1$ and $f_D T < 0.002$). Thus, Wang's metric suggests that an FOMM could be adopted, while our criterion indicates that an FOMM may not be useful to model β_i in very slow Rayleigh fading channels with $0.01 \leq \varepsilon \leq 0.65$. From Fig. 1(d) we note that, for a certain value of ε, the MAE of ACCs increases as the fading rate decreases (e.g., MAE$>$ 0.15 for $\varepsilon \approx 0.1$ and $f_D T < 0.002$). From this analysis, it is simple to verify that the proposed approach outperforms that based on Wang's criterion. Furthermore, unlike that suggested by Wang's criterion in Fig. 1(c), from Fig. 1(b) and (d) we infer that an FOMM could be adopted for slow–medium fading rates ($0.01 < f_D T < 0.1$) when the block error rate is small enough. In this case, the information about β_i carried by all past samples is negligible; thus the accuracy of an FOMM may be reasonable.

Note that higher order Markov chains could be needed for the range of ε used in this paper when $f_D T \leq 0.01$. However, as was suggested in [7], it is important to realize that an FOMM may be adequate for studying many problems of interest such as applications requiring fewer consecutive samples (e.g., performance of retransmission protocols with moderate round-trip delays in slow fading channels as in [1], [3], and [4]).

2) Analysis of FOMM in Rayleigh Fading Channels With MRC Diversity: In the following, we explore the accuracy of the first-order Markovian approximation for block errors in transmissions over Rayleigh fading channels with MRC diversity. Toward this end, we first analyze the mean time during which the SNR is below $1/F$, $T_{(1/F)}$, defined as

$$T_{\left(\frac{1}{F}\right)} = \frac{\Pr\left\{\gamma < \frac{1}{F}\right\}}{N_{\left(\frac{1}{F}\right)}} \tag{19}$$

[1] A simulator with identical statistical quality but using half as many inverse fast Fourier tranfer calls to generate the variates has been proposed in [12].

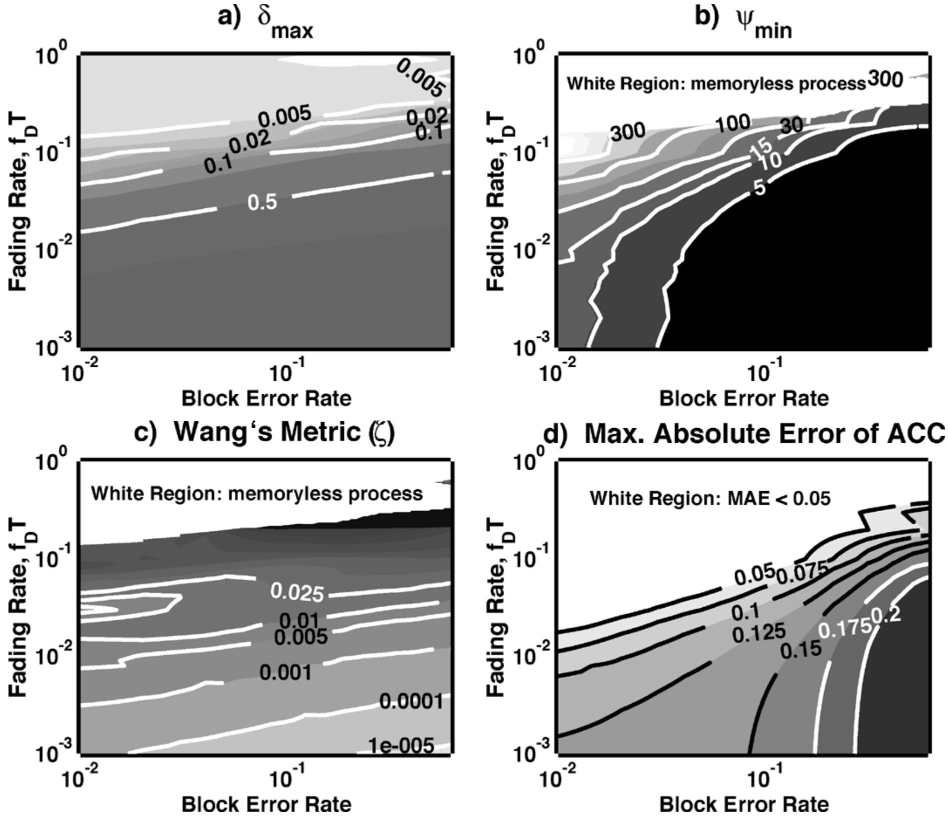

Fig. 1. Verification of the Markovian approximation for block error transmissions over Rayleigh fading channels without diversity ($L = 1$). (a) $\delta_{\max}$. (b) $\psi_{\min}$. (c) Wang's metric. (d) Maximum absolute error of the autocorrelation coefficients.

where $N_{(1/F)}$ is the level crossing rate [13], [14]. For MRC with identical average SNR at all branches, $N_{(1/F)}$ is given by [3], [13]

$$N_{\left(\frac{1}{F}\right)} = f_D f_\gamma \left(\frac{1}{F}\right) \sqrt{\frac{2\pi\bar{\gamma}}{F}} \tag{20}$$

where $f_\gamma(.)$ is the probability density function of the SNR at the MRC output and $\bar{\gamma}$ is the average SNR per branch. From (16), we observe that (19) yields

$$T_{\left(\frac{1}{F}\right)} = \frac{\varepsilon}{N_{\left(\frac{1}{F}\right)}}. \tag{21}$$

Note that $T_{(1/F)}$ represents the mean time that consecutive data blocks are received with error. Fig. 2 shows the product $T_{(1/F)} f_D$ versus ε for $L = 1$ and 3. For a certain value of ε, we observe that the mean time $T_{(1/F)}$ grows with L at low values of $\varepsilon(\sim 0.01)$. This fact indicates that, compared to transmission without diversity, the correlation of process β_i increases for $L = 3$, which suggests that the effects of channel memory maybe more important as the diversity order grows. This can be seen in Fig. 3(a) and (b) where we present $\delta_{\max}$ and $\psi_{\min}$, respectively, for $L = 3$. Comparisons of the proposed criteria (i.e., $\psi_{\min} \gg 1$ and $\delta_{\max} \rightarrow 0$) for $L = 1$ and 3 reveal that the accuracy of the first-order Markov assumption decreases with increasing diversity order, as we predicted from the previous analysis [compare Figs. 1(a) versus 3(a), and Figs. 1(b) versus 3(b)]. This result can be verified in Fig. 3(c), where we show

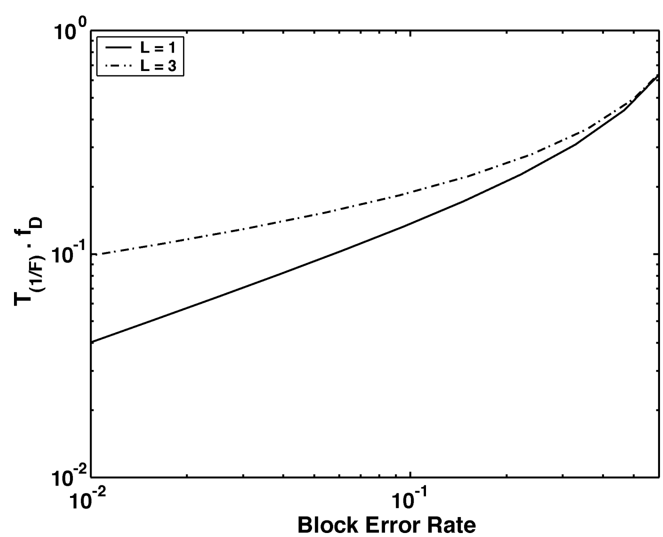

Fig. 2. Average fade duration for Rayleigh channels with MRC diversity.

$\psi_{\min}$ for $f_D T = 0.01$ and 0.1. Note also that the effect of the diversity order decreases as $f_D T$ does, which agrees very well with the corresponding MAE of ACCs depicted in Fig. 3(d).[2]

B. Analysis of Block Errors in Transmissions Over Ricean Fading Channels

Rayleigh fading is not always a good fading model [2]. In microcellular/picocellular environments, channel behavior is often

[2]In this paper, the ACCs for Rayleigh channels with diversity and Ricean fading are derived from computer simulations.

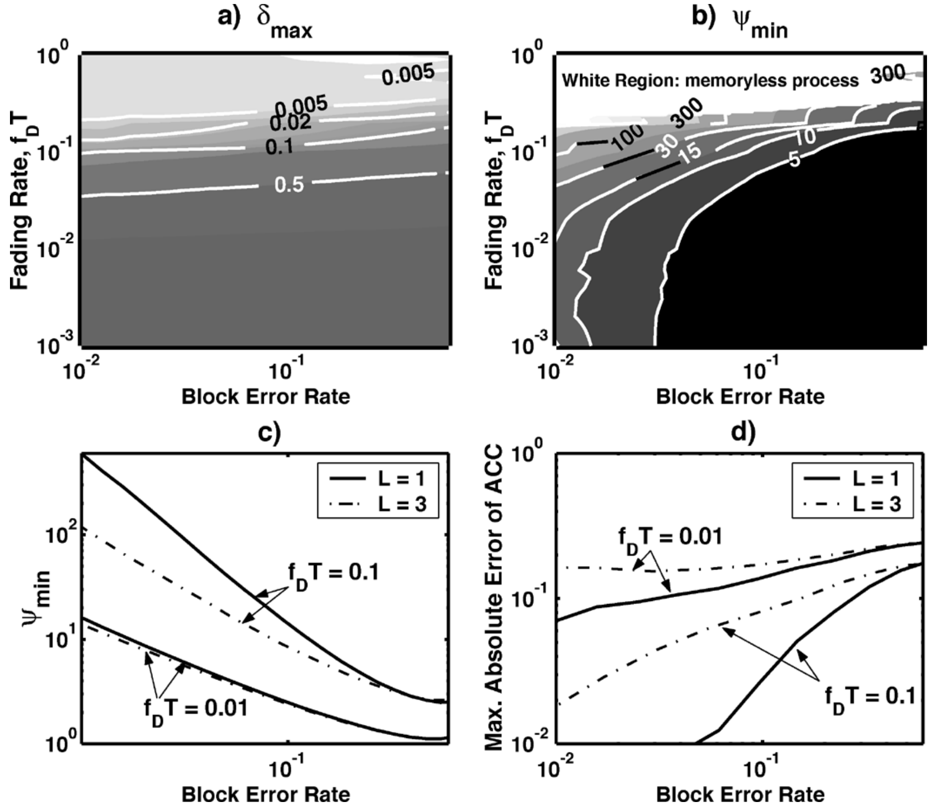

Fig. 3. Verification of the Markovian approximation for block error transmissions over Rayleigh fading channels with diversity $(L = 3)$. (a) $\delta_{\max}$. (b) and (c) $\psi_{\min}$. (d) Maximum absolute error of the autocorrelation coefficients.

Ricean due to the presence of a strong direct path between the fixed base and the mobile end.

Next, numerical results are derived adopting the threshold model for the block error process

$$\beta_i = \begin{cases} 0 & \gamma_i > \frac{1}{F} \\ 1 & \gamma_i \leq \frac{1}{F} \end{cases}. \tag{22}$$

Ricean fading has been simulated using the filtered method presented in [11] with $E\{\gamma_i\} = 1$. We consider 500 000 simulated blocks. Similar to the previous case, the ranges of CMMM are specified by the end points $\{-\infty, -10, -9, \ldots, 10, \infty\}$ dB, and the fade margin range is chosen to achieve a block error rate range of $0.01 \leq \varepsilon \leq 0.65$. The Rice factor used in this paper is five [2].

Fig. 4(a) and (b) shows $\delta_{\max}$ and $\psi_{\min}$, respectively, for transmission with no diversity $(L = 1)$. Unlike fast Rayleigh fading channels (where $\delta_{\max} \approx 0.005$), from Fig. 4(a) we note that a memoryless process may not be useful in fast Ricean fading channels $(\delta_{\max} \approx 0.1)$. Furthermore, from Fig. 4(b), we can infer that a first-order Markov approach may not be appropriate in Ricean channels, even in fast fading situations. Fig. 4(c) and (d) depicts ACCs of β_i for transmissions over Rayleigh and Ricean channels, respectively, with $\varepsilon = 0.04$ and $f_D T = 0.1$. Comparisons of both ACCs (i.e., exact and first-order Markov approach) confirm the reduction of the accuracy of FOMM in Ricean channels predicted from the information theoretic analysis introduced in this paper.

From Fig. 4(b) we can infer that higher order Markov chains may be needed. In this case, the new metric can be easily modified to explore the accuracy of higher order Markov chains. For example, a second-order Markov model requires evaluating (13) with

$$\lambda_{\min} = \frac{I(\beta_i; \beta_{i-1}\beta_{i-2})}{H(\beta_i) - H(\beta_i|\beta_{i-1}\cdots\beta_{i-n+1}\boldsymbol{S}_{i-n})}. \tag{23}$$

Based on (23), in Fig. 5(a) and (b) we show $\psi_{\min}$ for first- and second-order Markov models in fast Ricean fading channels $(0.1 \leq f_D T \leq 1)$, respectively. Our results indicate that the accuracy improvements of a second-order Markov approach with respect to the first-order case may be meaningful, particularly in very fast fading situations (i.e., $f_D T > 0.4$).

From Fig. 5(a), note that $\psi_{\min}$ decreases rapidly when $f_D T \approx 0.4$ because the correlation of two consecutive samples of the fading envelope is small at this value of the fading rate. In particular, two samples of the ISORA fading channel separated $T \approx 0.4/f_D$ s are not correlated (in this case, the correlation factor is $J_0(2\pi f_D T) = 0$) [7]. Since a complex Gaussian fading process is assumed in the ISORA model, two consecutive fading samples $T \approx 0.4/f_D$ seconds apart are independent. The corresponding Rayleigh/Ricean samples and block states are also independent, since they are related to the Gaussian samples by memoryless transformations (see [1, (8)]). Then, from (5) and (6) we verify that $I(\beta_i; \beta_{i-1}) = 0$; therefore $\psi = 0$ for first-Markov models, which agrees with the results depicted in Fig. 5(a). Note that this effect is not present for higher order Markov models since the zero-crossings of $J_0(.)$ are not uniformly spaced (e.g., $\psi > 0$ for second-Markov models since $I(\beta_i; \beta_{i-1}\beta_{i-2}) > 0$).

1) Discussion on the Validity of FOMM in Transmission Over Slow Fading Channels: In most previous contributions

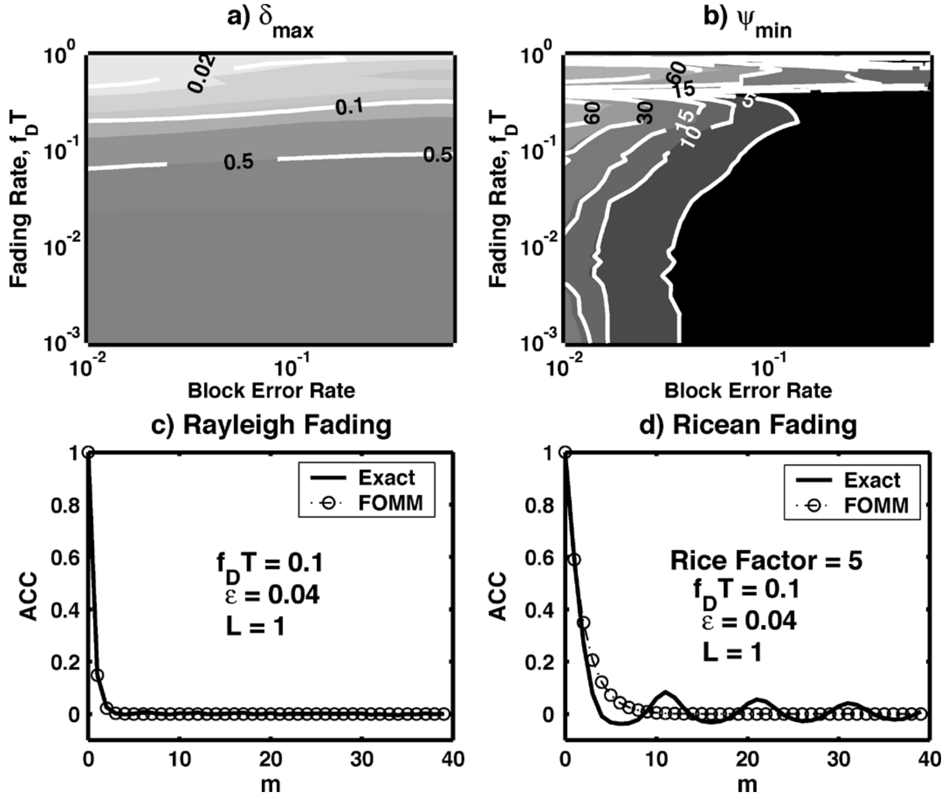

Fig. 4. Verification of the Markovian approximation for block error transmissions over Ricean fading channels without diversity ($L = 1$). (a) $\delta_{\max}$. (b) $\psi_{\min}$. (c) ACC in Rayleigh fading channels. (d) ACC in Ricean fading channels.

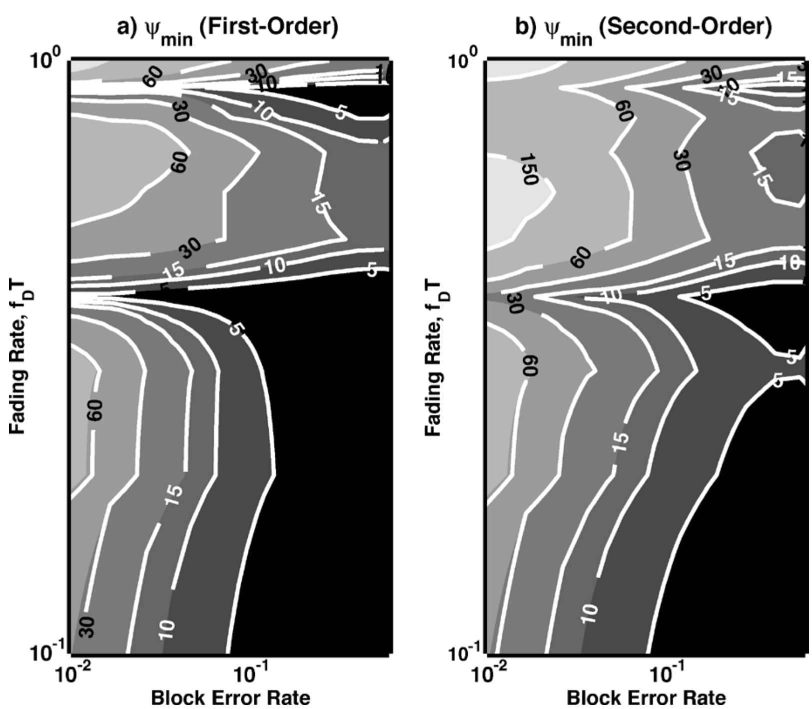

Fig. 5. Verification of the first- and second-order Markovian approximation for block error transmissions over Ricean fading channels. (a) $\psi_{\min}$ (first order). (b) $\psi_{\min}$ (second order).

(e.g., [1]–[3]), Wang's metric (2) was used to explore the accuracy of FOMM in transmission over slow fading channels, while the average error burst length was used to verify the results predicted by (2). Based on these parameters, it has been claimed that an FOMM could be used to model block errors in transmission over slow Rayleigh [1], [3] and Ricean [2] fading channels.

On the other hand, the results based on the metrics introduced in this paper [and depicted in Figs. 1(b), 3(b), and 4(b)], indicate that FOMM may not be suitable for block errors in transmis-

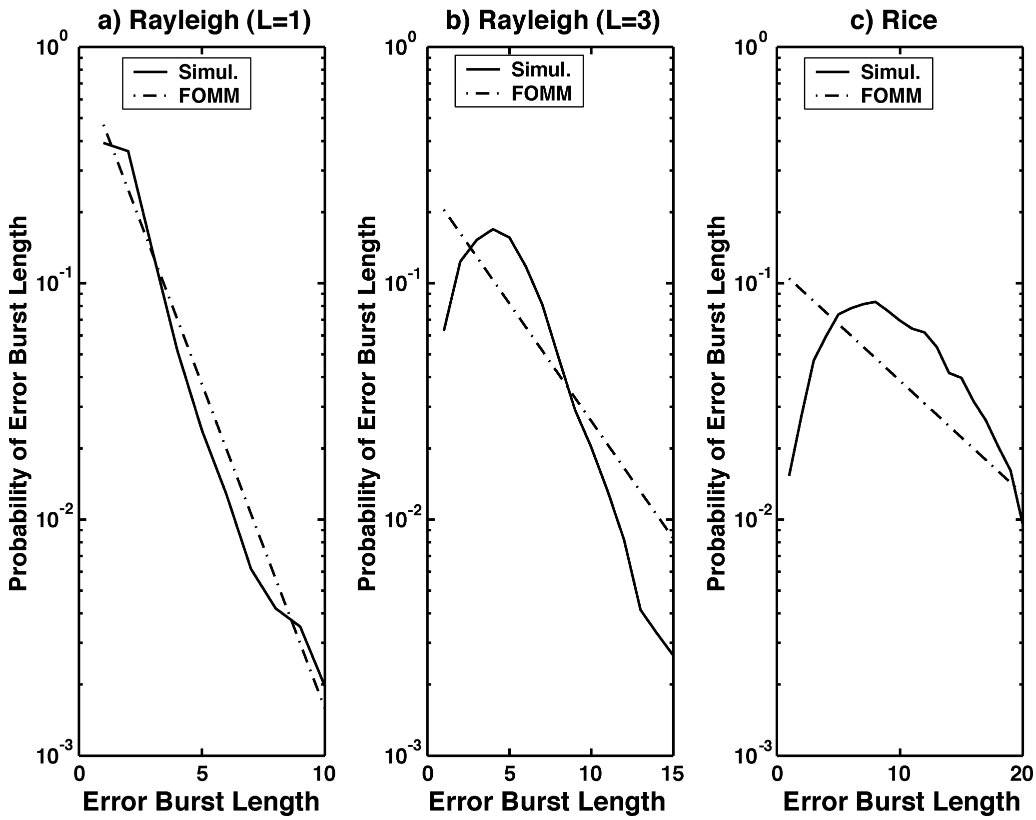

Fig. 6. Probability of error burst length in transmissions over slow Rayleigh and Ricean fading channels ($f_D T = 0.02$). Block error probability: $\varepsilon = 0.01$. (a) Rayleigh fading with no diversity ($L = 1$). (b) Rayleigh fading with diversity ($L = 3$). (c) Ricean fading channel (Rice factor $= 5$; $L = 1$).

sions over slow Ricean and Rayleigh fading channels with diversity when $0.01 \le \varepsilon \le 0.65$. Although this result has been confirmed previously by analysis of MAE of ACCs [see Figs. 1(d), 3(d), and 4(d)], in the following we use the probability of error burst length to verify the accuracy of our approach in transmission over slow fading channels.

Fig. 6 depicts the probability of error burst length in transmissions over slow Rayleigh (with/without diversity) and Ricean fading channels obtained from both 1) simulations of 10^7 blocks and 2) FOMM (details regarding the estimation of FOMM parameters appear in [1]–[3]). The fading rate is $f_D T = 0.02$ and $\varepsilon = 0.01$. Although the values of the average error burst length derived from FOMM and theoretical analysis are very close (see [1]–[3]), from Fig. 6 we observe that an FOMM may not be suitable for block errors in transmissions over slow Ricean and Rayleigh fading channels with diversity, as we predicted from the analysis based on the new information theoretic metrics presented in Sections III-A and -B. In particular, comparison of the curves depicted in Fig. 6(a) and (b) confirms that the accuracy of FOMM decreases with increasing diversity order, according to the results inferred from Fig. 2.

From the above, we conclude that the average error burst length may not be sufficient for certain applications such as fast simulation based performance evaluation of coded video transmission, where the characteristics of block errors (e.g., distribution of error burst lengths) may significantly affect system performance (e.g., the quality of the decoded video in wireless multimedia transmissions [15]).

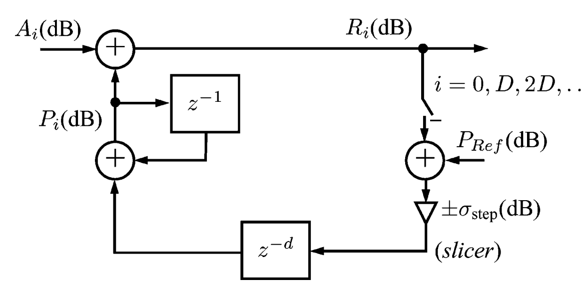

Fig. 7. CLPC log-model.

C. *Analysis of Block Errors in CDMA Transmissions Over Slow Fading Channels With Closed-Loop Power Control (CLPC) and Diversity*

Direct-sequence CDMA (DS-CDMA) is one of the most promising techniques proposed for next-generation networks [16]. Therefore, exploring the goodness of an FOMM in transmission over these systems is a topic of practical interest for researchers and designers [17]. Power control mechanisms are incorporated in DS-CDMA to limit transmitted power on the forward and reverse links, and improve the system performance against fading by compensating fading dips. If it followed the channel fading perfectly, power control would turn a fading channel into an additive white Gaussian noise channel by eliminating the fading dips completely. In this situation, the process β_i could be considered as an independent process. In practical situations, however, power control is imperfect, thus β_i may not be well approximated by a memoryless process.

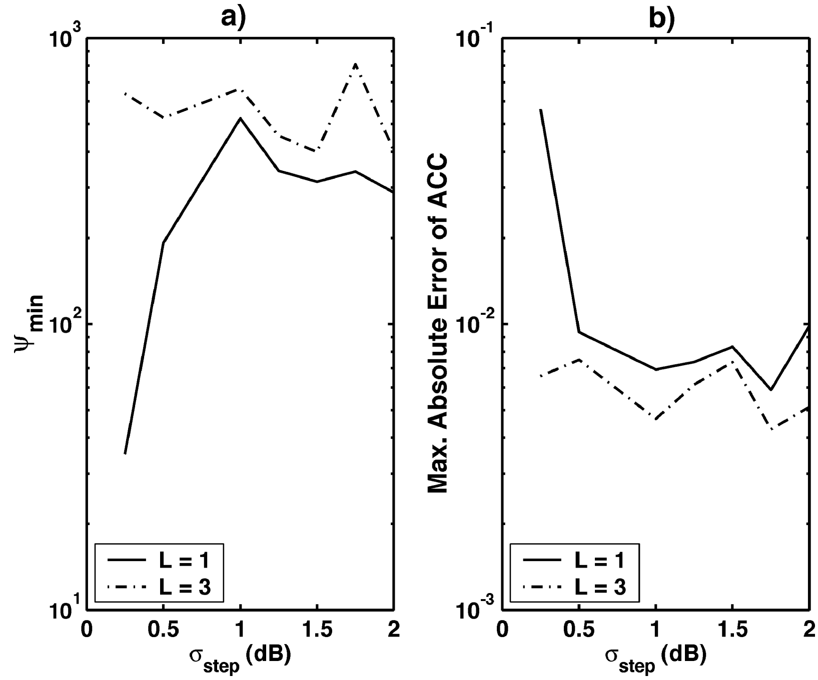

Fig. 8. Verification of the Markovian approximation for block error transmissions over Rayleigh fading channels with closed-loop power control. (a) $\psi_{\min}$. (b) Maximum absolute error of the autocorrelation coefficients.

Closed-loop power control (CLPC) is the most important mechanism to improve DS-CDMA system capacity in slow fading environments [18]. CLPC measures the signal-to-(interference plus noise) ratio (SINR), compares it with a reference value γ_{Ref} according to a desired performance, and sends commands to the transmitter at the other end to adjust the transmission power. CLPC algorithms have been investigated by various authors [18]–[20]. For instance, the performance of a single and a multiple step-size closed-loop power control scheme has been studied in [20]. A simplified single step-size feedback power control log-model is shown in Fig. 7 (we use "X (dB)" to denote X expressed in dB) [19]. This CLPC scheme is widely used by several CDMA standards such as IS-95B [21]. A_i denotes the channel variation caused by fading. The transmitted signal power at time interval i, P_i, is updated with a fixed step-size σ_{step} every D samples. The received signal power R_i is compared with a desired power level P_{ref} according to the required performance.[3] If the received signal level is higher (lower) than the desired value, a power control command will be sent to request the transmitter to decrease (increase) its power by σ_{step}. Extra loop delay Dd (d integer) is included to take into account two-way signal propagation delay and the time delay involved in generating, transmitting, and carrying out a power control command, or sometimes simply to reduce the update frequency to save bandwidth.

In the following, we present results obtained from computer simulations of the transmission system. We use $d = 1$ and $DT_b = 1.25$ ms (T_b is the bit duration). In addition, since transmission errors of the power control commands are not important for the single step-size CLPC [19], [20], we assume no feedback errors. Without loss of generality, we use the parameters, interleaver/deinterleaver, and convolutional code of the IS-95B standard [21]. Furthermore, we use soft-decision decoding of the rate 1/2, constraint length 9, and convolutional code. In this case, the SINR required at the channel decoder input to achieve a block error rate of 1% is $\gamma_{\mathrm{Ref}} \approx 0.9$. Carrier frequency is 1800 MHz and the data rate is 9600 bps (i.e., rate set 1 of IS-95B [21]). The block duration is $T = 20$ ms, and the fading rate is $f_D T = 0.04$. A 16-bit cyclic redundancy code (CRC) is used for block error detection. Let L denote the total number of RAKE fingers. In this section we consider $L = 1$ and $L = 3$ with identical average SINR at all fingers. The total number of multipath components of the channel is assumed equal to L. Since the rate variation of the channel is small, we have assumed ideal coherent demodulation at the RAKE receiver [22]. The Rayleigh channel has been simulated using Jakes' model [14]. We use the Gaussian approximation for the interference-plus-noise component [23]. Each simulation involved 100 000 data blocks. Since CLPC decreases the correlation of the power signal [19], for a value of n large enough it can be assumed that

$$H(\beta_i|\beta_{i-1}\beta_{i-2}\cdots\beta_{i-n+1}\boldsymbol{S}_{i-n}) \approx H(\beta_i|\beta_{i-1}\beta_{i-2}\cdots\beta_{i-n+1}). \quad (24)$$

Based on (24), in Fig. 8(a) we show $\psi_{\min}$ versus σ_{step} for MRC-RAKE receivers with $L = 1$ and 3. We infer that, for practical values of σ_{step} (i.e., $\sigma_{\mathrm{step}} \geq 1$ dB), the process β_i can be well modeled as a first-order Markov chain ($\psi_{\min} > 200$). These results agree very well with MAE of ACCs shown in Fig. 8(b). However, for $L = 1$ and low values of the step-size σ_{step} (i.e., $\sigma_{\mathrm{step}} < 1$ dB), we observe that the accuracy of the Markovian assumption decreases. This is because CLPC is not effective since a deep fading has long duration when diversity is not avail-

[3]Since a slow fading channel is considered, we assume that R_i can be accurately estimated by the receiver (e.g., by using the pilot signal incorporated in both forward and reverse links of *cdma2000*).

able, and the channel variation cannot be tracked with a small step-size [in a limit case, for $\sigma_{\mathrm{step}} \rightarrow 1$ (0 dB) the behavior of block errors tends to that shown in Fig. 1(b)]. Nevertheless, since values of σ_{step} smaller than 1 dB are not of practical interest [20], we conclude that a first-order Markov chain can be adopted to model the block error process in DS-CDMA transmissions with CLPC over slow Rayleigh fading channels.

IV. Concluding Remarks

This paper has introduced a new information theoretic test of the Markovian approximation for the block error process in transmission over fading channels. Using the proposed test, we investigated the accuracy of FOMMs in transmissions over Rayleigh fading channels with MRC diversity. Our results show that the accuracy of the first-order Markov model (for a certain value of the block error rate) decreases with increasing diversity order. We also found that, for the interval of block error rates considered here $(0.01 \leq \varepsilon \leq 0.65)$, an FOMM does not provide a good description of block errors in Ricean channels, even in fast fading situations. We also found that a second-order Markov model may be adequate on very fast Ricean fading channels. In addition, we explored the validity of an FOMM for block errors in transmissions over wireless CDMA cellular networks with CLPC over slow Rayleigh fading channels. Our results have shown that the block success/failure process in these systems is well modeled by a first-order Markov chain. The techniques presented in this paper are general (i.e., they can be used to test higher order models and different fading statistics) and they provide significant accuracy improvements over previously proposed methods.

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