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# OPTIMAL CONTROL BASED HEURISTIC FOR CONGESTION REDUCTION IN TRAFFIC NETWORKS

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***Abstract*— The principal purpose of this work is to test the TUC strategy in a simple case using a micro-simulator designed ad hoc, previous to its real implementation.**

**Using concepts of traffic engineering we describe a well known dynamic linear model of traffic flow in a urban traffic network that is controlled using the traffic-light times. This simplified model allows to obtain a Riccati feedback matrix and compute traffic-light times that will improve the congestion levels.**

**We present some numerical experiments made with the model on an academic example and we validated them with a microscopic simulator that we have created based on Car Following theory and discrete event models.**

***Keywords*— Optimal Control, LQ-Control, Urban traffic models**

## I. INTRODUCTION

The reduction of urban traffic congestion is nowadays obtaining more and more attention. The traffic demand increases continuously and it cannot be followed by the improvement of the offer because it would imply the enlargement of the road infrastructure. The only way to cope with it is to improve the efficiency of the urban network.

Many research lines are currently active inside Traffic Engineering, some of them to determine the average distribution of the traffic on some time interval and how to manage it. The classical notions of Wardrop equilibrium and social optimum help to analyze and optimize medium or long term planning problems. This can help to reduce the congestion on average, but when there is some fluctuations around the mean traffic it is important to be able to reduce the possible negative impact. A practical way to reduce congestion is through an adaptive traffic light setting strategy.

When there is a technology capable of coordinating traffic lights, with green times calculated through sophisticated mathematical algorithms, a moderate reduction of travel times (or congestion) can be experienced as it is stated by the references Diakaki *et al.* (2002), Dinopoulou *et al.* (2006). We have also verified this reduction at least in numerical examples made on microscopic simulators.

Our aim is to apply this methodology in the real case of the medium-sized city of Tandil, Argentina. This city is making up an Urban Traffic Control System which in its final stage will consist of a set of dynamic observers implemented through video-cameras in some junctions linked to a central computer that will compute the optimal green times and send them back to the linked junctions.

The methodology presented here is fundamentally based on the work of M. Papageorgiou and his coworkers and students (see Diakaki *et al.* (2002), Dinopoulou *et al.* (2006) and the references therein). It is called Traffic-responsive Urban Control (TUC) and is derived from a discrete time controlled model of the evolution of vehicle queues in each junction where the control of the system is made through the green times.

More precisely, we consider the macroscopic traffic model proposed by Gazis and Potts (1963) known as "store and forward" which is a linear model. The control variables are the traffic-light times at each intersection and the observed variables are the queue lengths on each arc. The desired objective is to minimize the total waiting time, so we consider as control objective the reduction of the number of cars actually present on the network during the analyzed period.

The TUC strategy can be described as follows. Thanks to the linearity of the system and the quadratic form of the objective function LQ methodology can be used to obtain a feedback matrix. Nevertheless, the restrictions imposed by traffic-light duration cannot be considered as LQ theory could not afford them, so the solution must be modified. This is done by a projection of the computed solution on the feasible solution set with an efficient algorithm that allows almost real time computation.

In the next sections we present the model (section II) and the control design and its projections over the feasible control set (section III). In the section IV we describe a micro simulator based on discrete event system theory, designed and developed to test the strategy. In section IV we present numerical results and tests and finally, we present the conclusion.

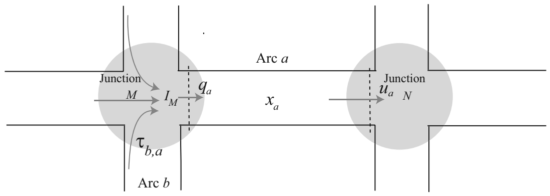


Figure 1: Variable definition.

## II. DYNAMIC MODEL

The dynamic equations for the mathematical model are based on, the now well established, Store and Forward model due to Gazis and Potts (1963). The choice of this model is due to the simplifications it imposes on the equations that will allow us to write them as linear equations on the number of vehicles and the green time of the junctions.

The network is represented by a directed graph composed of nodes and arcs. The nodes $j \in J$ represent intersections and the arcs $a \in A$ the unidirectional travel links. On every arc, the dynamic equation represents the progress of the total number of vehicles on the arc, expressed as private vehicle unit (PVU) (for example a bus equals 2,3 PVU).

The traffic dynamics on each arc $a$ is modeled using the vehicle-conservation equation (Diakaki *et al.*, 2002),

$$x_a(k+1) = x_a(k) + T[q_a(k) - u_a(k)], \tag{1}$$

where $x_a$ is the number of cars on the link expressed in PVU, $q_a$ and $u_a$ are the inflow and the outflow of link $a$ during $[kT, (k+1)T]$ where $k$ is the discrete time step and $T$ is the sampling time. See Fig. 1 to clarify the relations between the variables. Here we have neglected the traffic generated and consumed in each link, it would be easy to include them without substantially changing the current development.

In order to formulate the equations for $q$ and $u$ we will consider the saturation flow of each link $S_a$, that represents the maximum traffic flow that can exit the link, expressed in PVU/s. The *Store and Forward* model assumes that the vehicles reaching the arc's end are stored there and exit with rate $S_a$ during the green light. Hence, we can transform queue-legth quantities into flow ones writing:

$$u_a(k) = \frac{S_a.G_a(k)}{C}, \tag{2}$$

where $C$ is the cycle time and $G_a(k)$ is the effective green time of link $a$, i.e., the green light duration attributed to arc $a$ during the traffic light cycle $C$ of the intersection situated at the arc exit, and will be the control variable in our approach. If the green light periods are attributed to arc $a$ during different phases (see Fig. 2), $G_a(k)$ is equal to the sum of all of these green light durations,

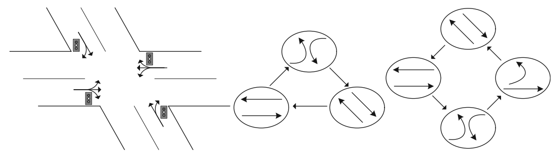

Figure 2: A typical junction with two possible phase definitions.

$$G_a(k) = \sum_{j,i \in P_j^a} G_{j,i}(k), \tag{3}$$

where $P_j^a$ is the set of the intersection phases $j$ during which arc $a$ has the green light. It also assumes that the outflow is distributed among the different following links according to the coefficients $\tau_{ab}$, called turning rates, that represent the proportion of outflow from $a$ entering in arc $b$.

If the link $a$ originates at the junction $M$, the inflow traffic rate entering arc $a$ can be written as the sum of the outflow traffic rates coming from the arcs entering junction $M$ (other than $a$). If the arc $b$ precedes arc $a$, the corresponding flow is $\tau_{ba}u_b$, so the total flow entering arc $a$ is

$$q_a(k) = \sum_{b \in I_M} \tau_{b,a} u_b(k), \tag{4}$$

where $I_M$ is the set of arcs entering junction $M$, and we have defined $\tau_{aa} = 0$.

Replacing all the previous definitions in the Eq. (1), we obtain the following model :

$$\begin{aligned} x_a(k+1) = x_a(k) + \frac{T}{C}[&\sum_{b \in I_M} \tau_{b,a} S_b G_{M,b}(k) \\ &- S_a \sum_{j,i \in P_j^a} G_{j,i}(k)] \end{aligned} \tag{5}$$

or in matrix form :

$$X(k+1) = X(k) + B.G(k), \tag{6}$$

where $B$ is a matrix of dimension $N_L \times N_P$, $N_L$ is the number of links and $N_P$ is the total number of phases on the network.

This modeling is possible under the following assumptions:

- the sampling time interval T is at least equal to the duration of the light cycle C, we will use $T = C$,
- the gaps between the intersections are not taken into account,
- variations in the queue are neglected, which means that the model considers that all the input flows on the arc have the green phase at the same time.

## III. OPTIMAL CONTROL PROBLEM

Here we describe the TUC methodology as it is presented in Diakaki *et al.* (2002), Dinopoulou *et al.* (2006). We pose the optimal control problem, the control means that we will be able to choose the green light times in order to modify the flows. The optimality will be measured in terms of the number of vehicles on the system. In this section, we will make explicit these definitions, when doing so, we keep in mind that we want to obtain a simply computable global green time. As we have linear dynamics, choosing a quadratic objective function and imposing no restrictions will make the optimal control problem over an infinite horizon belong to the LQ class. The importance of that relies in the fact that the optimal solution can be written as a feedback law and the matrix that defines this law is the solution of a matrix equation (Riccati equation) stated in terms of the given data.

From the viewpoint of the traffic regulation, our objective is to improve the traffic conditions on the network. The objective function need to be quadratic in terms of the state and control variables to rest in the LQ case, the general form of these functions is

$$J(x,u) = \int_0^\infty \alpha_x \|x\|^2_{Q_x} + \alpha_u \|u\|^2_{Q_u}, \tag{7}$$

where $Q_x$ and $Q_u$ are positive definite matrices that allow to weigh differently the components of $x$ and $u$, and $\alpha_x$ and $\alpha_u$ are non negative coefficients. These conditions guarantee that the function $J$ will be convex (strongly if $\alpha_{x,u} > 0$) which in turn guarantees the existence (and uniqueness) of the solution over the closed convex set defined by the linear dynamic Eqs. (1).

The choice of the matrices $Q_x$ and $Q_u$ could be used to obtain different relative improvements among the arcs. For example, bus traversed arcs could be given heavier weights in order to reduce congestion given priority to those arcs, and so to public transport. This and other strategies with public transport priority have been analyzed in Bhouri and Lotito (2005).

In our (discrete time) case we propose the following objective function

$$J(G) = \sum_{k=0}^{\infty} (\alpha \|X(k)\|^2 + \beta \|G(k)\|^2), \tag{8}$$

where $\alpha$ and $\beta$ are non-negative weighing parameters and the values of $X$ are given by the dynamic equations (5) and (6).

We can give an interpretation to each term in the objective function (8), the first term of the criteria aims at reducing the number of vehicles on every arc on the network and thus to equalize the congestion on every arc; the second term helps to avoid large variations of the control (green light times).

### A. Control Law

The problem of optimal control consists in minimizing the criteria given by Eq. (8) respecting the dynamics of the system given by the Eqs. (6). In order to avoid working with the input and exit flows we define a nominal green time $G^N$ that solves $BG^N = 0$, in such case the corresponding nominal state is constant and we can work with the following dynamic equation

$$X(k+1) = AX(k) + B\Delta G(k), \tag{9}$$

where $\Delta G(k) = G(k) - G^N$.

Using the LQ optimization method, the applied command law is given by the following equation

$$G(k) = G^N - F.X(k), \tag{10}$$

where $F$ is the Feedback matrix defined as $F = (R + B^T PB)^{-1} B^T PA$ and the matrix $P$ solves the Riccati matrix equation $P = Q + A^T PA - A^T PBF$ which depends on the coefficients $\alpha$, $\beta$, and $\gamma$ of the objective function through matrices $Q$ ad $R$.

Applying the Eq. (10) to $G(k)$ and to $G(k-1)$, by a simple substraction, one obtains

$$G(k) = G(k-1) - F.(X(k) - X(k-1)), \tag{11}$$

the use of this equation rather than of equation (11), avoids the estimation of the nominal values of the control.

It should be noted that the choice of an infinite time horizon in Eq. (8) implies that the Feedback matrix $F$ is time independent. This choice is justified by the will for a real time command of the intersection lights and thus by the simplification of the calculations for each command.

### B. The constraints

The solution of the optimal command problem by the LQ method doesn't enable us to take the constraints into account because the Riccati equation will no longer be valid. However, for operative needs, at every intersection $j$, the durations of green lights should comply with a certain number of constraints:

- the cycle duration ($C$),
- the phase diagram : all of phases $P_j$ should have their green light within the cycle,
- the clearance times between phases $R_j$,

which implies:

$$\sum_{i \in P_j} G_{j,i} + R_j = C, \quad \forall j. \tag{12}$$

On the other hand, the duration of every green light is limited by a maximum and a minimum. Indeed, a too long red light duration can be interpreted by users as a malfunction of the intersection lights and imply their non-compliance:

$$G_{j,i,min} \leq G_{j,i} \leq G_{j,i,max}. \tag{13}$$

We solved this problem through a projection of the obtained command values onto the set of feasible values defined by the above constraints. It means to obtain the closest (in some distance) values to the optimal but not feasible ones. The projection step means to solve the following quadratic optimization problem that includes the constraints (12) and (13),

$$\begin{array}{ll} \min\limits_{\overline{G}} & \sum_{i \in P_j} (G_{j,i} - \overline{G}_{j,i})^2, \\ \text{s.t.} & \quad (12) \text{ and } (13). \end{array} \tag{14}$$

This problem belongs to the class of Quadratic Knapsacks problems and the numerical solution was done according to the algorithm presented in Lotito (2006).

The authors of the TUC strategy (Diakaki *et al.*, 2002, Dinopoulou *et al.*, 2006) propose another algorithm to compute the projection, we have not tested in practice if it outperforms the one presented here, as the theoretical results show that their computational complexity is similar.

## IV. MICROSCOPIC SIMULATOR

The availability of mathematical models describing the dynamics of vehicles is fundamental in order to apply control theory. The model previously presented, stated in terms of continuous vehicle flows, is considered as a Macroscopic model, in comparison to Microscopic models, which consider the position of each vehicle.

In order to perform computational tests, it is crucial to use a model of a different nature from the one used to design the control strategy. Microscopic simulators are mostly based mostly in Cellular Automata (Nagel, 2002, Lotito *et al.*, 2005), or on the Car-following model (Papageorgiou, 1983). The last one was chosen to develop our simulator. Hence we considered a discrete event system such that at each time step there are vehicles entering at fixed rates and interacting following certain rules.

The position of the vehicles evolves according to the equations:

$$y_n - y_{n+1} = L + S\dot{y}_{n+1} \tag{15}$$

where $n$ is the precedent vehicle, $L$ is the vehicle length, and $S$ is a separation coefficient. In this formula, the vehicle $n+1$ is separated from the precedent by a fixed distance ($L$) plus a distance proportional to its speed.

After differentiating Eq. (15) it results:

$$\ddot{y}_{n+1} = \frac{1}{S}(\dot{y}_n(t) - \dot{y}_{n+1}(t)) \tag{16}$$

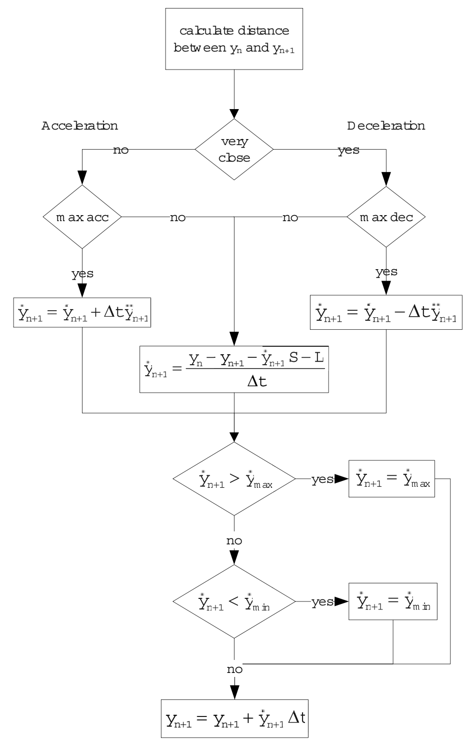


Figure 3: Flow diagram of the micro-simulator algorithmic kernel.

showing that the acceleration (or deceleration) is proportional to the relative speed between successive vehicles. Defining the factor $1/S$ and introducing a delay coefficient, the following formula for the speeds is obtained:

$$\ddot{y}_{n+1}(t+\tau) = \frac{1}{S}(\dot{y}_n(t) - \dot{y}_{n+1}(t)) \tag{17}$$

Even if these considerations are common in practice, a set of parameters that are randomly distributed among the different vehicles is used here. These parameters include: maximum speed, length of the vehicle, behavior, anxiety, etc. Now, in the proposed model, the vehicle position is given by a set of rules that includes: precedent vehicle position, relative speed, maximum speed, maximum acceleration (or deceleration).

In Fig. 3 we presented a flow diagram of the algorithmic kernel of the microsimulator.

With the aim to develop a microsimulator that fit as good as possible the real traffic dynamic, the car following model has been complemented with multilines and lane changing strategies (Hidas, 2002). A lane change may be necessary for a number of reasons, for example, when a driver wants to turn left or right at the end of the line, he should be located in the corresponding line. However, if the driver can not take

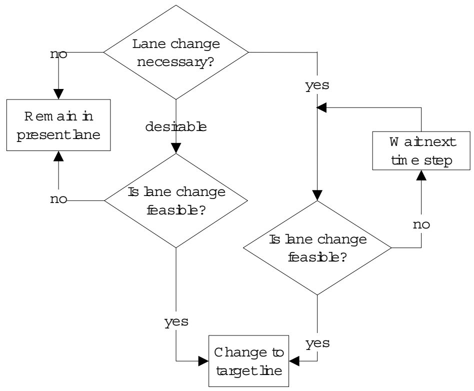


Figure 4: Flow diagram of the lane changing process.

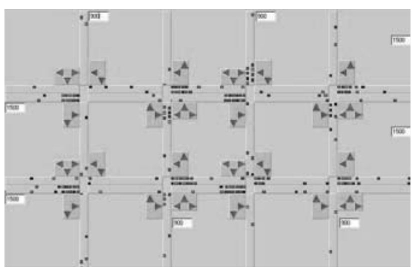

Figure 5: Screen-shot of the micro-simulator.

the correspond line, he should stop and wait for the next time step to located in the right line. This case is named compulsory lane change.

In the other hand, if the driver wants to change because he will overtake another driver, or take the correct line for a future turn, the lane change is called desirable. In this case, if the driver can not change, it keeps moving in the original lane.

In Fig. 4 we presented a flow diagram of the lane changing process used in the microsimulator. Another addition to the model was the inclusion of several vehicles types as an urban traffic network generally has. For example, trucks and public transport vehicles have very different dynamics. The developed microsimulator has a vehicle type modelling a public transport that interacts with private cars.

Consequently, at each time step new vehicles enter the system according to predefined rates of entering arcs, and interact with the existing ones following the described rules. The simulation is shown with the aid of a graphical interface, which also serves to enter the entering rates. A screen-shot of the simulator is shown in Fig. 5.

The calibration made for the city of Tandil (Argentina) in a study requested by the Transit Authorities (Lotito and Mayorano, 2008) has shown that the simulation really fit the dynamic behavior. Indeed, the

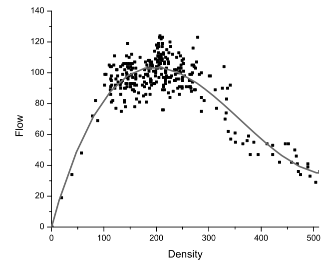


Figure 6: The resulting traffic fundamental diagram.

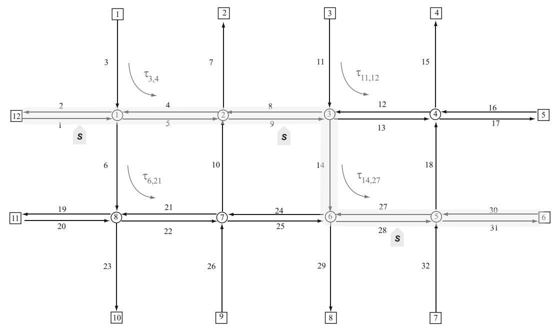

Figure 7: The example network

simulation results approach very well the Fundamental Diagram of Traffic provided by the city (see Fig. 6).

## V. NUMERICAL EXPERIMENTS

In this section we expose numerical tests made over a small example network. The numerical tests have been made using a micro simulator based on *car following models* (Gabard, 1991, Helbing *et al.*, 2002). This simulator has been validated with real data taken from the city of Tandil in Argentine (Lotito and Mayorano, 2008).

### A. Example Network

The chosen example network has 8 intersections, 32 links, and a bus line (highlighted) as shown in the Fig. 7.

Each intersection has the general form given in Fig. 8 and has three phases. In the nominal state, each one is given 50%, 10%, and 30% of the green time respectively, as shown in Fig. 9. In this figure the turning rates for each movement are also shown.

The saturation flow is 0.5 veh./s everywhere and the entering flows are given in the following table:

Arcs / Flow (veh./s)

| 1 | 3 | 11 | 16 | 20 | 26 | 30 | 32 |
|---|---|---|---|---|---|---|---|
| 0.25 | 0.15 | 0.15 | 0.25 | 0.25 | 0.15 | 0.25 | 0.15 |

The flow $d$ originated and consumed at each link is determined in such a way that $B * \bar{G} + d = 0$, thus guaranting that the proposed 'nominal' state is indeed nominal.

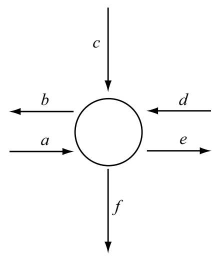


Figure 8: Diagram of a general junction

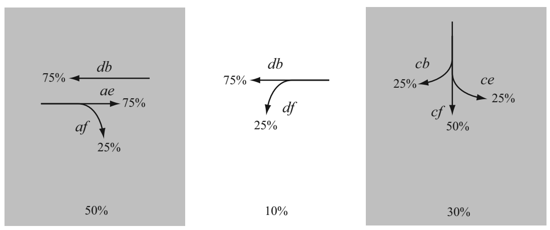


Figure 9: Diagram of the different phases with the proportion of green time and the turning rates, for a given junction

In this example, the bus line (see Fig. 7 in yellow) enters on node 12 and traverses intersections 1, 5, 9, 14, 28 and 31, making a stop before intersections 1, 9 and 28, (shown with an S in the Fig. 7). The frequency of the bus line is 1 bus every 3 time steps.

The feedback matrices are used to compute the variation on phases duration from the variation of the number of cars on every arc, as it is described mathematically by Eq. (11). Hence the rows of the feedback matrices correspond to phase durations and the columns to arc occupancy. Each phase allows for some movements (see Fig. 8 and Table 1).

| Phase | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Turning mov. | $1 \to 5$ | $4 \to 2$ | $3 \to 2$ | $8 \to 7$ |
| | $1 \to 6$ | $4 \to 6$ | $3 \to 6$ | $8 \to 4$ |
| | $4 \to 2$ | | $3 \to 5$ | $5 \to 9$ |
| Phase | 5 | 6 | 7 | 8 |
| Turning mov. | $5 \to 9$ | $10 \to 4$ | $9 \to 13$ | $12 \to 8$ |
| | $5 \to 7$ | $10 \to 7$ | $9 \to 14$ | $12 \to 14$ |
| | | $10 \to 9$ | $12 \to 8$ | |
| Phase | 9 | 10 | 11 | 12 |
| Turning mov. | $11 \to 8$ | $16 \to 12$ | $13 \to 17$ | $18 \to 17$ |
| | $11 \to 14$ | $16 \to 15$ | $13 \to 15$ | $18 \to 15$ |
| | $11 \to 13$ | $13 \to 17$ | | $18 \to 13$ |

Table 1: Turning movements corresponding to each phase

### B. Numerical results

In order to see the control in action, the following perturbation scheme was considered: From 0:00 to 1:00, the entering rates are the nominal ones. From 1:00 to 1:30, the entering rates on arcs 1, 11 and 16 are increased by 50%. From 1:30 to 5:00, the entering rates are reduced to nominal values.

The numerical results obtained by the described simulations are presented in Table 2. In order to reduce the impact of stochastic variations, the values correspond to an average of 100 runs of the simulator. The first one row, named None, corresponds to the base case in which no strategy is applied, and the second one with the base in which TUC strategy is applied. In the column Total, the total congestion is shown, it is computed as $\sum_a \sum_k X_a(k)$. The column Bus, is the sum of the number of cars only for the arcs traversed by bus, and column $X_b$ is the sum of the numbers of cars, which are at the same arc of a bus at the same time. The column BMT corresponds to the bus mean travel time measured in seconds.

| | Total | Bus | $X_b$ | BMT (s) | Sat. |
|---|---|---|---|---|---|
| None | 774.6 | 171.2 | 88.8 | 1106.6 | 100% |
| TUC | **762.0** | **152.5** | **77.0** | **1061.8** | **65%** |

Table 2: Simulation results

For the sake of comparison we have defined $\Delta X$ and $\Delta X_b$ by the formulae:

$$\Delta X = \left( \sum_{t=0}^{T} \|X(t) - \tilde{X}(t)\|^2 \right)^{\frac{1}{2}} \tag{18}$$

and

$$\Delta X_b = \left( \sum_{t=0}^{T} X_b(t) \|X(t) - \tilde{X}(t)\|^2 \right)^{\frac{1}{2}} \tag{19}$$

The value of $\Delta X$ measures the distance to the nominal state for the total number of vehicles in the system during the whole period. The value of $\Delta X_b$ measures the distance to the nominal state for the total number of vehicles that are present at the same time with a bus on the same arc during the whole period. Those values are shown in the corresponding columns in the table 3.

| | $\Delta X$ | $\Delta X_b$ |
|---|---|---|
| None | 81.7 | 39.9 |
| TUC | **62.3** | 16.7 |

Table 3: Simulation results

We call an arc ***saturated*** when there is a bus on it and the number of cars is greater than the nominal state increased by 25%. The definition is taken from

Diakaki (1999) but modified to take into account the bus. In the column Sat. the percentage of saturated arcs with respect to the base case is shown.

In Table 2, the numbers in bold represent the best value in each column. From the results shown we can see that the TUC strategy is the best for each particular criterium.

In Fig. 10 we can see the evolution of $\Delta X$ and $\Delta X_b$, for the uncontrolled case and the TUC strategy. We can observe that the TUC strategy restores the nominal flow over all the arcs faster than None (see left graph), and in the case of the flow together with buses it is too the strategy which better restores the nominal regime (see right graph).

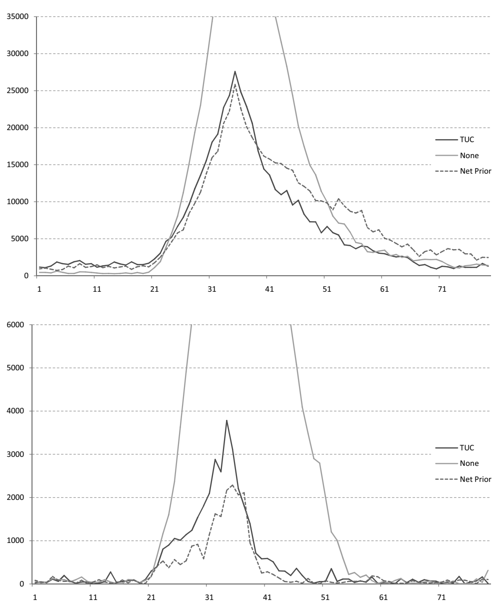


Figure 10: Evolution of $\Delta X$ (left) and $\Delta X_b$ (right) in the uncontrolled case (none) and when the TUC and the NetPrior Strategies are applied. The graphs of the uncontrolled case have been truncated, the peaks are 61307 and 15487.

## VI. CONCLUSION

In this work we have considered the problem of the reduction of congestion in an urban traffic network. The application of the TUC strategy to a simple example and the development of a microscopic simulator to test it have been presented. The obtained numerical results confirmed the results obtained previously by the authors of TUC and show that, at least for the micro simulations, TUC is a good option for congestion reduction. This strategy is realistic and easily implementable, facilitating the real time computation of green times. More advanced tests on the real traffic urban network of Tandil, with traffic real data will be achieved to validate it thoroughly. The application of this methodology can also be envisaged to reduce congestion giving priority to public transport as it has been done in the related work Farhi *et al.* (2006).

## ACKNOWLEDGE

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