International Journal of Control
Vol. 84, No. 3, March 2011, 432–444

Robust model predictive control of Wiener systems

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(*Received 28 May 2010; final version received 1 February 2011*)

Block-oriented models (BOMs) have shown to be appealing and efficient as nonlinear representations for many applications. They are at the same time valid and simple models in a more extensive region than time-invariant linear models. In this work, Wiener models are considered. They are one of the most diffused BOMs, and their structure consists in a linear dynamics in cascade with a nonlinear static block. Particularly, the problem of control of these systems in the presence of uncertainty is treated. The proposed methodology makes use of a robust identification procedure in order to obtain a robust model to represent the uncertain system. This model is then employed to design a model predictive controller. The mathematical problem involved in the controller design is formulated in the context of the existing linear matrix inequalities (LMI) theory. The main feature of this approach is that it takes advantage of the static nature of the nonlinearity, which allows to solve the control problem by focusing only in the linear dynamics. This formulation results in a simplified design procedure, because the original nonlinear model predictive control (MPC) problem turns into a linear one.

Keywords: Wiener system; MPC; IMI; uncertainty; optimisation

1. Introduction

There are very few controller design techniques that can be proven to stabilise processes in the presence of nonlinearities and constraints. Model predictive control (MPC) is one of these techniques. MPC refers to a class of computer control algorithms that control the future behaviour of a plant through the use of an explicit process model. At each control interval, the MPC algorithm computes an open-loop sequence of manipulated variable adjustments in order to optimise future plant behaviour. The first input in the optimal sequence is injected into the plant, and the entire optimisation is repeated at subsequent control intervals (Qin and Badwell 1997).

Though industrial processes are inherently nonlinear, the vast majority of MPC applications up to date are based on linear dynamic models, the step and impulse response models being the most common ones are derived from the convolution integral. There are several potential reasons for this, for example, by using a linear model and a quadratic objective function, the nominal MPC algorithm takes the form of a highly structured convex quadratic program (QP), for which reliable solution algorithms and software can be easily found (Wright 1997). This is important because the algorithm solution must reliably converge to the optimum value in no more than few tens of seconds in order to be useful in manufacturing applications.

Nevertheless, there are many cases where nonlinear effects are significant enough to justify the use of nonlinear model predictive control (NMPC). Those cases include at least two broad categories of applications: (a) regulator control problems where the process is highly nonlinear and subject to large frequent disturbances, and (b) servo control problems where the operating points change frequently and span a sufficiently wide range of nonlinear process dynamics.

With the introduction of a nonlinear dynamics model within the NMPC algorithm, the complexity of the predictive control problem increases significantly. This issue has been thoroughly dealt with in the review papers by Bequette (1991) and Henson (1998), where they presented various approaches for handling nonlinear systems via MPC.

In particular, Wiener models (WMs) have a special structure that facilitates their application to NMPC (Norquay, Palazoglu, and Romagnoli 1998; Gerkšič, Juričic, Strmčnic, and Matko 2000; Lussón-Cervantes, Agamennoni, and Figueroa 2003). These models consist in a process with linear dynamics followed by a nonlinear gain (Figure 1), and can represent many of the nonlinearities commonly encountered in industrial processes. Due to the static nature of the nonlinearities, they can be removed from the control problem, which allows solving the NMPC problem as a linear MPC (LMPC) one.

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ISSN 0020–7179 print/ISSN 1366–5820 online

DOI: 10.1080/00207179.2011.560191
http://www.informaworld.com

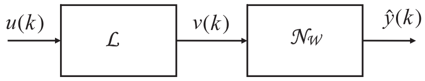

Figure 1. Wiener model.

The use of WMs has been treated in the literature in various contexts. WMs have proved to be useful for application in several fields, such as in chemical processes (Pajunen 1992; Kalafatis, Arifin, Wang, and Cluett 1995; Zhu 1999; Pearson and Pottmann 2000) biological processes (Korenberg 1973; Hunter and Korenberg 1986), communications (Kang, Cho, and Youn 1998, 1999; Cheong, Werner, Cousseau, and Laakso 2005) and control (Norquay et al. 1998; Gerkšič et al. 2000; Lussón-Cervantes et al. 2003; Biagiola, Agamennoni, and Figueroa 2004).

In this article, the design of robust model predictive controllers for Wiener systems is addressed. In general, robust model predictive control (RMPC) are MPC algorithms that explicitly consider the process model uncertainties in the control law computation. Since the seminal work by Campo and Morari (1987), other several approaches have been developed for the treatment of robustness in linear MPC. Some of these algorithms assume that the process model is described by an infinite set of linear models and propose a min–max optimisation to obtain the control law (Allwright and Papavasiliou 1992; Zheng and Morari 1993; Oliveira, Amaral, Favier, and Dumont 2000). Another approach for RMPC is based on state spaces models with polytopic uncertainties and cost function defined over an infinite horizon (Kothare, Balakrishnan, and Morari 1996; Cuzzola, Geromel, and Morari 2002; Wan and Kothare 2002; Mao 2003; Ding, Xi, and Li 2004), in which case the control is computed by a convex optimisation subject to a set of linear matrix inequalities (LMI). However, to the best of the authors' knowledge, no RMPC results are available in the literature for the case of WMs.

In this work a controller synthesis procedure is developed for WMs with bounded uncertainty. A model predictive-based strategy is used for the controller design, recalling the methodology presented in Kothare et al. (1996) and recently extended in Araújo and Oliveira (2009) to linear models represented by orthonormal basis functions (OBFs) (Heuberger, Van den Hof, and Wahlberg 2005). The uncertainty in the nonlinear model is explicitly considered in the controller synthesis. This is achieved by the inclusion of uncertainty bounds previously determined for the WM through the characterisation algorithm presented in Figueroa, Biagiola, and Agamennoni (2008) and generalised in Biagiola and Figueroa (2009).

This article is organised in the following way. In Section 2 some previous results on RMPC are revised. Section 3 deals with the problem of identifying a WM capable to reflect the observed uncertainty. Section 4 presents the RMPC synthesis for such identified WMs, which is the main contribution of this article. Two different examples are presented in Section 5 to illustrate the modelling and control strategies. This article concludes with some final remarks in Section 6.

2. RMPC for linear systems

In the seminal work by Kothare et al. (1996), the 'multi-model' paradigm for RMPC was introduced. The mathematical expression of this model is

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k), \\ [A \quad B] &\in \Omega, \end{aligned} \tag{1}$$

where $u(k) \in \Re^{n_u}$ is the control input, $x(k) \in \Re^{n_x}$ is the state of the plant, $y(k) \in \Re^{n_y}$ is the plant output and Ω is the set of uncertainty, described as a polytope

$$\Omega = Co\{[A_1 \quad B_1],\ [A_2 \quad B_2], \ldots, [A_L \quad B_L]\}, \tag{2}$$

where Co means convex hull. In other words, if $[A \quad B] \in \Omega$ then, for some $\lambda_i \geq 0$; $i = 1, \ldots, L$ with $\sum \lambda_i = 1$ we have

$$[A \quad B] = \sum_{i=1}^{L} \lambda_i [A_i \quad B_i]. \tag{3}$$

For this process description, the unconstrained RMPC performance objective can be posed as (Kothare et al. 1996)

$$\min_{u(k+i|k), i=0,1,\ldots,m} \ \max_{[A \quad B] \in \Omega} J_\infty(k) \tag{4}$$

with

$$\begin{aligned} J_\infty(k) = \sum_{i=0}^{\infty} \big[& x(k+i|k)^T Q_1 x(k+i|k) \\ & + u(k+i|k)^T R u(k+i|k) \big], \end{aligned} \tag{5}$$

where $x(k+i|k)$ and $u(k+i|k)$ are the state and control move, respectively, at time $k+i$, computed based on measurements at time k. In particular, $x(k|k)$ is the measured state at time k and $u(k|k)$ is the control move to be implemented at time k. It is assumed that there is no control action after time $k+m-1$ (i.e. $u(k+i|k) = 0$ for $i \geq m$), where m is the control horizon.

Equation (4) stands for a 'min–max' problem. The maximisation is over the set of possible plants, and corresponds to choosing a plant $[A \ B] \in \Omega$, which, if used as model for predictions, would lead to the worst case value of $J_\infty(k)$ among all plants in Ω. This worst case value is then minimised over the present and future control moves $u(k+i|k)$, $i=0,1,\ldots,m$.

To solve this problem, an upper bound on $J_\infty(k)$ can be developed in the form of

$$J_\infty(k) \le V(x(k|k)), \tag{6}$$

where $V(x)=x^TPx$ with $P>0$. Thus

$$\max_{[A\quad B]\in\Omega} J_\infty(k) \le V(x(k|k)). \tag{7}$$

This expression gives an upper bound on the robust performance objective. Thus solution of the problem can be obtained on the basis of the following theorem.

Theorem 2.1: *Let $x(k)=x(k|k)$ be the state of the uncertain system* (1). *Then the state feedback matrix F in the control law $u(k+i|k)=Fx(k+i|k)$, $i\ge 0$ that minimises the upper bound $V(x(k|k))$ on the robust performance objective function at sampling time k is given by*

$$F = YQ^{-1}, \tag{8}$$

where $Q>0$ and Y are obtained from the solution (*if it exists*) *of the following linear objective minimisation problem with LMI constraints*:

$$\min_{\gamma,Q,Y} \gamma \tag{9}$$

subject to

$$\begin{bmatrix} 1 & x(k|k)^T \\ x(k|k) & Q \end{bmatrix} \ge 0 \tag{10}$$

and

$$\begin{bmatrix} Q & QA_j^T + Y^TB_j^T & QQ_1^{1/2} & Y^TR^{1/2} \\ A_jQ+B_jY & Q & 0 & 0 \\ QQ_1^{1/2} & 0 & \gamma I & 0 \\ R^{1/2}Y & 0 & 0 & \gamma I \end{bmatrix} \ge 0, \quad j=1,2,\ldots,L. \tag{11}$$

Proof: See Kothare et al. (1996). □

This problem could be extended to consider constraints on the manipulated variables and on the process outputs. If constraints on the control variables are set by $\|u(k+i|k)\|_2 \le u_{\max}$, for $i\ge 0$, the following LMI should be included as a constraint in problem (9)–(11)

$$\begin{bmatrix} u_{\max}^2 I & Y \\ Y^T & Q \end{bmatrix} \ge 0. \tag{12}$$

In the presence of output constraints of the form

$$\max_{[A\quad B]\in\Omega} \| y(k+i|k)\|_2 \le y_{\max}, \tag{13}$$

the LMI constraint to be included in (9)–(11) is

$$\begin{bmatrix} Q & (A_jQ+B_jY)^TC^T \\ C(A_jQ+B_jY) & y_{\max}^2 I \end{bmatrix} \ge 0, \quad j=1,2,\ldots,L. \tag{14}$$

Based on these results, Araújo and Oliveira (2009) provided an extension of this approach for plants modelled with OBFs in a recent work. In this particular case, the model adopts the following expression:

$$\begin{aligned} \tilde{x}(k+1) &= \tilde{A}\tilde{x}(k) + \tilde{B}\tilde{u}(k), \\ \tilde{y}(k) &= \tilde{C}\tilde{x}(k), \end{aligned} \tag{15}$$

where matrices $\tilde{A}$, $\tilde{B}$ and vector $\tilde{x}(\cdot)$ are formed by an appropriate concatenation of the OBFs and input signals which are not 'contaminated' with uncertainty. Moreover, the states $\tilde{x}$ have no connections with the system states $x(k)$, but are the outputs of the OBF model when excited by the input u. In these models, the uncertainty is concentrated on matrix $\tilde{C}$ that gathers the parameters of the model.

The convex hull is defined as

$$\Omega_C = Co\{C_1, C_2, \ldots, C_L\}. \tag{16}$$

In other words, if $\tilde{C}\in\Omega_C$ then there exists any $\lambda_i \ge 0$; $i=1,\ldots,L$ with $\sum\lambda_i = 1$.

Therefore

$$\tilde{C} = \sum_{i=1}^{L} \lambda_i C_i. \tag{17}$$

Rewriting the model in deviation variables, and taking into account a constant setpoint (w) along the control horizon m, the following model is obtained (Araújo and Oliveira 2009):

$$\begin{aligned} \begin{bmatrix} \Delta\tilde{x}(k+1) \\ \tilde{y}(k+1)-w \end{bmatrix} &= \begin{bmatrix} \tilde{A} & 0 \\ \tilde{C}\tilde{A} & I \end{bmatrix} \begin{bmatrix} \Delta\tilde{x}(k) \\ \tilde{y}(k)-w \end{bmatrix} + \begin{bmatrix} \tilde{B} \\ \tilde{C}\tilde{B} \end{bmatrix} \Delta\tilde{u}(k) \\ \begin{bmatrix} \tilde{y}(k)-w \end{bmatrix} &= \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} \Delta\tilde{x}(k) \\ \tilde{y}(k)-w \end{bmatrix}, \end{aligned} \tag{18}$$

where $\Delta = 1-q^{-1}$ and q^{-1} is the delay operator.

Now, the following definitions are made:

$$A = \begin{bmatrix} \tilde{A} & 0 \\ \tilde{C}\tilde{A} & I \end{bmatrix}, \quad B = \begin{bmatrix} \tilde{B} \\ \tilde{C}\tilde{B} \end{bmatrix}, \quad C = \begin{bmatrix} 0 & I \end{bmatrix},$$

$$x = \begin{bmatrix} \Delta\tilde{x}(k) \\ \tilde{y}(k)-w \end{bmatrix}, \quad y(k) = \tilde{y}(k) - w$$

$$u(k) = \Delta\tilde{u}(k), \quad A_i = \begin{bmatrix} \tilde{A} & 0 \\ \tilde{C}_i\tilde{A} & I \end{bmatrix} \quad \text{and}$$

$$B_i = \begin{bmatrix} \tilde{B} \\ \tilde{C}_i\tilde{B} \end{bmatrix} \quad \text{for } i=1,\ldots,L.$$

Then, it is possible to solve the RMPC problem given the following objective function:

$$J_\infty(k) = \sum_{i=0}^{\infty}\big\{(\tilde{y}(k+i|k)-w)^T Q_1(\tilde{y}(k+i|k)-w) + \Delta\tilde{u}(k+i|k)^T R \Delta\tilde{u}(k+i|k)\big\}. \quad (19)$$

This can be accomplished using the result of Theorem 2.1.

The next section deals with the problem of robust identification of uncertain WMs, in which an ad hoc developed identification approach is described. The identification task is the previous stage to the controller design work, which is then treated in Section 4.

3. Uncertain WM identification

As it was introduced in Section 1, WMs structure consists of an LTI system $\mathcal{L}$ followed by a static nonlinearity $\mathcal{N}_W$ (Figure 1). Then, the linear model $\mathcal{L}$ maps the input sequence $\{u(k)\}$ into the intermediate sequence $\{v(k)\}$, and the overall model output is the output of the nonlinear block, i.e. $\hat{y}(k)=\mathcal{N}_W(v(k))$. In general, the following parameterisation have been adopted (Gómez and Baeyens 2004; Falugi, Giarré, and Zappa 2005):

- *Linear block*

$$\mathcal{L}(q) = \sum_{i=1}^{M} h_i B_i(q^{-1}), \quad h = [h_1, \ldots, h_M]^T, \quad (20)$$

where $B_i(q^{-1})$ is any rational basis (Laguerre, Kautz, orthonormal, etc.).

- *Nonlinear block*

$$\mathcal{N}(x) = \sum_{i=1}^{N} p_i g_i(x), \quad p = [p_1, \ldots, p_N]^T, \quad (21)$$

where $g_i(\cdot):\Re \to \Re$ are a set of specified basis functions such as polynomial, trigonometric, piecewise linear functions.

In the identification of block-oriented models (BOMs), there is a scale factor which can be arbitrarily distributed between the linear block and the memoryless one without affecting the input–output characteristics of the model (Pottmann and Pearson 1998). In the following we assume $h_1=1$, since that any other value of this gain can be included in the linear block. Therefore, the identification task will involve the determination of $M+N-1$ unknown parameters.

The signal $v(k)$ can be written as[1]

$$v(k) = \mathcal{N}_W^{-1}(\hat{y}(k)) = \sum_{i=1}^{N} p_i g_i(\hat{y}(k)). \quad (22)$$

Note that the nonlinear block has been represented as in Equation (21), where $\mathcal{N}=\mathcal{N}_W^{-1}$ and $x=\hat{y}(k)$. From Figure 1, this signal can also be written as the output of the linear block,

$$v(k) = \sum_{i=1}^{M} h_i B_i(q^{-1})u(k). \quad (23)$$

Equating both sides of these equations (with the inclusion of an error function $\epsilon(k)$ to allow for modelling error), the following equation is obtained:

$$\epsilon(k) = \sum_{i=1}^{N} p_i g_i(y(k)) - B_1(q^{-1})u(k) - \sum_{i=2}^{M} h_i B_i(q^{-1})u(k), \quad (24)$$

which is a linear regression in the parameters. Defining θ and ϕ_k as follows:

$$\theta = [p_1, \ldots, p_N, h_2, \ldots, h_M]^T \quad (25)$$

$$\phi_k = \big[g_1(y(k)), \ldots, g_N(y(k)), -B_2(q^{-1})u(k), \ldots, -B_M(q^{-1})u(k)\big]^T, \quad (26)$$

then, Equation (24) can be written as

$$\epsilon(k) = \theta^T\phi_k - B_1(q^{-1})u(k). \quad (27)$$

Now, an estimate $\hat{\theta}$ of θ can be computed by minimising a quadratic criterion on the prediction errors $\epsilon(k)$, i.e. the least squares estimate (Kalafatis et al. 1995; Gómez and Baeyens 2004). It is well known that this estimate is given by

$$\hat{\theta} = \big(\Phi_K\Phi_K^T\big)^{-1}\Phi_K\Gamma, \quad (28)$$

where $\Gamma=[B_1(q^{-1})u(1), \ldots, B_1(q^{-1})u(K)]^T$ and $\Phi_K=[\phi_1, \ldots, \phi_K]$ use the set of K input/output data available from the process. Note that the practical identifiability condition must be imposed to θ, which implies that Φ is full column rank. Two main advantages of this approach are the relative simplicity and the uniqueness of the solution.

Now, estimates of the parameters $\hat{p}_i$ $(i=1,\ldots,N)$ and $\hat{h}_i$ $(i=2,\ldots,M)$ can be computed by partitioning the estimate $\hat{\theta}$, according to the definition of θ in (25). It is important to remark that we are identifying the inverse of the nonlinearity, which is frequently used in many control applications.

Now, let us consider the more general case which is the uncertain WM. In order to develop a methodology to characterise the uncertainties, we use the nominal description. To perform it, let us introduce a set of parameters $\mathcal{H}$ for the linear dynamic block and a set $\mathcal{P}$ for the parameters of the nonlinear block we want to

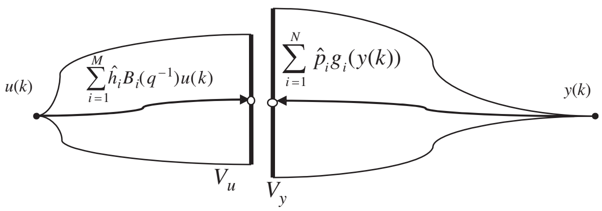

Figure 2. Uncertainty sets in a WM.

identify (i.e. $\mathcal{N}_W^{-1}$):

$$\mathcal{H} = \{h : h_i^l \leq h_i \leq h_i^u,\ 1 \leq i \leq M\} \tag{29}$$

$$\mathcal{P} = \{p : p_i^l \leq p_i \leq p_i^u,\ 1 \leq i \leq N\}, \tag{30}$$

where the subscript i means the ith entry of a vector.

Now, to determine the parameters bounds in the WM, let us first define some sets (Figueroa et al. 2008). Given the input datum u_k, the linear uncertain system defined by $\mathcal{H}$ maps at some specific time k over a set

$$\mathcal{V}_u = \left\{ v : v(k) = \sum_{i=1}^{M} h_i B_i(q^{-1})u(k),\ h \in \mathcal{H} \right\}. \tag{31}$$

Given an input $u(k)$, the basis term of order i, i.e. $h_iB_i(q^{-1})u(k)$, is a real number and the set $\mathcal{V}_u$ takes the form of $\mathcal{V}_u = \{v : v_l \leq v(k) \leq v_u\}$.

On the other hand, if we consider the uncertain description of the parameters in $\mathcal{P}$, a given output $y(k)$ maps at some specific time k over a set

$$\mathcal{V}_y = \left\{ v : v(k) = \sum_{i=1}^{N} p_i g_i(y(k)),\ p = [p_1 \dots p_N]^T \in \mathcal{P} \right\}. \tag{32}$$

This situation is illustrated in Figure 2. Remember that in order to obtain an uncertain model, every input data $u(k)$ should be mapped through the model to the corresponding $y(k)$. From this picture it is clear that the parameters set will match the uncertainties description if $\mathcal{V}_y \cap \mathcal{V}_u \neq \emptyset$ for all k. In this way, the input $u(k)$ is mapped onto $\mathcal{V}_u$ through the family of models generated by $\mathcal{H}$. Then, since $\mathcal{V}_y \cap \mathcal{V}_u \neq \emptyset$, the intermediate value (i.e. $v(k)$) will result in $y(k)$ through the inverse of $\mathcal{N}_w^{-1}$.

Now, let us analyse this situation in order to compute the parameters bounds to satisfy this condition. This determination is based on the whole input/output data available.

Note that the linear bases $B_i(q^{-1})$ are a set of real numbers for each input $u(k)$. Let $B(q^{-1})u(k)$ be the vector whose ith entry is the linear basis $B_i(q^{-1})u(k)$. Since the entries of $B(q^{-1})u(k)$ could be positive or negative, it is possible to split the vector. For this purpose, we define $B^+(u(k)) \triangleq \max(B(q^{-1})u(k), 0)$ and $B^-(u(k)) \triangleq \min(B(q^{-1})u(k), 0)$ and form the new vector $\gamma_B(k) \triangleq [(B^-(u(k)))^T,\ (B^+(u(k)))^T]^T$. Note that the construction of vector $B^-(u(k)) = \min(B(u(k)), 0)$ involves keeping all the negative elements in the vector and putting zero otherwise. Analogously, $B^+(u(k))$ keeps the positive elements and puts zero otherwise. Therefore, the resultant γ_B is a vector with all negative elements in the first rows and all positive elements in the last ones.

In a similar way, since the nonlinear bases $g_i(y(k))$ are real numbers for each output $y(k)$, it is possible to define $\gamma_g(k) \triangleq [(g^-(y(k)))^T, (g^+(y(k)))^T]^T$, where $g^+(y(k)) \triangleq \max(g(y(k)), 0)$ and $g^-(y(k)) \triangleq \min(g(y(k)), 0)$.

Theorem 3.1: *The bounds of the uncertain parameters h^l, h^u, p^l, p^u can be obtained by solving the following optimisation problem*:

$$\min_{h^l, h^u, p^l, p^u} \left(\alpha \sum_{i=2}^{M} (h_i^u - h_i^l) + (1-\alpha) \sum_{i=1}^{N} (p_i^u - p_i^l) \right) \tag{33}$$

subject to

$$\left[(h^l)^T, (h^u)^T, -(p^u)^T, -(p^l)^T\right] \begin{bmatrix} \gamma_B(k) \\ \gamma_g(k) \end{bmatrix} \geq 0; \quad k = 1, \dots, K \tag{34}$$

$$\left[(h^u)^T, (h^l)^T, -(p^l)^T, -(p^u)^T\right] \begin{bmatrix} \gamma_B(k) \\ \gamma_g(k) \end{bmatrix} \leq 0; \quad k = 1, \dots, K, \tag{35}$$

where the parameter $\alpha \in (0, 1)$ is a selected factor which allows to distribute the weight of the uncertainty between the linear and the nonlinear blocks. Note that the robust identification requirement $\mathcal{V}_y \cap \mathcal{V}_u \neq \emptyset$ must be ensured $\forall k$.

Proof: See Biagiola and Figueroa (2009). □

Note that the proposed approach for the identification problem allows to transform it into a linear programming problem with convex feasible region. The number of optimisation variables is twice the number of model parameters and the number of constraints is twice the number of the process data. Due to the suitable formulation of these problem, its solution is obtained in an efficient way.

4. RMPC for Wiener systems

The objective of this section is to design of an RMPC algorithm for uncertain WMs such as the ones identified in Section 3.

In order to develop a valid RMPC algorithm for this type of WMs with uncertainty, an optimisation approach is followed. The main tools involved in the controller design are LMI theory and Lyapunov functions.

The parametric representation introduced by Araújo and Oliveira (2009) for linear models with uncertainty is herein recalled. This mathematical formulation based on OBFs and dedicated to linear models is

$$\begin{aligned}\tilde{x}(k+1) &= \tilde{A}\tilde{x}(k) + \tilde{B}\tilde{u}(k),\\ \tilde{v}(k) &= \tilde{C}\tilde{x}(k),\end{aligned} \tag{36}$$

where matrices $\tilde{A}$, $\tilde{B}$ and vector $\tilde{x}(\cdot)$ are formed by an appropriate concatenation of the basis data (B_i) generated with the input signal. Note that none of these elements are 'contaminated' with uncertainty. The uncertainty is all concentrated in matrix $\tilde{C}$, and it is represented by the set of parameters $h \in \mathcal{H}$ (see Equation (29)). Therefore, the representation in Equation (36) is herein considered as a suitable model for the uncertain linear block in Figure 1.

In this context, the RMPC problem can be formulated in the line of the approach by Kothare et al. (1996) as

$$\min_{u(k+i|k), i=0,1,\ldots,m} \quad \max_{h\in\mathcal{H},\, p\in\mathcal{P}} J_\infty(k), \tag{37}$$

where

$$J_\infty(k) = \sum_{i=0}^{\infty}\{(\tilde{v}(k+i|k) - w_v)^T Q_1(\tilde{v}(k+i|k) - w_v) + \Delta\tilde{u}(k+i|k)^T R \Delta\tilde{u}(k+i|k)\}. \tag{38}$$

The following step is to analyse how to work out the maximisation in Equation (37) subject to the modelling approach followed in this work. This can be posed as

$$\max_{h\in\mathcal{H},\, p\in\mathcal{P}} J_\infty(k) = \max_{p\in\mathcal{P}}\left(\max_{h\in\mathcal{H}} J_\infty(k)\right), \tag{39}$$

and recalling the bound of Equation (7), it is possible to write

$$\max_{h\in\mathcal{H},\, p\in\mathcal{P}} J_\infty(k) \le \max_{p\in\mathcal{P}} V(x(k|k)), \tag{40}$$

where, as defined above, $x = \left[\begin{smallmatrix}\Delta\tilde{x}(k)\\ \tilde{v}(k)-w_v\end{smallmatrix}\right]$ and, in this expression, w_v is the set point signal translated to the output of the linear block. Now, since $V(\cdot)$ is a quadratic positive definite function, it is clear that $\max_{p\in\mathcal{P}} V(x(k|k))$ is equivalent to

$$\max_{p\in\mathcal{P}}\left\|\begin{bmatrix}\Delta\tilde{x}(k)\\ \tilde{v}(k)-w_v\end{bmatrix}\right\| = \left\|\begin{bmatrix}\Delta\tilde{x}(k)\\ \max_{p\in\mathcal{P}}(\tilde{v}(k)-w_v)\end{bmatrix}\right\|. \tag{41}$$

Let us analyse this last optimisation. From the optimisation proposed in Equation (41), taking into account the output measure $y(k)$ at the time k, and considering that the desired value for the output is w, we have the following result:

$$\max_{p\in\mathcal{P}}\|(\tilde{v}(k) - w_v)\| = \max_{p\in\mathcal{P}}\left\|\sum_{i=1}^{N} p_i(g_i(y(k)) - g_i(w))\right\|. \tag{42}$$

To obtain this maximum, we have to apply a procedure similar to the one used for the robust identification accomplished in Section 3. For this aim, we define $\gamma \triangleq [g^-,\ g^+]^T$ where $g^+ \triangleq \max(g(y(k)) - g(w), 0)$ and $g^- \triangleq \min(g(y(k) - g(w)), 0)$. Therefore, through the solution of the following optimisation problem

$$\max_{p\in\mathcal{P}}\left\|\sum_{i=1}^{N} p_i(g_i(y(k)) - g_i(w))\right\| = \max(|p^u g^+ + p^l g^-|, |p^l g^+ + p^u g^-|), \tag{43}$$

we obtain the parameters that, according to Equation (42), generate the worst case for $(\tilde{v}(k) - w_v)$, where $(\tilde{v}(k) - w_v)$ stands for the last entry of the vector x.

Now, if this vector x is used as a datum, the control input is given by the solution of the problem pointed in Theorem 2.1. In this way, an RMPC algorithm is provided for the uncertain WM, which is supported on the basis of LMI theory. The controller synthesis methodology takes into account the plant uncertainty, and the calculated control law is the one that minimises a worst-case objective function subject to the operation constraints.

The main feature of this approach is that it takes advantage of the static nature of the nonlinearity, which allows to solve the control problem by focusing only in the linear dynamics. This formulation results in a simplified design procedure, because the original nonlinear MPC problem turns into a linear one.

It should also be remarked that in this formulation, the only information required from the process is the controlled output $y(k)$ which is used in Equation (42). Therefore, the process states values are not necessary, which contrasts with the commonly used state space approaches for MPC.

In the next section the previous control strategy is applied to simple illustration example. The design procedure is applied and the controller performance is evaluated.

5. Simulation examples

In this section, two examples are presented to illustrate the implementation of the proposed RMPC algorithm for controlling a Wiener system. The goal of the first example is to show the method for a SISO system. In this way, and to avoid further complexity, a simple case is tackled to illustrate the methodology and implementation details.

In a second example, a MIMO Wiener system is addressed to illustrate further aspects of the identification and control proposal, such as implementation for a multivariable system, influence of measurement noise and conservatism of the robust controller approach.

5.1 *Example 1*

In this example we recall the widely cited WM which was first introduced by Wigren (1993) to describe the behaviour of a control valve for fluid flow. The mathematical description for this final control element is given by the following equations:

$$v(k) = \frac{0.1044q^{-1} + 0.0883q^{-2}}{1 - 1.4138q^{-1} + 0.6065q^{-2}}\,u(k) \qquad (44)$$

$$y(k) = \mathcal{N}(v(k)) = \frac{v(k)}{\sqrt{0.10 + 0.90v^2(k)}}, \qquad (45)$$

where $u(k)$ is the controller output, i.e. the signal applied to the stem. The stem position is denoted as $v(k)$, and the resulting flow through the valve is represented by $y(k)$.

In order to generate the input–output data of the system, it is necessary to define the input signal characteristics. Following the example proposed by Wigren (1993), a random zero-mean sequence between −0.5 and 0.5 was first generated. The input $u(k)$ was then constructed by adding a bias of 0.5 and holding each value in the sequence during six sampling intervals.

To perform the WM identification, we assume a WM formed by a Laguerre system followed by a polynomial-type nonlinearity. Three Laguerre terms with a dominant pole in −0.6 were considered for the linear block, and a third-order polynomial was proposed for the nonlinear block. Therefore, $M=3$ and $N=3$.

From the solution of the optimisation problem formulated in Equations (33)–(35), the parameter bounds of the uncertain WM were obtained. The structural identifiability constraint $h_1 = 1$ was specified. In this example, the weighting factor α was taken equal to 0.5. Therefore, the uncertainty was equally weighted between the linear and the nonlinear blocks, i.e. both terms of uncertainty were identically weighted in the optimisation cost function. The results for the parameter bounds are shown in Table 1.

Once the parametric uncertain WM is obtained, the RMP Controller discussed in Section 4 can be designed. For this purpose, the algorithm parameters are set equal to $Q_1 = 0.001$, $R=1$, $m=1$ and $\Delta u^2_{\max} = 0.005$.

Table 1. Bounds on WMs parameters.

Parameter	Lower bound	Upper bound
h_1	1.0000	1.0000
h_2	0.0115	0.1272
h_3	1.4783	1.4783
p_1	1.2644	2.6265
p_2	1.8124	1.8124
p_3	−2.9801	−0.7056

In this example, the optimisation problem subject to LMI constraints was solved using the LMI Control Toolbox (Gahinet, Nemirovski, Laub, and Chilali 1995) in the Matlab environment.

The performance of the RMP controller designed in Section 4 is depicted in Figures 3 and 4. Note that in this example, two setpoint changes take place (from 0 to 0.2 in the sample number 10 and from 0.2 to −0.3 in sample number 500). Figure 3 illustrates the manipulated variable and the achieved output for the first setpoint change. The second one is shown in Figure 4.

To achieve the control goal under the imposed conditions and operation constraint (i.e. $\Delta u^2_{\max} = 0.005$), the control action shown in Figures 3(a) and 4(a) has to be implemented. Note that both in Figures 3 and 4 deviation variables were depicted (i.e. the plotted controlled/manipulated variable is the difference between the real controlled/manipulated variable and its nominal value).

It must be remarked that the controller synthesis procedure proposed for system is formulated for a WM that explicitly takes into account the plant uncertainty. Moreover, the proposed control law is the one that minimises a worst-case infinite horizon objective function subject to the operation constraint using no other information from the process than the measured output.

5.2 *Example 2*

In this example we recall the model introduced by Ray and Majumder (1983) to describe the behaviour of a steam generating unit (SGU). The mathematical description for this MIMO nonlinear system is given by the following equations:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -0.00193SP^{1/8} + 0.000736w_c + 0.014524F - 0.00121L + 0.000176T_e \qquad (46)$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = 10c_vP^{1/2} - 0.78571S \qquad (47)$$

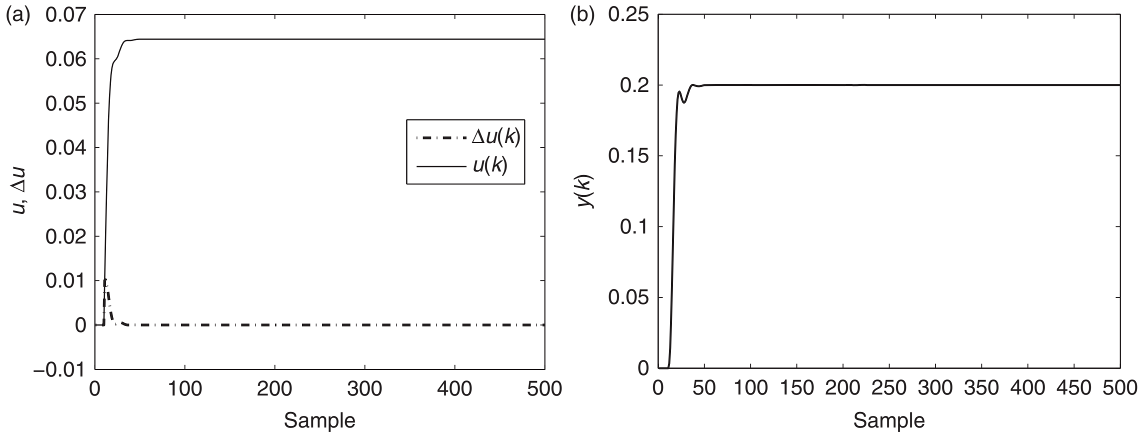

Figure 3. First setpoint change.

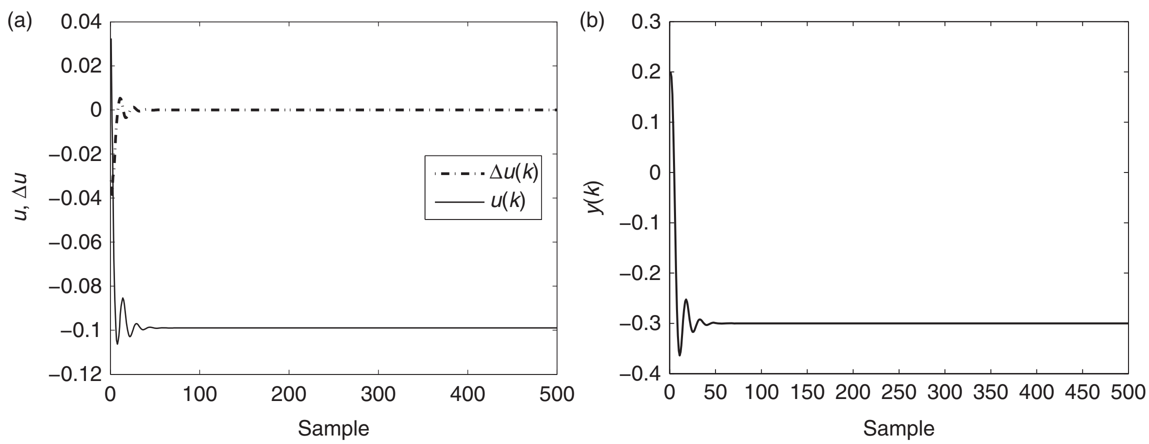

Figure 4. Second setpoint change.

$$\frac{\mathrm{d}L}{\mathrm{d}t} = 0.00893w_c + 0.002F + 0.463c_v - 610^{-6}P^2 - 0.00914L - 8.210^{-5}L^2 - 0.007328S. \quad (48)$$

The state variables in this nonlinear model are: the pressure (P), the steam flow (S) through the high pressure turbine and the drum level (L). The states P and L are the controlled variables. There are two manipulated variables: the fuel input (F) and the feed water input (w_c), and two disturbances: the feed water temperature (T_e) and the control valve setting (c_v). The steady state values for these variables are shown in Table 2. In the sequel, $y = [P, L]^T$ and $u = [F, w_c]^T$ will be the vectors of controlled and manipulated variables, respectively.

In order to illustrate how the presence of noise can be dealt with by the identification algorithm, we consider the measured outputs corrupted with noise.

Table 2. SGU variables.

Variable	Value
F (kg/s)	40
w_c (kg/s)	180
T_e (K)	290
c_v	0.8

The most direct and trivial approach would be to include the noisy outputs data in the identification method developed in Section 3. In such a way the identified robust model should be able to predict the whole noisy measurements. However, this approach is too conservative because the uncertainty bounds on the parameters would be increased drastically, which means we will be justifying with uncertainty the effect of noise.

On the other hand, if we consider that the output is disturbed as follows:

$$y'(k) = y(k) + e(k) \quad |e(k)| \leq \epsilon \in \Re^+, \tag{49}$$

the conservatism can be reduced by introducing a modification in the constraints (34)–(35) of the identification algorithm presented in Section 3. The change will consist in the inclusion of an error margin associated to the noise bounds. Therefore, taking into account that $p_{i,i,1}=1$ for $i=1,\ldots,N_s$, constraint (34) can be rewritten as

$$\left[(h_i^l)^T, (h_i^u)^T, -(p_i^u)^T, -(p_i^l)^T\right]\begin{bmatrix}\gamma_i^B(k)\\ \gamma_i^g(k)\end{bmatrix} - e(k) \geq 0 \tag{50}$$

and if the error bounds are set to their minimum value (i.e. to make the problem less conservative), we have

$$\left[(h_i^l)^T, (h_i^u)^T, -(p_i^u)^T, -(p_i^l)^T\right]\begin{bmatrix}\gamma_i^B(k)\\ \gamma_i^g(k)\end{bmatrix} \geq -\epsilon. \tag{51}$$

In a similar form, constraint (35) results in

$$\left[(h_i^u)^T, (h_i^l)^T, -(p_i^l)^T, -(p_i^u)^T\right]\begin{bmatrix}\gamma_i^B(k)\\ \gamma_i^g(k)\end{bmatrix} \leq \epsilon. \tag{52}$$

It must be remarked that this modification in the optimisation algorithm allows the inclusion of the known bounds of the measurement noise for reducing the estimated bounds of the model parameters.

Therefore, now the model's output prediction could differ from the measured output as much as ϵ. It is assumed this possible bias is due to the noise. Under this hypothesis, it is no longer possible to guarantee that the output sampled data will be completely justified by the model.

This approach based on noisy measurements is now applied to the identification of a WM for the SGU. For this purpose, a set of 1000 data is generated with a sample period of 20 s. The system is excited with uniformly distributed random manipulated variables (F and w_c). It is assumed both signals have a standard deviation of 1%. Additionally, it was considered the output data corrupted with noise bounded by $\epsilon=0.5$.

The linear block was modelled as a Laguerre system. Each basis was assumed to be integrated by three terms with dominant poles equal to $\xi_{1,1}=\xi_{1,2}=\xi_{2,1}=0.95$ and $\xi_{2,2}=0.05$. On the other hand, second-order polynomials were proposed for the nonlinear block. The values given to the parameter α were 0.05 for P and 0.35 for L, respectively.

Figures 5–8 illustrate the identification bounds for the sets $\mathcal{V}_i^y$ and $\mathcal{V}_i^u$ for both the pressure and level, respectively. The simulations shown in Figures 5 and 6 were carried out using samples corrupted with

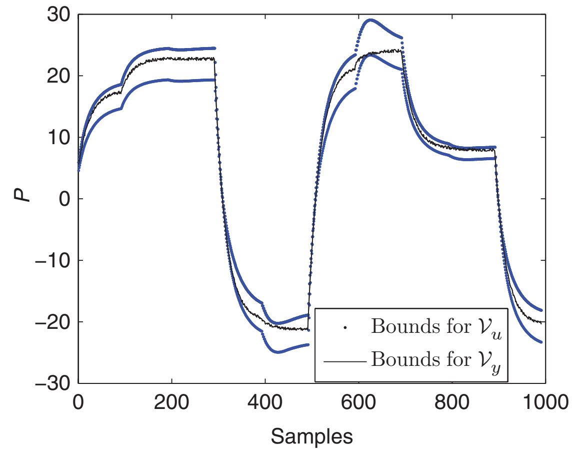

Figure 5. Robust identification without noise information (P).

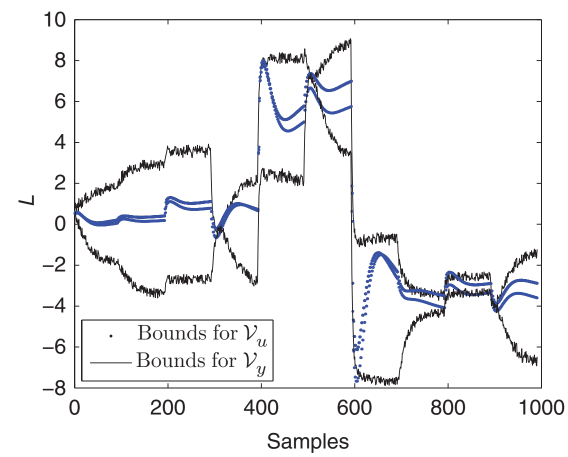

Figure 6. Robust identification without noise information (L).

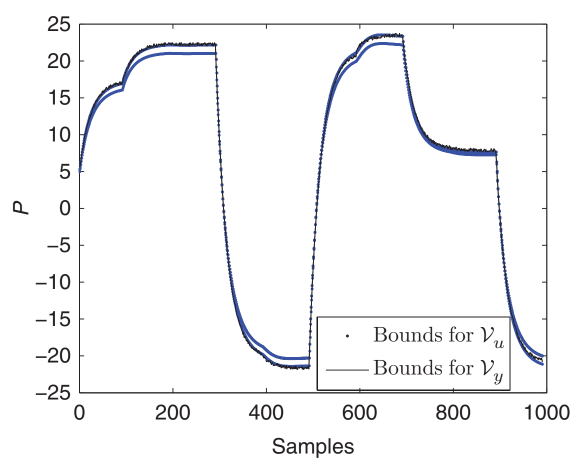

Figure 7. Robust identification with noise information (P).

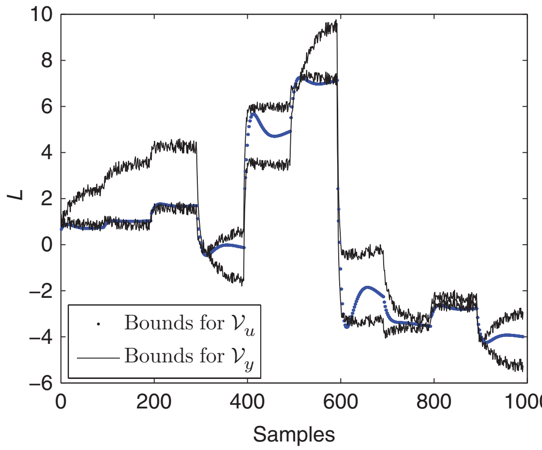

Figure 8. Robust identification with noise information (L).

Table 3. WM parameters (identification with noise).

Parameter	Lower bound	Upper bound
$h_{1,1,1}$	1.009391564821534	1.056215269911753
$h_{1,1,2}$	0.054602770292	0.054602770292
$h_{1,1,3}$	0.03504078065	0.03504078065
$h_{1,2,1}$	−0.01698209270	−0.01698209270
$h_{1,2,2}$	−0.00099619246	−0.00099619246
$h_{1,2,3}$	0.00166984650	0.00166984650
$h_{2,1,1}$	0.18155136496	0.18155136486
$h_{2,1,2}$	−0.108601038	−0.108601038
$h_{2,1,3}$	0.042514434	0.042514434
$h_{2,2,1}$	0.09459015674	0.09459015674
$h_{2,2,2}$	0.1178946005	0.1178946005
$h_{2,2,3}$	−0.11179579510	−0.11179579510
$p_{1,1,1}$	1.000	1.000
$p_{1,1,2}$	−0.000341816277861	0.000265422453944
$p_{1,2,1}$	0.000	0.000
$p_{1,2,2}$	−0.000341816277861	0.000265422453944
$p_{2,1,1}$	0.320687467210654	0.320687467204003
$p_{2,1,2}$	−0.001299202756650	0.004592308614066
$p_{2,2,1}$	1.000	1.000
$p_{2,2,2}$	0.001842907845663	0.001842907770726

measurement noise. However, this noise was ignored in the model's identification procedure. On the other side, Figures 7 and 8 show the results obtained when the knowledge on the measurement noise is incorporated in the identification algorithm by using constraints (51) and (52). It should be noted how the bounds amplitude decrease when the knowledge about the noise is used. Table 3 shows the parameters obtained when the noise information is included in the identification algorithm.

Once the identification of the SGU is accomplished, the algorithm introduced in Section 4 is used to develop a controller for this system. To evaluate the performance of the RMPC design algorithm, three different situations were considered. The first case consists in designing the controller based on the WM obtained as described in Section 3 but replacing Equations (34) and (35) by Equations (51) and (52). We will make reference to this approach as model based predictive control with noise consideration (*MBPCwn*). The second approach ignores the measurement noise present in the output data and it is referred to as model based predictive control without noise consideration (*MBPCwon*). The comparison between both cases allows to interpret the advantages of using the information about the noise. The third controller design (which is herein named as *MBPC*) is based on the identification of a nominal model. The performance of this last approach gives information about the conservatism of the robust control algorithm.

In all the situations the controller design parameters were set to $Q_1 = 10I$, $R = 0.1I$, $m = 1$ and $\Delta u^2_{\max} = 5$. The optimisation problem subject to LMI constraints is solved by means of the LMI control toolbox (Gahinet et al. 1995) in Matlab environment.

Figure 9 depicts the simulation results for several changes in the reference signal. The plots show the necessary control movements in both F and w_c to achieve the desired values for P and L. The three different strategies (*MBPC*, *MBPCwn* and *MBPCwon*) were accomplished. For performance assessment, the comparison criterion illustrated in Figure 10 was considered. It shows the calculated bounds for the objective function in Equation (38) for each controller (i.e. *MBPC*, *MBPCwn* and *MBPCwon*). These values are normalised with respect to the nominal MBPC. It should be remarked that the nominal controller design was accomplished taking into account the mean value for each of the identified parameters (i.e. the average between the upper and the lower identified bounds). From these results we conclude that the proposed algorithm does not present a relevant conservatism. We can also conclude that the use of the information about the measurement noise in the identification procedure reduces the conservatism significantly.

6. Conclusions

In this work the model-based control of uncertain Wiener systems has been dealt with. In this sense, a robust procedure is dedicated for controlling this system in the presence of uncertainty. Provided an input–output data set obtained from the system is available, the first stage consists in the identification of a WMs family able to reproduce the whole output information from the input data. This modelling problem is tackled using a parametric identification

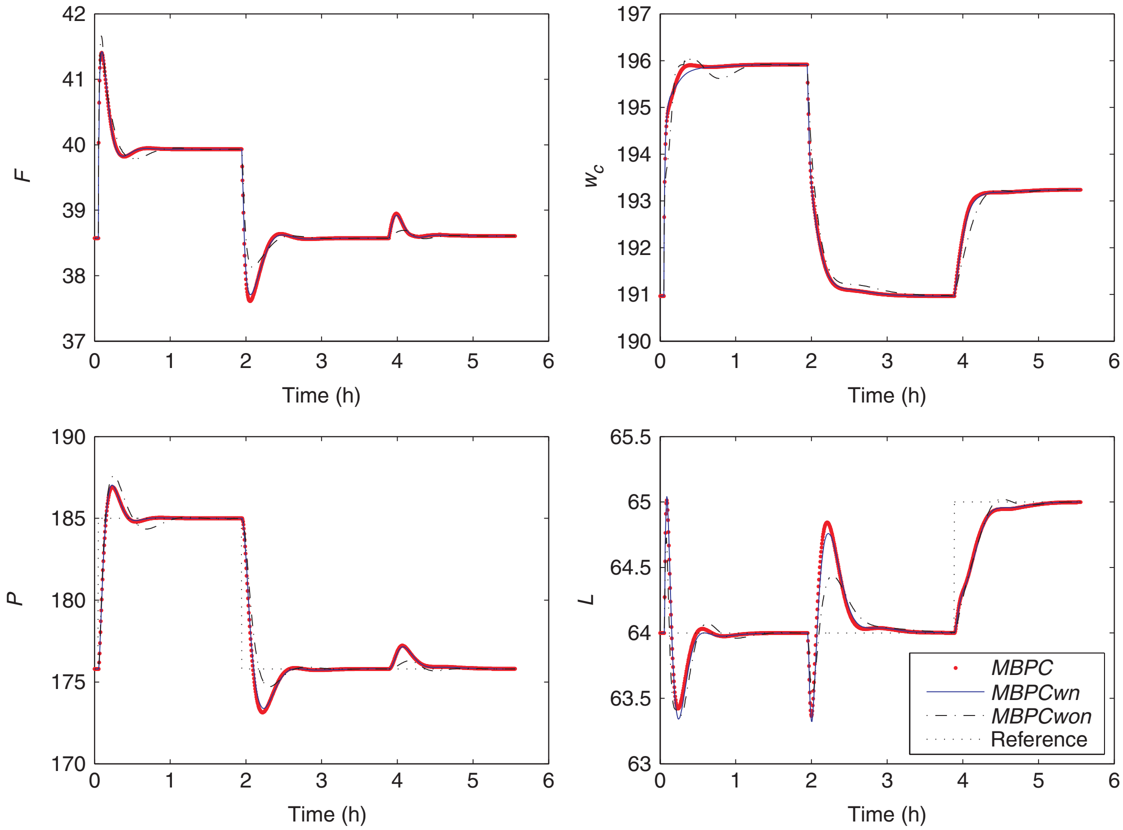

Figure 9. Simulation results for changes in the reference signal.

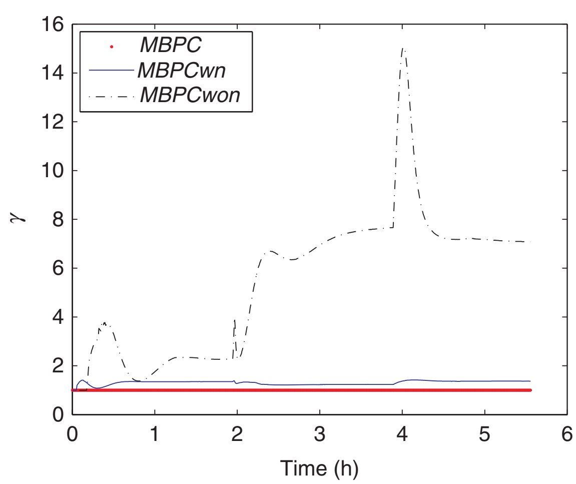

Figure 10. Objective function bound (normalised with respect to the nominal controller).

approach which is formulated and solved as a linear programming problem.

The special configuration of the Wiener structure (a linear dynamics followed by a static nonlinearity) in combination with the OBF supported parametric model approach, makes it possible to concentrate all the uncertainty in a unique output matrix. The main contribution of work relies in the development of an ad hoc robust model predictive-based methodology for controlling a Wiener system in the presence of uncertainty.

The proposed RMPC algorithm is built using Lyapunov functions, which is a well-known tool for stability assessment. The algorithm shows an appropriate performance in the tracking of changing setpoints. This results in an efficient robust control of uncertain Wiener systems.

Acknowledgements

This work was financially supported by the ANPCyT, CONICET and Universidad Nacional del Sur.

Note

1. The present identification algorithm requires that the static nonlinearity is invertible, for both the nominal and uncertain model.

References

Allwright, J.C., and Papavasiliou, G.C. (1992), 'On Linear Programming and Robust Model-predictive Control using Impulse Responses', *Systems and Control Letters*, 18, 159–164.

Araújo, H.X., and Oliveira, G.H.C. (2009), 'An LMI Approach for Output Feedback Robust Predictive Control using Orthonormal Basis Functions', in *Proceedings of the Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, 16–18 December, Shangai, PR China.

Bequette, B.W. (1991), 'Nonlinear Control of Chemical Processes: A Review', *Industrial and Engineering Chemistry Research*, 30, 1391–1413.

Biagiola, S.I., Agamennoni, O.E., and Figueroa, J.L. (2004), 'H_∞ Control of a Wiener Type System', *International Journal of Control*, 77, 572–583.

Biagiola, S.I., and Figueroa, J.L. (2009), 'Wiener and Hammerstein Uncertain Models Identification', *Mathematics and Computers in Simulation*, 11, 3296–3313.

Campo, P.J., and Morari, M. (1987), 'Robust Model Predictive Control', in *Proceedings of American Control Conference*, Minneapolis, USA, pp. 1021–1026.

Cheong, M.Y., Werner, S., Cousseau, J., and Laakso, T. (2005), 'Predistorter Identification using the Simplicial Canonical Piecewise Linear Function', in *Proceedings of 12th International Conference on Telecommunications*, 3–6 May 2005, Cape Town, South Africa.

Cuzzola, F.C., Geromel, J.C., and Morari, M. (2002), 'An Improved Approach for Constrained Robust Model Predictive Control', *Automatica*, 38, 1183–1189.

Ding, B., Xi, Y., and Li, S. (2004), 'A Synthesis Approach of On-line Constrained Robust Model Predictive Control', *Automatica*, 40, 163–167.

Falugi, P., Giarré, L., and Zappa, G. (2005), 'Approximation of the Feasible Parameter Set in Worst-case Identification of Hammerstein Models', *Automatica*, 41, 1017–1024.

Figueroa, J.L., Biagiola, S.I., and Agamennoni, O.E. (2008), 'An Approach for Identification of Uncertain Wiener Systems', *Mathematics and Computer Modelling*, 48, 305–315.

Gahinet, P., Nemirovski, A., Laub, A.J., and Chilali, M. (1995), *LMI Control Toolbox*, MA, USA: The MathWorks.

Gerkšič, S., Juričic, D., Strmčnic, S., and Matko, D. (2000), 'Wiener Model Based Nonlinear Predictive Control', *International Journal of Systems Science*, 31, 189–202.

Gómez, J.C., and Baeyens, E. (2004), 'Identification of Block-oriented Nonlinear Systems using Orthonormal Bases', *Journal of Process Control*, 14, 685–697.

Henson, M.A. (1998), 'Nonlinear Model Predictive Control: Current Status and Future Directions', *Computers and Chemical Engineering*, 23, 187–202.

Heuberger, P.S.C., Van den Hof, P.M.J., and Wahlberg, B. (2005), *Modelling and Identification with Rational Orthogonal Basis Functions*, London: Springer-Verlag.

Hunter, I.W., and Korenberg, M.J. (1986), 'The Identification of Nonlinear Biological Systems: Wiener and Hammerstein Cascade Models', *Biological Cybernetics*, 55, 135–144.

Kalafatis, A., Arifin, N., Wang, L., and Cluett, W.R. (1995), 'A New Approach to the Identification of pH Processes Based on Wiener Model', *Chemical Engineering Science*, 50, 3693–3701.

Kang, H.W., Cho, Y.S., and Youn, D.H. (1998), 'Adaptive Precompensation of Wiener Systems', *IEEE Transactions on Signal Processing*, 40, 2825–2829.

Kang, H.W., Cho, Y.S., and Youn, D.H. (1999), 'On Compensating Nonlinear Distortions of an OFDM System using an Efficient Adaptive Predistorter', *IEEE Transactions on Communications*, 47, 522–526.

Korenberg, M.J. (1973), 'Identification of Biological Cascades of Linear and Static Nonlinear Systems', in *Proceedings of the 16th IEEE Midwest Symposium on Circuit Theory*, New York, USA, pp. 1–9.

Kothare, M.V., Balakrishnan, V., and Morari, M. (1996), 'Robust Constrained Model Predictive Control using Linear Matrix Inequalities', *Automatica*, 32, 1361–1379.

Lussón-Cervantes, A., Agamennoni, O.E., and Figueroa, J.L. (2003), 'A Nonlinear Model Predictive Control Scheme Based on Wiener Piecewise Linear Models', *Journal of Process Control*, 13, 655–666.

Mao, W.J. (2003), 'Robust Stabilisation of Uncertain Time-varying Discrete Systems', *Automatica*, 39, 1109–1112.

Norquay, S.J., Palazoglu, A., and Romagnoli, J.A. (1998), 'Model Predictive Control Based on Wiener Models', *Chemical Engineering Science*, 53, 75–84.

Oliveira, G.H.C., Amaral, W.C., Favier, G., and Dumont, G. (2000), 'Constrained Robust Predictive Controller for Uncertain Processes Modelled by Orthogonal Series Functions', *Automatica*, 36, 563–572.

Pajunen, G.A. (1992), 'Adaptive Control of Wiener Type Nonlinear Systems', *Automatica*, 28, 781–785.

Pearson, R.K., and Pottmann, M. (2000), 'Gray-box Identification of Block-oriented Nonlinear Models', *Journal of Process Control*, 10, 301–315.

Pottmann, M., and Pearson, R.K. (1998), 'Block-oriented NARMAX Models with Output Multiplicities', *AIChE Journal*, 44, 131–140.

Qin, S.J., and Badwell, T.A. (1997), 'An Overview of Industrial Model Predictive Control Technology', in *Proceedings of the Fifth International Conference on Chemical Process Control, AIChE Symposium Series 316* (Vol. 93), eds. C.E. Garcia, J.C. Kantor, and B. Carnahan, California: American Institute of Chemical Engineering, pp. 232–256.

Ray, K.S., and Majumder, D. Dutta (1983), 'Simulation of a Nonlinear Steam Generating Unit', in *Proceedings of the*

International Conference on Systems, Man and Cybernetics, Mumbai, India, pp. 705–710.
Wan, Z., and Kothare, M.V. (2002), 'Robust Output Feedback Model Predictive Control using Off-line Linear Matrix Inequality', *Journal of Process Control*, 12, 763–774.
Wigren, T. (1993), 'Recursive Prediction Error Identification using the Nonlinear Wiener Model', *Automatica*, 29, 1011–1025.
Wright, S. (1997), 'Applying New Optimisation Algorithms to Model Predictive Control', in *Proceedings of the Fifth International Conference on Chemical Process Control, AIChE Symposium Series 316* (Vol. 93), eds. C.E. Garcia, J.C. Kantor, and B. Carnahan, California: American Institute of Chemical Engineering, pp. 147–155.
Zheng, Z.Q., and Morari, M. (1993), 'Robust Stability of Constrained Model Predictive Control', in *Proceedings of the American Control Conference*, June, San Francisco, CA, USA, pp. 379–383.
Zhu Y. (1999), 'Distillation Column Identification for Control using Wiener Model', in *Proceedings of the American Control Conference* (Vol. 5), 2–4 June, San Diego, California USA, pp. 3462–3466.