The KH stability of the supersonic magnetopause flanks modeled by continuous profiles for the transition

G. Gnavi[1], F. T Gratton[1], C. J. Farrugia[2], and L. Bilbao[1].

[1] *Instituto de Física del Plasma, CONICET-UBA, Argentina*
[2] *Space Research Center, Univ. New Hampshire, NH, USA*

Abstract. Two examples of stability analysis of the supersonic magnetopause (MP) flanks, based on compressible, ideal MHD theory are reported. The input parameters for local MP models are derived from spacecraft crossings data. The results emphasize the sensitivity of the Kelvin-Helmholtz instability on fine structure features of the boundary layer.

Keywords: Kelvin-Helmholtz instability; magnetopause; supersonic flanks, MHD theory.
PACS: 94.30.ch; 94.30.cq; 94.30.-d; 94.05.-a.

INTRODUCTION

Compressibility influences the stability of a velocity shear flow when $\Delta \boldsymbol{V}$ across the velocity gradient layer becomes supersonic. The KH modes with $\boldsymbol{k} \| \Delta \boldsymbol{V}$ of a tangential discontinuity (TD) in a uniform density gas flow are stable when $M>2$ ($M=|\Delta \boldsymbol{V}|/c_s$; Landau, 1944). However, modes for which the projection $V_\kappa \equiv \boldsymbol{\kappa}\cdot\Delta \boldsymbol{V}$ is subsonic (where $\boldsymbol{\kappa} \equiv \boldsymbol{k}/|\boldsymbol{k}|$) are still unstable. The MHD stability of the Earth magnetopause (MP) is a complex problem that depends also on several other features: the presence of a magnetic shear angle, the non uniform density transition, the ratio λ/Δ (Δ: scale length of the velocity shear layer) the possible existence of a multiple-scale structure, and so on.

The plasma flow at the MP flanks is supersonic $M>1$ and super-Alfvénic $M_A>1$. We deal here with the Kelvin-Helmholtz (KH) stability of the MP flanks that requires a compressible MHD treatment. The study focuses on the stability of particular events observed during spacecraft crossings, aiming at a survey of the stability of characteristic MP flank configurations currently under progress. Due to space limitations we report only two examples.

MHD Stability and boundary layer models

In the realm of ideal MHD the stability of a planar stratified flow, in which the physical quantities are uniform in the coordinates x, z, and vary with the coordinate y like $\boldsymbol{V}=(V_x(y),\ 0,\ 0)$, $\boldsymbol{B}=(B_x(y),\ 0,\ B_z(y))$, $\rho(y)$, $T(y)$, is governed by the following equation[1]

$$\frac{d}{dy}\left(H\left(1-\frac{1}{M}\right)\frac{d\zeta}{dy}\right)-k^2 H\zeta=0, \Xi=\zeta(y)exp\left(-i\omega t+ik_x x+ik_z z\right)$$

where Ξ is the y component of the Lagrangian displacement from the initial state of a plasma element, ζ is the amplitude of the Fourier mode with complex angular frequency $\omega=\omega_r+i\gamma$, and $k^2=k_x^2+k_z^2$. The coefficients H and M are defined by

$$H(y)=\rho(y)[(c-V_\kappa(y))^2-V_{A\kappa}^2(y)],$$

$$M(y)=1-\frac{(V_A^2+c_s^2)}{(c-V_k)^2}+\frac{(V_{Ak}^2 c_s^2)}{(c-V_k)^4},$$

where $c\equiv\omega/k$ is the complex phase velocity, and

$$V_\kappa^2(y)=(\mathbf{V}\cdot\boldsymbol{\kappa})^2,\ V_A(y)=\frac{B}{\sqrt{4\pi\rho}},V_{A\kappa}^2(y)=\frac{(\mathbf{B}\cdot\boldsymbol{\kappa})^2}{4\pi\rho}=\frac{B_\kappa{}^2}{4\pi\rho}$$

where V_A is the Alfvén speed, c_s is the speed of sound, $V\kappa$, $B\kappa$, are the projections of the velocity and the magnetic field on the $\boldsymbol{k}$ direction, all quantities that are functions of y.

We model locally the MP with a planar slab structure, and analyze its stability. This layer is set on the tangent plane of the boundary, at chosen positions on the MP flanks. One-scale and two-scale models of the MP interface are built with hyperbolic tangents for the equilibrium fields. The parameters that define the models are the strength of the vector fields, $|\boldsymbol{V}|$, $|\boldsymbol{B}|$, and the scalar fields $\rho=n\ m_p$, T, the angles between the vector fields $\boldsymbol{V}$ and $\boldsymbol{B}$, on both sides of the transition

CP875, *Plasma and Fusion Science*
edited by J. J. E. Herrera-Velázquez

(indices *1*, magnetosheath, and *2*, magnetosphere), the thickness $\Delta=2d$ of the boundary layer, and $\delta=2d'$ the current sheet width (scale lengths of tanh(y/d) and tanh(y/d')). Thus, setting $Y=y/d$, the basic field profiles are as follows,

$$V_k = \frac{U_1 \cos(\varphi)}{2} [1 + \tanh(Y)],$$

$$\rho = \frac{\rho_1}{2} [(1 + r_d) + (1 - r_d) \tanh(\frac{\Delta}{\delta} Y)],$$

$$B_a = \frac{1}{2}(B_{1a} + B_{2a}) + \frac{1}{2}(B_{1a} - B_{2a}) \tanh(\frac{\Delta}{\delta} Y),$$

while the temperature profile is obtained from the pressure balance equation, $p+|\boldsymbol{B}|^2/8\pi = const.$ The ratio $r_s=\delta/\Delta$ is 1 for the one scale model, or is suggested by data of spacecraft crossings for a two scale model. We may keep Δ as a free quantity, and infer a value *a posteriori* (see example below). The boundary value problem for *c* is solved numerically by a conventional shooting method.[2]

The case of January 11, 1997

The event happened at the tail of a large interplanetary magnetic cloud. From 1:00-2:00 UT a plasma plug of high density passed the Earth. There was a large increase (~30 × normal) of the dynamic pressure of the solar wind, while the interplanetary magnetic field (IMF) shifted to north. In the following hours the pressure subsided, and the clock angle decreased in well defined steps, as the IMF became strongly northward. Interball-tail, a Russian magnetospheric spacecraft, was left out in the magnetosheath by the huge compression of the boundary. It explored the outer region adjacent to the MP during some time, and then reentered in the magnetosphere as the size returned to normality, slowly crossing the low latitude boundary layer (LLBL) at a near dusk flank position, distant ~13 R_E in the anti-sunward direction. A description of the complex event and a discussion of the data fluctuations observed by the spacecraft, which were interpreted as KH waves, is given in ref.3.

Here we report a stability calculation with the one-scale model for a period of the event in which the basic parameters derived from Interball-tail data are summarized below. The local values of the main non dimensional numbers, based on magnetosheath quantities, are M=1.90, M_A=1.67, so that a KH calculation needs a compressible treatment (not given in ref.3). Fig. 1 shows the growth rate $g \equiv \gamma d/U_1$ versus the wave number *kd*, both as normalized quantities (γ=Im(ω), $k \equiv |\boldsymbol{k}|$). The result is for modes with a particular orientation of $\boldsymbol{k}$, which has an angle with $\boldsymbol{V}$, φ=- 4.33°, for the maximum growth rate of the instability. This is shown in Fig. 2 that illustrates the dependence of *g* on φ , while *kd*=0.65 remains fixed at the value associated to the mode of maximum growth rate, $\gamma d/U_1$=0.05 of Fig.1.

TABLE 1. Parameters for Jan.11, 1997.

	magnetosheath	magnetosphere
V_X	300 km/s	~ 0 km/s
B	30 nT	20 nT
θ	100°	30°
n	10 p/cm3	1 p/cm3
T	0.130 keV	1.9 keV

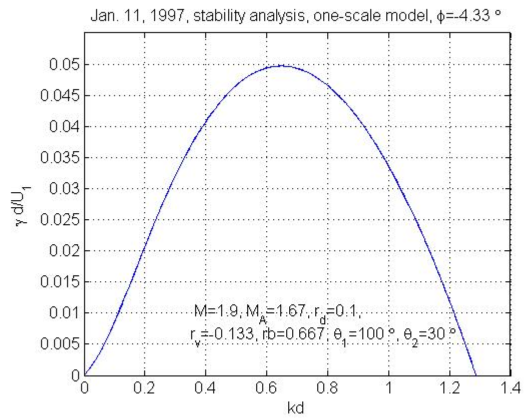

FIGURE 1. Growth rate as function of wave number, Jan. 11, 1997 case, one scale model.

Modes with changes $\Delta\varphi$ ≈±15° from the optimal $\boldsymbol{k}$ orientation are stable. The maximum growth rate mode has a wavelength λ≈4.8 R_E if we assume Δ≈1 R_E for the LLBL width (estimation often quoted for the flanks; see also next example). The corresponding *e-folding time* for the instability is τ_e ≈3.5 min.

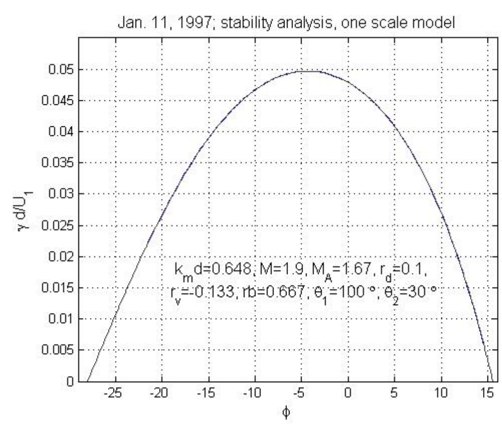

FIGURE 2. Angular dependence of the growth rate, Jan. 11, 1997 case, one scale model.

We performed another calculation on the same basic configuration of magnetic field, density, temperature, and relative field angles, but with a reduced value of the magnetosheath speed U_1, such that M=1.00, M_A=0.88. This could represent a possible upstream scenario of the MP (at a smaller anti-sunward distance) during the same event. The KH instability is convective and is carried downstream by the flow, continuing its growth during the transport if the background support is unstable. In the upstream scenario the growth rate g is smaller as expected, but the fastest growing mode appears at a longer λ.

The Flank on November 20, 2001.

The second example concerns an event in which, due to a combination of circumstances, the CLUSTER flotilla stayed in the LLBL of the near magnetospheric dusk flank (at a GSM X position ~-4 R_E) for about 10 hours. The IMF was northward during this interval, with long lapses of low clock angle. The spacecrafts recorded series of intermittent bursts (3 – 4) of nearly periodic, large amplitude, signals during most of the time of the long LLBL exploration. Particularly neat wavy features were seen in the data records of the magnetic field, and plasma density.[4]

For a stability analysis, the input parameters estimated from CLUSTER data for a period ~14:00-17:00 U, are the following

TABLE 2. Parameters for Nov. 20, 2001.

	magnetosh.		magnetosph.
n_1	10 p/cm^{-3}	n_2	1 p/cm^{-3}
B_1	20 nT	B_2	20 nT
U_1	300 km/s	U_2	0 km/s
T_1	0.3 kev	T_2	~2 kev
$\theta_1(B_1V)$	80°	$\theta(B_2V)$	30°

Several values are equal to the previous example, but here B_1=B_2, and the angles are different. The change in the results shows the sensitivity of the instability analysis to the input values. In this case M=1.36, M_A=1.97, and using the one scale model the fastest growing mode is found for φ=-28°. Fig. 3 gives a schematic of the configuration of vector fields and angles across the boundary layer. Fig. 4 shows the growth rate g as a function of kd. Interestingly, the limit of very long wavelengths, $\lambda >> \Delta$ is stable, while the calculation for $\lambda \sim \Delta$ shows instability at the CLUSTER site. This means that a stability analysis performed with a TD model for the MP, which implies the limit $\Delta/\lambda \to 0$, would arrive at a wrong conclusion, i.e., a stability verdict (on this matter see ref. 5).

The average period observed by CLUSTER was T =120 - 150 s.[4] We estimate both $\lambda = 2\pi/k$ and Δ, from the experimental value for $\omega_r = 2\pi/T$, assuming that the observed perturbations are due to the KH mode of largest growth ($kd \approx 0.7$). These should pass by CLUSTER with a phase velocity $Re(c)=v_{ph} \approx 2/3\ U_1$ (magnetosheet speed U_1=300 km/s). We find λ =3.71 – 4.64 R_E, and the LLBL thickness Δ = 0.8 – 1 R_E, depending on the value of T. The estimates are in agreement with values quoted for the near flank in other works.

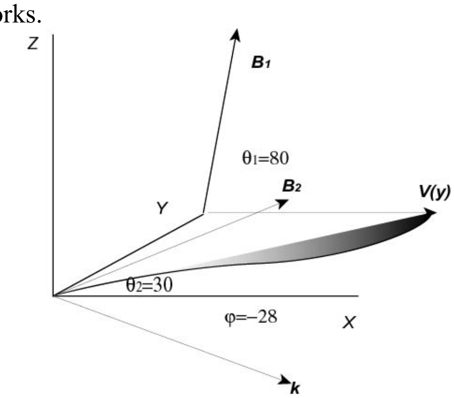

FIGURE 3. Sketch of vector fields and angles, Nov. 20, 2001 example.

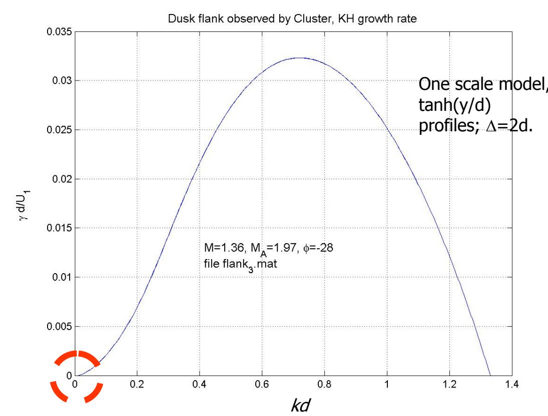

FIGURE 4. Growth rate vs. wave number, Nov. 20, 2001 case, one scale model.

Two-scale model for November 20, 2001

The data, however, show evidence that the MP structure was stratified with a scale length for ***B***, *n*, and a different one for ***V***. Another calculation was therefore done with the two scale model. Although it was not easy to ascertain, an estimate of 0.2 for the ratio $r_s = \delta/\Delta$ was employed. A striking result is that the growth rates of the KH modes with a ***k*** direction φ =-28° are negligible. This is the ***k*** direction of maximum growth rate for the one-scale model. The instability was found instead, changing the orientation of the ***k*** -vector until it was nearly perpendicular to the magnetospheric field, i.e., φ=-60°. The maximum growth rate is for *kd*=0.5 (1.4 × λ increase) reduced by a factor ≈3.25 with respect to the one scale model value. The direction of ***k*** most favorable for instability in the two-scale model is such that the magnetic field in the LLBL and the magnetosphere is fluted by the perturbation, so that the magnetic tensions are turned off on that side.

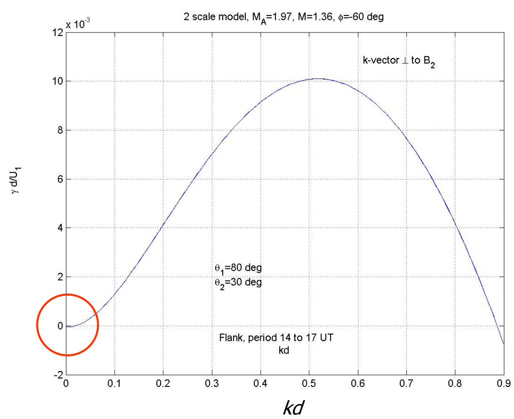

Figure 5. Growth rate vs. wave number, Nov. 20, 2001 case, two scale model.

Comparing the results of the one and two scale models, the important changes found show how responsive is the stability of the MP to fine structure properties of the LLBL. Moreover, the sensitivity of the ***k*** orientation on the structure of the boundary layer has important consequences for the subsequent nonlinear evolution of the KH instability.

ACKNOWLEDGMENTS

Research supported by Argentinean grants: CONICET PIP 5291, and UBACYT X291. USA, NASA grants: NAG5-13116, NAG5-12189, and NNG05GG25G. We are grateful for data and valuable discussions on Nov. 20, 2001, to D. Fairfield, Laboratory for Extraterrestrial Physics, NASA Goddard Space Flight Center, USA. Bilbao, Gratton, and Gnavi, are members of the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET).

REFERENCES

1. Gratton, J., A. G. González, and F. T. Gratton, *Plasma Phys. & Contr. Fusion*, **30**, 435, 1988.
2. Keller, H. B., *Numerical methods for two point boundary-value problems*, Dover, N.Y., 1992
3. Farrugia, C. J., F. T. Gratton, J. Contin, *et al.*, *J. Geoph. Res*, **105**, A4, 7639, 2000.
4. Fairfield, D., *et al*., Eos Trans. AGU, ***84*** (46), Fall Meet. Suppl. 2003; *EOS Trans. AGU,* ***85*** (17)*,* Jt. Assem. Suppl., 2004.
5. Gratton, F. T., L. Bender, C. J. Farrugia, and G. Gnavi, *J. Geoph. Res*, **109**, A04211, 2004.
6. F. T. Gratton, C. J. Farrugia, Bilbao, L. E., G. Gnavi and L. Lund, 2005, these Proceedings.