Controlled dephasing of Andreev states in superconducting quantum point contacts

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We have studied the relaxation and dephasing processes in a superconducting quantum point contact induced by the interaction with an electromagnetic environment. Based on a density matrix approach we obtain the rates for the dissipative dynamics as function of the transmission, the phase difference on the contact, and the external impedance. Our calculation allows us to determine the appropriate range of parameters for the observation of coherent oscillations in the current through the contact.

In a superconducting quantum-point contact (SQPC) the zero-voltage transport properties are determined by the so-called Andreev states within the superconducting energy gap Δ, which are given by[1]

$$E_{\pm}(\phi) = \pm \Delta \sqrt{1 - \tau \sin^2(\phi/2)}, \quad (1)$$

where ϕ is the phase difference between the superconducting electrodes and τ is the normal transmission coefficient for each conduction channel. Atomic contacts, produced by scanning tunneling microscope or the mechanically controllable break-junction (MCBJ) technique, have provided an almost ideal realization of an SQPC with a few conduction channels whose transmissions can be determined experimentally.[2] Different methods for Andreev level spectroscopy have been suggested.[3]

The Andreev states in a single channel SQPC constitute an interesting realization of a two-level system with well characterized parameters, which could be proposed as a solid state realization of a quantum qubit.[4] The phase difference through the contact can be fixed by the magnetic flux in a superconducting ring geometry like the one depicted in Fig. 1(a). In addition, by varying the external flux one could prepare the system in a given linear combination of the Andreev states, in close resemblance to the case of a spin-1/2 in a magnetic field. The phase would be, however, affected by fluctuations originated in the coupling of the ring with its electromagnetic environment, characterized by a finite impedance $Z(\omega)$. These fluctuations provide a source of relaxation and dephasing for the dynamics of our two-level system.[5]

The aim of this work is to investigate this dissipative dynamics in order to determine the conditions for the observation of quantum-coherence effects in this system.

For a single mode SQPC in the low bias regime one can neglect the continuous part of the quasiparticle spectrum and restrict the description to the subgap states, introducing the following Hamiltonian,[6]

$$\hat{H}_0 = \Delta \left\{ \cos\left(\frac{\phi}{2}\right) \hat{\sigma}_z + r \sin\left(\frac{\phi}{2}\right) \hat{\sigma}_x \right\}, \quad (2)$$

where $\hat{\sigma}_i$ are the Pauli matrix and $r = \sqrt{1-\tau}$ is the normal reflection amplitude of the contact. The Hamiltonian adopts

the form (2) in the basis of ballistic states and becomes diagonal $\hat{H}_0 = E_+(\phi)\,\hat{\sigma}_z$ in the basis of the Andreev states. The ballistic and the Andreev states are represented in Fig. 1(b).

In the ring geometry, the phase on the contact will be given by $\phi = \phi_0 - \delta\phi$, where $\phi_0 = (2e/\hbar)\Phi$ (being Φ the external magnetic flux) and $\delta\phi$ represents the phase fluctuations due to the electromagnetic environment. If the impedance viewed by the contact is small compared to the resistance quantum $R_Q = h/4e^2$ one can expand Eq. (2) to the first order in $\delta\phi$ to obtain the coupling between the subgap levels and the environment. Then, the Hamiltonian can be written as $\hat{H}_0(\phi) = \hat{H}_0(\phi_0) + \hat{H}_c$, where the coupling $\hat{H}_c = -(\hbar/2e)\hat{I}\,\delta\phi$ is proportional to the current operator $\hat{I}$. Performing a rotation to the basis of the Andreev states, the current operator is given by

$$\hat{I} = \frac{e\Delta^2}{\hbar E_+(\phi_0)} \left\{ \sqrt{1-\tau}\,\hat{\sigma}_x - \tau \sin\left(\frac{\phi_0}{2}\right) \cos\left(\frac{\phi_0}{2}\right) \hat{\sigma}_z \right\}. \quad (3)$$

On the other hand, a generic environment can be represented by a set of L-C circuits, with phases ϕ_n, such that $\delta\phi = \Sigma_n \phi_n$. The relevant information of the environment is contained in the phase-phase correlation function[7]
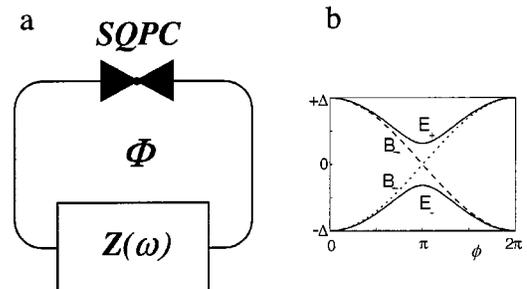
FIG. 1. (a) Schematic circuit: a superconducting quantum point contact (SQPC) inserted in a superconducting ring threaded by a magnetic flux Φ. $Z(\omega)$ denotes the effective impedance seen by the contact. (b) Andreev levels with phase dependent energies $E_{\pm}$ (full lines). The dotted and dashed lines indicate the ballistic states.

$$C(t) = \sum_{n,m} \langle \phi_n(t) \phi_m(0) \rangle$$

$$= \frac{1}{R_Q} \int_0^\infty d\omega \frac{\text{Re}Z_t(\omega)}{\omega} \left\{ \coth\left(\frac{\beta\hbar\omega}{2}\right) \cos\omega t - i\sin\omega t \right\}, \quad (4)$$

where $Z_t(\omega)$ is the effective impedance of the circuit as seen from the contact and β is the inverse temperature.

Projection operator techniques allow to derive a time-convolutionless generalized master equation[8] (GME) for the reduced density matrix $\hat{\rho}$ of the Andreev levels.[9] Up to second order in the coupling Hamiltonian this equation is given by

$$\dot{\hat{\rho}}(t) + \frac{i}{\hbar}[\hat{H}_0, \hat{\rho}(t)] = -\frac{1}{\hbar^2} \int_0^t d\tau \{ [\hat{I}, \hat{I}(-\tau)\hat{\rho}(t)] C(\tau)$$

$$- [\hat{I}, \hat{\rho}(t)\hat{I}(-\tau)] C^*(\tau) \}, \quad (5)$$

where $\hat{I}(\tau)$ denotes the current operator (3) in the interaction representation.

One can extract the evolution equations for the matrix elements of the reduced density projecting the GME (5) in the $\{|+\rangle, |-\rangle\}$ basis of the Andreev states. Taking into account that $\rho_{++} + \rho_{--} = 1$, and defining ρ_R and ρ_I as the real and imaginary part of the off-diagonal matrix element ρ_{+-}, we finally get the rate equations

$$\dot{\rho}_+ = -(W^+ + W^-)\rho_+ + W^+ - \Lambda\rho_R, \quad (6)$$

$$\dot{\rho}_R = \Omega\rho_I + \eta\dot{\rho}_+, \quad (7)$$

$$\dot{\rho}_I = -(W^+ + W^- + \eta\Lambda)\rho_I - (\Omega + 2\Omega_r)\rho_R$$

$$+ \eta\Omega_r(2\rho_+ - 1) + \Upsilon, \quad (8)$$

where $\rho_+ = \rho_{++}$, $\eta = \tau\sin(\phi_0/2)\cos(\phi_0/2)/\sqrt{1-\tau}$ and $\Omega = (E_+ - E_-)/\hbar$.

The coefficients $W^\pm$ entering in Eqs. (6) to (8) are the upward and downward transition rates which determine the system relaxation. The dephasing, i.e., the decay of the off diagonal elements, is also controlled by the coefficient Λ, while Ω_r plays the role of a level shift as discussed below.

The explicit form of all these coefficients depends on the actual impedance $Z_t(\omega)$. Typically, this impedance would be determined by the measuring circuit, which could consist of a SQUID inductively coupled to the ring with the SQPC. We shall denote by M and L the mutual inductance between the ring and the SQUID and the self-inductance of the SQUID respectively. The SQUID is also characterized by a finite resistance R. The charging effects associated with the contact capacitance can be neglected in the low impedance regime that we are considering.[7] Under these conditions the real part of Z_t will be given by

$$\text{Re}Z_t(\omega) = \frac{\omega^2 M^2 R}{R^2 + \omega^2 L^2}. \quad (9)$$

If the environment has a large cutoff frequency ($\gamma = R/L \gg \Delta/\hbar$), we can make the Markov approximation in the evolution equations (6)–(8). In this case, the involved coefficients can be written as

$$W^{\mp} = W_0^- \begin{cases} n(\Omega) + 1, \\ n(\Omega), \end{cases} \quad (10)$$

$$\Lambda = W_0^- \eta \frac{2}{\beta\hbar\Omega}, \quad (11)$$

$$\Omega_r = W_0^- \left\{ \frac{1}{2}\cot\left(\frac{2}{\beta\hbar\gamma}\right) - \frac{2\gamma^2}{\beta\hbar} \sum_{k=1}^\infty \frac{\nu_k}{[\gamma^2 - \nu_k^2][\Omega^2 + \nu_k^2]} \right\}, \quad (12)$$

$$\Upsilon = W_0^- \eta \frac{\Omega}{\gamma}, \quad (13)$$

where

$$W_0^- = \frac{\pi}{4} \frac{\Delta}{\hbar} \frac{\text{Re}Z_t(\Omega)}{R_Q} \frac{1-\tau}{[1 - \tau\sin^2(\phi/2)]^{3/2}} \quad (14)$$

is the downward transition rate at zero temperature, $n(\Omega)$ is the Bose mean occupation number, and $\nu_k = 2\pi k/\beta\hbar$ are the Matsubara frequencies.

The actual value of the coefficients is controlled by the environment parameters M, L, and R. The experimental setup can be designed in order to fix these parameters in the appropriate range to observe quantum-coherence effects. Longer decoherence times would be obtained by reducing M as much as possible. However, SQUID parameters must satisfy certain constraints, imposed by the need to avoid hysteresis and thermal fluctuations.[12] These considerations permit to estimate a minimal value of M of the order of 0.5 nH, with $L = 0.1$ nH and $R = 20$ Ω.[13] Assuming that the superconducting material is aluminum ($\Delta_{Al} \sim 0.18$ meV), one obtains values of W_0^- in the $n\text{sec}^{-1}$ range. This rate fixes the order of magnitude of the coefficients (10)–(13).

The system of differential equations (6)–(8) is characterized by a set of eigenvalues which determine the typical times for the dissipative dynamics. In the limit of small impedance these eigenvalues are given by the following simple expressions:

$$\lambda_1 = -(W^+ + W^-),$$

$$\lambda_{2,3} = -\frac{1}{2}(W^+ + W^- + 2\eta\Lambda) \pm i(\Omega + \Omega_r). \quad (15)$$

Notice that due to the presence of the environment, the Andreev levels are shifted according to $\tilde{E}_\pm \simeq E_\pm \pm \Omega_r/2$. However, for the range of parameters we are considering this renormalization is small and can be neglected.

Solving Eqs. (6)–(8) at the lowest order in Ω^{-1} (which is equivalent to implement the rotating wave approximation[14]) one realizes that the eigenvalues (15) are directly connected to the relaxation and dephasing rates by the relations $\Gamma_R \simeq -\lambda_1$ and $\Gamma_D \simeq -\text{Re}\{\lambda_{2,3}\}$.

The approximate evolution equations can then be written as

$$\rho_+ = \frac{W^+}{\Gamma_R}(1 - e^{-\Gamma_R t}) + \rho_+(0)e^{-\Gamma_R t}, \qquad (16)$$

$$\rho_R = e^{-\Gamma_D t}[\rho_R(0)\cos\Omega t + \rho_I(0)\sin\Omega t], \qquad (17)$$

$$\rho_I = e^{-\Gamma_D t}[\rho_I(0)\cos\Omega t - \rho_R(0)\sin\Omega t], \qquad (18)$$

which clearly shows that the diagonal elements of the density matrix decay exponentially towards their equilibrium values on a time scale given by $1/\Gamma_R$ while the nondiagonal elements perform oscillations with frequency Ω which are damped on a typical time $1/\Gamma_D$.

This behavior is reflected in the evolution of the current mean value which is given by

$$\langle I(t)\rangle = I_0\left(1 - 2\frac{W^+}{\Gamma_R}\right) + 2I_0 e^{-\Gamma_R t}\left(\frac{W^+}{\Gamma_R} - \rho_+(0)\right)$$
$$+ 4e\left(\frac{\Delta}{\hbar}\right)^2\frac{\sqrt{1-\tau}}{\Omega}e^{-\Gamma_D t}[\rho_R(0)\cos\Omega t$$
$$+ \rho_I(0)\sin\Omega t], \qquad (19)$$

where $I_0 = e(\Delta/\hbar)^2\tau\sin\phi/\Omega$ is the equilibrium current at zero temperature.

According to Eq. (19) quantum coherence would manifest as damped Rabi oscillations in the mean current through the contact. These oscillations appear for an initial condition in which both the lower and the upper Andreev state are populated. An arbitrary initial condition can be in principle reached from the equilibrium situation by imposing a given time-dependent evolution to the phase. The amplitude of the oscillations are maximal for $\phi = \pi$ where they become transmission independent. However, by varying the transmission, the frequency of the oscillations could be tuned in order to reach an experimentally accessible range. For instance, in the case of aluminum and transmission $\tau = 0.99$ the frequency Ω would be of the order of 3×10^{10} Hz.

The decay of these oscillations is controlled by the dephasing rate. Figure 2 illustrates the dependence of the relaxation and dephasing rates on the phase ϕ_0, the contact transmission τ and the temperature. At zero temperature we have $\Gamma_R = 2\Gamma_D = W_0^-$, which means that both rates reach a maximum at $\phi_0 = \pi$ regardless of the contact transmission. This maximum decrease with transmission as $\sqrt{1-\tau}$. At finite but low temperatures we have $\Gamma_R = W_0^-(1 + 2e^{-\hbar\Omega/kT})$ and $\Gamma_D = W_0^-(\frac{1}{2} + 2\eta^2 kT/\hbar\Omega)$. As expected, both rates in-
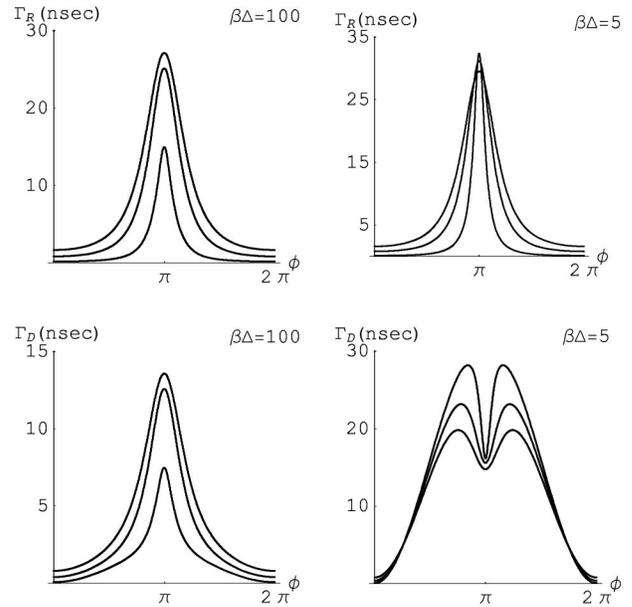
FIG. 2. Relaxation and dephasing rates Γ_R and Γ_D as function of the phase ϕ for transmission coefficients $\tau = 0.9$, 0.95, and 0.99 (from top to bottom) and two different values of the parameter $\beta\Delta$. The downward transition rate at zero temperature W_0^- coincides in the graphic scale with Γ_R for $\beta\Delta = 100$.

crease with temperature but the dephasing rate increases linearly instead of exponentially. The maximum of Γ_R is located at $\phi_0 = \pi$ for all range of temperatures and behave as $\sqrt{1-\tau}\coth\beta\Delta\sqrt{1-\tau}$. On the other hand, Γ_D develops a dip at $\phi_0 = \pi$ and exhibits a double peaked structure located $\phi_0 = 2\arccos\pm\sqrt{1 + [\sqrt{(1-\tau)/2} - 1]/\tau}$ for high temperatures. In this classical limit we get $\Gamma_R = W_0^- 2kT/\hbar\Omega$ and $\Gamma_D = \Gamma_R(\frac{1}{2} + \eta^2)$.

In summary, we have studied the dissipative dynamics of the Andreev states in a SQPC coupled to an electromagnetic environment characterized by a generic frequency-dependent impedance. Our results show that with the appropriate choice of the environment parameters one can reach values of the dephasing and relaxation rates within the $n\sec^{-1}$ range. A SQPC in a ring geometry thus provides an interesting realization of a solid-state two-level system in which quantum interference effects could be observed.

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[9] Notice that our analysis is restricted to the subgap states neglecting coupling with the continuous part of the spectrum. Other interactions, such as electron-phonon coupling, would introduce a broadening of the subgap states (Ref. 10). For aluminum atomic contacts this broadening is, however, small (Ref. 11) and would not introduce important corrections to the relaxation and dephasing rates.

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