

**<sub>1</sub> A Vortical Boundary Layer for Near-Radial IMF:  
<sub>2</sub> Wind Observations on October 24, 2001**

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3 **Abstract.** We present an example of a boundary layer tailward of the  
4 dawn terminator which is entirely populated by rolled-up flow vortices. Ob-  
5 servations were made by Wind on October 24, 2001 as the spacecraft moved  
6 across the region at  $X \sim -13 R_E$ . Interplanetary conditions were steady with  
7 a near-radial IMF. Approximately 15 vortices were observed over the 1.5 hr  
8 duration of Wind's crossing, each lasting  $\sim 5$  min. The rolling-up is inferred  
9 from the presence of a hot tenuous plasma being accelerated to speeds higher  
10 than in the adjoining magnetosheath, a circumstance which has been shown  
11 to be a reliable signature of this in single-spacecraft observations [*Takagi et*

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12 *al.*, 2006]. A blob of cold dense plasma was entrained in each vortex, at whose  
13 leading edge abrupt polarity changes of field and velocity components at cur-  
14 rent sheets were regularly observed. In the frame of the average boundary  
15 layer velocity, the dense blobs were moving predominantly sunward and their  
16 scale size along  $X$  was  $\sim 8.4 R_E$ . Inquiring into the generation mechanism  
17 of the vortices, we analyze the stability of the boundary layer to sheared flows  
18 using compressible magnetohydrodynamic Kelvin–Helmholtz theory with con-  
19 tinuous profiles for the physical quantities. We input parameters from (i) the  
20 exact theory of magnetosheath flow under aligned solar wind field and flow  
21 vectors [Spreiter and Rizzi, 1974] near the terminator, and (ii) the Wind data.  
22 It is shown that the configuration is indeed KH unstable. This is the first  
23 reported example of KH-unstable waves at the magnetopause under a ra-  
24 dial IMF.

## 1. Introduction

25 There is a long history of observations of waves at the boundary between the magneto-  
26 sphere and the magnetosheath (e.g. [Lepping and Burlaga, 1979]; [Sckopke et al., 1981],  
27 [Chen and Kivelson, 1993]; [Farrugia et al., 2001], and references therein). In view of the  
28 velocity shear that exists between these two plasma regimes, the Kelvin-Helmholtz (KH)  
29 instability has often been invoked to explain these waves.

30 From theoretical studies of the KH instability, two main points to keep in mind are  
31 the following: (i) the magnetic tension force (analogous to the surface tension force in  
32 hydrodynamics); and (ii) the compressibility of the plasma. Both are stabilizing factors.  
33 Thus KH instability depends on the magnetic field configurations, in particular their  
34 orientations with respect to the flow, and the speed of the plasma, which increases with  
35 distance down the flanks.

36 KH waves are thought to be one way of transferring solar wind momentum and energy  
37 to the magnetosphere. The KH instability forms part of the so-called “viscous-type” solar  
38 wind–magnetosphere interactions, to distinguish them from reconnection between the  
39 magnetosheath and magnetosphere fields. The contribution of viscous-type interactions  
40 to the cross-polar cap potential is often estimated as  $\sim 30$  kV [Cowley, 1982]. The  
41 question of magnetosheath mass entry goes beyond considerations of ideal MHD stability  
42 since other processes are required to break the associated frozen-in condition. However,  
43 the large vortices generated by the KH instability may set up conditions favorable to  
44 small-scale tearing of magnetic field lines inside the structures and, as a consequence, to

45 mass diffusion (see e.g. *Otto and Fairfield*, [2000]; *Smets et al.*, [2002]; *Otto and Nykyri*,  
46 [2003]).

47 Many of the data examples of KH instability in a magnetospheric context have used the  
48 capability of multiple spacecraft observations, such as Cluster, to confirm the presence of  
49 waves and their features, in particular if they have reached a non-linear phase and started  
50 to roll over. However, we do not often have this luxury and there are many tabulated  
51 crossings of a wavy magnetopause boundary made by single spacecraft. Can we somehow  
52 infer the presence of rolled-up vortices from single-spacecraft observations?

53 A key advance in this direction was made by *Takagi et al.* [2006]. Their MHD simu-  
54 lations showed that *in situ* observations of a low density magnetospheric plasma moving  
55 tailward at speeds higher than that of the adjacent magnetosheath is a very good indica-  
56 tor of rolled-up vortices. This opens new possibilities. First to apply this criterion were  
57 *Hasegawa et al.* [2006], who confirmed results on rolled-up KH vortices obtained earlier  
58 by *Hasegawa et al.* [2004] with a multi-spacecraft analysis. While the simulations were  
59 done for a northward IMF and specific parameters characterizing the ambient regions,  
60 *Nakamura et al.* [2004] had already given the physical origin of the signature of a rolled-  
61 up vortex. For pressure balance to hold across the vortex (same centrifugal forces at a  
62 given radial distance from the center), the hot tenuous plasma must revolve at a higher  
63 speed than the cold dense plasma.

64 Figure 1 presents a schematic to help visualize this point. The upper panel illustrates  
65 the perturbed magnetopause (MP) at the equatorial dawn flank that begins to roll over  
66 into a vortex by the KH instability. The magnetosheath flow is tailward ( $V_x < 0$ ), while  
67 the magnetosphere is stagnant. Accordingly, across the boundary layer there is a velocity

68 gradient. The drawing is shown in the frame of the vortex, so that the cold dense mag-  
 69 netosheath tongue (blue) protruding to the left is slowing down relative to the average  
 70 flow, while the related hot tenuous magnetosphere protuberance (red) points to the right  
 71 and accelerates toward the tail. The cold dense plasma intermingles with the hot tenuous  
 72 plasma. The thick arrowed lines give an indication of the plasma motion. The thin blue  
 73 lines are the conjectured deformation of the magnetosheath magnetic field projected into  
 74 the XY plane. Hence (1) we expect an alternation of high and low density cycles in the  
 75 data recorded by a spacecraft crossing the structure. Besides, (2) we anticipate that a  
 76 scatter plot of  $V_x$  versus the plasma density  $N$  during the passage of the whirling flow  
 77 should show the statistical trend indicated in the bottom panel. Features (1) and (2) are  
 78 the basic elements of a criterion that permits the identification of a boundary roll-over in  
 79 the observations.

80 Aside from (i) a case study addressing an interval of southward-pointing interplanetary  
 81 magnetic field (IMF) [Hwang *et al.*, 2012a], and (ii) another study with a dawnward-  
 82 pointing IMF [Hwang *et al.*, 2012b], most of the works on vortical structures at the  
 83 magnetopause/boundary layer have concentrated on a strongly northward-pointing IMF,  
 84 which is parallel to the Earth's field at low latitudes. If this lasts for several hours,  
 85 it is typically associated with the northward-pointing phase of interplanetary magnetic  
 86 clouds [Burlaga *et al.*, 1981]. A northward orientation favors a KH instability development  
 87 because when the wave vector  $\vec{k}$  of the perturbation is orthogonal to the average direction  
 88 of the two magnetic fields (“*flute*” modes), or normal to the stronger one, the restraining  
 89 magnetic forces are nearly canceled. At the same time, a substantial part of the velocity  
 90 shear effect is retained. This argument applies equally well to southward IMF.

91 Here, by contrast, we focus on a situation where the IMF is oriented in a radial direction  
92 pointing approximately opposite to the solar wind flow. Under this configuration we  
93 present an example of rolled-up flow vortices making up the entire boundary layer at  
94 low latitudes a few  $R_E$  (Earth radii) tailward of the dawn terminator. The criterion  
95 for inferring the rolling-up stage, which was mentioned above, is satisfied. Furthermore,  
96 the Wind probe that recorded the rolling motion on October 24, 2001, was traveling  
97 orthogonal to the bulk motion of these structures, an ideal circumstance and one which is  
98 much superior to magnetopause-skimming orbits, which do not sample the whole structure  
99 of the vortices. In addition, the external field, too, was exceptionally steady and smooth  
100 in a plasma of low beta. In particular, there were no significant variations in the solar  
101 wind dynamic pressure. As noted by *Farrugia et al.* [2007] the magnetosphere was in a  
102 very quiescent state. Reconnection processes were at best weak and patchy (in time).

103 We then inquire into the possibility that the vortices are of KH origin. We adopt two  
104 approaches. In the first approach, we input parameters to the theoretical stability analysis  
105 taken from the exact MHD solution derived by *Spreiter and Rizzi* [1974] and appropriate  
106 for collinear field and flow. This theory was applied to the present event during the  
107 later time when Wind was crossing the magnetosheath [*Farrugia et al.*, 2010]. In the  
108 second approach we input to the theoretical calculations the observations made by the  
109 Wind spacecraft. In both cases we work with compressible MHD equations, using for the  
110 physical quantities continuous profiles across a thick boundary layer. This avoids pitfalls  
111 in the use of the stability condition for a thin boundary model (Appendix A, formula13),  
112 pointed out by *Gratton et al.* [2004a]. In both cases we find the region to be KH-unstable.

113 This is thus the first reported instance of rolled-up KH vortices populating a boundary  
114 layer under a near-radial IMF.

115 A magnetic field aligned with the flow is, of course, *prima facie* the most unfavorable  
116 configuration for the Kelvin-Helmholtz instability because the magnetic tension exerts a  
117 stabilizing action, which cannot be avoided by modes of the “*flute*” type. The stabilizing  
118 action of the field tension is precisely that avoided by “*flute*” modes. With a wave vector  
119  $\vec{k}$  perpendicular to the magnetic field, these eliminate its operation, but in field-aligned  
120 flows they eliminate also the instability driver. In this paper we discuss how, nonetheless,  
121 the configuration can be KH unstable.

122 A distinctive aspect of the case we present is that a radial magnetic field is forced to  
123 be drawn along by the billows when they arise. This constitutes a substantial difference  
124 from the KH instability for northward-pointing fields, where vortices can grow in a “*flute*  
125 *mode*” configuration, with only small changes in the orientation of the field lines.

126 The layout of the paper is as follows. After discussing the interplanetary data, we  
127 describe the observations in the boundary layer made by Wind. We then discuss elements  
128 of the KH instability relevant to our work. A summary and discussion follows. We give  
129 some technical details on the KH instability in the two appendices.

## 2. Observations

### 2.1. Interplanetary: ACE

130 Interplanetary conditions during the period we study consisted of a structure which  
131 formed the last in a set of interacting interplanetary coronal mass ejections (ICMEs).  
132 The ICMEs passed Earth during the five-day period from October 21 to October 25,  
133 2001 (see *Farrugia et al.*, 2007) and were preceded by a strong shock. The state of the



134 magnetosphere went from being strongly disturbed (October 21-October 23) to being  
 135 almost quiescent. In the first period, two intense geomagnetic storms (Dst <-150 nT)  
 136 were recorded. Then, on October 24-25, all organized activity subsided. This very quiet  
 137 period ended when a trailing shock was seen advancing into the ICME sequence.

138 Observations over that part of this interval which is relevant to our study are shown  
 139 in Figure 2. The interplanetary plasma and magnetic field observations are from the  
 140 ACE spacecraft in orbit around the L1 Lagrangian point. They were acquired by the  
 141 SWEPAM [*McComas et al.*, 1998] and MAG [*Smith et al.*, 1998] instruments, and are at  
 142 64 s (plasma) and 16 s (magnetic field) temporal resolution. The time interval shown is  
 143 18–21 UT, October 24, 2001. From top to bottom the panels display the proton density,  
 144 temperature, (in red: the expected temperature after the statistical analysis of *Lopez*,  
 145 1987), bulk speed, the GSM components of the magnetic field (color-coded), the total  
 146 field strength, the IMF cone angle, i.e. the angle made by the magnetic field to the Earth-  
 147 Sun line, the dynamic pressure, the angle ('shear') between the field and flow vectors, the  
 148 proton beta, and the sonic and Alfvén Mach numbers ( $M_s$  and  $M_A$ , respectively).

149 This period is marked by steady conditions and very smooth field and plasma temporal  
 150 profiles. The temporal variations, which were a leading feature of the previous three  
 151 days, have died down completely. The data show a slow (average and standard deviation:  
 152  $\langle V \rangle = 372.5 \pm 2.5 \text{ km s}^{-1}$ ) and very cold ( $\langle T \rangle = 4187 \pm 375 \text{ K}$ ) ICME, the proton  
 153 temperature being about eight times less than the expected temperature. Compared to  
 154 the normally dense slow solar wind, the density ( $\langle N \rangle = 3.74 \pm 0.46 \text{ cm}^{-3}$ ) is about  
 155 one-half of a typical value of 7-10  $\text{cm}^{-3}$ , leading to below-average dynamic pressure of  
 156  $1.0 \pm 0.10 \text{ nPa}$ . As a consequence of this, the proton  $\beta$  is also very low, whence the

157 smooth magnetic field profile. Because of the low  $T_p$ , the sonic Mach number  $M_s$  is very  
 158 high, of order 35. At the position of Wind tailward of the dawn terminator, we therefore  
 159 expect that effects due to compressibility of the plasma will be accentuated. The Alfvén  
 160 Mach number is not particularly small ( $\sim 7$ ) and so the magnetic forces should not have a  
 161 dominating influence on the instability. Importantly, the magnetic field has a near-radial  
 162 orientation (panel 4) with a cone angle of  $17.7^\circ \pm 4.1^\circ$ . It makes an angle with the plasma  
 163 flow vector of  $162.0^\circ \pm 4.7^\circ$ , so that it points almost opposite to the solar wind.

## 2.2. Observations in the dawnside boundary layer: Wind

164 In October 2001 the Wind spacecraft was orbiting the magnetosphere, reaching perigee  
 165 in the near-geomagnetic tail region. Figure 3 shows its orbit from 19 UT, October 24 to  
 166 02 UT, October 25, after which time it exited into the solar wind [Farrugia *et al.*, 2010].  
 167 The red segments indicate the time when Wind was traversing the dawnside low-latitude  
 168 boundary layer (LLBL) (19-20:30 UT, see below) downstream of the terminator at  $X \sim$   
 169  $-13.5 R_E$  and at somewhat northerly GSM latitudes ( $Z \sim 5.5 R_E$ ). This orbit cuts across  
 170 any structures which are propagating downstream in this region. This is an ideal situation  
 171 for our purposes.

172 Examples of the structures encountered in the period 19:00-19:30 UT are shown in  
 173 Figure 4. The data are from the 3D Plasma Analyzer [3DP, Lin *et al.*, 1995] and the  
 174 magnetic field investigation [Lepping *et al.*, 1995], both plotted at 3 s resolution. Shown  
 175 from top to bottom are the proton density, bulk speed, temperature, the total field and  
 176 its GSM components, and the GSM components of the flow vector. The dashed blue line  
 177 in panel 2 gives the average magnetosheath velocity in the first half-hour after Wind's  
 178 entry at 20:30 UT ( $= 314 \text{ km s}^{-1}$ ). The averages of the bulk flow velocity components

179 are marked in the last three panels by the horizontal red lines. The average negative  
180  $V_y$ -component ( $= -46 \text{ km s}^{-1}$ ) results from the dawnward flaring of the magnetopause  
181 ( $\approx 7.5^\circ$ ) at Wind's dawnside locale. Marked by vertical guidelines are the times when sharp  
182 increases in density occur and when simultaneously impulsive changes in the magnetic  
183 field and/or plasma parameters are evident. It should be noted that uncertainties in the  
184 plasma moment results do increase as the proton density drops below  $\sim 0.1 \text{ cm}^{-3}$ . However,  
185 nowhere are the interpretation and conclusions of the paper affected by this.

186 We note the following features:

187 (i) Repetitive high-speed bursts of a hot tenuous plasma reaching speeds (up to  $\sim 650$   
188  $\text{ km s}^{-1}$ ) which are well in excess of the magnetosheath speed.

189 (ii) After the discontinuities (vertical lines), intervals of a cold dense (magnetosheath)  
190 plasma each lasting for  $\sim 2$ -3 min are encountered.

191 (iii) In the Earth's frame, the cold dense plasma is moving more slowly tailward than  
192 the average flow. In the average velocity frame, its motion is thus predominantly sunward.

193 (iv) By contrast, in the average velocity frame the hot tenuous plasma is moving an-  
194 tisuward. Note the repeated overshoot of this plasma with respect to the antisunward  
195 velocity.

196 These last two points may be seen very well from the clear anti-correlated behavior of  
197 the density  $N$  and the antisunward velocity,  $-V_x$ .

198 (v) Sharp changes in the field and flow vectors, including abrupt polarity reversals,  
199 tend to occur at the leading edges of the cold dense (magnetosheath) structures. With  
200 one exception (that at 19:11 UT) the leading edges are thus simultaneously current and  
201 vortex sheets.

(vi) Considering only the largest changes (indicated by the vertical guidelines) there are six intervals of roughly repetitive structures with an average duration of  $\sim 4.5$  min. This average periodicity is retained throughout the entire 1.5 hr traversal of the LLBL, as we discuss below (section 2.3).

From (i) and (iv) we see that the criterion for identifying rolled-up vortices given in the Introduction, namely, a hot tenuous plasma flowing at speeds higher than that of the magnetosheath, is well satisfied. We now illustrate these features by focusing on one typical cycle of the plasma and field behavior.

Plasma and magnetic field data for a single cycle, corresponding to the interval marked by the horizontal red bar in the top panel of Figure 4, are shown in Figure 5. The same quantities as in Figure 4 are plotted in the first seven panels (note, however, the linear scale for the density). The last three panels show the plasma velocity in the average velocity frame. The approximate duration of the cycle is from  $\sim 19:12:30$  UT to  $\sim 19:18:00$  UT ( $\sim 5.5$  min). The cold dense plasma interval is bracketed by the two vertical dashed red lines and lasts for  $\sim 2:25$  min. Immediately preceding the leading edge of the cold dense plasma at 19:12:20 UT, a plasma of low density and elevated temperature is moving at a speed exceeding that of the solar wind. The plasma there is flowing mainly perpendicular to the local magnetic field (not shown). The same may be seen from  $\sim 19:16$  to 19:18 UT ahead of the next cold dense plasma burst.

Relative to the average velocity, the cold dense plasma is moving mainly sunward ( $\Delta V_x > 0$ ). With an average speed of  $371 \text{ km s}^{-1}$  and a duration of 2:25 min, the scale size of the cold dense plasma in the X-direction is estimated as  $8.4 R_E$ . So it is very stretched in the X-direction, compared, say, to the distance around the magnetopause

225 from the nose which is of order 20-25  $R_E$ . After the cold dense plasma there follows a  
 226 stage ( $\sim 19:15:10$ - $19:16:00$  UT) in which the plasma has acquired a dawnward velocity  
 227 component ( $\Delta V_y < 0$ ) and its sunward speed has decreased. Then comes a burst of hot  
 228 tenuous plasma moving antisunward and northward and which ends up moving strongly  
 229 antisunward and duskward. A rotational motion superposed on the antisunward and  
 230 dawnward bulk flow is thus evident.

231 To visualize the flow rotation in the average velocity frame, we show in Figure 6 the  
 232 residual flow vectors  $\Delta V_x$ ,  $\Delta V_y$  for the period 19:12:30–19:18 UT. Time runs from the  
 233 bottom to the top, and the labels 'S' and 'E' refer to the start and end of the interval.  
 234 The arrows show the coordinates of the residual vectors. The blue arrows refer to the  
 235 cold dense plasma, the red arrows to the hot tenuous plasma, and the green arrows to  
 236 an intermediate state in  $(N, T)$ . It is seen that, in the average velocity frame, the cold  
 237 dense plasma is flowing mainly sunward with generally a very small duskward component  
 238 ( $\Delta V_y > 0$ ). The hot tenuous plasma (red) first flows tailward and then tailward and  
 239 duskward. The flow direction of the plasma in between (green) starts rotating from  
 240 a sunward and dawnward orientation and finishes in an antisunward orientation. This  
 241 provides clear evidence of rolling-up (see Introduction).

### 2.3. General features of the Vortical Structures

242 The fact that the hot tenuous magnetospheric plasma is moving at speeds above those  
 243 of the solar wind is strong evidence that the structure we are dealing with in Figure 6 is  
 244 a rolled - up vortex [*Nakamura et al.*, 2004; *Takagi et al.*, 2006; *Hasegawa et al.*, 2004,  
 245 2006]. We wish now to confirm this for all the quasi-periodic structures seen by Wind in  
 246 the interval 19:00-20:30 UT. In Figure 7 we plot in the bottom panel  $V_x$  versus  $N$  for the

247 whole interval. The top panel show the same quantities for the one plasma and field cycle  
 248 we have just been discussing. The color scheme indicates the temperature (red for hot  
 249 and blue for cold). The dashed horizontal lines show the corresponding quantities when  
 250 Wind entered the magnetosheath, (not shown; but see *Farrugia et al.*, 2010, their Figure  
 251 7), averaged over the first half-hour.

252 From the bottom panel it is seen that the bulk of the hot tenuous plasma is moving in  
 253 an antisunward direction faster than the magnetosheath. Clearly also, the figure shows  
 254 that the origin of the cold dense plasma is the magnetosheath. This is the same trend as  
 255 seen in the single cycle plotted in the top panel although the highest speeds recorded there  
 256 were  $\sim 580 \text{ km s}^{-1}$ . Following *Takagi et al.* [2006], we conclude that Wind is observing  
 257 an LLBL populated entirely by a sequence of rolled-up vortices.

258 Figure 8 depicts the motion of the plasma in the dawn-dusk direction in the form of a  
 259 scatter plot of the residual  $\Delta V_y$  versus  $\Delta V_x$  for the whole interval 19:00–20:30 UT. The  
 260 color is proportional to  $\log T$  (red = hot) and the size of the squares to  $N$ . The figure  
 261 shows a continuous distribution of  $\Delta V_y$  values spanning across zero. There is no strong  
 262 preference for positive or negative  $\Delta V_y$ . The spread in  $\Delta V_y$  of the hot tenuous plasma  
 263 is wider. This overall picture confirms the persistence of the rotational motion in the  
 264 average velocity frame quite clearly.

265 We recall from Figure 2 that interplanetary conditions were steady. Specifically, there  
 266 were no significant variations in the dynamic pressure,  $P_{dyn}$ . But, in fact, there are  
 267 large-amplitude, quasi-periodic fluctuations of this quantity generated by the vortices  
 268 themselves. To show this, we consider in Figure 9 the temporal variation of  $P_{dyn}$  at  
 269 Wind. The 1.5 - hour traversal is split into three  $\sim 0.5$ -hour segments which are plotted

270 underneath each another. Each panel shows the thermal plasma pressure (green trace)  
 271 and the dynamic pressure (black trace). The blue and red traces are 21-point ( $\sim 1$  min)  
 272 running averages of these two quantities, respectively. In computing the dynamic pressure,  
 273 we took into account the average flaring of the dawn magnetopause, given by  $V_y$  in Figure  
 274 4. This gives a flaring angle of  $\theta \approx 7.5^\circ$  and the dynamic pressure has been multiplied  
 275 by  $\sin(\theta)$ . Underneath each plasma pressure panel, we plot the magnetic pressure for the  
 276 corresponding interval. A linear scale is used for this.

277 One can see that the average dynamic pressure is subject to large-amplitude oscillations  
 278 of period  $\sim 5.0$  min. Note that there are six clear waves corresponding to the vortices in  
 279 the top panel. These are the ones identified in Figure 4 except for the small one between  
 280 19:11 and 19:12.2 UT, which might indicate some ongoing coalescence. The thermal  
 281 plasma pressure behaves as the dynamic pressure, only at much reduced amplitude. The  
 282 magnetic pressure is variable and its size is bigger than that of the plasma pressures.  
 283 Overall pressure balance is not maintained. The fluctuations of the thermal pressure can  
 284 produce ion acoustic waves along the geomagnetic field. The variation of the magnetic  
 285 pressure can radiate magnetosonic waves across the magnetic field. The vortices can thus  
 286 give rise to large scale effects in the plasma sheet.

### 3. Generating Mechanism: The Kelvin-Helmholtz Instability Source

287 The configuration of October 24, 2001 appears not to favor the onset of the KH in-  
 288 stability: (i) the restraining magnetic forces are strong in field-aligned flows and (ii) the  
 289 large  $M_s$  ushers in the other stabilizing factor, compressibility.

290 We now examine the issue more closely. We model the LLBL transition by continuous  
 291 functions for the physical parameters. We call this “*thick model*” for short, to distinguish

292 it from a “*thin*” approximation where the quantities suffer a discontinuous change across  
 293 the boundary layer. (Appendix A, 13; see also *Gratton et al.*, [2004a], [2004b]; *Gnavi et*  
 294 *al.*, [2009]). For the stability analysis we work in a flow-aligned coordinate system defined  
 295 as follows. The  $x$ -axis points in the direction of the local  $\vec{V}$ . The  $y$ -axis points across the  
 296 LLBL, normal to the local magnetopause and directed outward. The  $z$ -axis completes the  
 297 right-handed Cartesian triad, and is oriented in the same sense as geomagnetic north.

Scalar and vector quantities in the LLBL are represented by hyperbolic tangent func-  
 tions with a scale length  $d$ , for example:

$$V_x = V_1(1 + \tanh(y/d))/2, \quad (1)$$

298 for the velocity, and with similar expressions for  $\vec{B}$  and  $N$  (Appendix A). Subscripts ‘1’  
 299 and ‘2’ refer to magnetosheath and magnetosphere quantities, respectively. The temper-  
 300 ature profile  $T(y)$  follows from the pressure balance equation across the layer (Appendix  
 301 B). We take  $D = 4d$  as a representative value for the LLBL thickness, which ranges ap-  
 302 proximately from  $y = -2d$  to  $+2d$ . The normalized quantities contain  $d$  and  $V_1$  implicitly  
 303 such as, for example, in the normalized growth rate  $g = \gamma d/V_1$ . An estimated value of  
 304  $D$ , and a measured value of  $V_1$ , can be introduced in the discussion at the end of the  
 305 theoretical calculation; it is not necessary to assume them beforehand. The compressible  
 306 MHD stability theory used here is summarized in Appendix A, where some details of the  
 307 procedure can be found. For every  $k$ -mode the KH instability is driven by the intensity  
 308 of the velocity projection  $V_k$  in the  $\vec{k}$ -direction ( $V_k = \vec{V} \cdot \hat{k}$ , where  $\hat{k} = \vec{k}/|\vec{k}|$ ). The  
 309 magnetic tension that opposes the instability depends on the magnetic field projection  $B_k$   
 310 in the  $\vec{k}$ -direction ( $B_k = \vec{B} \cdot \hat{k}$ ).



311 We follow two approaches regarding the physical parameters which we input into the  
 312 theory. In the first approach, the Mach numbers are based on the *Spreiter and Rizzi*  
 313 [1974] theory, that gives an approximate representation of the solar wind – magnetosheath  
 314 transition for collinear MHD flows. We use the Spreiter – Rizzi solution with solar wind  
 315 input from ACE. Close to the terminator this theory predicts approximately:  $M_s = 7.7$ ,  
 316  $M_A = 4.9$ , which corresponds to a magnetosheath plasma  $\beta_1 = 0.97$ .

317 The geomagnetic field on the dayside is assumed to be perpendicular to  $\vec{V}$ , i.e. the  
 318 magnetic shear angle in this model is  $90^\circ$ . Presumably, this angle was not exactly  $90^\circ$ ,  
 319 and it varies with the distance from the subsolar point. But we think that near Earth  
 320 deviations from  $90^\circ$  could not have been substantial. Anyway, the chosen shear angle is  
 321 not critical to decide on the instability because we intend to switch-off – or, at least, much  
 322 reduce – the magnetic tensions on the magnetosphere side of the LLBL by considering  
 323  $k$  -vectors normal to the local geomagnetic field. This choice of  $\vec{k}$  favors  $V_k$ , the driver  
 324 of the instability (and maximizes it when the magnetic shear angle is exactly  $90^\circ$ ), but  
 325 exposes the  $k$  -mode to the full stabilizing influence of the magnetosheath field projection  
 326  $B_k$ . For the dayside we assume a typical particle density ratio  $N_2/N_1 \sim 0.1$ .

327 The pressure balance equation imposes an upper limit on the magnetic field ratio  
 328  $B_2/B_1 < 1.4$ . (About this requirement see condition 20 in Appendix B.) In approach  
 329 (1) we computed with  $B_2/B_1 = 1$ ,  $n_2/n_1 = 0.1$ , and (as a consequence of eq.(19), Ap-  
 330 pendix B) a temperature ratio  $T_2/T_1 \sim 10$ . The choice reflects expected values at the MP  
 331 away from the subsolar point, but still near Earth, as the terminator.

332 The mode considered is with  $\vec{k}$  parallel to the flow. (Computation shows it to be the  
 333  $\vec{k}$  orientation of fastest growth.) Figure 10 shows the (normalized) imaginary part of the

334 characteristic value,  $c_i = \gamma/kV_1$ , as a function of  $kd$ . Quantity  $c_i$  is a linear function of  $kd$   
 335 in most of the interval (for  $kd \geq 0.15$ ), so that the growth rate  $g = \gamma d/V_1$  as a function of  
 336  $kd$  is approximated by a parabola (not shown). The maximum of the normalized growth  
 337 rate  $g = c_i kd = \gamma d/V_1$  occurs at  $kd = 0.245$  ( $\lambda = 6.41D$ ) for  $g = 0.0832$ . From this value we  
 338 may estimate an e-folding time  $\tau_e = 1/\gamma \sim 48$  s for LLBL sites near the terminator, with  
 339  $D = 0.5R_E$ , assuming that the LLBL thickness is not yet broadened by the instability, and  
 340 with  $V_1 \sim 200$  km/s. Therefore, in a boundary layer not yet widened by perturbations,  
 341 the KH instability can grow quite fast. We think that the vortices observed by Wind are  
 342 generated in the LLBL closer to Earth.

343 We now discuss the second approach. Here we input to the model data acquired during  
 344 the Wind's LLBL traversal. The scenario is a composition of averages of measurements:  
 345 (a) made in the magnetosphere before, but close to, the entrance to the LLBL (including  
 346 early times of the passage through it) with (b) data recorded – albeit later – in the adjacent  
 347 magnetosheath.

348 At Wind's position, with a magnetosheath average velocity of  $314 \text{ km s}^{-1}$ , the Mach  
 349 numbers computed with data for that region are  $M_s = 5.6$  and  $M_A = 6.8$ . They lead to  
 350 a plasma of  $\beta_1 = 3.5$ . The magnetic field  $\vec{B}$  in the magnetosheath is still approximately  
 351 collinear with the flow [Farrugia et al., 2010]. A difference from approach 1 is that the  
 352 magnetic shear angle is not  $90^\circ$  (as hypothesized for near-Earth positions) but  $71^\circ$  with  
 353 respect to the sunward direction (taken from an average  $\vec{B}$  on the magnetosphere side).

354 To sum up, the input parameters for the stability analysis are as follows.

355 Table 1

356

Magnetosheath	Magnetosphere
$M_s = 5.6, M_A = 6.8$	-----
$V_1 = 314 \text{ km/s}$	$V_2 = 0$
$N_1 = 5 \text{ cm}^{-3}$	$N_2 = 0.05 \text{ cm}^{-3}$
$B_1 = 4.5 \text{ nT}$	$B_2 = 9.5 \text{ nT}$
$\chi_1 = 180^\circ$	$\chi_2 = 109^\circ$

357 where  $\chi$  denotes the angle that the magnetic field makes with the  $x$ -axis. Inside the  
358 magnetosphere the total pressure is almost purely magnetic, but in the magnetosheath  
359 thermal and magnetic pressures are of the same order. The  $B_2/B_1$  ratio satisfies condition  
360 (20) Appendix B close to the limiting value 2.13, but the quantities still conform to an  
361 approximate pressure balance across the BL (the upper bound of 20 could be a bit larger  
362 with a correction for flaring).

363 Figure 11 shows the normalized growth rate  $g$  a function of  $kd$ . It reaches a still  
364 significant maximum value  $g = 0.033$  at  $kd = 0.43$  and goes to zero at  $kd \approx 0.07$  (long  
365  $\lambda$ ) and  $kd = 0.84$  (short  $\lambda$ ). The angle of  $\vec{k}$  with the  $x$  axis is  $\phi = 19^\circ$ . With a ratio  
366  $N_2/N_1 = 10^{-2}$  the growth rate  $\gamma$  is zero in the  $kd \rightarrow 0$  limit. This is a case of stability at  
367 long wavelengths.

368 The second approach intends to show that a steady-state LLBL model, endowed with  
369 equilibrium quantities represented by continuous functions that connect mean values of  
370 magnetospheric and magnetosheath data, is unstable. The averages include long stretches  
371 of time on either side of the LLBL, because the instability is found with wavelengths of  
372 several  $R_E$ , and the penetration depth of the KH perturbation is expected to be large.  
373 However, the LLBL and the magnetosheath are both perturbed already. The former by  
374 the passage of vortical structures as discussed in section 2, which we conclude are formed  
375 at some place near Earth; the latter by the turbulence after the bow shock, and by large

376 amplitude oscillations of long period [*Farrugia et al.*, 2010]. Nonetheless the result is  
377 an indication that the LLBL, at the Wind's orbit position, has amplifying properties  
378 regarding the KH instability mechanism. In other words, if by reason of intermittency  
379 the passage of vortical perturbations is temporarily suspended, the (unstable) background  
380 LLBL still maintains the capability to grow perturbations, and eventually to roll-over the  
381 velocity gradient layer again.

#### 4. Summary and Discussion

382 We took advantage a rare coincidence of a long interval of radial IMF, steady solar wind  
383 conditions, and a spacecraft taking observations along a path that cuts perpendicularly  
384 through the near-Earth flank of the magnetosphere. The Wind observations through the  
385 LLBL at X=-13 Re showed the LLBL to be full of rolled-up vortices. These were shown  
386 to arise from the KH instability. The new result here is not so much the observation of  
387 KH-like oscillations but that they occur under a radial IMF, which should suppress the  
388 growth of KH waves. So, while rolled-up vortices for northward IMF have been reported  
389 before, this is first reported case of observations of KH rolled-up vortices for radial IMF.

390 In some ways this was a continuation of work started in *Farrugia et al.*, [2007], [2010].  
391 In those papers we focussed on the extremely quiet state of the magnetosphere after a  
392 three-day long period of strong disturbance. The cause was a series of ICMEs, and the  
393 period we concentrated on here constituted the last of these where very steady conditions  
394 prevailed. The *Farrugia et al.* [2010] study of this event dealt with the entire magne-  
395 tosheath showing that the near-parallel alignment of field and flow held throughout the  
396 magnetosheath and that the data matched a relevant theory that treats flow-aligned field  
397 in the magnetosheath.

398 In a 1.5 hr traversal we identified approximately 15 vortices. We argued they had  
399 reached the non-linear stage and had started to roll up. The rolling-up process was inferred  
400 from the repeated presence of a low density, magnetosphere plasma moving antisunward  
401 at speeds greater than in the magnetosheath, which recent studies have shown to be a  
402 reliable indicator of such structures based on single-spacecraft *in situ* observations.

403 We then presented two KH instability calculations using different inputs for the theory  
404 model of the transitions. In the first we inputted data from the theory of *Spreiter and*  
405 *Rizzi* [1974], which gives an exact MHD solution for field-aligned flows. In the second, we  
406 inputted direct measurements made by Wind in the magnetosphere and in the magneto-  
407 sphere at the beginning and end of the LLBL crossing, respectively. In both cases the  
408 LLBL was found to be unstable.

409 Although the solar wind dynamic pressure was very steady, the passage of the struc-  
410 tures gave rise to large-amplitude modulations of the magnetic pressure and the dynamic  
411 pressure in the boundary layer. This could set up waves travelling along (ion acoustic  
412 waves) and perpendicular (magnetosonic waves) to the magnetic field. This shows that  
413 the passage of the large vortices at dawn could influence large parts of the plasma sheet.

414 We now discuss various aspects of the observations of the October 24, 2001 event. In  
415 previous work, *Farrugia et al.* [2007] concentrated on the very low level of geomagnetic  
416 disturbances which prevailed on this day. Such were, for example, an average Kp index =  
417 0+, polar caps which had very weak electron precipitation without any consistent north-  
418 south asymmetries, and patchy and weak reconnection at low latitudes or poleward of the  
419 cusp. In particular, the authors noted a cross-polar cap potential of  $\sim 20$  kV. This would  
420 be of about the same magnitude as that commonly ascribed to the contribution to the

421 CPCP of viscous-type interactions. However, *Farrugia et al.* [2007] argued against the  
422 KH instability. This conclusion was essentially based on the lack of ULF pulsations of the  
423 geomagnetic field in the Pc 5 range (2-7 mHz), which the KH instability is often thought  
424 to give rise to via the field line resonance theory [*Chen and Hasegawa, 1974; Southwood,*  
425 *1974*]. They thus could not find a solar wind driver for the weak and patchy convection.

426 Was the instability a result of the (radial) direction of the IMF, or was it favored by  
427 specific values of the Mach numbers in this case? This is an important question that  
428 deserves further attention. We think that in this case  $M_A$  values (estimated at the ter-  
429 minator, and measured at Wind's orbit) were clearly helping the onset, and development  
430 of the instability. The radial field orientation is unfavorable, in general, to the KH insta-  
431 bility. However, even in the case of normal solar wind and Parker's spiral field, there is  
432 the possibility that KH waves may develop inside the boundary layer, and grow thereafter  
433 tailwards. Further work is necessary to test other cases to see if it was the radial IMF  
434 orientation that did this, or rather other favorable parameters.

435 From the theory it was concluded that (i) in both approaches the boundary of this  
436 wide LLBL was KH unstable and (ii) the long wavelength limit is stable. That is, a thin  
437 layer would be stable. The instability appears only with the thick boundary layer. We  
438 also concluded that the generating site was well upstream of the observation locale. As  
439 a consequence of the stability study, we assume, as seems reasonable, that the lifetime of  
440 each member of the vortex sequence is similar because they are generated approximately  
441 at the same position upstream (closer to the Earth).

442 If a magnetic field collinear with the flow is generally unfavorable to the development  
443 of the KH instability, and the magnetosheath motion was supersonic, why is the LLBL

444 unstable in this event? This is so because the physical conditions of the inner edge of the  
445 LLBL are very different from those of the outer edge (adjacent to the magnetosheath).  
446 That the inner edge of the LLBL, in general, may be prone to the KH instability was also  
447 considered by other authors, among them *Ogilvie and Fitzenreiter*, [1989], and *Miura*,  
448 [1992].

449 Behind the bow shock, the magnetosheath is frequently in a turbulent state. The plasma  
450 of the boundary layer is pulled along by the solar wind. A velocity shear flow parallel  
451 to that of the magnetosheath is established across this layer. The motion is subsonic  
452 inside the LLBL, because of the decreased speed with respect to the magnetosheath, and  
453 the higher temperature of the magnetospheric plasma. At the equatorial dayside, the  
454 geomagnetic field is mainly normal to the flow. At the inner edge side, even if moving at  
455 reduced speed, the obstacles to the growth of the KH instability are attenuated. Flute  
456 modes with a wave vector normal to the local geomagnetic field, and parallel to the internal  
457 flow direction, are not restrained by magnetic tension forces, and the low compressibility  
458 reduces additional stabilizing effects. Downstream,  $M_A$  increases and magnetic tensions  
459 are further reduced, so that conditions for instability improve.

460 A major reason why people are interested in the non-linear stage of the KH instability  
461 in the first place is that, by breaking the frozen-in condition, it offers the possibility for  
462 mass transfer. This transfer would happen at current sheets where oppositely-directed  
463 magnetic fields have been brought next to each other during the rolling-up process (see  
464 Figure 4). We found several instances of current and vortex sheets. Indeed, most field  
465 and flow cycles contained one of these.

466 Current sheets are prone to magnetic line tearing, and hence are a possible way to mass  
 467 transport across the MP (see e.g. *Otto and Fairfield*, [2000], and *Otto and Nykyri*, [2003]  
 468 for computer simulation studies of field lines coiled-up inside vortices). From the data  
 469 we cannot tell to what extent the field lines are entrained by the vortices. But we find  
 470 repeated evidence of current sheet formation. Whether mass transport is actually taking  
 471 place in our case will be pursued in a further study.

472 Our work and that of *Hwang et al.* [2012a], [2012b] show that it is not necessary to  
 473 have a northward-pointing IMF to excite the KH instability. Neither is it necessary for  
 474 the IMF to point north to produce field configurations conducive to reconnection and, by  
 475 implication, mass entry.

## 5. Appendix A. KH Theory

476 The LLBL model with continuous functions used in section 3 describes a MHD parallel  
 477 flow with a local  $x$  axis directed along the velocity field. (The flow does not change  
 478 direction.) The physical quantities are constant over  $(x, z)$  planes, and vary only in the  
 479 transverse  $y$  direction, chosen to be normal to the MP. However, in general, the magnetic  
 480 field may change both in direction and strength. The unperturbed (or average state)  
 481 LLBL model is given by a set of functions:  $\vec{V} = (V_x(y), 0, 0)$ ,  $\vec{B} = (B_x(y), 0, B_z(y))$ ,  
 482  $\rho(y) = m_p N(y)$ , for velocity, magnetic fields, and mass density  $\rho$  or particle density  $N$ ,  
 483 respectively. The temperature function  $T(y)$  results from this set of functions and the  
 484 pressure balance equation (Appendix B).

Across the LLBL, the physical quantities have hyperbolic function profiles:

$$V_x = V_1(1 + \tanh(y/d))/2, \tag{2}$$



$$B = (B_1 + B_2)/2 + (B_1 - B_2) \tanh(y/d)/2 \quad (3)$$

$$\theta = (\theta_1 + \theta_2)/2 + (\theta_1 - \theta_2) \tanh(y/d)/2 \quad (4)$$

$$B_x = B \cos(\theta), \quad B_z = B \sin(\theta) \quad (5)$$

$$N = (N_1 + N_2)/2 + (N_1 - N_2) \tanh(y/d)/2 \quad (6)$$

485 where  $d$  is a scale length. The width  $D$  of the thick LLBL model is taken as  $D = 4d$ .

The perturbation modes of the KH instability are of the form

$$\Xi = \zeta(y) \exp(-i\omega t + ik_x x + ik_z z), \quad (7)$$

486 where  $\Xi$  is the  $y$ -component of the Lagrangian displacement of a plasma element from a  
 487 steady state position, and  $\zeta(y)$  is the corresponding amplitude. The (complex) angular  
 488 frequency of the modes is denoted by  $\omega = \omega_r + i\gamma$ . The real part  $\omega_r$  gives the frequency of  
 489 the oscillations, and the imaginary part is the growth rate of the instability when  $\gamma > 0$ ;  
 490 the e-folding time is  $\tau_e = 1/\gamma$ . The wavevector is represented by  $\vec{k} = (k_x, 0, k_z)$ ,  $k = |\vec{k}|$   
 491 is the wavenumber;  $\lambda = 2\pi/k$  is the wavelength.

The amplitude of the Fourier modes of the KH perturbation is governed by the second order differential equation,

$$\frac{d}{dy} \left[ H \left( 1 - \frac{1}{M} \right) \frac{d\zeta}{dy} \right] - k^2 H \zeta = 0, \quad (8)$$

derived from the linearized equations of ideal (non-resistive), compressible MHD [*Gratton et al.*, 1988]. A complex phase velocity  $c = \omega/k$  is introduced so that the functions  $H(y)$  and  $M(y)$  of the differential equation for  $\zeta$  can be written as,

$$H(y) = \rho \left[ (c - V_k)^2 - V_{Ak}^2 \right], \quad (9)$$

$$M = 1 - \frac{c_s^2 + V_A^2}{(c - V_k)^2} + \frac{c_s^2 V_{Ak}^2}{(c - V_k)^4} \quad (10)$$

492 where  $V_A = B/\sqrt{4\pi\rho}$  is the Alfvén speed,  $V_{Ak} = B_k/\sqrt{4\pi\rho}$ , is a projected Alfvén speed,  
 493  $c_s$  is the speed of sound, and  $V_k$ ,  $B_k$  are projections of the velocity and magnetic fields  
 494 in the  $\vec{k}$  direction. All these quantities are functions of  $y$ . The analysis is of a temporal  
 495 type, that is, a (real) wavenumber  $\vec{k}$  is given (as a Fourier component of the initial  
 496 perturbation), and the response of the system determines the unknown (complex) value  
 497 of  $c$ . To obtain  $c$  a boundary value problem for equation 8 must be solved.

When  $c_s \rightarrow \infty$ , the coefficient  $M \rightarrow \infty$ , and equation (8) reduces to

$$\frac{d}{dy} \left[ H \frac{d\zeta}{dy} \right] - k^2 H \zeta = 0, \quad (11)$$

that represents the incompressible MHD approximation. When the transition layer is very thin with respect to the wavelength, that is when  $kd \ll 1$ , an approximate dispersion relation can be derived:

$$H_1 + H_2 = 0, \quad (12)$$

where  $H_1$  and  $H_2$ , are the values taken by  $H$  on each side of the BL; labels 1 and 2 refer to the magnetosheath and magnetosphere, respectively. This is the “*thin model*” result for incompressible plasma flows. From equation (12) a well known stability condition follows,

$$\rho_R (\Delta \vec{V} \cdot \hat{k})^2 \leq \frac{1}{4\pi} \left[ (\vec{B}_1 \cdot \hat{k})^2 + (\vec{B}_2 \cdot \hat{k})^2 \right]. \quad (13)$$

498 where  $\rho_R$  is defined by  $1/\rho_R = (1/\rho_1 + 1/\rho_2)$ , and  $\Delta \vec{V} \equiv V_1 - V_2$ . The “*thin model*”  
 499 condition, often used in the current literature, ensures stability when it holds for all  
 500 directions of  $\vec{k}$ . The thin model stability does not depend on the wavelength.

501 The intricacy of the boundary value problem for equation (8) with finite wavelengths  
 502 derives from the fact that  $c$  is not an eigenvalue but a characteristic value entangled in a  
 503 non-linear fashion in the functions  $H$  and  $M$ . Moreover, when the direction of  $\vec{k}$  changes,

504 the functions  $V_k(y)$ ,  $B_k(y)$  (and other functions, such as  $c_s(y)$ , etc.) also change. Thus  
 505 the analysis requires the solution of separate differential equations for every  $\vec{k}$  direction.  
 506 In this paper we solved the boundary value problem for  $c$  using a conventional shooting  
 507 method. The compact form of equation (8) facilitates the use of shooting methods.

## 6. Appendix B. Pressure Balance Condition

The field functions of the local LLBL model,  $B(y)$ ,  $\rho(y)$ ,  $T(y)$ , etc., must satisfy pressure balance,

$$p_1 + \frac{B_1^2}{8\pi} = p(y) + \frac{B(y)^2}{8\pi} = p_2 + \frac{B_2^2}{8\pi}. \quad (14)$$

508 Here  $p = nk_B(T_i + T_e)$  is the thermal pressure ( $k_B$  is Boltzmann's constant). We assume  
 509 a common temperature value  $T_i = T_e = T$  (in our case the proton temperature is from  
 510 spacecraft data).

It is convenient to write equation (14) in terms of  $M_s$  and  $M_A$ , both computed with magnetosheath parameters adjacent to the local LLBL. Quantity  $M_s = V_1/c_{s1}$ , where  $c_{s1} = \sqrt{(\gamma k_B T_e/m_i)} = \sqrt{(\gamma k_B T_1/m_p)}$  ( $\gamma = 5/3$ , and  $m_i = m_p$  is the proton mass). Similarly,  $M_A = V_1/V_{A1}$ , with  $V_{A1} = B_1/\sqrt{4\pi\rho_1} = B_1/\sqrt{4\pi n_1 m_p}$ . Then eq. 14 can be written in the form

$$\frac{1}{\rho_1 V_1^2} \left[ p(y) + \frac{B(y)^2}{8\pi} \right] = \frac{\rho}{\rho_1 V_1^2} \left( \frac{4}{\gamma} c_s^2(y) + V_A^2(y) \right) = \left( \frac{4}{\gamma} \frac{1}{M_s^2} + \frac{1}{M_A^2} \right). \quad (15)$$

When  $M_s$  and  $M_A$  are known, the plasma beta is fixed because

$$\beta = \frac{2nk_B T}{B^2/8\pi} = \frac{4}{\gamma} \frac{c_s^2}{V_A^2}, \quad (16)$$

and since we are interested in the magnetosheath beta,

$$\beta_1 = \frac{4}{\gamma} \frac{M_A^2}{M_s^2}. \quad (17)$$

From these, the temperature function across the boundary layer can be written as,

$$\frac{T(y)}{T_1} = \frac{n_1}{n(y)} \left[ 1 + \frac{1}{\beta_1} \left( 1 - \frac{B(y)^2}{B_1^2} \right) \right]. \quad (18)$$

The local magnetosphere-to-magnetosheath temperature ratio is therefore:

$$\frac{T_2}{T_1} = \frac{n_1}{n_2} \left[ 1 + \frac{1}{\beta_1} \left( 1 - \frac{B_2^2}{B_1^2} \right) \right], \quad (19)$$

which implies that the magnetosheath beta,  $\beta_1$ , together with the magnetic field intensity ratio  $B_2^2/B_1^2$ , set a limit to the steady state boundary layer models. A local pressure balance does exist when

$$B_2^2/B_1^2 < 1 + \beta_1, \quad (20)$$

511 and we see that  $T_2$  becomes zero when  $B_2^2/B_1^2 = 1 + \beta_1$ . Under ordinary conditions, the  
 512 magnetosheath  $\beta_1$  is much larger than unity, so that this limitation is not important.  
 513 But in the October 24, 2001 event the values of  $\beta_1$  are comparable to, or even smaller  
 514 than, unity. Hence, when setting stability models the constraint (20) must be taken into  
 515 account.

516 Condition (20) needs a correction when the boundary is flared with respect to the solar  
 517 wind flow due to the presence of a normal component of the momentum flux. In practice,  
 518 this can be approximately assumed as an increment of the effective  $B_1^2$ , and then (20)  
 519 becomes a less severe bound.

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## References

- 525 Burlaga, L. F., Sittler, E., Mariani, F., and Schwenn, R. (1981), Magnetic loop behind an  
526 interplanetary shock: Voyager, Helios and IMP 8 observations, *J. Geophys. Res.*, *86*,  
527 6673.
- 528 Chen, L., and A. Hasegawa (1974), A theory of long-period magnetic pulsations, 1. Steady  
529 state excitation of field line resonance, *J. Geophys. Res.*, *79(7)*, 1024.
- 530 Chen, S. -H., and M. G. Kivelson (1993), On nonsinusoidal waves at the Earth's magne-  
531 topause, *Geophys. Res. Lett.*, *20(23)*, 2699, doi:10.1029/93GL02622.
- 532 Cowley, S. W. H. (1982), The causes of convection in the Earth's magnetosphere: A review  
533 of developments during the IMS, *Rev. Geophys.*, *20*, 531.
- 534 Farrugia, C. J., F. T. Gratton, and R. B. Torbert (2001), The role of viscous-type pro-  
535 cesses in solar wind-magnetosphere interactions, in Challenges to long-standing unsolved  
536 problems in space physics in the 20th century, *Space Science Rev.*, *95*, 443.
- 537 Farrugia, C. J., A. Grocott, P. E. Sandholt, S. W. H. Cowley, Y. Miyoshi, F. J. Rich, V.  
538 K. Jordanova, R. B. Torbert, and A. Sharma (2007), The magnetosphere under weak  
539 solar wind forcing, *Ann. Geophys.*, *25*, 1.
- 540 Farrugia, C. J., N. V. Erkaev, R. B. Torbert, H. K. Biernat, F. T. Gratton. A. Szabo, H.  
541 Kucharek, H. Matsui, R. P. Lin, K. W. Ogilvie, R. P. Lepping. C. W. Smith (2010),  
542 Magnetosheath for almost-aligned solar wind magnetic field and flow vectors: Wind  
543 observations across the dawnside magnetosheath at  $X = -12 R_E$  *J. Geophys. Res.* *115*,

544 A08227, doi:10.1029/2009JA015128.

545 Gnavi, G., F. T. Gratton, C. J. Farrugia and L. E. Bilbao (2009), Supersonic mixing  
546 layers: stability of magnetospheric flanks models. *Journal of Physics: C. S.* **166** 012022,  
547 doi:10.1088/1742-6596/166/1/012022.

548 Gratton, F. T., L. Bender, C. J. Farrugia, and G. Gnavi (2004a), Concerning a problem  
549 of the Kelvin-Helmholtz instability of the thin magnetopause, *J. Geophys. Res. A* **109**  
550 doi: 10.102 V2003JA010146.

551 Gratton, F. T., G. Gnavi, C. J. Farrugia, and L. Bender (2004b), On the MHD boundary  
552 of Kelvin-Helmholtz stability diagrams at large wavelengths, *Brazilian J. Phys.* **34** 1804-  
553 1813.

554 Gratton J., F. T. Gratton, and A. G. González (1988), Convective instability of internal  
555 modes in accelerated compressible plasmas, *Plasma Phys. Contr. Fusion* **30** 435-456.

556 Hasegawa, H., M. Fujimoto, K. Takagi, Y. Saito, T. Mukai, and H. Rème (2006), Single-  
557 spacecraft detection of rolled-up Kelvin-Helmholtz vortices at the flank magnetopause,  
558 *J. Geophys. Res.*, *111*, A09203, doi:10.1029/2006JA011728.

559 Hasegawa, H., M. Fujimoto, T.-D. Phan, H. Rème, A. Balogh, M. W. Dunlop, C.  
560 Hashimoto, and R. TanDokoro (2004), Transport of solar wind into Earth's magne-  
561 tosphere through rolled-up Kelvin-Hemholtz vortices, *Nature*, *430*, 755.

562 Hwang, K.-J., M. M. Kuznetsova, F. Sahraoui, M. L. Goldstein, E. Lee, and G. K. Parks  
563 (2012a), Kelvin-Helmholtz waves under southward interplanetary magnetic field, *J.*  
564 *Geophys. Res.*, *116*, A08210, doi:10.1029/2011JA016596.

565 Hwang, K.-J., M. L. Goldstein, M. M. Kuznetsova, Y. Yang, and A. Viñas (2012b), The  
566 first in situ observation of Kelvin-Helmholtz waves at high latitude magnetopause during

567 strongly dawnward interplanetary magnetic field conditions (2012), *J. Geophys. Res.*,  
568 117, A08233, doi:10.1029/2011JA017256.

569 Lepping, R., and L. Burlaga (1979), Geomagnetopause surface fluctuations observed by  
570 Voyager 1, *J. Geophys. Res.*, 84(A12), 7099-7106.

571 Lepping, R. P., et al. (1995), The Wind Magnetic Field Investigation, *Space Sci. Rev.*,  
572 71, 207.

573 Lin, R. P., et al. (1995), A Three-Dimensional Plasma and Energetic Particle Investigation  
574 for the Wind spacecraft, *Space Sci. Rev.*, 71, 125.

575 Lopez, R. E. (1987), Solar cycle invariance in solar wind proton temperature relationships,  
576 *J. Geophys. Res.*, 92, 11,187.

577 Miura, A. (1992), Kelvin-Helmholtz instability at the magnetospheric boundary: Depen-  
578 dence on sonic Mach number, *J. Geophys. Res.*, 97, 10655.

579 McComas, D. J., et al. (1998), Solar wind electron proton alpha monitor (SWEPAM) for  
580 the advanced composition explorer, *Space Sci. Rev.*, 86, 563.

581 Nakamura R., W. Baumjohann, T. Nagai, M. Fujimoto, T. Mukai, B. Klecker, R.  
582 Treumann, A. Balogh, H. Rème, J. A. Sauvaud, L. Kistler, C. Mouikis, C. J. Owen, A.  
583 N. Fazakerley, J. P. Dewhurst and Y. Bogdanova (2004), Flow shear near the boundary  
584 of the plasma sheet observed by Cluster and Geotail, *J. Geophys. Res.*, 109, A18, 5204,  
585 doi:10.1029/2003JA010174.

586 Ogilvie, K. W., and R. J. Fitzenreiter (1989), The Kelvin-Helmholtz Instability at the  
587 Magnetopause and Inner Boundary Layer Surface, *J. Geophys. Res.*, 94 (A11), 15, 113,  
588 123.

- 589 Otto, A., and D. Fairfield (2000), Kelvin-Helmholtz instability at the magnetotail bound-  
590 ary: MHD simulation and comparison with Geotail observations, *J. Geophys. Res.*,  
591 *105(A9)*, 21175.
- 592 Otto, A. and K. Nykyri (2003), Earth's Low-Latitude Boundary Layer Magnetopause  
593 *Geophys. Monog. Ser. vol 133* ed. P. T. Newell and T. Onsager (Washington DC: AGU),  
594 53.
- 595 Sckopke, N., G. Paschmann, G. Haerendel, B. Sonnerup, S. Bame, T. Forbes, E. Hones,  
596 and C. Russell (1981), Structure of the low-latitude boundary layer, *J. Geophys. Res.*,  
597 *86, A4*, doi:10.1029/JA086iA04p02099.
- 598 Smets R., D. Delcourt, G. Chanteur, and T. E. Moore (2002), On the incidence of Kelvin-  
599 Helmholtz instability for mass exchange process at the Earth's magnetopause, *Ann.*  
600 *Geophys. 20*, 757.
- 601 Smith, C. W., J. L'heureux, N. F. Ness, M. H. Acuna, L. F. Burlaga, and J. Scheifele  
602 (1998), The ACE magnetic fields experiment, *Space Sci. Rev.*, *86*, 613.
- 603 Southwood, D. J. (1974), Some features of field line resonances in the magnetosphere,  
604 *Planetary and Space Science*, *22, Issue 3*, p. 483-491.
- 605 Spreiter, J. R., and A. W. Rizzi (1974), Aligned magnetohydrodynamic solution for solar  
606 wind flow past the earth's magnetosphere, *Acta Astron.*, *1*, 15, 1974.
- 607 Takagi, K., C. Hashimoto, H. Hasegawa, M. Fujimoto, and R. TanDokoro (2006), Kelvin-  
608 Helmholtz instability in a magnetotail flanklike geometry: Three-dimensional MHD  
609 simulations, *J. Geophys. Res.*, *111, A08202*, doi:10.1029/2006JA011631.



## 8. Figure Captions

### Figure 1

A schematic to help interpret the Wind data. The wavy magnetopause at the equatorial dawn flank is shown in the top panel as it begins to roll over into a vortex by the KH instability. The magnetosheath flow is tailward ( $V_x < 0$ ), while the magnetosphere is stagnant. The drawing is shown in the frame of the vortex, so that the cold dense magnetosheath tongue (blue) protruding to the left is slowing down relative to the average flow, while the related hot tenuous magnetosphere is moving faster. The bottom panel shows expectations drawn from this for the scatter plot of  $V_x$  versus  $N$ .

**Figure 2.** ACE plasma and field observations during 18-21 UT, October 24, 2001. The panels show the proton density, temperature (in red: the expected temperature for normal solar wind expansion), bulk speed, the total field and (colored) its GSM components, the IMF cone angle, the dynamic pressure based on the protons, the angle between the field and flow vectors, the proton beta, and the sonic and Alfvén Mach numbers.

**Figure 3.** A segment of Wind's orbit for the time interval 19 UT, October 24–02 UT, October 25. The plot shows the trajectory projected into the X-Y and X-Z planes. Tick marks are shown at each hour. The red segment (19:00 - 21:30 UT) refers to the time Wind was crossing the LLBL, thus moving predominantly downward.

**Figure 4.** Proton plasma and magnetic field observations from Wind for the period 19:00-19:30 UT. From top to bottom: the proton density, bulk speed, temperature, the total field and its GSM components, and the GSM velocity components. The dashed blue line in panel 2 gives the average magnetosheath speed. Dashed red lines in last three panels show the average values

631 of the respective quantities over the interval plotted. Note the speeds of the hot tenuous plasma,  
 632 which exceed the solar wind.

633 **Figure 5.** Wind plasma and field data for the time interval  $\sim 19:12$  UT to  $\sim 19:18$  UT.  
 634 The format is the same as for Figure 4, except that the last three panels show the velocity  
 635 components in the average velocity frame, i.e. when the average velocity computed over this  
 636 interval is subtracted.

637 **Figure 6.**

638 Residual vectors in the  $XY$  plane for the structure in Figure 5. Symbols S and E mark the  
 639 start and end of the structure. The labels CD and HT refer to "cold dense" and "hot tenuous",  
 640 respectively. Time runs from bottom to top. The blue, green, and red vectors represent differing  
 641 plasma parameters, as explained in the text. In the average velocity frame shown, the flows  
 642 start moving sunward and slightly duskward (blue). They then rotate downward and become  
 643 progressively antisunward (green), and finish flowing antisunward and duskward (red).

644 **Figure 7.**

645 Scatter plots of  $V_x$  versus  $N$  for the vortex at 19:12:30–19:18:00 UT (upper panel) and for the  
 646 whole LLBL crossing (lower panel). The logarithm of the temperature is indicated by the colors,  
 647 where red = hot and blue = cold. Velocities are plotted in the Earth's frame. The horizontal  
 648 dashed line marks the average magnetosheath speed observed when Wind crossed into this region  
 649 at 20:30 UT (not shown; see *Farrugia et al.*, 2010; their Figure 7). Both panels show the presence  
 650 of (i) a hot tenuous plasma moving at high speeds tailward and (ii) a dense cold plasma moving  
 651 antisunward at speeds close to that of the magnetosheath. The figure shows clearly that the  
 652 origin of the cold dense plasma is the magnetosheath.

**Figure 8**

The figure shows a scatter plot of  $\Delta V_y$  against  $\Delta V_x$  for the whole interval. Color is proportional to  $\log T$  and size is proportional to  $N$ . The plot is in the average velocity frame. The distribution in the dawn-dusk direction  $\Delta V_y$  shows a wide spread across zero. It is wider for the hot tenuous plasma.

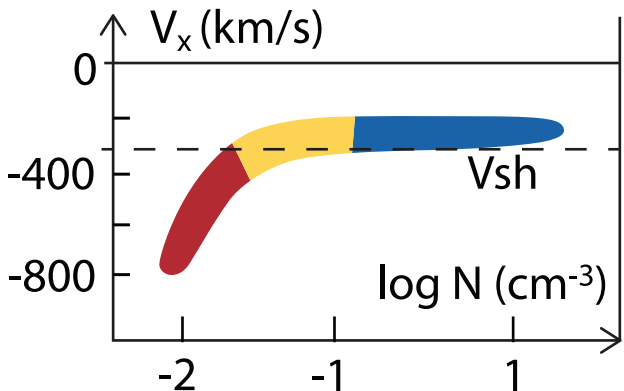
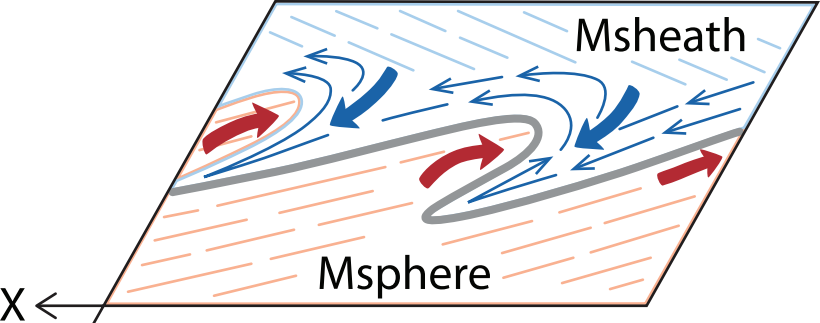
**Figure 9** For three sub-intervals, the plot shows pairwise the proton thermal pressure and dynamic pressure and below the magnetic pressure. The smoothed average of the thermal and dynamic pressure are shown by a blue and red traces, respectively.

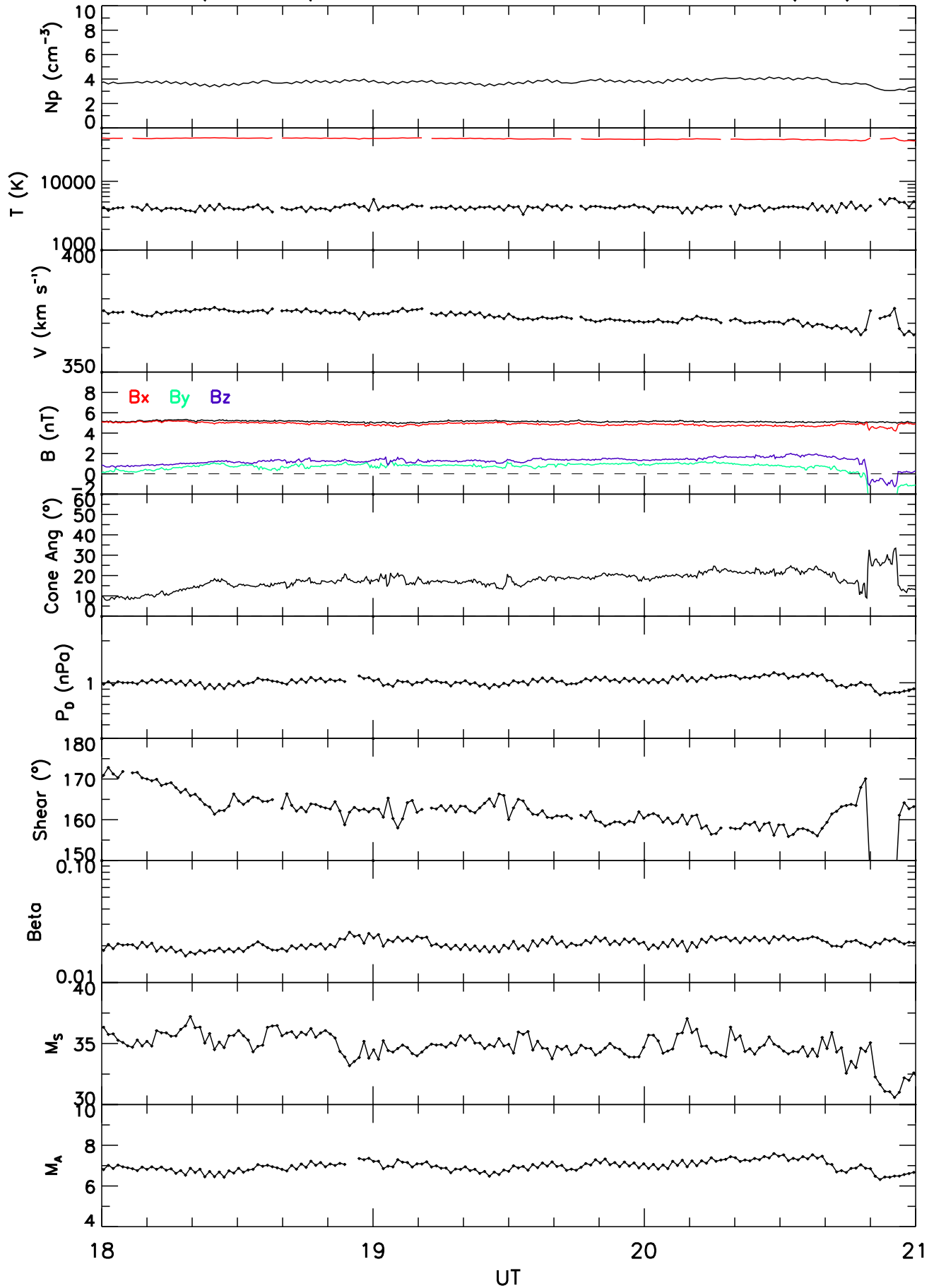
**Figure 10**

Normalized imaginary part of  $c$ , i.e., the complex phase velocity  $\gamma/kV_1$ , versus  $kd$ . Hyperbolic tangent model with input parameters from the Spreiter-Rizzi theory [1974] for a boundary layer at the terminator. Quantities  $M_s = 7.7$ ,  $M_A = 4.9$ .

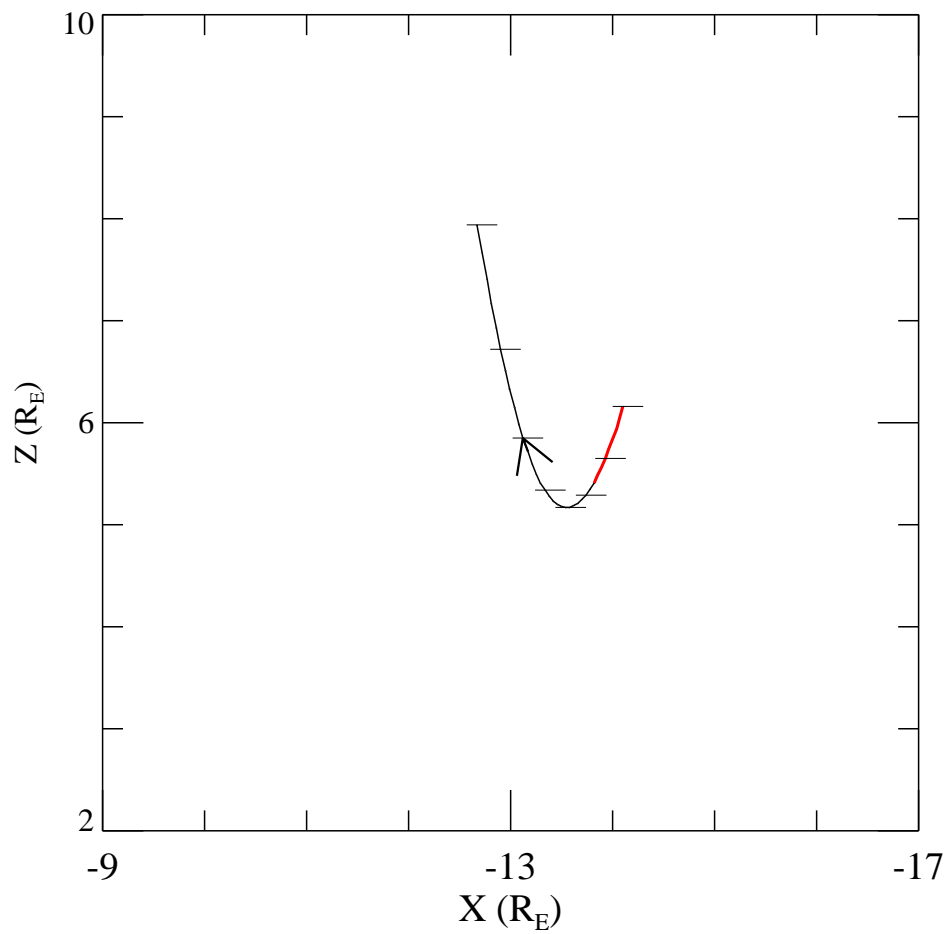
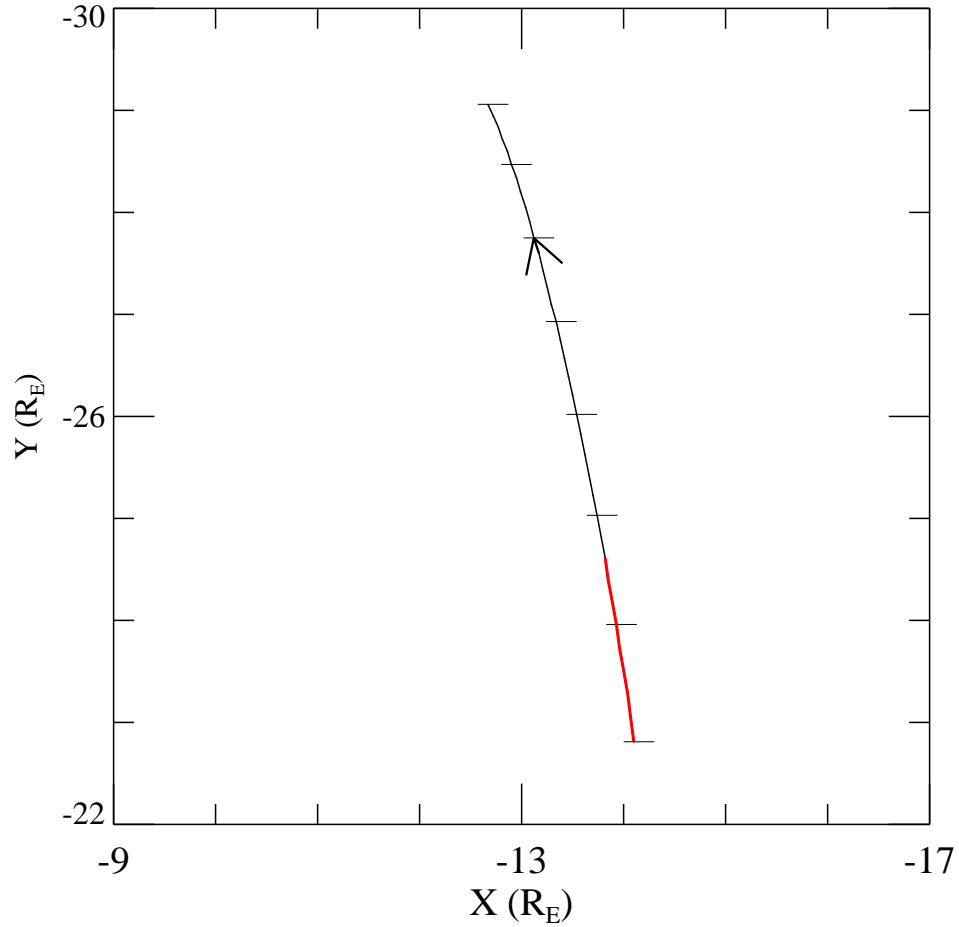
**Figure 11**

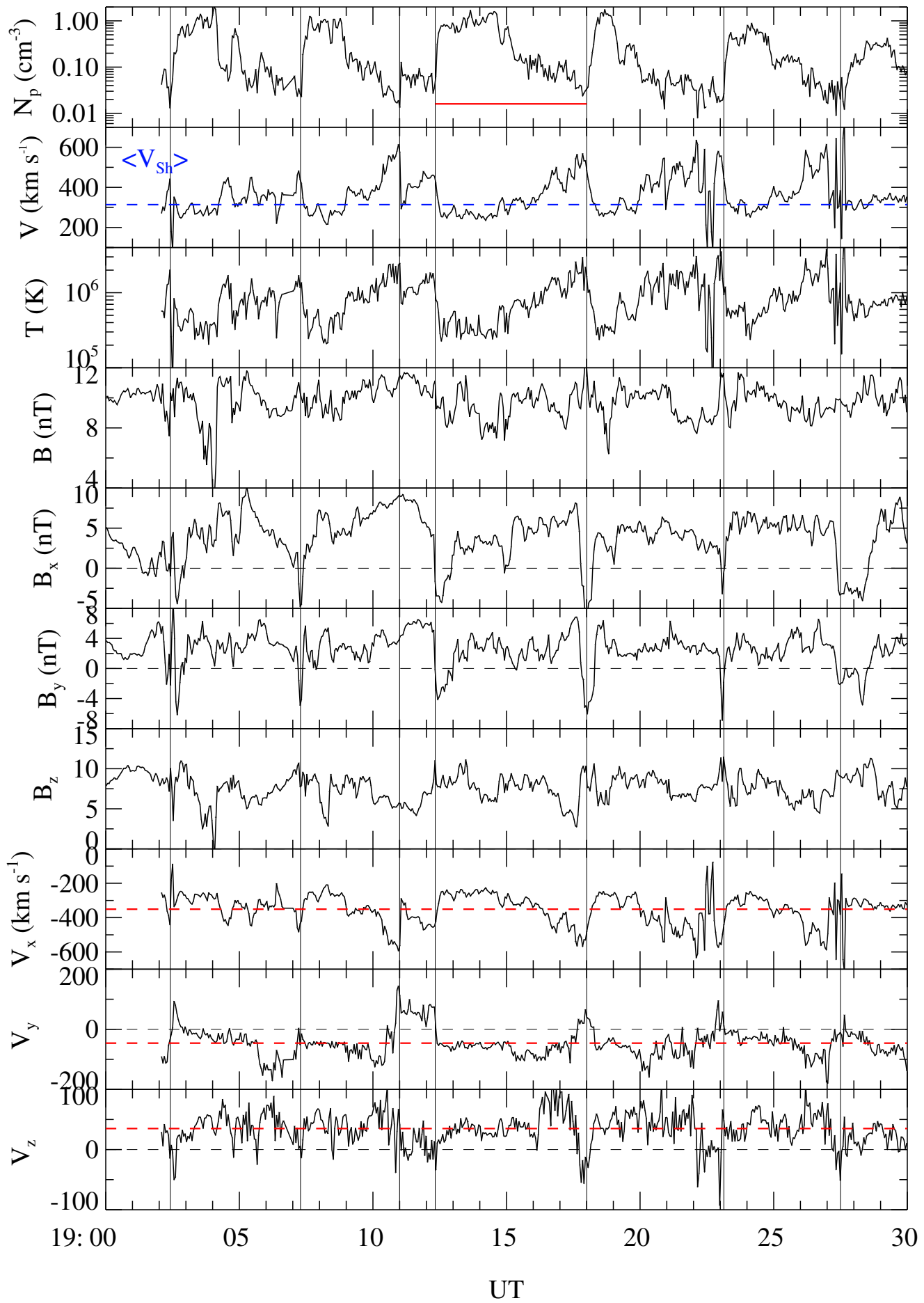
Normalized growth rate as a function of  $kd$ . The input parameters for the stability calculations are based on Wind data. Maximum growth rate is reached at  $kd = 0.43$ . For further details, see text.

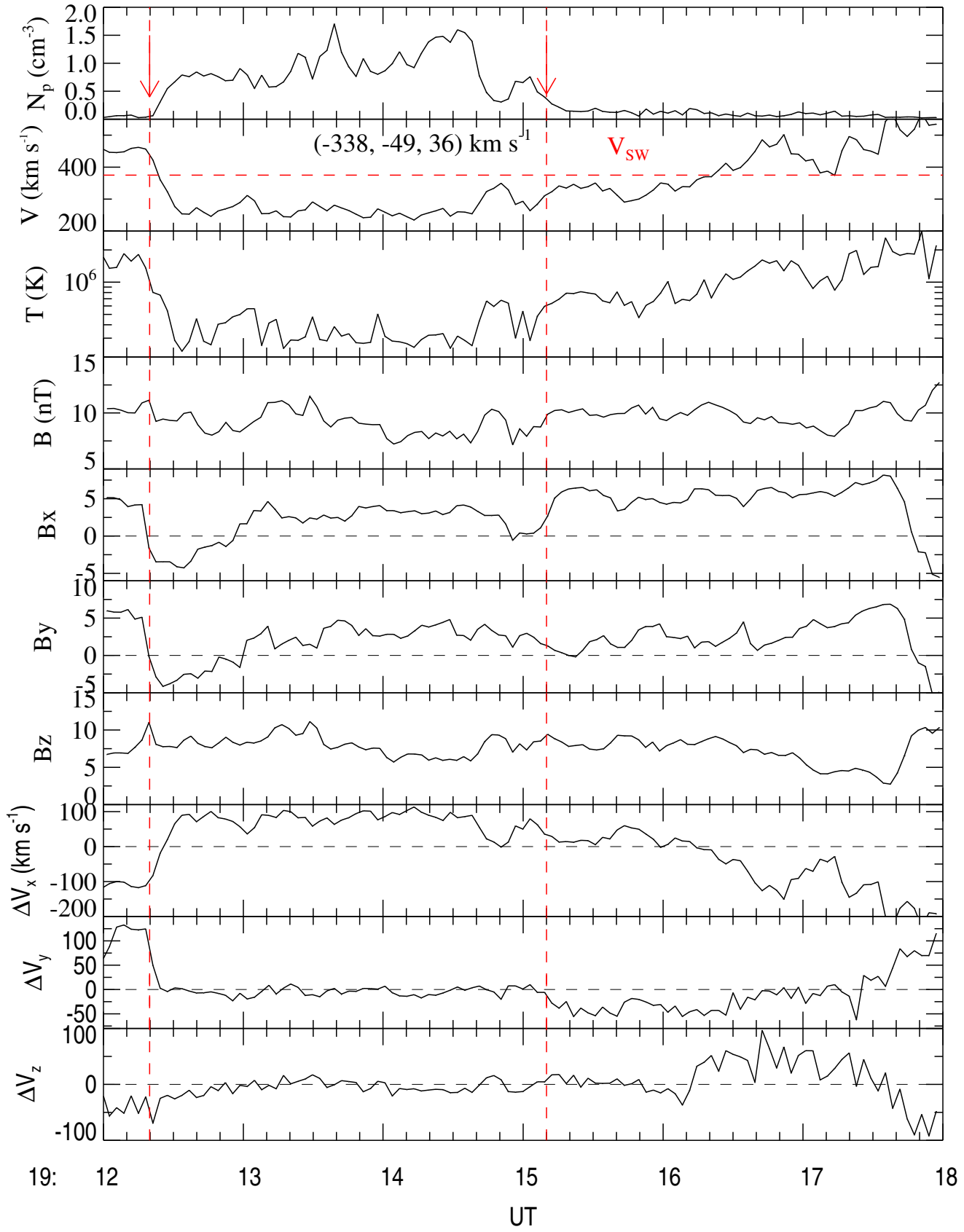




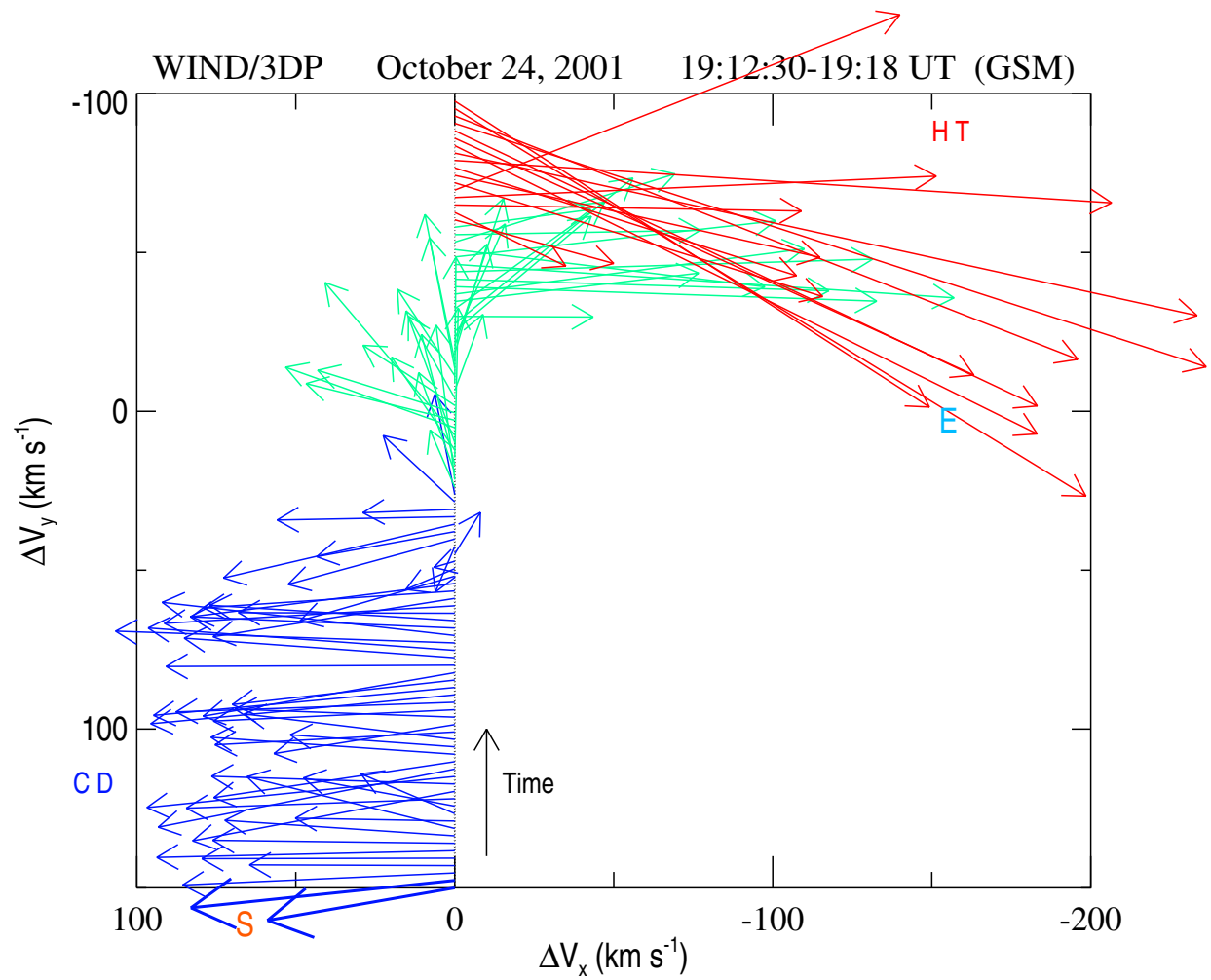
WIND 19 UT, OCT 24--02 UT, OCT 25, 2001 (GSM)



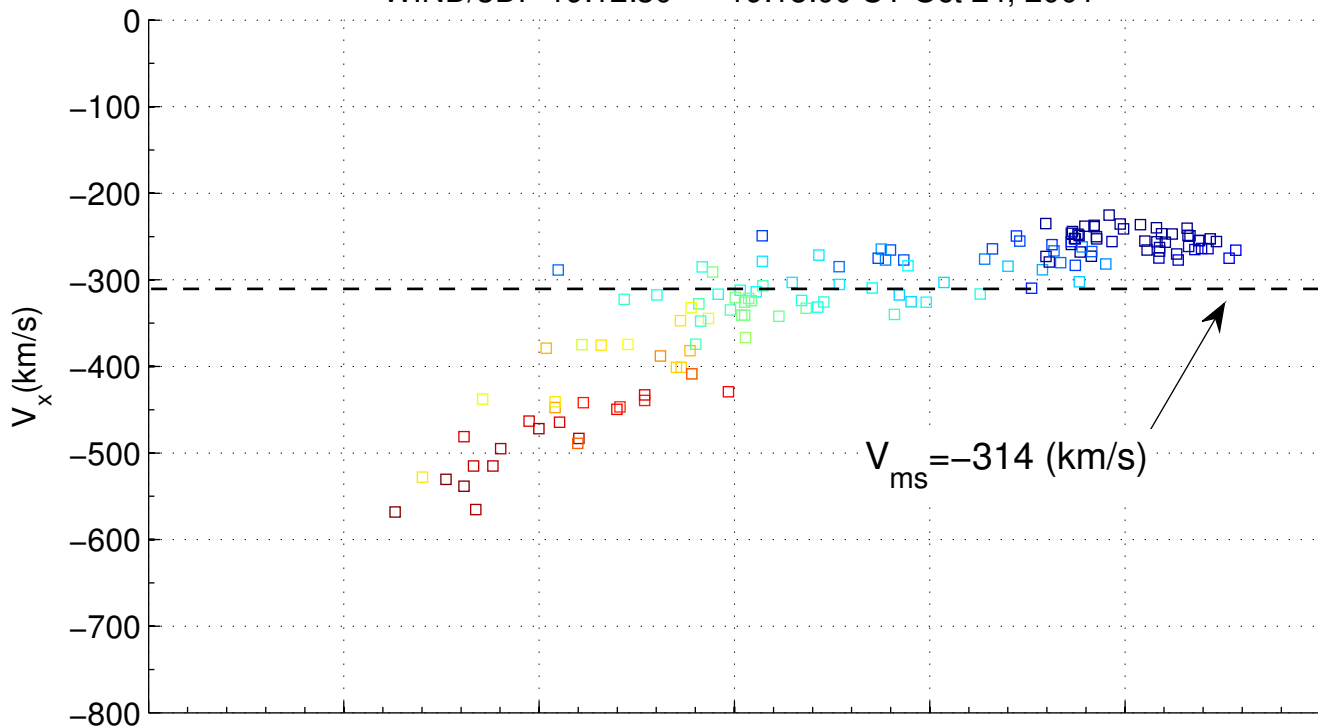




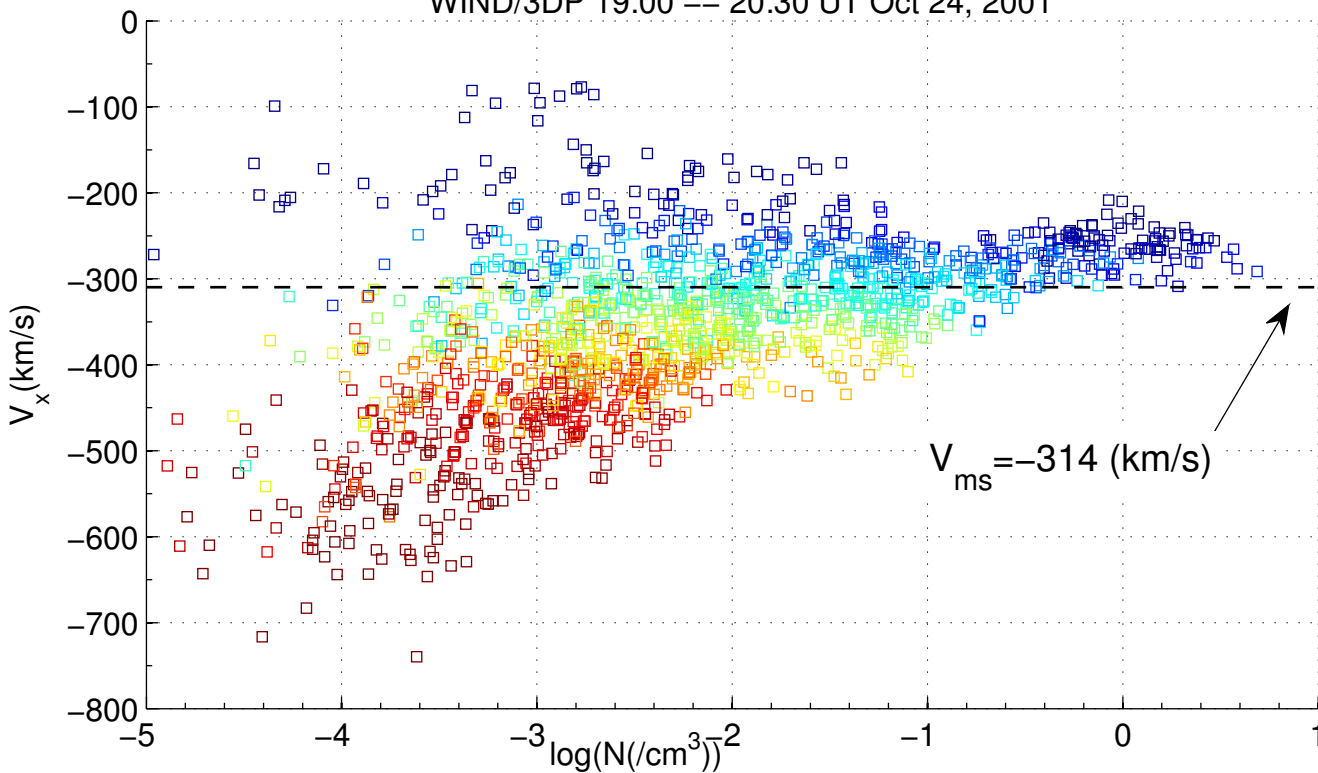




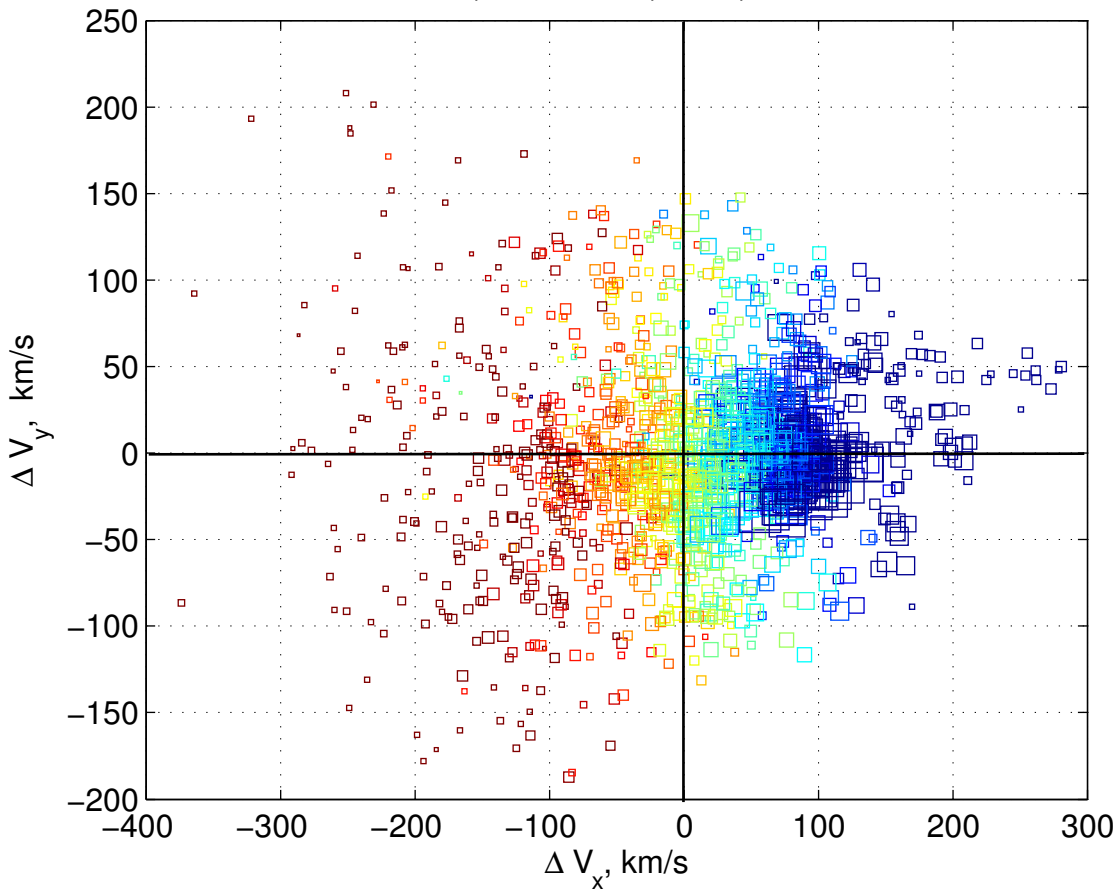
WIND/3DP 19:12:30 -- 19:18:00 UT Oct 24, 2001

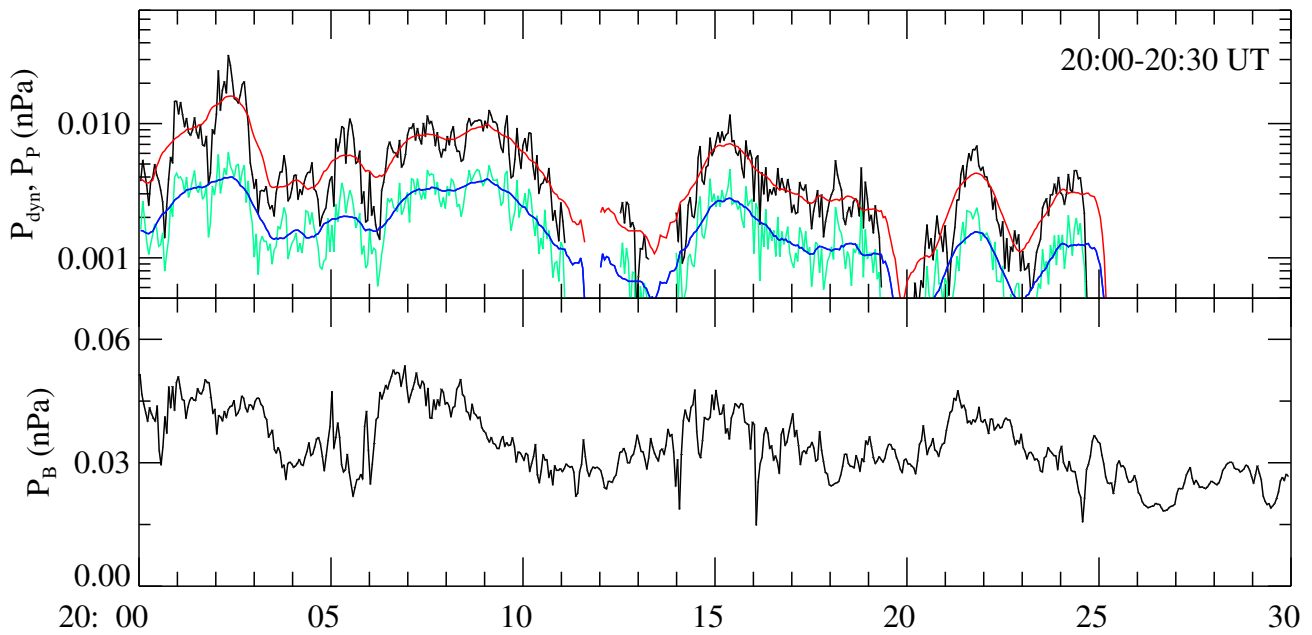
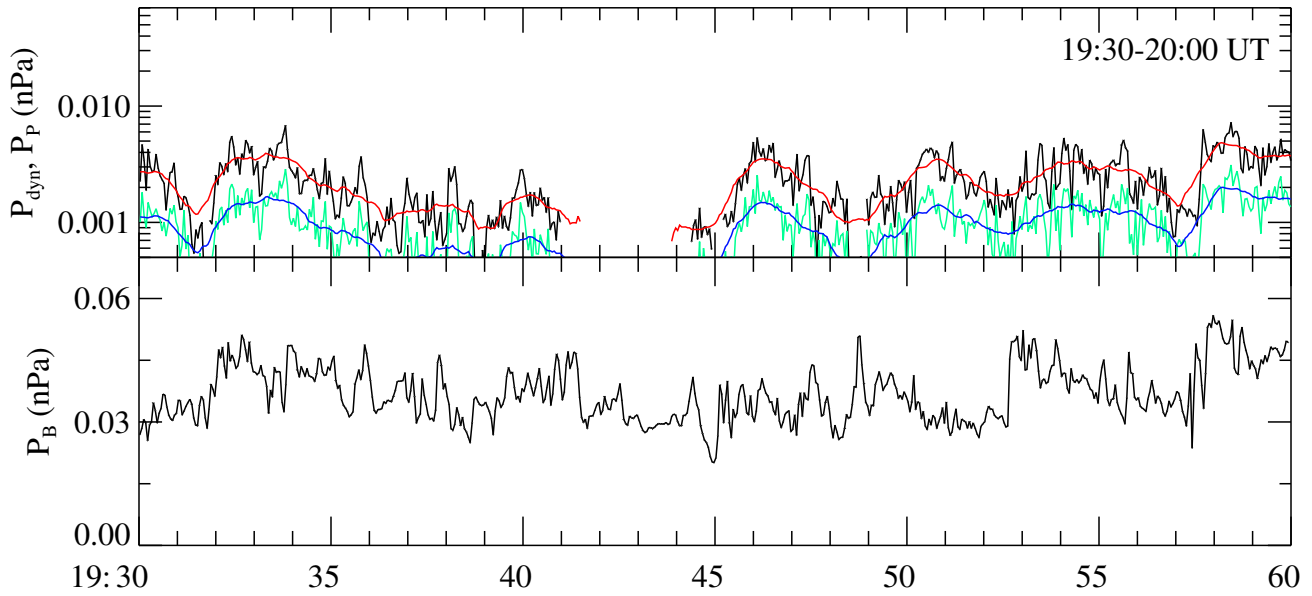
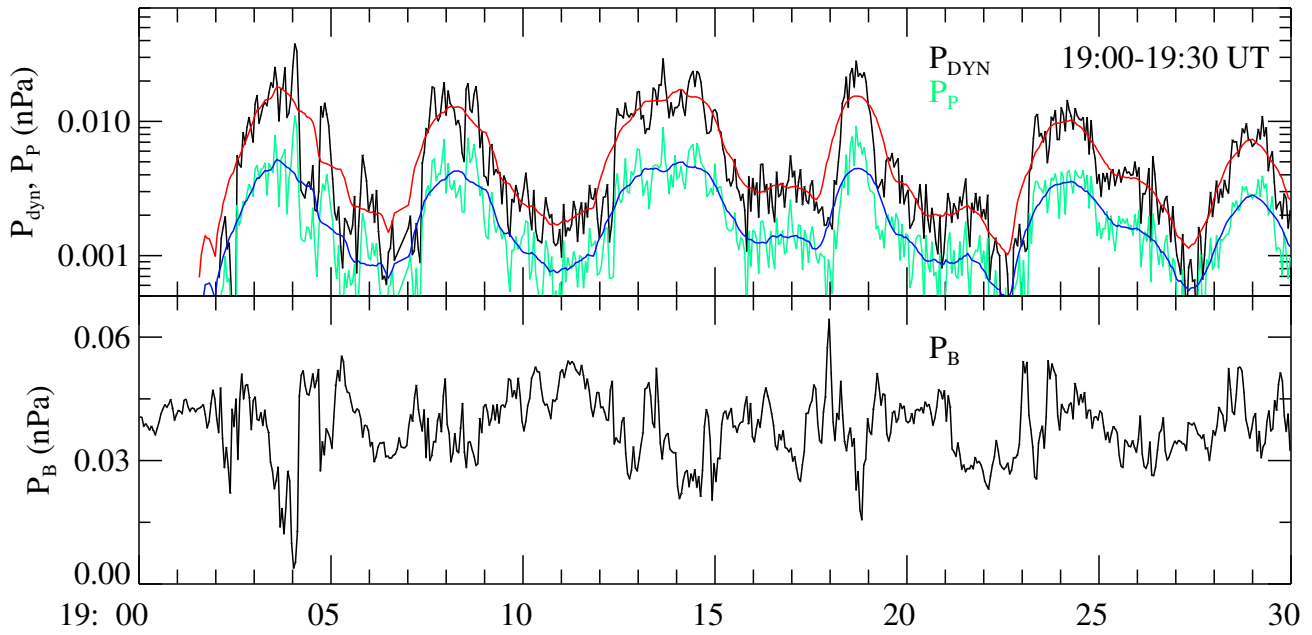


WIND/3DP 19:00 -- 20:30 UT Oct 24, 2001

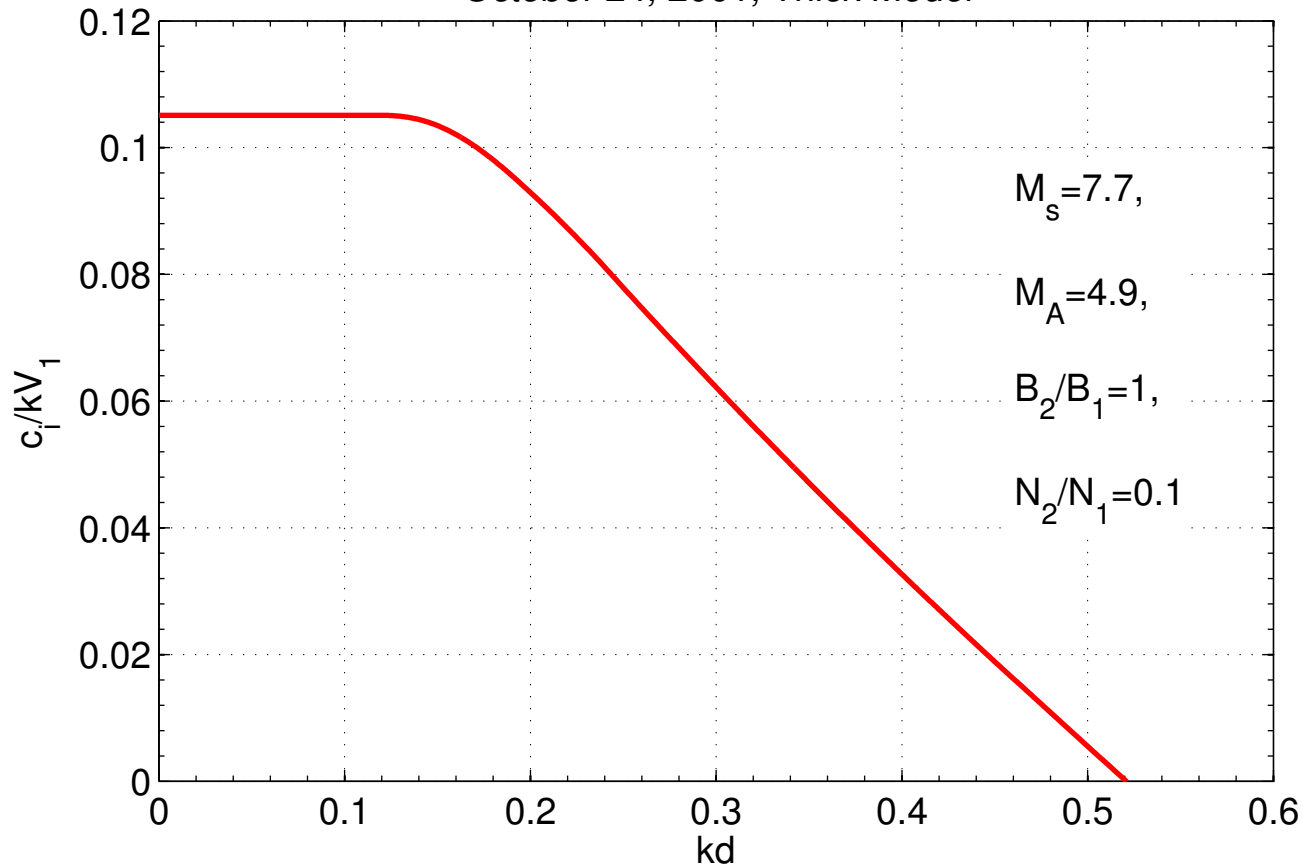


*LLBL Wind data, October 24, 2001, 19:00 – 20:30 UT*





*October 24, 2001, Thick Model*



*October 24, 2001, Thick Model*

